Language Modeling

Introduction to N-grams

Probabilistic Language Models

Today's goal: assign a probability to a sentence

- Machine Translation:
  - P(**high** winds tonite) > P(**large** winds tonite)

this one makes more sense

- Spell Correction
  - The office is about fifteen **minuets** from my house
    - P(about fifteen **minutes** from) > P(about fifteen **minuets** from)

Why?

- Speech Recognition
  - P(I saw a van) >> P(eyes awe of an)
- \+ Summarization, question-answering, etc., etc.!!

Probabilistic Language Modeling

Goal: compute the probability of a sentence or sequence of words:

$P(W) = P(w_1,w_2,w_3,w_4,w_5 \dots w_n)$

Related task: probability of an upcoming word:

$P(w_5|w_1,w_2,w_3,w_4)$

A model that computes either of these:

$P(W)$ or $P(w_n|w_1,w_2 \dots w_{n-1})$ is called a **language model**.

Better: **the grammar** But **language model** or **LM** is standard

How to compute P(W)

How to compute this joint probability:

- P(its, water, is, so, transparent, that)

Intuition: let's rely on the Chain Rule of Probability

Reminder: The Chain Rule

Recall the definition of conditional probabilities

$p(B|A) = P(A,B)/P(A)$ Rewriting: **$P(A,B) = P(A)P(B|A)$**

More variables:

$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$

The Chain Rule in General

$$P(x_1,x_2,x_3,\ldots,x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1,x_2)\ldots P(x_n|x_1,\ldots,x_{n-1})$$

The Chain Rule applied to compute joint probability of words in sentence

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

P("its water is so transparent") =

P(its) × P(water|its) × P(is|its water)

× P(so|its water is) × P(transparent|its water is so)

How to estimate these probabilities

Could we just count and divide?

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$

No! Too many possible sentences!

We'll never see enough data for estimating these

Markov Assumption

Andrei Markov

Simplifying assumption:

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$$

Or maybe

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$$

select the previous one/two words, and assuming the probability is similar.

Markov Assumption

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

In other words, we approximate each component in the product

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-k} \ldots w_{i-1})$$

Simplest case: Unigram model

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model

```
fifth, an, of, futures, the, an, incorporated, a,
a, the, inflation, most, dollars, quarter, in, is,
mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the
```

Bigram model = taking a single previous word

- Condition on the previous word:

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-1})$$

texaco, rose, one, in, this, issue, is, pursuing, growth, in, a, boiler, house, said, mr., gurria, mexico, 's, motion, control, proposal, without, permission, from, five, hundred, fifty, five, yen

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november

N-gram models = taking N-1 previous words

We can extend to trigrams, 4-grams, 5-grams

In general this is an insufficient model of language

- because language has **long-distance dependencies:**

  “The computer which I had just put into the machine room on the fifth floor crashed.”

But we can often get away with N-gram models

Language Modeling

Introduction to N-grams

Language Modeling

Estimating N-gram Probabilities

Estimating bigram probabilities

The Maximum Likelihood Estimate

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

the number of w_{i-1} and w_i

the number of w_{i-1}

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An example

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$P(\texttt{I} \mid \texttt{<s>}) = \frac{2}{3} = .67$ $\quad P(\texttt{Sam} \mid \texttt{<s>}) = \frac{1}{3} = .33$ $\quad P(\texttt{am} \mid \texttt{I}) = \frac{2}{3} = .67$

$P(\texttt{</s>} \mid \texttt{Sam}) = \frac{1}{2} = 0.5$ $\quad P(\texttt{Sam} \mid \texttt{am}) = \frac{1}{2} = .5$ $\quad P(\texttt{do} \mid \texttt{I}) = \frac{1}{3} = .33$

More examples: Berkeley Restaurant Project sentences

can you tell me about any good cantonese restaurants close by

mid priced thai food is what i’m looking for

tell me about chez panisse

can you give me a listing of the kinds of food that are available

i’m looking for a good place to eat breakfast

when is caffe venezia open during the day

Raw bigram counts

Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities

Normalize by unigrams:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

Result:

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

P(<s> I want english food </s>) =

P(I|<s>)

× P(want|I)

× P(english|want)

× P(food|english)

× P(</s>|food)

= .000031

ずっとかけ算をすると，数が小さすぎる.

What kinds of knowledge?

P(english|want) = .0011

P(chinese|want) = .0065

ppl want Chinese food more than English food.

