TRIANGULAR

$$\Delta(t) = g(t) = \begin{cases} A) \ t+1 & -1 < t < 0 \\ B) \ -t+1 & 0 < t < 1 \end{cases}$$

Está cierto $D_n = \frac{1}{2} \cdot \operatorname{Sinc}\left(\frac{n\omega_0}{2}\right)^2$?

$$D_nA = \frac{1}{T_0}\int_{-1}^{0} (t+1)\, e^{-j\omega_0 n t}\, dt =$$

$u = t+1 \Rightarrow du = dt$

$dv = e^{-j\omega_0 n t} dt \Rightarrow v = \frac{e^{-j\omega_0 n t}}{-j\omega_0 n}$

$$D_nA = \frac{1}{2} \cdot \left[\frac{(t+1)e^{-j\omega_0 n t}}{-j\omega_0 n}\Bigg|_{-1}^{0} - \int_{-1}^{0} \frac{\left(e^{-j\omega_0 n t}\right)}{-j\omega_0 n} dt \right] = \frac{1}{2}\left[\frac{(t+1)e^{-j\omega_0 n t}}{-j\omega_0 n} - \frac{e^{-j\omega_0 n t}}{(-j\omega_0 n)^2} \right]\Bigg|_{-1}^{0}$$

$$= \frac{1}{2}\left[\frac{(0+1)e^{0} - \cancel{(-1+1)}^{0} e^{j\omega_0 n}}{-j\omega_0 n} + \frac{e^{0} - e^{j\omega_0 n}}{(+1)\,\omega_0^2 n^2} \right]$$

$$= \frac{1}{2}\left[\frac{-e^{0}}{j\omega_0 n} + \frac{e^{0} - e^{j\omega_0 n}}{\omega_0^2 n^2} \right] = \frac{1}{2}\left[-\frac{1}{j\omega_0 n} + \frac{1}{\omega_0^2 n^2} - \frac{e^{j\omega_0 n}}{\omega_0^2 n^2} \right]$$

$$D_nB = \frac{1}{T_0}\int_{0}^{1} (-t+1)\, e^{-j\omega_0 n t}\, dt \quad \begin{cases} u = (-t+1) \Rightarrow du = -dt \\ dv = e^{-j\omega_0 n t} dt \Rightarrow v = \frac{e^{-j\omega_0 n t}}{-j\omega_0 n} \end{cases}$$

$$= \frac{1}{2}\left[\frac{(-t+1)\, e^{-j\omega_0 n t}}{-j\omega_0 n}\Bigg|_{0}^{1} + \int_{0}^{1} \frac{e^{-j\omega_0 n t}}{-j\omega_0 n} (+1)\, dt \right]$$

$$= \frac{1}{2}\left[\frac{\cancel{(-1+1)}^{0} e^{-j\omega_0 n} + (0+1)e^{0}}{+j\omega_0 n} + \frac{e^{-j\omega_0 n t}}{(-j\omega_0 n)^2}\Bigg|_{0}^{1} \right] = \frac{1}{2}\left[\frac{1}{j\omega_0 n} + \frac{e^{-j\omega_0 n} - e^{0}}{-\omega_0^2 n^2} \right]$$

$$= \frac{1}{2}\left[\frac{1}{j\omega_0 n} + \frac{1}{\omega_0^2 n^2} - \frac{e^{-j\omega_0 n}}{\omega_0^2 n^2} \right]$$

$$D_n = D_{nA} + D_{nB}$$

$$= \frac{1}{2}\left[\cancel{-\frac{1}{j\omega_0 n}} + \frac{1}{\omega_0^2 n^2} - \frac{e^{j\omega_0 n}}{\omega_0^2 n^2} + \cancel{\frac{1}{j\omega_0 n}} + \frac{1}{\omega_0^2 n^2} - \frac{e^{-j\omega_0 n}}{\omega_0^2 n^2}\right]$$

