EE209 HW1

October 17, 2018

In [1]:
```
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
import matplotlib.patches as patches
from matplotlib.collections import PatchCollection
import math
from timeit import default_timer as timer
```

In [2]:
```
##################################################################################
# 1(a): The size of N_S is R x C x num_directions = 6x6x12 = 432 for our example #
##################################################################################
class StateSpace:
    def __init__(self, num_directions, num_cols, num_rows):
        self.statespace = []
        for i in range(num_directions):
            self.statespace.append([[State(direction=i, col=j, row=k, reward=0) for k 

    def setRewardOfSpecificCoordinate(self, d,c,r, reward):
        self.statespace[d][c][r].reward = reward

    def setRewardOfAllDirectionsAtCoordinate(self, c,r, reward):
        for i in range(len(self.statespace)):
            self.statespace[i][c][r].reward = reward

class State:
    col = 0
    row = 0
    direction = 0
    reward = 0

    def __init__(self, direction, col, row, reward):
        self.col = col
        self.row = row
        self.direction = direction
        self.reward = reward

    def __repr__(self):
        return("(" + str(self.col) + "," + str(self.row) + ")d:" + str(self.direction)
```

```python
            #return(str(self.col) + "," + str(self.row))

        def copy(self):
            return State(self.direction, self.col, self.row, self.reward)

        def equals(self, toCompare):
            if(self.direction == toCompare.direction and self.col == toCompare.col and sel
                return True
            return False
```

In [3]:
```python
##############################
# 1(b): The size of N_A is 7 #
##############################
class Action: #lawsuit
    move_direction = 0 # 1 = forward, 0 = no movement, -1 = backward
    rotate_direction = 0 # post-movement rotation: -1 = left turn, 0 = no turn, 1 = ri

    def __init__(self, in_m, in_r):
        self.move_direction = in_m
        self.rotate_direction = in_r

    def __repr__(self):
        if(self.move_direction == 0):
            return("no movement")
        elif(self.move_direction == -1):
            m = "backward then "
        elif(self.move_direction == 1):
            m = "forward then "
        if(self.rotate_direction == 0):
            return(m + "no turn")
        elif(self.rotate_direction == -1):
            return(m + "left turn")
        elif(self.rotate_direction == 1):
            return(m + "right turn")

    def equals(self, toCompare):
        if (self.move_direction == toCompare.move_direction and self.rotate_direction =
            return True
        return False
```

In [4]:
```python
########
# 1(c) #
########
def getProbNextState(currState, action, nextState, error_prob, statespace):
    #check to make sure states are adjacent before continuing (should save some comput
    if(abs(currState.col - nextState.col) + abs(currState.row - nextState.row) > 1):
        return 0
    #catch no movement:
```

```python
    if(action.move_direction == 0):
        if(nextState.equals(currState)):
            return 1
        else:
            return 0

    # Three cases to consider. error left, error right, and no error. If NextState is
    # then we return 0. Otherwise, we return error_prob for an error and 1-2*error_pro

    ########################
    # Case one: Error Left: #
    ########################
    tempState = currState.copy()
    tempState.direction = (tempState.direction + LEFT) % NUM_DIRECTIONS
    tempState = getNextState(tempState, action,statespace)
    tempState = statespace[tempState.direction][tempState.col][tempState.row].copy()
    if(nextState.equals(tempState)):
        return error_prob

    #########################
    # Case two: Error Right: #
    #########################
    tempState = currState.copy()
    tempState.direction = (tempState.direction + RIGHT) % NUM_DIRECTIONS
    tempState = getNextState(tempState, action, statespace)
    tempState = statespace[tempState.direction][tempState.col][tempState.row].copy()
    if(nextState.equals(tempState)):
        return error_prob

    ########################
    # Case three: No Error: #
    ########################3
    tempState = currState.copy()
    tempState = getNextState(tempState, action, statespace)
    tempState = statespace[tempState.direction][tempState.col][tempState.row].copy()
    if(nextState.equals(tempState)):
        return 1-(2*error_prob)

    #otherwise, we could not have reached nextState
    return 0

#helper function for 1(c)
def getNextState(currState, action, statespace):
    tempState = currState.copy()

    #quick catch if robot isnt programmed to move
    if(action.move_direction == 0):
        return currState
```

```
        #a shortcut to reduce amount of if statements i need to do
        if(action.move_direction == BACKWARD):
            tempState.direction = (tempState.direction + 6) % NUM_DIRECTIONS

        #move the robot's position
        if(tempState.direction in [11,0,1]): #move up, so increase row
            if(tempState.row + 1 < len(statespace[0])): #if it is still in bounds
                tempState.row = tempState.row + 1
        elif(tempState.direction in [2,3,4]): #move right, so increase col
            if(tempState.col + 1 < len(statespace[0][0])): #if it is still in bounds
                tempState.col = tempState.col + 1
        elif(tempState.direction in [5,6,7]): #move down, so decrease row
            if(tempState.row - 1 >= 0): #if it is still in bounds
                tempState.row = tempState.row - 1
        elif(tempState.direction in [8,9,10]): #move left, so decrease col
            if(tempState.col - 1 >= 0): #if it is still in bounds
                tempState.col = tempState.col - 1

        #post-rotate the robot, if necessary
        tempState.direction = (tempState.direction + action.rotate_direction) % NUM_DIRECTI

        #reverse the shortcut i used
        if(action.move_direction == BACKWARD):
            tempState.direction = (tempState.direction + 6) % NUM_DIRECTIONS

        return statespace[tempState.direction][tempState.col][tempState.row].copy()
```