P(to|want) = .66

P(eat | to) = .28

they are more likely to say "I want to"

P(food | to) = 0

P(want | spend) = 0

P (i | <s>) = .25

Practical Issues

We do everything in log space

- Avoid underflow
- (also adding is faster than multiplying)

確率に logをかける. 足し算にする.

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

Language Modeling Toolkits

SRILM

- http://www.speech.sri.com/projects/srilm/

KenLM

- https://kheafield.com/code/kenlm/

Google N-Gram Release, August 2006

AUG
3

All Our N-gram are Belong to You

Posted by Alex Franz and Thorsten Brants, Google Machine Translation Team

Here at Google Research we have been using word n-gram models for a variety of R&D projects,

…

That's why we decided to share this enormous dataset with everyone. We processed 1,024,908,267,229 words of running text and are publishing the counts for all 1,176,470,663 five-word sequences that appear at least 40 times. There are 13,588,391 unique words, after discarding words that appear less than 200 times.

Google N-Gram Release

```
serve as the incoming 92
serve as the incubator 99
serve as the independent 794
serve as the index 223
serve as the indication 72
serve as the indicator 120
serve as the indicators 45
serve as the indispensable 111
serve as the indispensible 40
serve as the individual 234
```

http://googleresearch.blogspot.com/2006/08/all-our-n-gram-are-belong-to-you.html

Google Book N-grams

http://ngrams.googlelabs.com/

Language Modeling

Estimating N-gram Probabilities

Language Modeling

Evaluation and Perplexity

Evaluation: How good is our model?

Does our language model prefer good sentences to bad ones?

- Assign higher probability to “real” or “frequently observed” sentences
  - Than “ungrammatical” or “rarely observed” sentences?

We train parameters of our model on a **training set**.

We test the model’s performance on data we haven’t seen.

- A **test set** is an unseen dataset that is different from our training set, totally unused.
- An **evaluation metric** tells us how well our model does on the test set.

Extrinsic evaluation of N-gram models

Best evaluation for comparing models A and B

- Put each model in a task
  - spelling corrector, speech recognizer, MT system
- Run the task, get an accuracy for A and for B
  - How many misspelled words corrected properly
  - How many words translated correctly
- Compare accuracy for A and B

W

Difficulty of extrinsic (in-vivo) evaluation of N-gram models

Extrinsic evaluation

- Time-consuming; can take days or weeks

So

- Sometimes use **intrinsic** evaluation: **perplexity**
- Bad approximation
  - unless the test data looks **just** like the training data
  - So **generally only useful in pilot experiments**
- But is helpful to think about.

Intuition of Perplexity

The **Shannon Game**:

- How well can we predict the next word?

  I always order pizza with cheese and ____

  The 33rd President of the US was ____

  I saw a ____

- Unigrams are terrible at this game. (Why?)

mushrooms 0.1

pepperoni 0.1

anchovies 0.01

….

fried rice 0.0001

….

and 1e-100

A better model of a text

- is one which assigns a higher probability to the word that actually occurs

Perplexity

The best language model is one that best predicts an unseen test set

- Gives the highest P(sentence)

Perplexity is the inverse probability of the test set, normalized by the number of words:

$$PP(W) = P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 \ldots w_N)}}$$

Chain rule:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_1 \ldots w_{i-1})}}$$

For bigrams:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_{i-1})}}$$

Minimizing perplexity is the same as maximizing probability

The Shannon Game intuition for perplexity

From Josh Goodman

Perplexity is weighted equivalent branching factor

How hard is the task of recognizing digits '0,1,2,3,4,5,6,7,8,9'

- Perplexity 10

How hard is recognizing (30,000) names at Microsoft.

- Perplexity = 30,000

Let's imagine a call-routing phone system gets 120K calls and has to recognize

- "Operator" (let's say this occurs 1 in 4 calls)
- "Sales" (1in 4)
- "Technical Support" (1 in 4)
- 30,000 different names (each name occurring 1 time in the 120K calls)
- What is the perplexity? Next slide

The Shannon Game intuition for perplexity

Josh Goodman: imagine a call-routing phone system gets 120K calls and has to recognize

- "Operator" (let's say this occurs 1 in 4 calls)
- "Sales" (1in 4)
- "Technical Support" (1 in 4)
- 30,000 different names (each name occurring 1 time in the 120K calls)

