Summative Assignment 3 - Solutions
Propositional Logic – Sequent Calculus and Natural Deduction

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1. The correct answer is that it is provable and that there is an infinite number of ways to prove this formula. Here are ways we can prove this formula:

$$\dfrac{\overline{A}^{\,1}}{A \to A}\;1\;[\to I]$$

or

$$\dfrac{\dfrac{\dfrac{\overline{A}^{\,1} \quad \overline{A}^{\,1}}{A \wedge A}\;[\wedge I]}{A}\;[\wedge E_L]}{A \to A}\;1\;[\to I]$$

or

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{A}^{\,1} \quad \overline{A}^{\,1}}{A \wedge A}\;[\wedge I]}{A}\;[\wedge E_L] \quad \overline{A}^{\,1}}{A \wedge A}\;[\wedge I]}{A}\;[\wedge E_L]}{A \to A}\;1\;[\to I]$$

etc.

2. The correct answer is that the proof rules of Natural Deduction and the proof rules of the Sequent Calculus are completely different. One major difference between the two sets of rules is that the rules of Natural Deduction allow deriving individual propositions, while the rules of the Sequent Calculus allow deriving sequents.

3. The correct answer is that $A \to B \to A$ is provable in both Natural Deduction and in the Sequent Calculus. As mentioned in the lectures, if a proposition is provable in Natural Deduction then it is also provable in the Sequent Calculus, and vice versa. Here is a Natural Deduction proof:

$$\dfrac{\dfrac{\overline{A}^{\,1}}{B \to A}\;2\;[\to I]}{A \to B \to A}\;1\;[\to I]$$

Here is a Sequent Calculus proof:

$$\dfrac{\dfrac{\dfrac{}{A, B \vdash A}\;[Id]}{A \vdash B \to A}\;[\to R]}{\vdash A \to B \to A}\;[\to R]$$

4. The formula is derivable as follows:

$$
\dfrac{\dfrac{}{A, B \vdash A}[Id] \quad \dfrac{\dfrac{}{A, B \vdash B}[Id]}{\neg B, A, B \vdash C}[\neg L]}{\dfrac{A \to \neg B, A, B \vdash C}{\dfrac{A \to \neg B, A \land B \vdash C}{\dfrac{A \to \neg B \vdash (A \land B) \to C}{\vdash (A \to \neg B) \to (A \land B) \to C}[\to R]}[\to R]}[\land L]}[\to L]
$$

The other answers are not correct because one is a Natural Deduction proof, and the other one did not use the rule $[\to L]$ correctly.