In [5]:
```
########
# 1(d) #
########
def getNextStateWithErrorProb(currState, action, error_prob, statespace):
    #catch no movement
    if(action.move_direction == STILL):
        return currState

    num = np.random.uniform()
    if(num <= error_prob): #pre-turn left
        print('left error')
        currState.direction = (currState.direction + LEFT) % NUM_DIRECTIONS
        return getNextState(currState, action, statespace)
    elif(num <= 2*error_prob): #pre-turn right
        print('right error')
        currState.direction = (currState.direction + RIGHT) % NUM_DIRECTIONS
        return getNextState(currState, action, statespace)
    return getNextState(currState, action, statespace)
```

In [6]:
```
########
```

```
    # 2(a) #
    ########
    def getReward(state):
        return state.reward
```

In [15]:

```
NUM_DIRECTIONS = 12
NUM_MOTIONS = 3 #forward, backward, no movement
NUM_TURNS = 3 # left, right, no turn

FORWARD = 1
BACKWARD = -1
RIGHT = 1
LEFT = -1
STILL = 0

class GameBoard:
    C = 0
    R = 0
    ss = None
    robot = None
    error_prob = 0
    policy_matrix = 0
    value_matrix = 0

    def __init__(self, C, R, error_prob):
        self.C = C
        self.R = R
        self.error_prob = error_prob
        self.ss = StateSpace(NUM_DIRECTIONS, self.C, self.R)
        self.robot = Robot(self.ss, 0,0,0)
        self.initializePolicyMatrix(State(0,3,4,0)) #this is specific for class
        self.initializeValueMatrix()


    # USER methods:
    def getValue(self, state):
        return self.value_matrix[state.direction][state.col][state.row]

    def getPolicy(self, state):
        return self.policy_matrix[state.direction][state.col][state.row]

    ########
    # 3(g) #
    ########
    def runPolicyIteration(self, gamma):
        start = timer()
        self.initializeValueMatrix()
        stable = False
```

```
        count = 0
        while(stable == False):
            self.setValueMatrix(self.policy_matrix, gamma)
            stable = self.improvePolicy(gamma)
            count = count + 1
            #print(count)
        end = timer()
        print('Policy Iteration took ' + str(end-start) + ' seconds to run.\n')


    ########
    # 4(a) #
    ########
    def runValueIteration(self, gamma):
        start = timer()
        self.initializeValueMatrix()
        delta = 1
        epsilon = 0.1
        actionspace = [Action(0,0),Action(1,-1),Action(1,0),Action(1,1),Action(-1,-1)
        count = 0

        while(delta >= epsilon):
            delta = 0
            for i in range(NUM_DIRECTIONS):
                for j in range(self.C):
                    for k in range(self.R):
                        v = self.value_matrix[i][j][k]
                        currState = self.ss.statespace[i][j][k]
                        m = -100000
                        bestAction = actionspace[0]
                        for a in range(len(actionspace)):
                            action = actionspace[a]
                            temp = self.calculateSummation(currState,action,gamma)
                            if(temp > m):
                                m = temp
                                bestAction = action
                        self.value_matrix[i][j][k] = m
                        self.policy_matrix[i][j][k] = bestAction
                        delta = max(delta, abs(v - m))
            count = count + 1
            #print(delta)
        end = timer()
        print('Value Iteration took ' + str(end-start) + ' seconds to run.\n')


    def moveRobotManually(self, d, c, r):
        self.robot = Robot(self.ss, d, c, r)
```

```
    def setSpecificGameBoardForClass(self):
        if(self.C != 6 or self.R != 6):
            raise Exception('incorrect board size')
        #add in the red squares
        self.setEdgePenalty()
        #add in the yellow squares
        for i in range(2,5):
            self.ss.setRewardOfAllDirectionsAtCoordinate(2, i, -10)
            self.ss.setRewardOfAllDirectionsAtCoordinate(4, i, -10)
        #add in the green square
        self.ss.setRewardOfAllDirectionsAtCoordinate(3, 4, 1)

    ##############################################
    # 3(b): runs the robot until it stops moving #
    ##############################################
    def run(self):
        state_trajectory = [gb.robot.state]
        values = self.value_matrix[gb.robot.state.direction][gb.robot.state.col][gb.ro
        still_moving = True
        while(still_moving):
            still_moving = self.haveRobotTakePolicy()
            state_trajectory.append(gb.robot.state)
            values = values + self.value_matrix[gb.robot.state.direction][gb.robot.sta

        self.drawSeries(state_trajectory)
        return [state_trajectory, values]

    #######################
    ## END USER FUNCTIONS #
    #######################

    def initializeValueMatrix(self):
        self.value_matrix = []
        for i in range(NUM_DIRECTIONS):
            self.value_matrix.append([[0 for k in range(self.R)] for j in range(self.C

    ########
    # 3(a) #
    ########
    #initializes a policy that always points towards a goal square
    def initializePolicyMatrix(self, goalState):
        temp = []
        for i in range(NUM_DIRECTIONS):
            temp.append([[getDirectAction(State(i,j,k,0), goalState) for k in range(se
        self.policy_matrix = temp

    ##############################################################
    # 3(d): evaluate the value matrix based on the input policy #
```

```python
##############################################################
def setValueMatrix(self, policy_matrix, gamma):
    delta = 1
    #count = 0
    epsilon = 0.1
    while(delta > epsilon): #convergence check
        delta = 0
        #for each state
        for i in range(NUM_DIRECTIONS):
            for j in range(self.C):
                for k in range(self.R):
                    currState = self.ss.statespace[i][j][k]
                    v = self.getValue(currState)