We get the perplexity of this sequence of length 120Kby first multiplying 120K probabilities (90K of which are 1/4 and 30K of which are 1/120K), nd then taking the inverse 120,000th root:

Perp = (¼ * ¼ * ¼* ¼ * ¼ * …. * 1/120K * 1/120K * ….)^(-1/120K)

But this can be arithmetically simplified to just N = 4: the operator (1/4), the sales (1/4), the tech support (1/4), and the 30,000 names (1/120,000):

Perplexity= ((¼ * ¼ * ¼ * 1/120K)^(-1/4) = 52.6

Perplexity as branching factor

Let's suppose a sentence consisting of random digits

What is the perplexity of this sentence according to a model that assign P=1/10 to each digit?

$$
\begin{aligned}
\mathrm{PP}(W) &= P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}} \\
&= \left(\frac{1}{10}^{N}\right)^{-\frac{1}{N}} \\
&= \frac{1}{10}^{-1} \\
&= 10
\end{aligned}
$$

Lower perplexity = better model

Training 38 million words, test 1.5 million words, WSJ

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

Language Modeling

Evaluation and Perplexity

Language Modeling

Generalization and zeros

The Shannon Visualization Method

Choose a random bigram

(<s>, w) according to its probability

Now choose a random bigram
(w, x) according to its probability

And so on until we choose </s>

Then string the words together

```
<s> I
    I want
      want to
           to eat
              eat Chinese
                  Chinese food
                          food  </s>
I want to eat Chinese food
```

Approximating Shakespeare

1 gram	–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have –Hill he late speaks; or! a more to leg less first you enter
2 gram	–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. –What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, ’tis done. –This shall forbid it should be branded, if renown made it empty.
4 gram	–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv’d in; –It cannot be but so.

Shakespeare as corpus

N=884,647 tokens, V=29,066

Shakespeare produced 300,000 bigram types out of V^2= 844 million possible bigrams.

- So 99.96% of the possible bigrams were never seen (have zero entries in the table)

Quadrigrams worse: What's coming out looks like Shakespeare because it ***is*** Shakespeare

The Wall Street Journal is not Shakespeare (no offense)

1 gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives
2 gram	Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her
3 gram	They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

Can you guess the training set author of the LM that generated these random 3-gram sentences?

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and gram Brazil on market conditions

This shall forbid it should be branded, if renown made it empty.

“You are uniformly charming!” cried he, with a smile of associating and now and then I bowed and they perceived a chaise and four to wish for.

The perils of overfitting

N-grams only work well for word prediction if the test corpus looks like the training corpus

- In real life, it often doesn’t
- We need to train robust models that generalize!
- One kind of generalization: Zeros!
  - Things that don’t ever occur in the training set
    - But occur in the test set

Zeros

Training set:

… denied the allegations
… denied the reports
… denied the claims
… denied the request

P("offer" | denied the) = 0

- Test set

  … denied the offer
  … denied the loan

Zero probability bigrams

Bigrams with zero probability

- mean that we will assign 0 probability to the test set!

And hence we cannot compute perplexity (can't divide by 0)!