$$= \frac{1}{2}\left[\frac{2}{\omega_0^2 n^2} - \frac{\left(e^{j\omega_0 n} + e^{-j\omega_0 n}\right)}{\omega_0^2 n^2}\right]$$

$$e^{j\omega_0 n} + e^{-j\omega_0 n} = \cos(\omega_0 n) + \cancel{j\,\mathrm{sen}(\omega_0 n)} + \cos(\omega_0 n) - \cancel{j\,\mathrm{sen}(\omega_0 n)} = 2\cos(\omega_0 n)$$

$$D_n = \frac{1}{2}\left[\frac{2}{\omega_0^2 n^2} - \frac{2\cos(\omega_0 n)}{\omega_0^2 n^2}\right] = \frac{1-\cos(\omega_0 n)}{\omega_0^2 n^2}$$

$$\cos a \cos b - \mathrm{sen}\, a\, \mathrm{sen}\, b$$

$$\cos\left(\frac{\omega_0 n}{2} + \frac{\omega_0 n}{2}\right) = \cos\left(\frac{\omega_0 n}{2}\right)^2 - \mathrm{sen}\left(\frac{\omega_0 n}{2}\right)^2$$

$$\cos \omega_0 n = \cos\left(\frac{\omega_0 n}{2}\right)^2 - \mathrm{sen}\left(\frac{\omega_0 n}{2}\right)^2$$

$$\cos \omega_0 n = 1 - \mathrm{sen}\left(\frac{\omega_0 n}{2}\right)^2 - \mathrm{sen}\left(\frac{\omega_0 n}{2}\right)^2$$

$$\cos(\omega_0 n) = 1 - 2\,\mathrm{sen}\left(\frac{\omega_0 n}{2}\right)^2$$

$$2\,\mathrm{sen}\left(\frac{\omega_0 n}{2}\right)^2 = 1 - \cos(\omega_0 n)$$

$$D_n = \frac{2\,\mathrm{sen}(\omega_0 n/2)^2}{4\,\frac{\omega_0^2 n^2}{4}} = \frac{1}{2}\,\frac{\mathrm{sen}(\omega_0 n/2)^2}{(\omega_0 n/2)^2} = \frac{1}{2}\left(\frac{\mathrm{sen}(\omega_0 n/2)}{(\omega_0 n/2)}\right)^2$$

$$= \frac{1}{2}\,\mathrm{Sinc}\left(\frac{\omega_0 n}{2}\right)^2$$

Está certo

Sinal em $g(t) = ?$, repete-se de forma ininterrupta com mesma duração, determine a banda p/ 95% de potência ser transmitida

$$g(t) = \Delta(t-1) - \Delta(t-3)$$

$$g(t) \begin{cases} t-1+1 - (t-3+1) \\ -(t-1)+1 - (-(t-3)+1) \end{cases}$$

$$g(t) \begin{cases} +2 & -1<t<0 \\ -t+2 + t - 4 = -2 & 0<t<1 \end{cases}$$

$$g(t) \begin{cases} +2 & -1<t<0 \\ -2 & 0<t<1 \end{cases}$$

$$= \frac{1}{T_0}\left(\int_{-1}^{0} 2e^{-j\omega_0 n t}\,dt + \int_{0}^{1} -2e^{-j\omega_0 n t}\right)$$

$$= \frac{1}{2}\left(2\frac{e^{-j\omega_0 n t}}{-j\omega_0 n}\Big|_{-1}^{0} + 2\frac{e^{-j\omega_0 n t}}{+j\omega_0 n}\Big|_{0}^{1}\right) = \frac{-(e^0 - e^{+j\omega_0 n}) + e^{-j\omega_0 n} - e^0}{j\omega_0 n}$$

$$= \frac{-1 + e^{j\omega_0 n} + e^{-j\omega_0 n} - 1}{j\omega_0 n} = \frac{2\cos(\omega_0 n) - 2}{j\omega_0 n} = \frac{j\cos(\omega_0 n) - 2j}{-\omega_0 n}$$