                    action = policy_matrix[currState.direction][currState.col][cu
                    self.value_matrix[i][j][k] = self.calculateSummation(currState
                    delta = max(delta, abs(v-self.value_matrix[i][j][k]))
        #count = count + 1
########
# 3(f) #
########
def improvePolicy(self, gamma):
    stable = True
    actionspace = [Action(0,0),Action(1,-1),Action(1,0),Action(1,1),Action(-1,-1)
    #update policy for each state
    for i in range(NUM_DIRECTIONS):
        for j in range(self.C):
            for k in range(self.R):
                prevAction = self.policy_matrix[i][j][k]
                currState = self.ss.statespace[i][j][k]
                #find best action
                m = -10000
                bestAction = actionspace[0]
                for a in range(len(actionspace)):
                    action = actionspace[a]
                    temp = self.calculateSummation(currState, action, gamma)
                    if(temp > m):
                        bestAction = action
                        m = temp
                if(bestAction.equals(prevAction) == False):
                    stable = False
                self.policy_matrix[i][j][k] = bestAction
    return stable

#helper function for policy iteration
def calculateSummation(self, currState, action, gamma):
    temp = 0
    for i in range(NUM_DIRECTIONS):
        for j in range(self.C):
```

```
            for k in range(self.R):
                nextState = self.ss.statespace[i][j][k]
                p = getProbNextState(currState, action, nextState, self.error_prob
                r = getReward(nextState)
                v = self.getValue(nextState)
                temp = temp + (p*(r+(gamma*v)))
    return temp

#set all the red squares
def setEdgePenalty(self):
    for i in range(self.C):
        self.ss.setRewardOfAllDirectionsAtCoordinate(i, 0, -100)
        self.ss.setRewardOfAllDirectionsAtCoordinate(i, self.C-1, -100)
    for i in range(self.R):
        self.ss.setRewardOfAllDirectionsAtCoordinate(0, i, -100)
        self.ss.setRewardOfAllDirectionsAtCoordinate(self.R-1, i, -100)

def haveRobotTakeAction(self, action):
    self.robot.move(action, self.error_prob)
    self.draw()

#have the robot take the action defined by the current policy
def haveRobotTakePolicy(self):
    temp = self.getPolicy(self.robot.state)
    self.robot.move(temp, self.error_prob)
    if(temp.equals(Action(0,0))):
        return False
    return True

#draw the current board
def draw(self):
    x_lims = (-0.5, self.C-0.5)
    y_lims = (-0.5, self.R-0.5)
    rewardMatrix = self.ss.statespace[6]
    %matplotlib inline

    fig = plt.figure(figsize=(5,5))
    ax = fig.add_subplot(111, aspect='equal')
    plt.yticks(np.arange(0, self.R, 1))
    plt.xticks(np.arange(0, self.C, 1))
    plt.ylim(y_lims)
    plt.xlim(x_lims)

    myPatches = []
    for i in range(self.C):
        for j in range(self.R):
            temp = patches.Rectangle((-0.5+i, -0.5+j), 1, 1)
            temp.set_edgecolor('black')
```

```
            temp.set_linewidth(3)

            if(rewardMatrix[i][j].reward == -100):
                temp.set_facecolor('red')
            elif(rewardMatrix[i][j].reward == -10):
                temp.set_facecolor('yellow')
            elif(rewardMatrix[i][j].reward == 1):
                temp.set_facecolor('green')
            else:
                temp.set_facecolor('white')

            ax.add_patch(temp)

    radius = 0.4
    robotPatch = patches.Circle((self.robot.state.col, self.robot.state.row), radi
    robotPatch.set_edgecolor('black')
    robotPatch.set_linewidth(3)
    robotPatch.set_facecolor('cyan')
    ax.add_patch(robotPatch)

    triangle = getDirectionTriangle(self.robot.state.direction, self.robot.state.
    directionPatch = patches.Polygon(triangle)
    ax.add_patch(directionPatch)

    plt.show()

#draw a series of boards
def drawSeries(self, states):
    x_lims = (-0.5, self.C-0.5)
    y_lims = (-0.5, self.R-0.5)
    rewardMatrix = self.ss.statespace[6]
    %matplotlib inline

    fig =plt.figure(figsize=(20,20))
    length = len(states)
    square = math.ceil(math.sqrt(length))

    for i in range(length):
        ax1 = fig.add_subplot(square,square,i+1,aspect='equal')
        myPatches = []
        for j in range(self.C):
            for k in range(self.R):
                temp_patch = patches.Rectangle((-0.5+j, -0.5+k),1,1)
                temp_patch.set_edgecolor('black')
                temp_patch.set_linewidth(3)

                if(rewardMatrix[j][k].reward == -100):
                    temp_patch.set_facecolor('red')
```

```
                    elif(rewardMatrix[j][k].reward == -10):
                        temp_patch.set_facecolor('yellow')
                    elif(rewardMatrix[j][k].reward == 1):
                        temp_patch.set_facecolor('green')
                    else:
                        temp_patch.set_facecolor('white')

                    ax1.add_patch(temp_patch)

            radius = 0.4
            robotPatch = patches.Circle((states[i].col,states[i].row),radius)
            robotPatch.set_edgecolor('black')
            robotPatch.set_linewidth(3)
            robotPatch.set_facecolor('cyan')
            ax1.add_patch(robotPatch)

            triangle = getDirectionTriangle(states[i].direction,states[i].col,states[i
            directionPatch = patches.Polygon(triangle)
            ax1.add_patch(directionPatch)
            plt.yticks(np.arange(0,self.R,1))
            plt.xticks(np.arange(0,self.C,1))
            plt.ylim(y_lims)
            plt.xlim(x_lims)
        plt.show()

class Robot:
    state = None
    ss = None

    def __init__(self, statespace, direction, col, row):
        self.ss = statespace
        self.state = self.ss.statespace[direction][col][row].copy()

    def move(self, action, error_prob):
        temp = getNextStateWithErrorProb(self.state.copy(), action, error_prob, self.s
        print(action)
        self.state = self.ss.statespace[temp.direction][temp.col][temp.row].copy()
```