Language Modeling

Generalization and zeros

Language Modeling

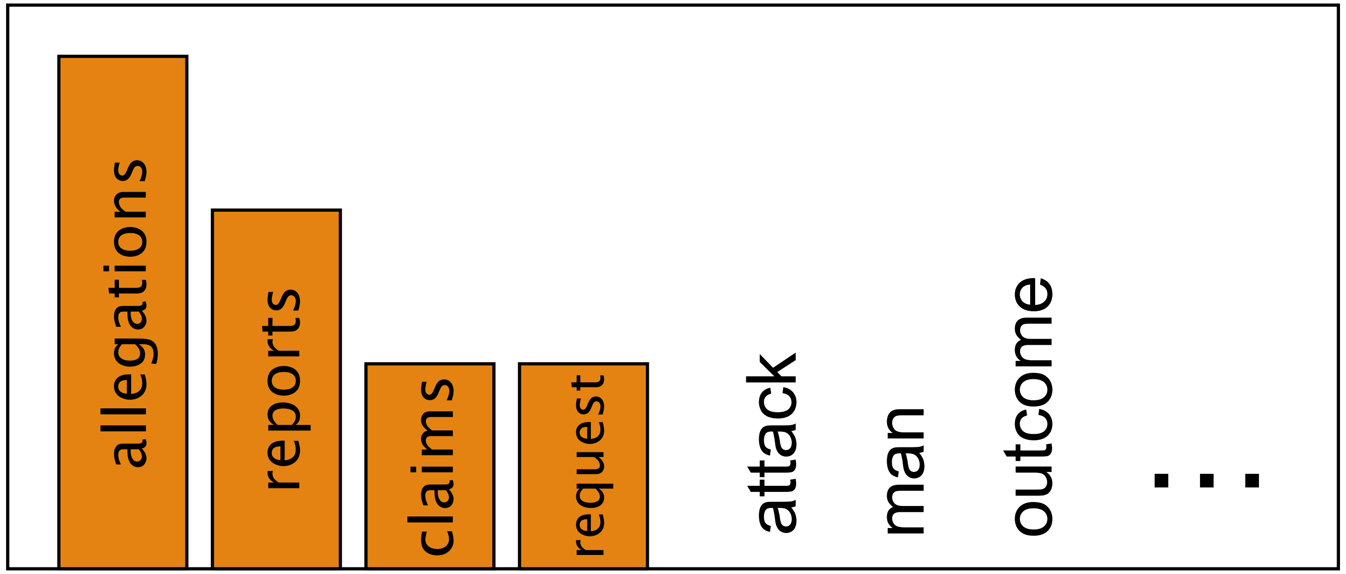
Smoothing: Add-one (Laplace) smoothing

The intuition of smoothing (from Dan Klein)

When we have sparse statistics:

P(w | denied the)
3 allegations
2 reports
1 claims
1 request

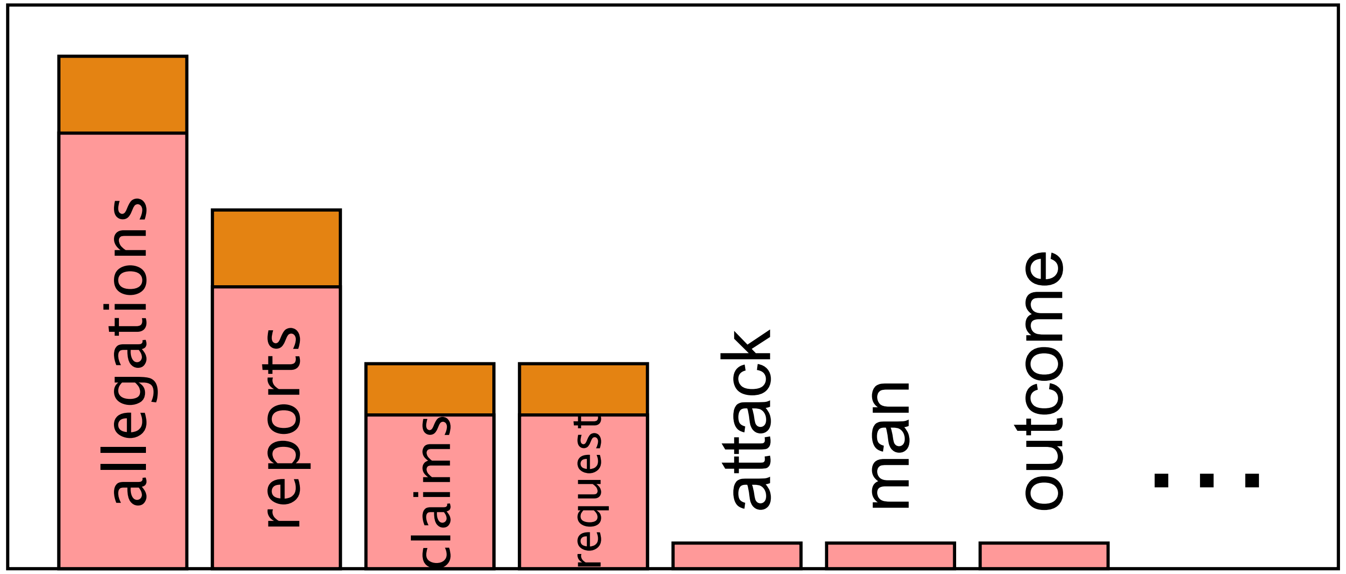
7 total



Steal probability mass to generalize better

P(w | denied the)
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other

7 total



Add-one estimation

Also called Laplace smoothing

Pretend we saw each word one more time than we did

Just add one to all the counts!

