Exercise 1:

1.

Main tape $w = a^n$
Aux tape $x = a^m$

M1:

There are two scenarios

1. $n < m$

$4(n+1) + 1 + 2n + 1 + 1$
$= 4n + 4 + 3 + 2n$
$= 6n + 7$ steps

2. $n > m$
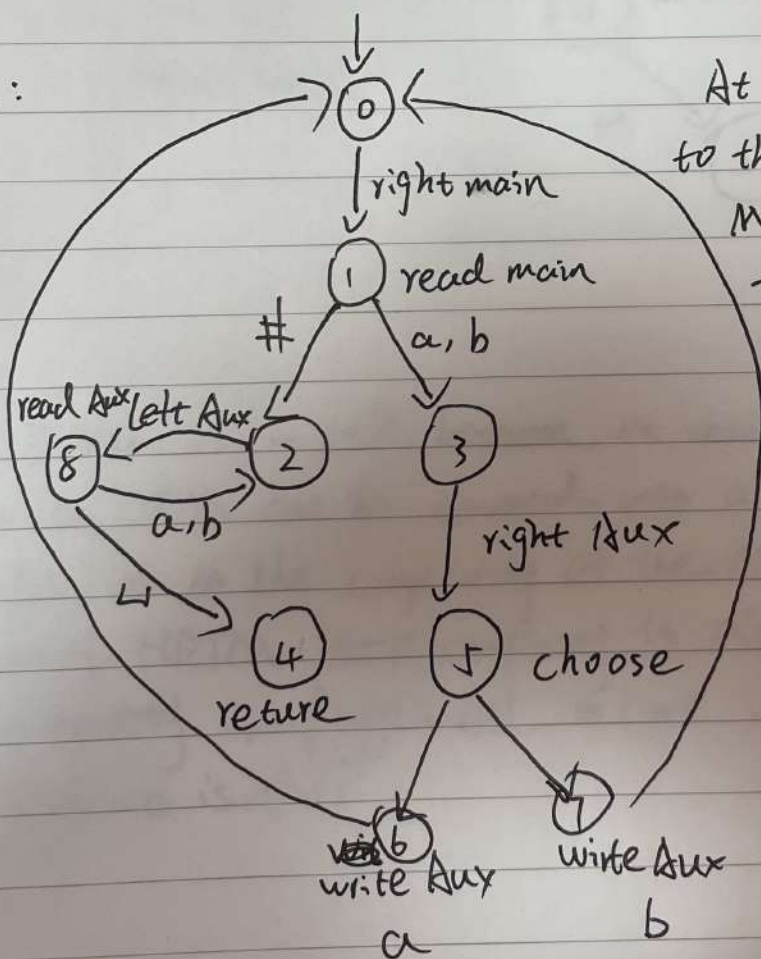
$4m + 2 + 1 + 2m + 1 + 2(n+1-m) + 2$
$= 6m + 4 + 2n + 2 - 2m + 2$
$= 4m + 2n + 8$ steps.

2.

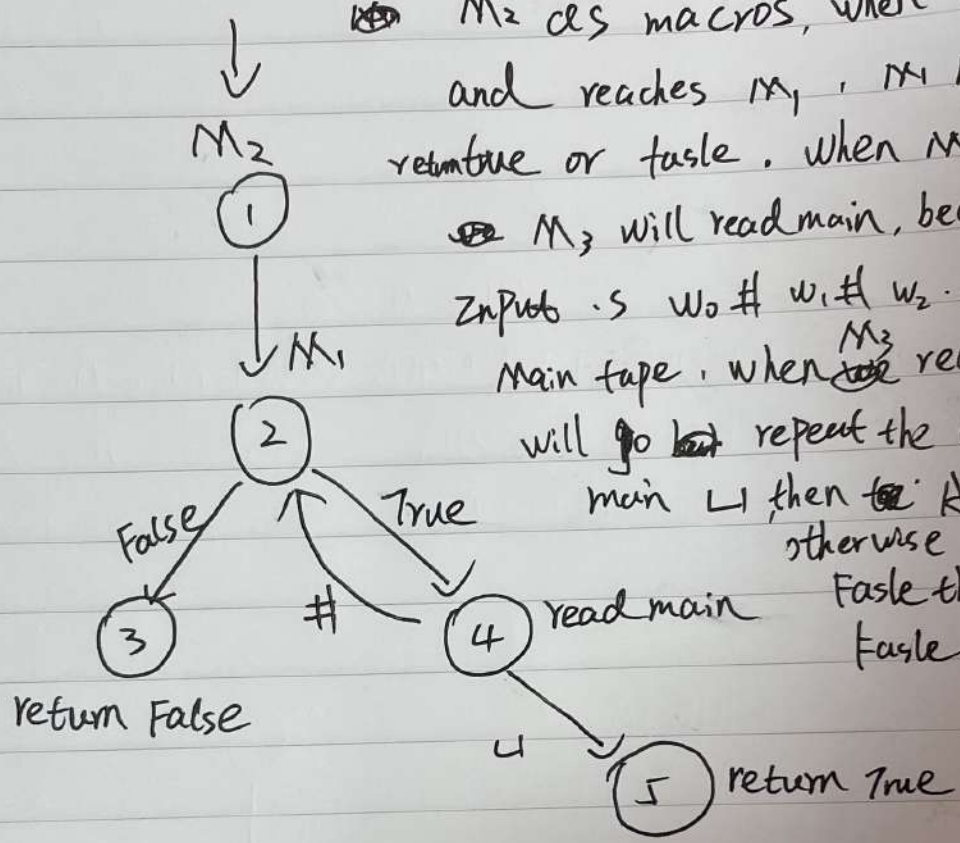$M_2:$

At the beginning, Head is to the left of Main and Aux
M2 will move right Main and then ~~use~~ read main, it it is read a or b, then right Aux, and choose to write a or b to Aux Repeat this loop until read main reaches #, and then left Aux, read Aux a or b until read Aux ⊔, finally return.

3.

M3 :

M_2
①

$\downarrow M_1$

②

False / True

③ #

④ read main

return False

⑤ return True

At the begining , use machine M_1 and M_3
M_2 as macros, when m_2 run out
and reaches M_1 , M_1 has two case:
return true or fasle . when M_1 return true
M_3 will read main , because the
input is w_0 # w_1 # w_2 .. # w_k . 0 in
main tape , when M_3 read main #/
will go back repeat the loop unitl read
main ⊔ then Accept (retuve T)
otherwise when M_1 return
Fasle then M_3 return
Fasle too .

4.

The problem is in NP , because , we need check the two tape
nondeterministy TM can be converted into a polytime one tape NDTM
For the M3 , the complexity of the M_2 is $O(n)$, and the
M3 remaining NDTM is a complexty is also $O(n)$, so the both
time complexty is polytimeal time , In conclusion this
decision problem is in NP .