

$$P(B|A) = \frac{P(A, B)}{P(A)}$$

$A \cap B : A \hookrightarrow B$

$$\Leftrightarrow P(A, B) = P(A) \cdot P(B|A)$$

$P(\text{"its water is so transparent"})$

$= P(\text{its})$

$\times P(\text{water} | \text{its})$

$\times P(\text{is} | \text{its water})$

$\times P(\text{so} | \text{its water is})$

$\times P(\text{transparent} | \text{its water is so})$

$$P(w_1, w_2, \dots, w_k) \approx \prod_i P(w_i | w_{i-k} \dots w_{i-1})$$

$P(\text{the} \mid \text{its water is so transparent})$

$\approx P(\text{the} \mid \text{that})$  Unigram

$\approx P(\text{the} \mid \text{transparent that})$  Bigram.

$$P(w_1, w_2, \dots, w_k)$$

$$= P(w_1) \times P(w_2 | w_1)$$

$$\times P(w_3 | w_1, w_2) \dots$$

$$\times P(w_k | w_{k-2}, w_{k-1})$$

$$\prod_i P(w_i | w_{i-2}, w_{i-1})$$

Trigram:

$$P(w_1, w_2, \dots, w_k)$$

$$= P(w_1) \times P(w_2 | w_1)$$

$$\times \prod_{i=3}^k P(w_i | w_{i-2}, w_{i-1})$$

You can continue to longer N-grams,  
but the combinations of words  
become more specific, so  
might be possible that N-grams  
not being present in the  
training data.

$$P(I | \langle s \rangle) = \frac{2}{3}$$

$$P(am | I) = \frac{C(I \text{ am})}{C(I)} = \frac{2}{3}$$

$$P(w_i | w_{i-1}) = \frac{C(w_{i-1}, w_i)}{\underbrace{C(w_{i-1})}}$$

In practice,  
these probabilities should be calculated under log  
to avoid underflow

$$\log(p_1 \times p_2 \times p_3 \times p_4)$$

$$= \log p_1 + \log p_2 + \log p_3 + \log p_4$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs  
and ham

Applying Trigram

$$P = P(w_1) \times P(w_2 | w_1) \\ \times \prod_{i=2}^k P(w_k | w_{k-2}, k-1)$$

$$P(<s>) = \frac{3}{3} = 1$$

$$P(I | <s>) = \frac{2}{3} = 0.67$$

$$P(am | <s> I) = \frac{1}{2} = 0.5$$



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$$P(\text{Sam} \mid \text{I am}) = \frac{1}{2} = 0.5$$

$$P(</s> \mid \text{am Sam}) = \frac{1}{1} = 1$$

$$P(<s> \mid \text{Sam </s>}) = \frac{1}{1} = 1$$

$$P(W_3 | W_1, W_2) \\ = \frac{C(W_1, W_2, W_3)}{C(W_1, W_2)}$$

P

(20000)

N

$$\left(\frac{1}{4}\right)^{90000} \times \left(\frac{1}{20000}\right)^{3000}$$

4

N

$$\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{20000}$$

$$\underbrace{0000 \dots 0}_{91} \quad \underbrace{123456789}_9$$

$$P_{(w)} = \sqrt[10]{\left(\frac{1}{100}\right)^1 \times \left(\frac{91}{100}\right)^9}$$

$$\approx 1.725$$


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$P(i \text{ want chinese food})$

$= P(i | <s>) \times P(\text{want} | i)$

$\times P(\text{chinese} | \text{want}) \times P(\text{food} | \text{chinese})$

$\times P(</s> | \text{food})$

$= 0.19 \times 0.33 \times 0.0065 \times 0.52 \times 0.40$

$| = 0.19 \times 0.21 \times 0.0029 \times 0.052 \times 0.40$

(2) The un-smoothed one is higher because the smoothed probability will provide some prob to the situation that will never happen, (probability = 0), so the probability will be more even in the smoothed probability.

[3]

$$P(\text{Sam} | \text{am})$$

$$= \frac{C(\text{am Sam}) + 1}{C(\text{am}) + \underset{\uparrow}{V}}$$

the number of  
vocab

$$= \frac{2+1}{3+V}$$

$$= \frac{2+1}{3+11} = \frac{3}{14} = 0.214$$

$$P(w_3 | w_1, w_2)$$

$$= \frac{C(w_1, w_2, w_3) + 1}{C(w_1, w_2) + V}$$

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$$P(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$$P(I | <s>) = \frac{c(I, <s>)}{c(<s>)} = \frac{2}{3} = \underline{\underline{\frac{2}{3}}}$$

$$P(am | I) = \frac{c(am, I)}{c(I)} = \frac{2}{3} = \underline{\underline{0.67}}$$

$$P(a | b) = P(b) \cdot P(a, b)$$

=

$$P(A) = 0.4$$

$$P(B) = 0.3$$

$$P(C) = 0.2$$

$$P(D) = 0.1$$

$$N \sqrt{\prod_{i=1}^N \frac{1}{P(W_i)}}$$

$$= 4 \sqrt{\frac{1}{0.4 \times 0.3 \times 0.4 \times 0.2}}$$

$$= \frac{10000}{4 \times 9 \times 3 \times 2} = \sqrt[4]{\frac{10000}{96}}$$

"A C"

$$P(A | \langle s \rangle) \times P(C | A) \times P(\langle s \rangle | C)$$

$$= 0.6 \times 0.7 \times 1.0$$

$$= \frac{6}{10} \times \frac{7}{10}$$

$$\sqrt[3]{\frac{100}{42}} =$$

$$\frac{1}{0.5 \times 0.3 \times 0.1 \times 0.1 \times 0.1}$$

$$= \frac{100000}{15}$$

$$5 \sqrt{\frac{10000}{15}} =$$

