

Exponência Negativa

$$g(t) = \begin{cases} 1 - e^{-t} & 0 < t < 0.5 \\ e^{-(t-0.5)} - e^{-0.5} & 0.5 < t < 1.0 \end{cases}$$

$$P_{gA} = \frac{1}{T_0} \int_{T_0} (1 - e^{-t})^2 dt = \frac{1}{1} \int_0^1 1 - 2e^{-t} + e^{-2t} dt = 2t - 4(-e^{-t}) - e^{-2t} \Big|_0^1$$

$$= (2(0.5 - 0) + 4(e^{-0.5} - e^0) - e^{-1} + e^0) / 2$$

$$= (1 - 4 + 4 - 1 + 1) / 2 = 0.02912 \text{ W}$$

$$P_{gB} = \frac{1}{T_0} \int_{T_0} (e^{0.5-t} - e^{-0.5})^2 dt = \frac{1}{1} \int_{0.5}^1 e^{(0.5-t)^2} - 2e^{(0.5-t)} \cdot e^{-0.5} + e^{-0.5 \cdot 2} dt$$

$$= \int_{0.5}^1 e^{1-2t} - 2e^{-t} + e^{-1} dt = \frac{-e^{1-2t}}{2} + 2e^{-t} + e^{-1}t \Big|_{0.5}^1$$

$$= \frac{-1}{2} (e^{-1} - e^0) + 2(e^{-1} - e^{-0.5}) + \frac{1}{2e} = 0.022697 \text{ W}$$

$$P_g = P_{gA} + P_{gB} = 0.051818 \text{ W}$$

$$D_{NA} = \frac{1}{T_0} \int_0^{0.5} (1 - e^{-t}) e^{-j\omega_{NA} t} dt$$

$$= \int_0^{0.5} e^{-j\omega_{NA} t} - e^{(-j\omega_{NA}-1)t} dt = \frac{e^{-j\omega_{NA} t}}{-j\omega_{NA}} - \frac{e^{(-j\omega_{NA}-1)t}}{-j\omega_{NA}-1} = \frac{e^{-j\omega_{NA} t}}{-j\omega_{NA}} + \frac{e^{(-j\omega_{NA}-1)t}}{j\omega_{NA}+1}$$

$$D_{NA} = \frac{e^{-j\omega_{NA} 0.5}}{-j\omega_{NA}} - \frac{e^{-j\omega_{NA} 0}}{-j\omega_{NA}} + \frac{e^{(-j\omega_{NA}-1)0.5}}{j\omega_{NA}+1} - \frac{e^{(-j\omega_{NA}-1)0}}{j\omega_{NA}+1}$$

$$D_{NA} = \frac{e^{-j\omega_{NA} 0.5}}{-j\omega_{NA}} - \frac{e^{-j\omega_{NA} 0.5}}{-j\omega_{NA}} + e^{-1} - e^{-0.5} =$$

$$D_{NA} = \frac{1}{T_0} \int_0^{0.5} y_1(t) dt + \frac{1}{T_0} \int_0^1 y_2(t) dt$$

$$= \frac{1}{1} \left(\int_0^{0.5} 1 - e^{-t} dt + \int_{0.5}^1 e^{0.5-t} - e^{-0.5} dt \right)$$

$$= \left. t + e^{-t} \right|_0^{0.5} - \left. t e^{-0.5} - e^{0.5-t} \right|_{0.5}^1$$

$$= (0.5 - 0) + (e^{-0.5} - e^0) - (0.5 - 0.5) \cdot e^{-0.5} - (e^{-0.5} - e^0)$$

$$= 0.5 + e^{-0.5} - e^0 - e^{-0.5} - e^{-0.5} + e^0 = 0.5 - \frac{1}{2\sqrt{e}} = 0.19673V$$

$$P_0 = 0.19673^2 = 0.038703W$$

$$P_0 / P_g = 74.69\%$$

$$D_{1A} = \frac{e^{-j\omega_0 \cdot 0.5}}{-j\omega_0} - \frac{e^{-j\omega_0 \cdot 1}}{-j\omega_0} + \frac{e^{-j\omega_0 \cdot 1.5}}{j\omega_0 + 1} - \frac{e^{-j\omega_0 \cdot 2}}{j\omega_0 + 1}$$

$$= \frac{e^{-j\pi \cdot 0.5}}{-j\omega_0} - \frac{1}{-j\omega_0} + \frac{e^{-j\pi \cdot 1.5} \cdot e^{-0.5}}{j\omega_0 + 1} - \frac{1}{j\omega_0 + 1}$$