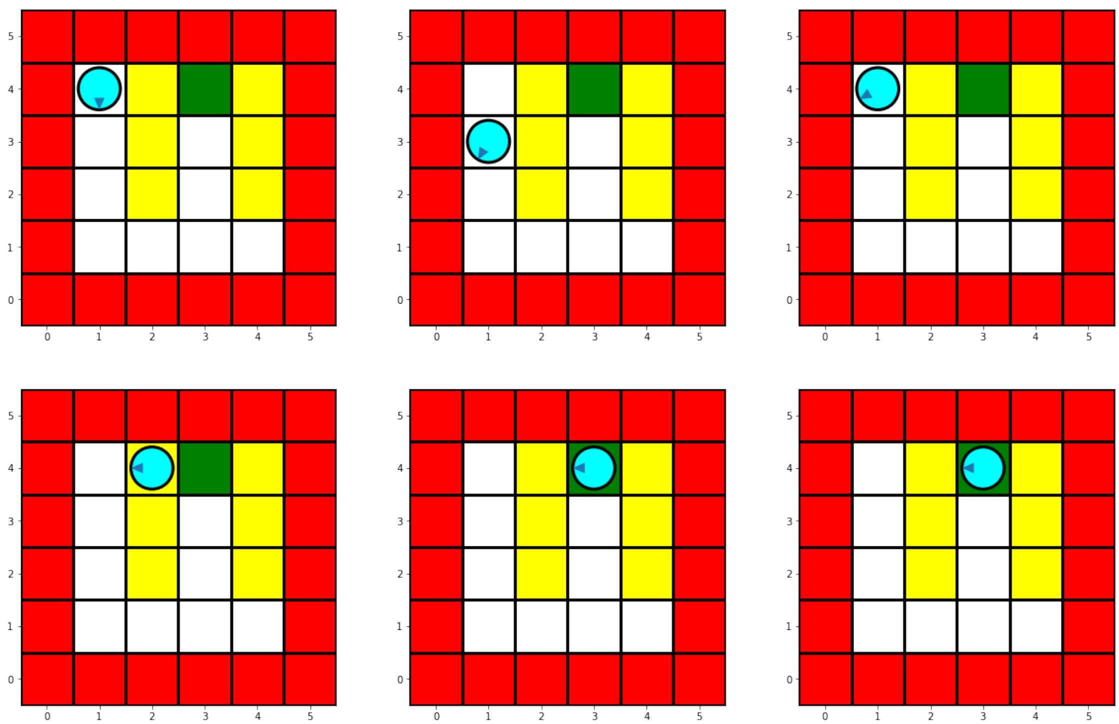
In [16]:
```
#helper method to draw the robot's direction
def getDirectionTriangle(direction, col, row, radius):
    theta = math.pi/6
    top_point = [radius*math.sin(theta*direction) + col, radius* math.cos(theta*direct
    left_direction = (direction-1) % NUM_DIRECTIONS
    left_point = [radius*math.sin(theta*left_direction)/2 + col, radius*math.cos(theta
    right_direction = (direction+1) % NUM_DIRECTIONS
    right_point = [radius*math.sin(theta*right_direction)/2 + col, radius*math.cos(the
    return[top_point, left_point, right_point]
```

```
################################################################################
# 3(a): this is the function that calculates the direction to move toward goal state
################################################################################
def getDirectAction(robotState, goalState):
    #check to see if same state
    if(robotState.col == goalState.col and robotState.row == goalState.row):
        return Action(STILL,STILL)

    #first, translate the goalState so that the robot is centered
    g_x = goalState.col - robotState.col
    g_y = goalState.row - robotState.row

    #create a vector that points in the direction of the robot
    direction = robotState.direction
    theta = -1*(math.pi/6)*(direction) + math.pi/2
    d_x = math.cos(theta)
    d_y = math.sin(theta)

    #now, rotate the vectors so that the goal vector lies on the +x axis
    if(g_x > 0 and g_y > 0): #rotate normally
        theta = math.atan(1.0*g_y/g_x)
    elif(g_x > 0 and g_y < 0): #rotate with an extra 3pi/2
        theta = math.atan(-1.0*g_x/g_y) + 3*math.pi/2
    elif(g_x < 0 and g_y > 0): #rotate with an extra pi/2
        theta = math.atan(-1.0*g_x/g_y) + math.pi/2
    elif(g_x < 0 and g_y < 0): #rotate with an extra pi
        theta = math.atan(1.0*g_y/g_x) + math.pi
    elif(g_x == 0 and g_y > 0): #rotate exactly pi/2
        theta = math.pi/2
    elif(g_x == 0 and g_y < 0): #rotate exactly 3*pi/2
        theta = 3*math.pi/2
    elif(g_x < 0 and g_y == 0): #rotate exactly pi
        theta = math.pi
    else: #no need to rotate
        theta = 0

    #now, rotate our direction vector. then we can decide which way to move and then
    rotation_matrix = np.matrix([[math.cos(theta), math.sin(theta)], [-1*math.sin(the
    vector = np.matrix([[d_x],[d_y]])
    new_vector = np.matmul(rotation_matrix,vector)
    n_x = new_vector[0]
    n_y = new_vector[1]

    delta = 0.000000001

    if(abs(n_x) < delta and n_y > 0):
        return Action(FORWARD,LEFT)
```

```
        elif(abs(n_x) < delta and n_y < 0):
            return Action(FORWARD,RIGHT)
        elif(n_x > 0 and abs(n_y) < delta):
            return Action(FORWARD,STILL)
        elif(n_x < 0 and abs(n_y) < delta):
            return Action(BACKWARD,STILL)
        elif(n_x > 0 and n_y > 0):
            return Action(FORWARD,RIGHT)
        elif(n_x < 0 and n_y > 0):
            return Action(BACKWARD,LEFT)
        elif(n_x < 0 and n_y < 0):
            return Action(BACKWARD,RIGHT)
        elif(n_x > 0 and n_y < 0):
            return Action(FORWARD,LEFT)
        else:
            return Action(STILL,STILL)
```

```
In [9]: ########
        # 3(c) #
        ########

        gb = GameBoard(C=6,R=6,error_prob=0)
        gb.setSpecificGameBoardForClass()
        gb.moveRobotManually(d=6,c=1,r=4)
        gb.setValueMatrix(gb.policy_matrix,0.9)

        [trajectory, value] = gb.run()

        ########
        # 3(e) #
        ########
        print('Value for 3(c): ' + str(value))
```

```
forward then right turn
backward then right turn
backward then right turn
backward then no turn
no movement
```

