Unfortunately not really, that would make it more interesting though. I think to best explain it would be to explain the context its in, so if we start with the basic building block a "set" which im sure you are aware of, is just what we call "any collection of distinct elements". Going one level deeper, any set that's elements fulfill these properties:

- Associativity of addition and multiplication: $a + (b + c) = (a + b) + c$, and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
- Commutativity of addition and multiplication: $a + b = b + a$, and $a \cdot b = b \cdot a$.
- Additive and multiplicative identity: there exist two different elements 0 and 1 in F such that $a + 0 = a$ and $a \cdot 1 = a$.
- Additive inverses: for every a in F, there exists an element in F, denoted $-a$, called the *additive inverse* of a, such that $a + (-a) = 0$.
- Multiplicative inverses: for every $a \neq 0$ in F, there exists an element in F, denoted by a^{-1} or $1/a$, called the *multiplicative inverse* of a, such that $a \cdot a^{-1} = 1$.
- Distributivity of multiplication over addition: $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$.

is then a "field". It's important to note though that "plus" and "multiplies" in these definitions don't really have any special inherent meaning, there just have to be two operations "+" and "x" that are defined and fulfill the properties listed. For example, look at the first axiom, if I take a set with every positive number in it, and define "+" as "a + b = a*b" (where a*b is our standard understanding of multiplication) we can see that this weird definition of "+" still fulfills the associativity definition (since a+(b + c) = a*b*c = (a + b) + c). The reason I bring up this aside is that it'll be hard to understand why something like a Hilbert space is an important definition without seeing that it is necessary to have these distinctions, because the type of fields you may be working with might have weird properties.

Then the next level of abstraction isn't exactly another level deeper, but more a step to the side: a "vector space" is a set of objects called "vectors" that has an associated "field" of objects called "scalars". This "vector space" must fulfill these:

In order for V to be a vector space, the following conditions must hold for all elements $\mathbf{X}, \mathbf{Y}, \mathbf{Z} \in V$ and any scalars $r, s \in F$:

1. Commutativity:

$$\mathbf{X} + \mathbf{Y} = \mathbf{Y} + \mathbf{X}.$$

2. Associativity of vector addition:

$$(\mathbf{X} + \mathbf{Y}) + \mathbf{Z} = \mathbf{X} + (\mathbf{Y} + \mathbf{Z}).$$

3. Additive identity: For all $\mathbf{X}$,

$$\mathbf{0} + \mathbf{X} = \mathbf{X} + \mathbf{0} = \mathbf{X}.$$

4. Existence of additive inverse: For any $\mathbf{X}$, there exists a $-\mathbf{X}$ such that

$$\mathbf{X} + (-\mathbf{X}) = \mathbf{0}.$$

5. Associativity of scalar multiplication:

$$r\,(s\,\mathbf{X}) = (r\,s)\,\mathbf{X}.$$

6. Distributivity of scalar sums:

$$(r + s)\,\mathbf{X} = r\,\mathbf{X} + s\,\mathbf{X}.$$

7. Distributivity of vector sums:

$$r\,(\mathbf{X} + \mathbf{Y}) = r\,\mathbf{X} + r\,\mathbf{Y}.$$

8. Scalar multiplication identity:

$$1\,\mathbf{X} = \mathbf{X}.$$

which may not seem all that different from the field axioms, but an important difference is multiplication isn't defined between the vectors, only between a scalar and a vector.

The next level would then be an "inner product space" that is a "vector space" that also lets you have an operation called the "inner product" that take in two elements of the set of "vectors" and returns an element in the field of "scalars". These are the axioms for an "inner product":

More precisely, for a real vector space, an inner product $\langle \cdot, \cdot \rangle$ satisfies the following four properties. Let u, v, and w be vectors and α be a scalar, then:

1. $\langle u + v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$.
2. $\langle \alpha v, w \rangle = \alpha \langle v, w \rangle$.
3. $\langle v, w \rangle = \langle w, v \rangle$.
4. $\langle v, v \rangle \geq 0$ and equal if and only if $v = 0$.

These aren't very interesting flat out, but basically, having a vector space with an inner product lets us do stuff with vectors like we are used to, like dot products ((1,2) · (3,4) = 1x3 + 2x4 = 11).

Now, if we let the scalar "field" that is associated with our "inner product space" be either the real numbers (all numbers as you know them, including irrationals, negatives, decimals etc.) or the complex numbers (essentially the real numbers but also with imaginary numbers), and we define something called the "norm" be the square root of the inner product of a vector with itself or $\sqrt{< x, x >}$ then we have a "norm", which is important because its gets us the "length" of real or complex vectors.

If we have an inner product space, which is close, but we also have to have it be a "normed vector space", which first requires it being a "metric space". First, for an "inner product space" to also be a "metric space" you have to have a "metric". A "metric" is a function that takes in two vectors from the space, and gives a "distance" between them. If we choose a metric such that these "distances" are only positive real numbers, we have our inner product space is now also a "metric space". Thus, we take the "norm" we talked about above and define the "metric" as norm(x - y), which gives us our usual distance formula we are used to.

Now, we have a "inner product + metric space" (only with associated scalar fields that are real or complex numbers), so we have the concept of "length" (norms), "distance" (metric from norm), and we also have all of the fun properties of the associated scalar fields, but we need one last piece to have a Hilbert Space: "completeness".

"Completeness" has means that "all cauchy sequences converge" which, without getting into more definitions, means: If you have an sequence of numbers a,b,c,.... where the numbers are getting closer and closer together, you have a "cauchy sequence", if your space is such that all of these "cauchy sequences" converge its "complete". For example, the real numbers are definitely not a vector space, but they are "complete" since all "cauchy sequences" converge (this doesn't mean all limits converge, since not all sequences are cauchy). The reason this is important is because essentially all of calculus is built on this idea.

Now we have a "complete + metric + inner product space", or just a Hilbert Space. The reason "Hilbert Spaces" are important is because they are our best formulation to let us to take alot of math, which was built by mathematicians on the "Euclidian Space" and generalize it to infinite dimensions. For example, all basic math classes start dealing in R (real numbers) such as 1 or 2, when you get to graphing and functions you go to R^2, coordinate pairs (a,b) and if you get far enough you do stuff in 3d, R^3 (a,b,c). Mathematicians understanding of calculus etc. was built in these kinds of spaces, Hilbert spaces were accepted as the best way to generalize it to R^{n} or even R^{∞} .

Unfortunately though, the applications are not as immediately exciting. Basically, for mathematicians and physicist's, being able to represent whatever thing you are working on as a hilbert space allows you to then treat it as a hilbert space. When you can just treat it as a hilbert space, there is a bunch of linear algebra theorems and such you can apply to it without having to care about the true nature of the problem you are working on.