Problems for Week 1

January 26, 2022

Problem 1. *Consider an experiment of rolling a die twice. The outcome of this experiment is an ordered pair whose first element is the first value rolled and whose second element is the second value rolled.*

1. *Find the sample space.*
2. *Find the event A that the value on the first roll is greater than or equal to the value on the second roll.*
3. *Find the event B that the first roll is a six.*
4. *Let C be the event that the first valued rolled and the second value rolled differ by two. Find $A \cap C$.*

Problem 2.

1. *Show that $\mathbb{P}(\cup_{k=1}^{n} A_k) \leq \sum_{k=1}^{n} \mathbb{P}(A_k)$*
2. *Show that $\mathbb{P}(\cap_{k=1}^{n} A_k) \geq 1 - \sum_{k=1}^{n} \mathbb{P}(A_k^c)$*

Problem 3. *Prove that*

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B).$$

Problem 4. *Consider tossing a coin. The event space is*

$$\mathcal{F} = \big\{\emptyset, \{H\}, \{T\}, \Omega\big\}.$$

We define two functions as follows

$$\begin{aligned} &\mathbb{P}_1[\emptyset] = 0, \quad \mathbb{P}_1[\{H\}] = 1/2, \quad \mathbb{P}_1[\{T\}] = 1/2, \quad \mathbb{P}_1[\Omega] = 1 \\ &\mathbb{P}_2[\emptyset] = 0, \quad \mathbb{P}_2[\{H\}] = 1/3, \quad \mathbb{P}_2[\{T\}] = 1/3, \quad \mathbb{P}_2[\Omega] = 1 \end{aligned}$$

1. *Is $\mathbb{P}_1$ a probability law?*
2. *Is $\mathbb{P}_2$ a probability law?*

Problem 5. *You toss a fair coin 5 times. What is the probability that you see at least two heads.*

Problem 6. *Let the events A and B have*

$$\mathbb{P}(A) = x, \quad \mathbb{P}(B) = y, \quad \mathbb{P}(A \cup B) = z.$$

Find the probabilities $\mathbb{P}(A \cap B), \mathbb{P}(A \cap B^c)$ and $\mathbb{P}(A^c \cap B^c)$.