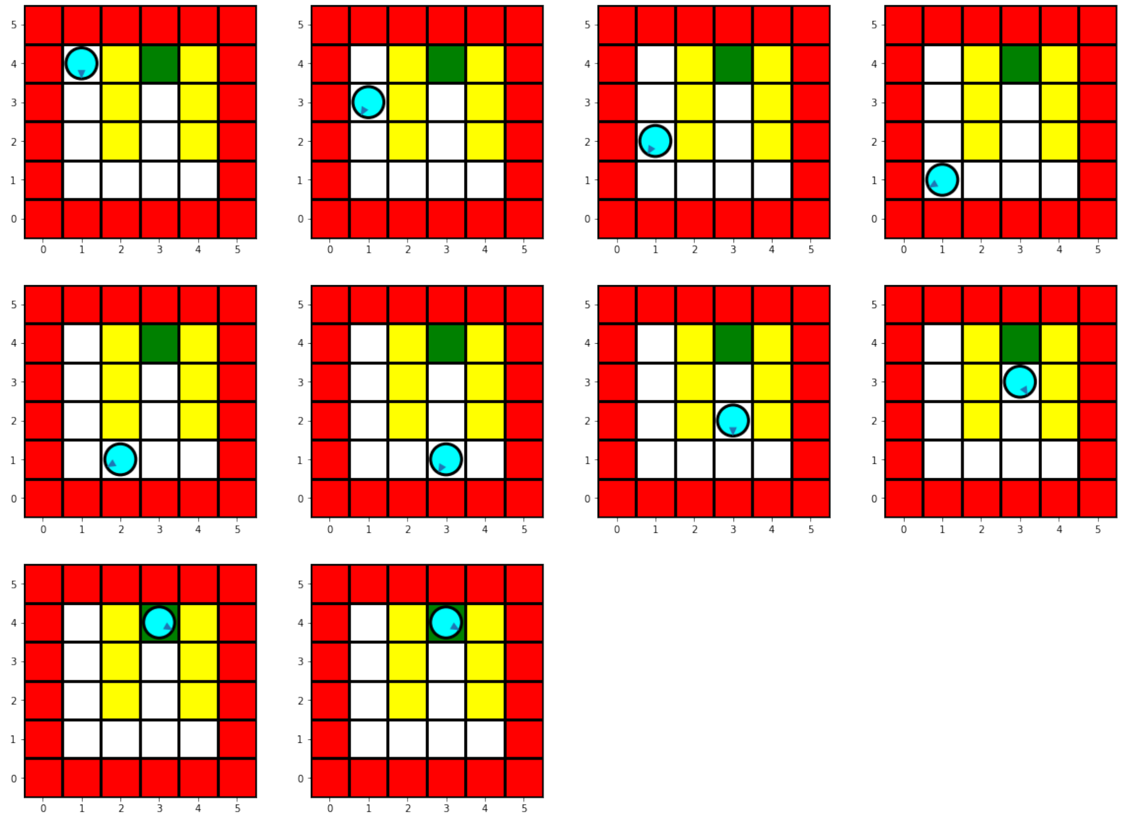


```
Value for 3(c): 21.9722371282
```

```
In [10]: ##############
         # 3(h), 3(i) #
         ##############
         gb = GameBoard(C=6,R=6,error_prob=0)
         gb.setSpecificGameBoardForClass()
         gb.moveRobotManually(d=6,c=1,r=4)
         gb.runPolicyIteration(gamma=0.9)

         [trajectory, value] = gb.run()

         print('Value for 3(h): ' + str(value))
```

```
Policy Iteration took 48.646041541 seconds to run.

forward then right turn
forward then no turn
forward then right turn
backward then no turn
backward then left turn
backward then left turn
backward then left turn
```

```
backward then left turn
no movement
```

