

233T100

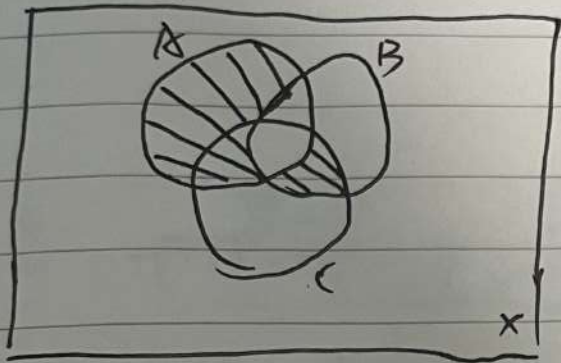
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~~Math~~ Question 1 [Numbers and Set theory]

(a)

i $(A \setminus B) \cup ((B \cap C) \setminus A)$

Venn diagram



ii $(A \cap B) \cup ((B \cap C) \cup A)$

 \therefore use $\cup \cap -$

$(A \cap B) \cup ((B \cap C) \cap \bar{A})$

iii $[x \in \mathbb{Z} \mid x^2 \leq 40] = D$

$D = \{0, 1, 2, 3, 4, 5, 6, -1, -2, -3, -4, -5, -6\}$

$[x \in \mathbb{Z} \mid \exists y \in \mathbb{Z} \mid y = x]$

$E = \{0, 3, 6, -3, -6\}$

$D \cap E = \{0, 3, 6, -3, -6\}$

$D = \{0, 1, 2, 3, 4, 5, 6, -1, -2, -3, -4, -5, -6\}$

(b) a

i. \mathbb{Z}_6 doesn't satisfy the inverse multiplication rule, Because

 \mathbb{Z}_6

$x \backslash y$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

ii

 $x \leftarrow 0$ $s \leftarrow 0$ while ($x < n$) { $x \leftarrow x + 1$ $s \leftarrow s + 2x - 1$ }return s

① check basis statement

$$P(0) = x = 0, s = 0, s = x^2$$

$$P(1) = x = 1, s = 1, s = x^2$$

assume $P(k)$ is true, $P(k+1)$

if P is true before the loop starts and true after loop iterates

$$x_{k+1} =$$

$$s : 0 + 2(1) - 1 + 2(2) - 1 + 2(3) -$$

$$\dots + 2(k) - 1 + 2(k+1)$$

$$= k^2 + 2k + 1$$

$$\downarrow (k+1)^2$$

if $P(k)$ is true $P(k+1)$ is also true so $s = x^2$

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In JAVA

(C) The cardinality of this set is countable, and the set is countable with both the list set and number stream that we defined, arrays are more efficient, but its capacity is fixed and cannot be ~~to~~ change dynamically. The set list can make capacity can grow dynamically, But ~~a~~ sacrificing efficiency.

Question 2 [Propositional Logic]

(a)

$$F: (\neg A \vee \neg B) \rightarrow C \quad C \rightarrow A \wedge B \rightarrow \neg C$$

F:

$$\begin{array}{c}
 \frac{C, C \rightarrow A \wedge B \rightarrow [E]}{A \wedge B} [E] \\
 \frac{A \wedge B}{B} [\wedge E] \\
 \frac{B}{\neg B} [I] \\
 \frac{\neg B}{\neg(B \rightarrow B)} [I] \\
 \frac{\neg(B \rightarrow B)}{\neg C} [I] \\
 \frac{\neg C}{(C \rightarrow A \wedge B) \rightarrow \neg C} [\rightarrow I] \\
 \frac{(C \rightarrow A \wedge B) \rightarrow \neg C}{(\neg A \vee \neg B) \rightarrow (C \rightarrow A \wedge B) \rightarrow \neg C} [\rightarrow I]
 \end{array}$$

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b) $\neg A$

4) $\neg \neg A \rightarrow \neg A$

$$\begin{array}{c}
 \frac{\neg \neg A}{\neg A} \quad \frac{\neg A}{\neg A} \quad \begin{array}{l} (IE) \\ (IZ) \end{array} \\
 \hline
 \neg(\neg \neg A) \quad \neg(\neg A) \quad (IE) \\
 \hline
 \neg \neg \neg A \rightarrow \neg A \quad (C-IZ)
 \end{array}$$

(ii) $\neg \neg \neg A \rightarrow \neg A$

$$\begin{array}{c}
 \frac{\neg A \vdash \neg A}{\vdash \neg \neg A, A} \quad (IZ) \\
 \hline
 \neg \neg \neg A \vdash \neg A \quad (IR) \\
 \hline
 \neg \neg \neg A \rightarrow \neg A \quad (C-IZ)
 \end{array}$$

(c)

H: $(\neg P \rightarrow Q \wedge R) \rightarrow P \vee R$

$$\begin{array}{c}
 \frac{P, R \vdash P}{P \vdash R \vdash \neg P \rightarrow Q \wedge R} \quad \begin{array}{l} (IZ) \\ (IR) \end{array} \\
 \hline
 P \vdash R, \neg P \rightarrow Q \wedge R \quad (IR) \\
 \hline
 \vdash P, R, (\neg P \rightarrow Q \wedge R) \quad (C) \\
 \hline
 (\neg P \rightarrow Q \wedge R) \vdash P, R \quad (C) \\
 \hline
 \vdash (\neg P \rightarrow Q \wedge R) \rightarrow P \vee R \quad (C-IR) \\
 \hline
 (\neg P \rightarrow Q \wedge R) \rightarrow P \vee R
 \end{array}$$