ID 2335100

Exam for Mathematical and Logical Foundations of Computer Science

Math

$4^0 = 1$ $\quad 1 \bmod 3 = 1$

Q1. $4^1 = 4$ $\quad 4 \bmod 3 = 1$

a(i) $4^2 = 16$ $\quad 16 \bmod 3 = 1$ $\qquad P(n) = 4^n \bmod 3$

$4^3 = 64$ $\quad 64 \bmod 3 = 1$

$\therefore 1 \in N$

$4^4 = 256$ $\quad 256 \bmod 3 = 1$

$\therefore 4^n \bmod 3$

Base case: $4^n \bmod 3 = 1,\ 1 \in N$

induction case: assume $4^2 \bmod 3 = n \in N$ is true

show the $\underbrace{4^n \bmod 3}_{P(n)} = n \in N$ is true too

Q2(ii)

$[x \in N \mid x \geq 0] \to [y \in R \mid y \geq 0]$

$sqrt(x) = y \quad x = y^2 \qquad x^2 > 0$

~~ever element only has one output~~

sqrt is injective because ~~$x = y$: $x = y^2$ two different value has same answer~~

sqrt is not ~~not~~ surjective because every possible output is not obtained

sqrt is ~~not~~ injective and is not surjective so is not bijective.

(iii) (1) for sqrt has infinitely ~~number~~ many elements, int has 2^{32} many

Javasqrt (2) If the parameter is ~~pos~~ postive infinity, then the result is postive infinity

Javasqrt (3) If the argument passed is postive or negative zero the result will be same as the ~~of the argument~~ but

(4) for normal positive numbers, to return the double value closest to the true math square root of the argument value. If the input was perfect square int, the result will be an integer valued double that is in int range

(5) R contains uncomputable elements, whereas ever output is the possible result of computation for sqrt $\in R$

Q (b)

(i)

$P_1\begin{pmatrix}1\\2\\4\end{pmatrix}$ $P_2=\begin{pmatrix}-1\\0\\4\end{pmatrix}$ $P_3\begin{pmatrix}-1\\2\\2\end{pmatrix}$

$\overrightarrow{P_1P_2}$ $\overrightarrow{P_2P_3}$ $\overrightarrow{P_1P_3}$

$\begin{pmatrix}-2\\-2\\0\end{pmatrix}$ $\begin{pmatrix}0\\2\\-2\end{pmatrix}$ $\begin{pmatrix}-2\\0\\-2\end{pmatrix}$

$\sqrt{\overrightarrow{P_1P_2}} = \sqrt{(-2)^2+(-2)^2+0^2} = 2\sqrt{2}$

$\sqrt{\overrightarrow{P_2P_3}} = \sqrt{0^2+2^2+(-2)^2} = 2\sqrt{2}$

$\sqrt{\overrightarrow{P_1P_3}} = \sqrt{(-2)^2+0+(-2)^2} = 2\sqrt{2}$

so that these is equilateral triangle

(ii)

$M=\begin{pmatrix}1\\2\\4\end{pmatrix}+s\begin{pmatrix}-2\\-2\\0\end{pmatrix}+t\begin{pmatrix}-2\\0\\-2\end{pmatrix}$

Parametic represent

nomal from:

$ax_1+bx_2+cx_3=d$

$4x_1 - 4x_2 - 4x_3 = -20$

$x_1 - x_2 - x_3 = -5$

$a = 4-0 = 4$

$b = 0-4 = -4$

$c = 0-4 = -4$

$d = \begin{pmatrix}1\\2\\4\end{pmatrix}\cdot\begin{pmatrix}4\\-4\\-4\end{pmatrix}$

$= 4-8-16 = -20.$

(iii)

$N \quad \begin{pmatrix}1\\0\\3\end{pmatrix} + q\begin{pmatrix}1\\1\\1\end{pmatrix} + r\begin{pmatrix}2\\0\\1\end{pmatrix}$