MLE estimate:

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

Add-1 estimate:

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

Maximum Likelihood Estimates

The maximum likelihood estimate

- of some parameter of a model M from a training set T
- maximizes the likelihood of the training set T given the model M

Suppose the word “bagel” occurs 400 times in a corpus of a million words

What is the probability that a random word from some other text will be “bagel”?

MLE estimate is 400/1,000,000 = .0004

This may be a bad estimate for some other corpus

- But it is the **estimate** that makes it **most likely** that “bagel” will occur 400 times in a million word corpus.

Berkeley Restaurant Corpus: Laplace smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Laplace-smoothed bigrams

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)+1}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Reconstituted counts

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n)+1] \times C(w_{n-1})}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Compare with raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Add-1 estimation is a blunt instrument

So add-1 isn't used for N-grams:

- We'll see better methods

But add-1 is used to smooth other NLP models

- For text classification
- In domains where the number of zeros isn't so huge.

Language Modeling

Smoothing: Add-one (Laplace) smoothing

Language Modeling

Interpolation, Backoff, and Web-Scale LMs

Backoff and Interpolation

Sometimes it helps to use **less** context

- Condition on less context for contexts you haven't learned much about

Backoff:

- use trigram if you have good evidence,
- otherwise bigram, otherwise unigram

Interpolation:

- mix unigram, bigram, trigram

Interpolation works better

Linear Interpolation

Simple interpolation

$$
\begin{aligned}
\hat{P}(w_n|w_{n-2}w_{n-1}) \;=\; & \lambda_1 P(w_n|w_{n-2}w_{n-1}) \\
& +\lambda_2 P(w_n|w_{n-1}) \\
& +\lambda_3 P(w_n)
\end{aligned}
$$

$$\sum_i \lambda_i = 1$$

Lambdas conditional on context:

$$
\begin{aligned}
\hat{P}(w_n|w_{n-2}w_{n-1}) \;=\; & \lambda_1(w_{n-2}^{n-1})P(w_n|w_{n-2}w_{n-1}) \\
& +\lambda_2(w_{n-2}^{n-1})P(w_n|w_{n-1}) \\
& +\lambda_3(w_{n-2}^{n-1})P(w_n)
\end{aligned}
$$

How to set the lambdas?

Use a **held-out** corpus

Training Data	Held-Out Data	Test Data

Choose λs to maximize the probability of held-out data:

- Fix the N-gram probabilities (on the training data)
- Then search for λs that give largest probability to held-out set:

$$\log P(w_1 \ldots w_n \mid M(\lambda_1 \ldots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \ldots \lambda_k)}(w_i \mid w_{i-1})$$

Unknown words: Open versus closed vocabulary tasks

If we know all the words in advanced
- Vocabulary V is fixed
- Closed vocabulary task

Often we don’t know this
- **Out Of Vocabulary** = OOV words
- Open vocabulary task

Instead: create an unknown word token <UNK>
- Training of <UNK> probabilities
  - Create a fixed lexicon L of size V
  - At text normalization phase, any training word not in L changed to <UNK>
  - Now we train its probabilities like a normal word
- At decoding time
  - If text input: Use UNK probabilities for any word not in training

Huge web-scale n-grams

How to deal with, e.g., Google N-gram corpus

Pruning

- Only store N-grams with count > threshold.
  - Remove singletons of higher-order n-grams
- Entropy-based pruning

Efficiency

- Efficient data structures like tries
- Bloom filters: approximate language models
- Store words as indexes, not strings
  - Use Huffman coding to fit large numbers of words into two bytes
- Quantize probabilities (4-8 bits instead of 8-byte float)

Smoothing for Web-scale N-grams

"Stupid backoff" (Brants *et al*. 2007)

No discounting, just use relative frequencies

$$S(w_i \mid w_{i-k+1}^{i-1}) = \begin{cases} \dfrac{\text{count}(w_{i-k+1}^{i})}{\text{count}(w_{i-k+1}^{i-1})} & \text{if} \quad \text{count}(w_{i-k+1}^{i}) > 0 \\ 0.4 S(w_i \mid w_{i-k+2}^{i-1}) & \text{otherwise} \end{cases}$$

$$S(w_i) = \frac{\text{count}(w_i)}{N}$$

N-gram Smoothing Summary

Add-1 smoothing:

- OK for text categorization, not for language modeling

The most commonly used method:

- Extended Interpolated Kneser-Ney

For very large N-grams like the Web:

- Stupid backoff

Advanced Language Modeling

Discriminative models:

- choose n-gram weights to improve a task, not to fit the training set

Parsing-based models

Caching Models

- Recently used words are more likely to appear

$$P_{CACHE}(w \mid history) = \lambda P(w_i \mid w_{i-2} w_{i-1}) + (1-\lambda)\frac{c(w \in history)}{\mid history \mid}$$

- These turned out to perform very poorly for speech recognition (why?)

