

Problems for Week 2

January 26, 2022

Problem 1. The table below shows some information about 200 students and their most frequent method of travel to school.

	Walk	Not walk	Total
Male	56	40	96
Female	72	32	104
Total	128	72	200

1. Find the probability that a randomly chosen student is female
2. Find the probability that a randomly chosen student is female and walks to school
3. Given the student is female what is the probability that they walk to school?

Problem 2. There are 8 counters in a bag. Five of the counters are red and 3 of the counters are blue. Two counters are taken at random from the bag. Compute the probability that one counter of each colour are taken.

Problem 3. For three events A, B and C , we know that

- A and C are independent
- B and C are independent
- A and B are disjoint
- $\mathbb{P}(A \cup C) = \frac{2}{3}, \mathbb{P}(B \cup C) = \frac{3}{4}, \mathbb{P}(A \cup B \cup C) = \frac{11}{12}$.

Compute $\mathbb{P}(A), \mathbb{P}(B)$ and $\mathbb{P}(C)$.

Problem 4. Consider two independent fair coin tosses, and the following events:

$$\begin{aligned}H_1 &= \{\text{1st toss is a head}\} \\H_2 &= \{\text{2nd toss is a head}\} \\D &= \{\text{the two tosses have different results}\}\end{aligned}$$

Show that these events are pairwise independent but not independent.

Problem 5. A computer manufacturer uses chips from three sources. Chips from source A, B and C are defective with probabilities 0.005, 0.001 and 0.01, respectively. The proportions of chips from A, B, C are 0.5, 0.1, 0.4, respectively. A randomly selected chip is found to be defective. Compute the probability that the chips are from A .

Problem 6. Let A_1 and A_2 be a partition of the sample space Ω . Assume $\mathbb{P}(B) > 0$ and $\mathbb{P}(A_1|B) < \mathbb{P}(A_1)$. Prove that $\mathbb{P}(A_2|B) > \mathbb{P}(A_2)$.