$M \quad \begin{pmatrix}1\\2\\4\end{pmatrix} + s\begin{pmatrix}-2\\-2\\0\end{pmatrix} + t\begin{pmatrix}-2\\0\\-2\end{pmatrix}$

$M = \begin{pmatrix}1-2s-2t\\2-2s\\4-2t\end{pmatrix} \quad N = \begin{pmatrix}1+q+2r\\q\\3+q+r\end{pmatrix}$

1: $1-2s-2t = 1+q+2r$

$-2s-2t-q-2r = 0$

2: $2-2s = q$

$q+2s = 2$

3: $4-2t = 3+q+r$

$q+r+2t = 1$

$$\left[\begin{array}{cccc|c} -2 & -2 & -1 & -2 & 0 \\ 2 & 0 & 1 & 0 & 2 \\ 0 & 2 & 1 & 1 & 1 \end{array}\right]$$

row1 + row2

$$\left[\begin{array}{cccc|c} -2 & -2 & -1 & -2 & 0 \\ 0 & -2 & 0 & -2 & 2 \\ 0 & 2 & 1 & 1 & 1 \end{array}\right]$$

row2 + row3

$$\left[\begin{array}{cccc|c} -2 & -2 & -1 & -2 & 0 \\ 0 & -2 & 0 & -2 & 2 \\ 0 & 0 & 1 & -2 & 3 \end{array}\right]$$

row2 − row1

$$\left[\begin{array}{cccc|c} -2 & 0 & 1 & 0 & 2 \\ 0 & -2 & 0 & -2 & 2 \\ 0 & 0 & 1 & -2 & 3 \end{array}\right] \rightarrow$$

row3 − row1

$$\left[\begin{array}{cccc|c} -2 & 0 & 0 & -2 & 1 \\ 0 & -2 & 0 & -2 & 2 \\ 0 & 0 & 1 & -2 & 3 \end{array}\right]$$

$$\begin{cases} -2s - \cancel{2r} = 1 \\ -2t - \cancel{2r} = 2 \\ q - \cancel{2r} = 3 \end{cases} \rightarrow \begin{matrix} -2s = 1 \\ -2t = 2 \\ q = 3 \end{matrix} \quad \begin{matrix} s = -\frac{1}{2} \\ t = -1 \\ q = 3 \end{matrix}$$

Date. /

2 when $q=3$

$$\begin{pmatrix}1\\0\\3\end{pmatrix}+3\begin{pmatrix}1\\1\\1\end{pmatrix}+r\begin{pmatrix}2\\0\\1\end{pmatrix}$$

$$l=\begin{pmatrix}4\\3\\6\end{pmatrix}+r\begin{pmatrix}2\\0\\1\end{pmatrix}$$

Q2

(i)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A}{\vdash A, A}[\to R] \qquad \dfrac{B \vdash B}{\vdash B, B}[\to R]}{\vdash A \vee B,\ B, A}(\vee R)}{A \vee B \vdash B, A}[\to L]}{(A \vee B) \vdash (B \vee A)}(\vee R)}{(A \vee B) \to (B \vee A)}[\to R]}{}$$

(the axioms are labelled [id])

(ii)

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{B \to (\neg B \vee \neg A)}^{2}}{(\neg B \vee \neg A)} \qquad \dfrac{\overline{A}^{1}}{A}(\vee I) \qquad \dfrac{\dfrac{\dfrac{\overline{A}^{1} \quad \overline{B}^{4}}{B \vee A}[\vee I]}{\bot}[\neg E]}{\neg B}(\neg I)^{4}}{\dfrac{\bot}{\neg B}[\neg I]}[\neg E]}{(B \to (\neg B \vee \neg A)) \to \neg B}[\to I]^{2}}{A \to (B \to (\neg B \vee \neg A)) \to \neg B}[\to I]^{1}$$

(iii) it is satisfiable, it can use Natural Deduction to Proof so G is satisfiable