Language Modeling

Interpolation, Backoff, and Web-Scale LMs

Language Modeling

Advanced:

Kneser-Ney Smoothing

Absolute discounting: just subtract a little from each count

Suppose we wanted to subtract a little from a count of 4 to save probability mass for the zeros

How much to subtract ?

Church and Gale (1991)'s clever idea

Divide up 22 million words of AP Newswire

- Training and held-out set
- for each bigram in the training set
- see the actual count in the held-out set!

It sure looks like c* = (c - .75)

Bigram count in training	Bigram count in heldout set
0	.0000270
1	0.448
2	1.25
3	2.24
4	3.23
5	4.21
6	5.23
7	6.21
8	7.21
9	8.26

Absolute Discounting Interpolation

Save ourselves some time and just subtract 0.75 (or some d)!

$$P_{\text{AbsoluteDiscounting}}(w_i \mid w_{i-1}) = \underbrace{\frac{c(w_{i-1}, w_i) - d}{c(w_{i-1})}}_{\text{discounted bigram}} + \underbrace{\lambda(w_{i-1})}_{\text{Interpolation weight}} \underbrace{P(w)}_{\text{unigram}}$$

- (Maybe keeping a couple extra values of d for counts 1 and 2)

But should we really just use the regular unigram P(w)?

Kneser-Ney Smoothing I

Better estimate for probabilities of lower-order unigrams!

- Shannon game: *I can't see without my reading* Kong / glasses___________?
- "Kong" turns out to be more common than "glasses"
- … but "Kong" always follows "Hong"

The unigram is useful exactly when we haven't seen this bigram!

Instead of P(w): "How likely is w"

$P_{continuation}(w)$: "How likely is w to appear as a novel continuation?

- For each word, count the number of bigram types it completes
- Every bigram type was a novel continuation the first time it was seen

$$P_{CONTINUATION}(w) \propto \left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|$$

Kneser-Ney Smoothing II

How many times does w appear as a novel continuation:

$$P_{CONTINUATION}(w) \propto \left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|$$

Normalized by the total number of word bigram types

$$\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|$$

$$P_{CONTINUATION}(w) = \frac{\left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|}{\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|}$$

Kneser-Ney Smoothing III

Alternative metaphor: The number of # of word types seen to precede w

$$|\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

normalized by the # of words preceding all words:

$$P_{CONTINUATION}(w) = \frac{|\{w_{i-1} : c(w_{i-1}, w) > 0\}|}{\sum_{w'} |\{w'_{i-1} : c(w'_{i-1}, w') > 0\}|}$$

A frequent word (Kong) occurring in only one context (Hong) will have a low continuation probability

Kneser-Ney Smoothing IV

$$P_{KN}(w_i \mid w_{i-1}) = \frac{\max(c(w_{i-1}, w_i) - d, 0)}{c(w_{i-1})} + \lambda(w_{i-1}) P_{CONTINUATION}(w_i)$$

λ is a normalizing constant; the probability mass we've discounted

$$\lambda(w_{i-1}) = \frac{d}{c(w_{i-1})} \left| \{ w : c(w_{i-1}, w) > 0 \} \right|$$

the normalized discount

The number of word types that can follow w_{i-1}
= # of word types we discounted
= # of times we applied normalized discount

Kneser-Ney Smoothing: Recursive formulation

$$P_{KN}(w_i \mid w_{i-n+1}^{i-1}) = \frac{\max(c_{KN}(w_{i-n+1}^{i}) - d, 0)}{c_{KN}(w_{i-n+1}^{i-1})} + \lambda(w_{i-n+1}^{i-1}) P_{KN}(w_i \mid w_{i-n+2}^{i-1})$$

$$c_{KN}(\bullet) = \begin{cases} count(\bullet) & \text{for the highest order} \\ continuationcount(\bullet) & \text{for lower order} \end{cases}$$

Continuation count = Number of unique single word contexts for •

Language Modeling

Advanced:

Kneser-Ney Smoothing