

Problems for Week 3

January 26, 2022

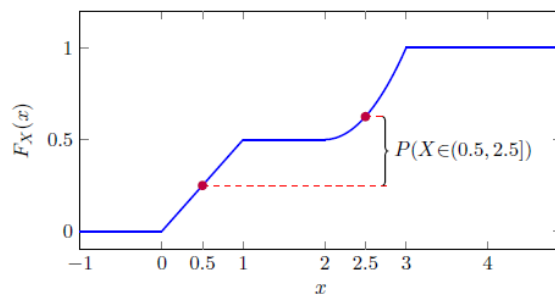
Problem 1. Two dice are tossed. Let X be the absolute difference in the number of dots facing up.

1. Find the PMF of X .
2. Find the CDF of X .

Problem 2. Consider a continuous random variable X with a CDF given by

$$F_X(x) = \begin{cases} 0, & \text{if } x < 0 \\ 0.5x, & \text{if } 0 \leq x \leq 1 \\ 0.5, & \text{if } 1 \leq x \leq 2 \\ 0.5(1 + (x - 2)^2), & \text{if } 2 \leq x \leq 3 \\ 1, & \text{otherwise.} \end{cases}$$

1. Compute $\mathbb{P}(X \in (0.5, 2.5))$
2. Compute the PDF



Problem 3. Let X have a CDF F_X . Denote $Y_1 = \max\{X, 0\}$ and $Y_2 = \min\{X, 0\}$. Compute the CDF of Y_1 and Y_2 .

Problem 4. The lifetime, X years, of a certain type of battery has probability density function given by

$$f_X(x) = \begin{cases} \frac{k}{x^2}, & \text{if } 1 \leq x \leq a \\ 0, & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

1. Compute the value of k .
2. Compute the CDF.
3. Compute the probability of $X \in (a/4, a/2)$.

Problem 5. Suppose that X has PDF

$$f_1(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{1}{(1+x)^2}, & \text{otherwise.} \end{cases}$$

Suppose Y has PDF

$$f_2(y) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{1}{1+y}, & \text{otherwise.} \end{cases}$$

1. Is f_1 a well-defined PDF?
2. Is f_2 a well-defined PDF?