

Signature: $\langle \langle \text{zero}^0, \text{succ}^1, *^2 \rangle, \langle \text{even}^1, \text{odd}^1, =^2 \rangle \rangle$

$$F = \forall x. \text{even}(x) \rightarrow \text{odd}(\text{succ}(x))$$

any model for the above signature will be of the form:

$$M = \langle D, \langle F_{\text{zero}}, F_{\text{succ}}, F_* \rangle, \langle R_{\text{even}}, R_{\text{odd}}, R_= \rangle \rangle$$

$$\begin{aligned} \text{where } F_{\text{zero}} &\in D & R_{\text{even}} &\subseteq D^1 \\ F_{\text{succ}} &\in D^1 \rightarrow D & R_{\text{odd}} &\subseteq D^1 \\ F_* &\in D^2 \rightarrow D & R_= &\subseteq D^2 \end{aligned}$$

Here is an example of such a model

$$\text{Let } D = \mathbb{N}$$

$$F_{\text{zero}} = 0$$

$$F_{\text{succ}} = \langle n \rangle \mapsto n+1 \text{ (which we write as } +1)$$

$$F_* = \langle n, m \rangle \mapsto n \times m \text{ (which we write as } *)$$

$$R_{\text{even}} = \{ \langle 0 \rangle \}$$

$$R_{\text{odd}} = \emptyset$$

$$R_= = \emptyset$$

we call this model M_2 : $M_2 = \langle \mathbb{N}, \langle 0, +1, * \rangle, \langle \{ \langle 0 \rangle \}, \emptyset, \emptyset \rangle \rangle$

According to a model of the form M , the meaning of F is:

for all $n \in D$, if $\langle n \rangle \in R_{\text{even}}$ then $\langle F_{\text{succ}} \langle n \rangle \rangle \in R_{\text{odd}}$

Therefore the meaning of F according to M_2 is:

for all $n \in \mathbb{N}$, if $\langle n \rangle \in \{ \langle 0 \rangle \}$ then $\langle n+1 \rangle \in \emptyset$

which is equivalent to

for all $n \in \mathbb{N}$, if $n=0$ then $n+1 \in \emptyset$

which is false: take $n=0$, $n+1$ is not in \emptyset (nothing is in \emptyset)

by replacing $D \rightsquigarrow \mathbb{N}$
 $R_{\text{even}} \rightsquigarrow \{ \langle 0 \rangle \}$
 $R_{\text{odd}} \rightsquigarrow \emptyset$
 $F_{\text{succ}} \rightsquigarrow +1$