$$D_n = \frac{2j - 2j\cos(\omega_0 n)}{\omega_0 n}$$

$$P_g = P_{gA} + P_{gB} = \frac{1}{T_0}\left(\int_{-1}^{0} (2)^2\,dt + \int_{0}^{+1} (-2)^2\,dt\right)$$

$$= \frac{1}{2}\left(4t\Big|_{-1}^{0} + 4t\Big|_{0}^{1}\right) = 2\cdot 0 + 2 + 2 - 2\cdot 0 = \underline{4W}$$

$$P_r = 95\% P_g = 3{,}8W$$

$$D_0 = \frac{0 - j\omega_0 n \cancel{(\cos(\omega_0 n))}}{\omega_0 n} = 0 \qquad \omega_0 = \frac{2\pi}{T_0} = \pi$$

$$D_1 = \frac{2j - 2j\cos(\pi)}{\pi} = \frac{2j + j + j}{\pi} = \frac{4}{\pi} j = 0 + 1{,}2732j$$

$$D_{-1} = 0 - 1{,}2732j$$

$$P_1 = P_0 + (D_1)^2 + (D_{-1})^2 = 0 + 2\cdot(1{,}2732)^2 = 3{,}24228\,W \to 81\%\,P_g$$

$$D_2 = \frac{\cancel{2}j - 2j\cancel{\cos(2\pi)}}{\pi \cancel{2}} = 0 + 0j$$

$$D_{-2} = 0 + 0j$$

$$P_2 = P_1$$

$$\therefore P_n \{ n = 2k / k \in \mathbb{N} \therefore D_n = 0 \wedge P_n = P_{n-1} \}$$

$$D_3 = \frac{2j - 2j\cos(3\pi)}{3\pi} = \frac{4j}{3\pi} = 0 + 0{,}4244j$$

$$D_{-3} = 0 - 0{,}4244j$$

$$P_3 = P_1 + (D_3)^2 + (D_{-3})^2 = 3{,}24228 + (D_3)^2 + (D_{-3})^2$$
$$= 3{,}24228 + (0{,}42244)^2 \cdot 2 = 3{,}24228 + 0{,}36025$$
$$P_3 = 3{,}6025 \to 90\%\,P_g$$

$$P_4 = P_3$$

$$D_5 = \frac{2j - 2j\cos(5\pi)}{5\pi} = \frac{4j}{5\pi} = 0 + 0{,}25465j$$

$$D_{-5} = 0 - 0{,}25465j$$

$$P_5 = P_4 + (D_5)^2 + (D_{-5})^2 = 3{,}6025 + 0{,}25465^2 \cdot 2 = 3{,}6025 + 0{,}12969$$
$$P_5 = 3{,}7322\,W \to 93{,}3\%\,P_g$$

$$P_6 = P_5$$

$$D_7 = \frac{2j - 2j\cos(7\pi)}{7\pi} = \frac{4j}{7\pi} = 0 + 0{,}18189j$$

$$D_{-7} = 0 - 0{,}18189j$$

$$P_7 = P_6 + (D_7)^2 + (D_{-7})^2 = 3{,}7322 + 2\cdot 0{,}18189^2 = 3{,}7322 + 0{,}06617$$

$$P_7 = 3{,}7984\ W \rightarrow 94{,}96\%\ P_g$$

$$P_8 = P_7$$

$$D_9 = \frac{2j - 2j\cos(9\pi)}{9\pi} = \frac{4j}{9\pi} = 0 + 0{,}14147j$$

$$D_{-9} = 0 - 0{,}14147j$$

$$P_9 = P_8 + (D_9)^2 + (D_{-9})^2 = 3{,}7984 + 2\cdot 0{,}14147^2$$

$$= 3{,}7984 + 0{,}0400 =$$

$$P_9 = 3{,}8384\ W \rightarrow 95{,}96\%\ P_g$$

$$\underline{B = 9\ Hz}$$