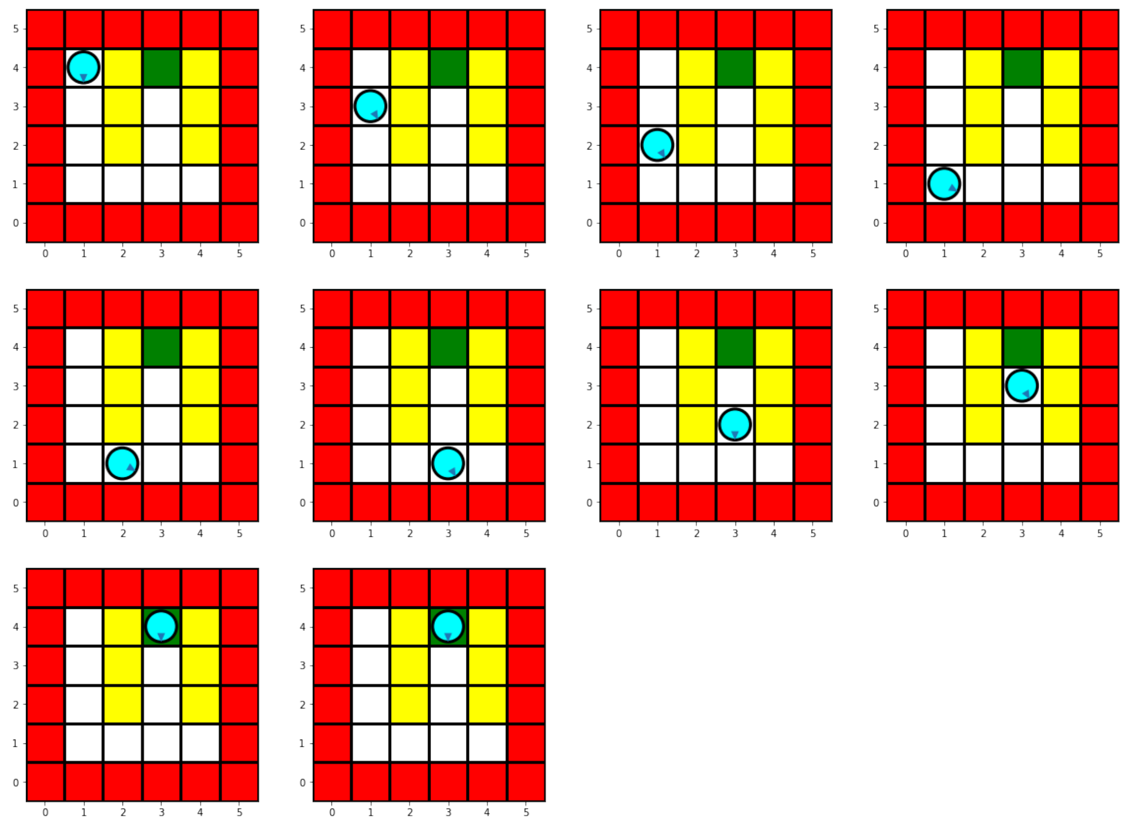


```
Value for 3(h): 76.9505141806
```

In [11]:
```python
##############
# 4(b), 4(c) #
##############
gb = GameBoard(C=6,R=6,error_prob=0)
gb.setSpecificGameBoardForClass()
gb.moveRobotManually(d=6,c=1,r=4)
gb.runValueIteration(gamma=0.9)

[trajectory, value] = gb.run()

print('Value for 4(b): ' + str(value))
```

```
Value Iteration took 55.2186033892 seconds to run.

forward then left turn
```

```
forward then no turn
forward then left turn
forward then no turn
forward then right turn
backward then right turn
backward then left turn