Date. /

Q3

(i)

$$\dfrac{\dfrac{\dfrac{\forall x.\ x < succ(x)^{\ 2}}{x\ 0 < 0}[\forall z]}{\exists x \forall y.\ x < y}[\exists z] \qquad \dfrac{\dfrac{\forall y\ (\exists x\ 0 < 0) \Rightarrow x < y^{\ 1}}{\forall x.\ \neg(x < 0) \Rightarrow x < y}[\forall z]}{}[\exists z]}{\dfrac{\dfrac{\bot}{\neg \exists x.\ \forall y.\ x < y}[\neg z]}{S_1 \to \neg \exists x.\ \forall y.\ x < y}[\to z]}$$

(ii) (M_1)

$\models_{M_1} \neg(\forall x.\ \forall y.\ x < y \to x \leq y)$

$M_1 : \langle D, \langle F_{zero}, F_{succ} \rangle, \langle R_< \rangle$

$D = \mathbb{N}$ $F_{zero} = 0$ $F_{succ} = \langle n \rangle \to n+1$

$R_< = \{ \langle n, m \rangle \mid n < m \}$

$M = \langle \mathbb{N}, 0, +1 \rangle \langle n, m \rangle \mid n < m \rangle$

where +1 is the function that give a number increment it by 1

for all $n, m \in D$ if $\langle n, m \rangle \in R_<$ then $\langle F_{succ} \langle n \rangle\ m \rangle \in R_<$

Therefor for all $n, m \in \mathbb{N}$ if $\langle n, m \rangle \in \langle n, m \rangle \{ \langle n, m \rangle \mid n < m \}$ then $\langle n+1, m \rangle \in \{ \langle n, m \rangle \mid n < m \}$

for all $n\ m \in \mathbb{N}$ if $n < m$ then $n+1 < m$

Which is false for Model $M_1\ \neg(\forall x.\ \forall y.\ x < y \to x \leq y)$

Date. /

$\mathcal{M}_2$

$\models_{\mathcal{M}_2} \forall x \cdot \forall y\ x \leq y \to x < y$

$\mathcal{M}_2 : \langle D, \langle F_{zero}, F_{succ} \rangle, R_< \rangle$

$D = \mathbb{N}$ $\quad F_{zero} = 0 \quad F_{succ} = \langle n \rangle \to n+1$

$R_< = (\langle n, m \rangle \mid n \leq m \rangle$

$\mathcal{M}_2 = \langle \mathbb{N}, \langle 0, +1 \rangle, \langle \{ n, m \mid True \rangle$

for all $\underset{N}{n} \in D$ if $n, \underset{m}{\in} R$ then $\langle F_{succ}(n)\ m \rangle \in R_<$

There for for all $n, m \in N$ if $\langle n, m) \in \langle n, m \rangle$ then $\langle n+1, m)$

$\in \{ \langle n, m \rangle \mid n < m \}$

for all $n, m \in N$ if $n < m$ then $n+1 \leq m$

Which is true for modle $\mathcal{M}_2 \models_{\mathcal{M}_2} \forall x \cdot \forall y\ x \leq y \to x < y$

Statement of good academic conduct

By submitting this assignment, I understand that I am agreeing to the following statement of good academic conduct:

- I confirm that this assignment is **my own work** and I have not worked with others in preparing this assignment.
- I confirm this assignment was written by me and is in my own words, except for any materials from published or other sources which are clearly indicated and acknowledged as such by appropriate referencing.
- I confirm that this work is not copied from any other person's work (published or unpublished), web site, book or other source, and has not previously been submitted for assessment either at the University of Birmingham or elsewhere.
- I confirm that I have not asked, or paid, others to prepare any part of this work for me.
- I confirm that I have read and understood the University regulations on plagiarism (https://intranet.birmingham.ac.uk/as/registry/policy/conduct/plagiarism/index.aspx).