BLOCK DOWNSAMPLING USING MODAL VALUES

Problem. Consider a d-dimensional array of size $2^{L_1} \times 2^{L_2} \times \cdots \times 2^{L_d}$, which will be called the "original image." The l-downsampled image of size $2^{L_1-l} \times 2^{L_2-l} \times \cdots \times 2^{L_d-l}$ is defined by dividing the original image into blocks of size $2^l \times 2^l \times \cdots \times 2^l$, and replacing each block by a single pixel that is a most common value in that block, i.e., a mode of pixels in that block. If there is a tie, any of the modes is considered correct. Assume that the pixel values are unsigned integers. Write C++ code that outputs all l-downsamplings of the original image, where $l=1,\ldots,\min\{L_1,\ldots,L_d\}$. The code should be multithreaded for maximum speed with a multicore processor. Make efficient use of RAM, which is large enough to contain the image and intermediate results.

Example 1. The $d = 2, L_1 = 2, L_2 = 3$ image

yields the 1-downsampled image:

and the 2-downsampled image:

Note that 11 is the wrong answer because mode-downsampling is based on blocks of the original image, rather than the previous downsampled image. The first value (1) of the 2-downsampled image is the mode of this block of the original image:

The second value (2) is the mode of this block of the original image:

Example 2. This case illustrates that any of the multiple modes is considered correct if there are ties for the most frequent value. The image

```
00000000
10101010
11001100
11101110
000011111
10101111
11101100
11001101
yields the 1-downsampled image
0000
1010
0011
1010
the 2-downsampled image
00
01
and the 3-downsampled image
```

Although 0 is the mode of the 2-downsampled image, the correct answer for the 3-downsampled image is 1, the mode of the original 8×8 image.

1. Implementation

You can either use C style arrays, or Boost.MultiArray (http://www.boost.org/doc/libs/1_45_0/libs/multi_array/doc/index.html). The boost library is provided with zi_lib (https://github.com/zlateski/zi_lib), which we use in the lab (subdirectory: https://github.com/zlateski/zi_lib/tree/master/external/include).

For multithreading, you can use the *zi_lib* concurrency library (https://github.com/zlateski/zi_lib/tree/master/zi/concurrency). Examples of usage are given here: https://github.com/zlateski/zi_lib/tree/master/zi/concurrency/test. The library is header only, so you don't need to compile anything. Just git-clone it, and include its path (and path of the external boost libraries). Also, if you are using Linux, don't forget to link against pthread and rt:

```
g++ your_file.cpp -Ipath/to/zi_lib -Ipath/to/zi_lib/external/include
-lpthread -lrt -o your_binary
```

2. EFFICIENCY

Your code should use the fastest possible parallel algorithm. Answer the following questions about computational complexity. Let N be the number of pixels in the original image, n the number of unique pixel values in the original image, and M the number of parallel threads.

- (1) How does the execution time of the fastest parallel algorithm scale with *N*, *n*, and *M*?
- (2) How does memory usage scale with N, n, and M?

3. Style

The style of your code will be evaluated, including aspects such as

- organization into functions object-oriented design
- memory management