2335100 (Relations . functions , induction linear equation)

Q1:

(a) $V = \{1, 2, 3, 4, 5, 6, 7\}$

$Z = \{(1,7)\ (2,1),\ (4,1)\ (6,5)(6,6)\}$.

Equivalence closure:

Reflexivity: $(x,x) \in Z$ $x: [1, 2, 3, 4, 5, 6, 7]$

$Z_R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\ (7,7)\}$

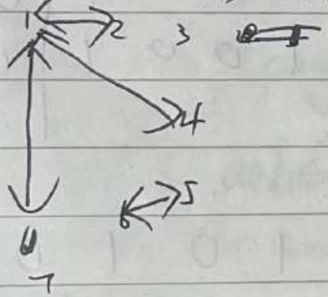
Symmetric: $(x,y) \in Z \wedge (y,x) \in E$ $Z((1,7)(2,1)\ (4,1)\ (6,5)(6,6))$

$Z_S = \{(7,1)\ (1,2)\ (1,4)\ (5,6)\ (6,6)\}$

Transitive: $(x,y) \in E \wedge (y,z) \in E \Rightarrow (x,z) \in E$

$[(1,7)\ (7,1)\ (2,1)\ (1,2)\ (1,4)\ (4,1)\ (6,5)\ (5,6)]$

$E_T = [(2,4)\ (4,2)\ (2,7)\ (7,2),\ (4,7)\ (7,4)]$



Equivalence closures is R

R: $[(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\ (7,7)\ (1,7)\ (7,1)\ (1,2)$
$(2,1),\ (4,1)\ (1,4)\ (6,5)\ (5,6)\ (2,4)\ (4,2)\ (2,7)\ (7,2)$
$(4,7)\ (7,4)]$

Equivalence classes: K

K: [1: [1, 2, 4, 7]   3: [3]   5: [5, 6]   7: [1, 2, 4, 7]
2: [1, 2, 4, 7]   4: [1, 2, 4, 7]   6: [5, 6]]

(b) in GF(2)

Gaussian elimination

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \xrightarrow{l_3 \leftrightarrow l_5} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right]$$

$l_4 - l_1$, $l_2 - l_1$, $l_3 - l_1$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right]$$

$l_5 \leftrightarrow l_4$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$l_4 - l_3$ $l_3 - l_2$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] \xrightarrow{l_5 - l_4} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$l_3 \leftrightarrow l_4$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & \boxed{0} & \boxed{0} \end{array}\right]$$

$x_5 =$ Any value

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$l_4 - l_3 \quad l_4 - l_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow{l_3 - l_2} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\downarrow l_2 - l_1$$

$x_1 = 1 \quad x_2 = 0$

$x_3 = 1 \quad x_4 = 1$

$x_5 =$ any value

$$\Leftarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

(c)

① $S \in \{a, b\}^*$

$\varepsilon \in S$

$w \in S$ $aabw \in S$

$w \in S$ any anagram of w is also in S.

(i) ∵ ($w \in S$ $aab \in S$). and any anagram of w is also in S

~~and~~ ∴ and The anagram of w is a string of letters arraged by the letter w.

∴ $aab \in S$, $baa \in S$

∴ ~~$aba \in S$~~ $aabbaa \in S$. $aba \in S$

∴ $ababaa \in S$

∵ ($w \in S$ $aab \in S$)

∴ ~~$aababaa \in S$~~ $wababaa \in S$

Threfor: $aabababaa \in S$

(ii)

According to the $w \in S$ then abb $w \in S$ and aababababaa $\in S$, it has basic ${}_{\text{rule}}$ 2 elements for "a" and 1 element for "b", in this stucture and as the length increase this stucture grow as basic ~~text~~ ${}_{\text{rule}}$ 2(elemnts+1) for "a" and (element+1) for "b", so aababababaa is follow this rule. ~~Accordin~~

Accordin to this conclusion ababab does not conform to the rule so ababab $\notin S$

(d)

In the Jave methods

Def: ① int doubleInt (int number) {reture number * 2}

This doubleInt is a injective function. Including number * 2 because we can * every number ~~that you *~~ by 2, ~~so~~ but we could have a number that's not just any number that you * by 2 so the injetive is satisfied.

② ~~int~~ float addOneTofloat (float number) {return number + 1.0f}

Def: This addonetofloat is Surjective function, each of these number represents a domain, In this function, when number is +1.0f, each number can be any possible number. So the Sujective condition is satisfied

Date /

Q2: (SAT, Predicate Logic)

(a)

(i) $(p \vee \neg q) \wedge (p \vee r) \wedge (p \vee s) \wedge (q \vee \neg p) \wedge (q \vee r) \wedge (q \vee s) \wedge (\neg r \vee \neg p)$
$\wedge \ ) \wedge (\neg r \vee \neg q) \wedge (\neg r \vee s) \wedge (\neg s \vee \neg p) \wedge (\neg s \vee \neg q) \wedge (\neg s \vee r)$

select $p = T$

remove $(p \vee \neg q)$ $(p \vee r)$ $(p \vee s)$ $(\neg p)$ $(\neg p)$ $(\neg p)$

obtain $(q) \wedge (q \vee r) \wedge (q \vee s) \wedge (\neg r) \wedge (\neg r \vee \neg q) \wedge (\neg r \vee s) \wedge$
$(\neg s) \wedge (\neg s \vee \neg q) \wedge (\neg s \vee r)$

Select $q = T$

remove: (q) $(q \vee r)$ $(q \vee s)$ $(\neg q)$ $(\neg q)$.

obtain: $(\neg r) \wedge (\neg r) \wedge (\neg r \vee s) \wedge (\neg s) \wedge (\neg s) \wedge (\neg s \vee r)$

Selet $r = F$

remove: $(\neg r)$ $(\neg r)$ $(\neg r \vee s)$ (r)

obtain: $(\neg s) \wedge (\neg s) \wedge (\neg s)$

selet $s = F$

remove: $(\neg s)$ $(\neg s)$ $(\neg s)$

obtain: no more clauses.

∴ SAT

(ii) The formula is valid. because it's an "OR" in Parentheses, and the parenthese are connectal by "AND" like DNF.

(iii) The property is p and q is filled and r and s is empty

(b)

(i) Function symbols: zero (arity 0); succ (arrity 1)

Predicate symbols: < (arity 2)

$S_1: \forall y . (\exists x . x<y) \to 0<y$

$S_2: \forall x . x < succ(x)$

$(S_1 \to (S_2 \to 0<2))$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{S_2}\,1}{\forall x . x < succ(x)}}{1 < succ(1)}[\forall E] \quad (1<2 \to 0<2)_2}{0<2}[\to E]$$

$$\dfrac{\dfrac{}{S_1}\,3}{\forall y . (\exists x . x<y \to 0<y)}(\forall E)$$

$$\exists x . x<2 \to 0<2$$

$$\dfrac{0<2 \qquad \exists x . x<2 \to 0<2}{0<2}[\exists E]_2$$

$$\dfrac{0<2}{S_1 \to S_2 \to 0<2}[\to I]\ 1,3$$

(ii)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{S_2}}{x < succ(x)}(\forall E)}{\forall y . y < succ(x)}\cancel{[\forall I]}}{\exists x . \forall y . y < x}[\exists I]}{S_2 \to \exists x \forall y . y < x}[\to I]$$

In this $[\forall I]$, we need to subsition all the x. For example, if $x<2$. $succ(x)<2$. so All of the natural number here are <2. so the tree is not Natural Deduction Proof.