backward then right turn
no movement
```

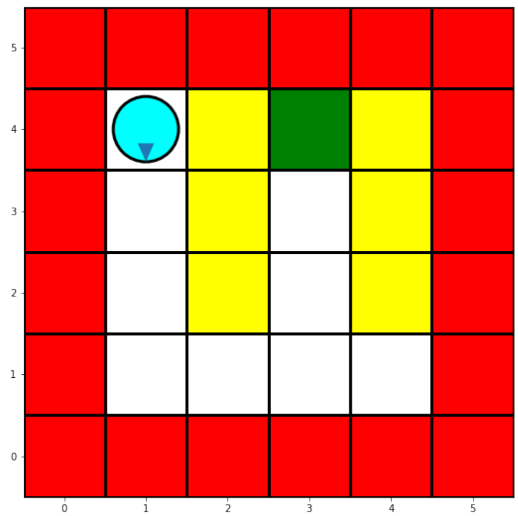


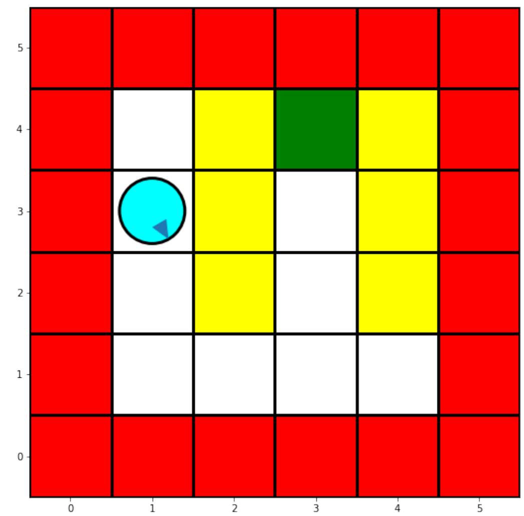
```
Value for 4(b): 70.3402478056
```

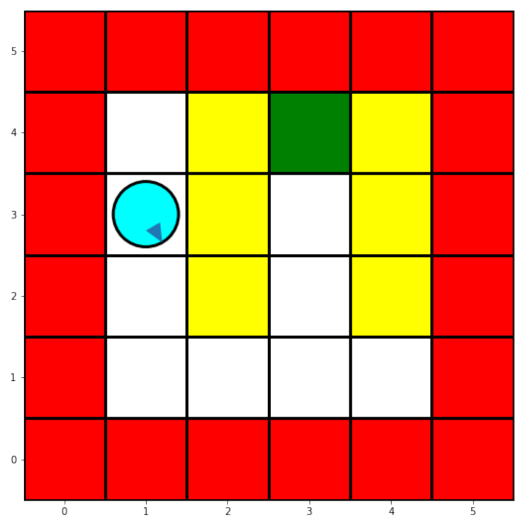
In [12]:
```
########
# 5(a) #
########

gb = GameBoard(C=6,R=6,error_prob=0.25)
gb.setSpecificGameBoardForClass()
gb.moveRobotManually(d=6,c=1,r=4)
gb.runPolicyIteration(0.9)
```

```
[trajectory, value] = gb.run()

print('Value for 5(a): ' + str(value))
```

```
Policy Iteration took 79.7900058277 seconds to run.

left error
forward then no turn
no movement
```



