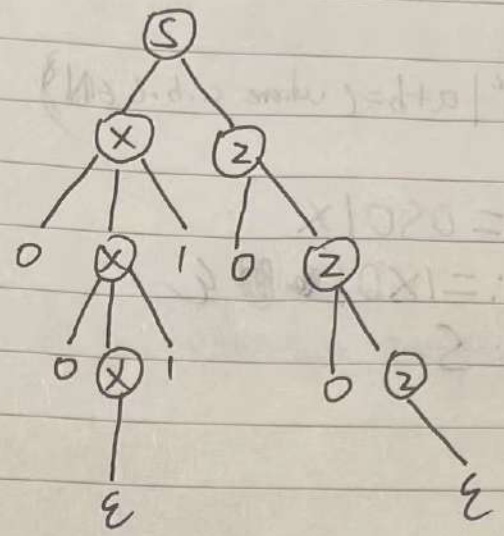
Ex1:

1) a derivation tree




left-most

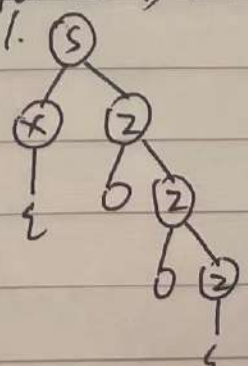
$S \Rightarrow \underline{X}Z$
$\Rightarrow 0\underline{X}1Z$
$\Rightarrow 00\underline{X}11Z$
$\Rightarrow 00\underline{\varepsilon}11Z$
$\Rightarrow 0011\underline{Z}$
$\Rightarrow 00110\underline{Z}$
$\Rightarrow 001100\underline{Z}$
$\Rightarrow 001100\underline{\varepsilon}$
$\Rightarrow 001100$

(2) The grammar G is ambiguous because this grammar G has more than one derivation tree for L(G). for example String "00"

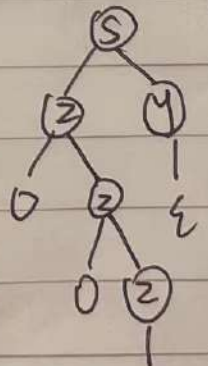
1. $S \Rightarrow \underline{X}Z$
$\Rightarrow \underline{\varepsilon}Z$
$\Rightarrow \underline{Z}$
$\Rightarrow 0\underline{Z}$
$\Rightarrow 00\underline{Z}$
$\Rightarrow 00$

2. $S \Rightarrow \underline{Z}Y$
$\Rightarrow 0\underline{Z}Y$
$\Rightarrow 00\underline{Z}Y$
$\Rightarrow 00\underline{\varepsilon}Y$
$\Rightarrow 00$

1.




2.




The exist multiple right-most or left most from this grammar L(G) so is ambiguous

(3)

(a) 01000

$S \Rightarrow \underline{X}2$

$\Rightarrow 0\underline{X}12$

$\Rightarrow 0\underline{1}2Z$

$\Rightarrow 01\underline{Z}$

$\Rightarrow 010\underline{Z}$

$\Rightarrow 0100\underline{Z}$

$\Rightarrow 01000\underline{Z}$

$\Rightarrow 01000\varepsilon$

$\Rightarrow 01000$ (Yes)

(b) 1 (NO)

(c) 01100

$S \Rightarrow \underline{Z}y$

$\Rightarrow 0\underline{Z}y$

$\Rightarrow 0\underline{1}3y$

$\Rightarrow 0\underline{y}$

$\Rightarrow 01\underline{y}0$

$\Rightarrow 011\underline{y}00$

$\Rightarrow 011\underline{\varepsilon}00$

$\Rightarrow 01100$ (Yes)

(d) $\varepsilon$

$S \Rightarrow XZ$

$\Rightarrow \varepsilon Z$

$\Rightarrow \varepsilon\varepsilon$

$\Rightarrow \varepsilon$ (Yes)

(4)

$$L(G_1) = \{ (0^a 1^b 0^c) \mid (a, b, c \in \mathbb{N}) \mid (a, b, c \geq 0) \}$$

Ex 2:

$$\Sigma = (0, 1)$$

$$\{ L = \{ 0^a 1^b 0^c \mid a+b=c \text{ (where } a, b, c \in \mathbb{N}) \} \}$$

Design CFG

$$\Rightarrow S ::= 0S0 \mid X$$

$$X ::= 1X0 \mid \varepsilon$$

start : S