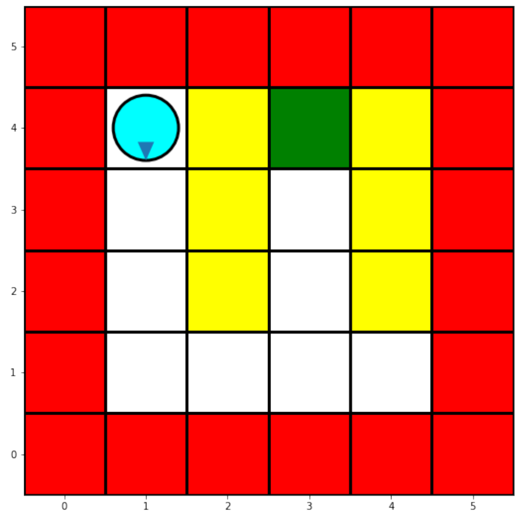


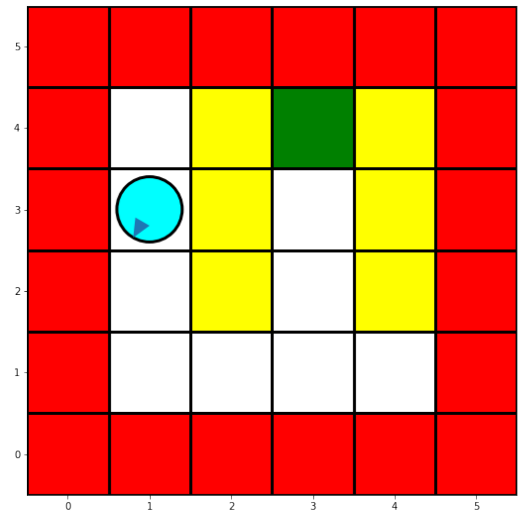


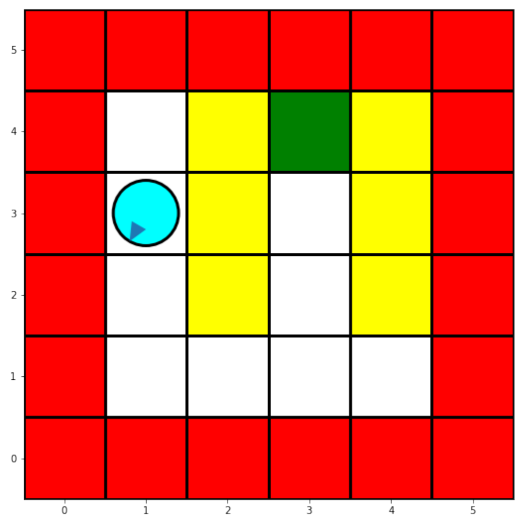
```
Value for 5(a): -0.101993904429
```

```
In [13]: ########
         # 5(b) #
         ########

         gb = GameBoard(C=6,R=6,error_prob=0.25)
         gb.setSpecificGameBoardForClass()
         gb.moveRobotManually(d=6,c=1,r=4)
         gb.ss.setRewardOfAllDirectionsAtCoordinate(3,4,0) #
         gb.ss.statespace[6][3][4].reward = 1
         gb.ss.statespace[5][3][4].reward = 1
         gb.ss.statespace[7][3][4].reward = 1
         gb.runValueIteration(0.9)

         [trajectory, value] = gb.run()

         print('Value for 5(b): ' + str(value))
```

```
Value Iteration took 54.8200043382 seconds to run.

right error
forward then no turn
no movement
```








```
Value for 5(b): 0.0652362442768
```

0.1 5(c): When there is no error probability, the robot is able to deftly navigate the penalty squares and make it to the goal square. However, if the penalty is too low, as it was when it was -1, then the robot ignores it and travels straight to the goal square. However, if the discount factor was lowered instead of increasing the penalty, the robot also managed to avoid the penalties, because taking an early penalty was far too costly to get to a lowered reward

0.2 When an error probability is introduced (especially a high one of 25% that leads to a total probability of 50% errors), the robot ceases to be able to navigate to the goal square, without being extremely lucky. This makes sense, because the benefit of a measely +1 (discounted by many gamma-steps), can never overcome the massive penalty of -100 or -10. Even at a 25% chance of hitting these things, the robot decides that it is more beneficial to give up than to continue on