$$= \frac{\cancel{e^{-j\pi}}}{-j2\pi} + \frac{1}{j2\pi} + \frac{\cancel{e^{-j\pi}} \cdot e^{-0.5}}{j2\pi + 1} - \frac{1}{j2\pi + 1}$$

$$= \frac{1}{j2\pi} + \frac{1}{j2\pi} - \frac{e^{-0.5} + 1}{j2\pi + 1} \cdot \frac{(1 - j2\pi)}{(1 - j2\pi)}$$

$$= \frac{2}{j2\pi} + \frac{-1 - e^{-0.5}}{j2\pi + 1} \cdot \frac{(1 - j2\pi)}{(1 - j2\pi)} = \frac{1}{j\pi} + \frac{-1 + j2\pi - e^{-0.5} - j2\pi e^{-0.5}}{1 + 4\pi^2}$$

$$= \frac{-1j}{\pi} + \frac{-1 - e^{-0.5}}{1 + 4\pi^2} + \frac{j2\pi(e^{-0.5} + 1)}{1 + 4\pi^2} =$$

$$= -0.039688 - 0.06894j$$

$$D_{1A} = D_{1A}^* = -0.039688 + 0.06894j$$

$$D_{nB} = \frac{1}{T_0} \int_{0.5}^1 (e^{0.5-t} - e^{-0.5}) e^{-j\omega_n t} dt$$

$$= \frac{1}{1} \int_{0.5}^1 (e^{0.5-t-j\omega_n t} - e^{-0.5-j\omega_n t}) dt = \int_{0.5}^1 (e^{0.5} e^{+(-1-j\omega_n)t} - e^{-0.5} e^{-j\omega_n t}) dt$$

$$= \left. \frac{e^{0.5} e^{+(-1-j\omega_n)t}}{-1-j\omega_n} + \frac{e^{-0.5} e^{-j\omega_n t}}{-j\omega_n} \right|_{0.5}^1 =$$

$$= \frac{e^{0.5} \cdot e^{-\omega_n} - e^{0.5} \cdot e^{-j\omega_n \cdot 0.5}}{-1-j\omega_n} + \frac{e^{0.5} \cdot e^{-j\omega_n} - e^{0.5} \cdot e^{0.5(-1-j\omega_n)}}{-1-j\omega_n}$$

$$= \frac{e^{0.5} (e^{-\omega_n})}{-1-j\omega_n} - \frac{e^{0.5} \cdot (e^{j\pi n})}{-1-j\omega_n} - \frac{e^{0.5} \cdot (e^{-j2\pi n})}{-1-j\omega_n} + \frac{e^{0.5} \cdot e^{-0.5} (e^{j\pi n})}{-1-j\omega_n}$$

$$D_{nB} = \frac{e^{-0.5}}{j\omega_n} - \frac{e^{-0.5} \cdot (-1)^n}{j\omega_n} - \frac{e^{-0.5}}{1+j\omega_n} + \frac{(-1)^n}{1+j\omega_n}$$

$$D_{1B} = \frac{e^{-0.5}}{j\omega_0} + \frac{e^{-0.5}}{j\omega_0} - \frac{e^{-0.5}}{1+j\omega_0} - \frac{1}{1+j\omega_0}$$

$$= \frac{2e^{-0.5}}{j2\pi} - \frac{(1+e^{-0.5})}{(1+j\omega_0)} \frac{(1-j\omega_0)}{(1-j\omega_0)}$$

$$= -\frac{e^{-0.5}}{\pi} j - \frac{(1-j\omega_0 + e^{-0.5} - j\omega_0 e^{-0.5})}{1+\omega_0^2}$$

$$= -\frac{e^{-0.5}}{\pi} j - \frac{(1+e^{-0.5})}{1+4\pi^2} \frac{j2\pi(1+e^{-0.5})}{1+4\pi^2}$$

$$D_{1B} = -0.039688 + 0.05630j$$

$$D_{1B}^* = D_{-1B} = -0.039688 - 0.05630j$$

$$D_1 = D_{1A} + D_{1B} = -0,039688 - 0,05894j - 0,039688 + 0,05630j$$

$$D_{-1} = D_{-1A} + D_{-1B} = -0,039688 + 0,05894j - 0,039688 - 0,05630j$$

$$D_1 = -0,07938 - 0,01264j$$

$$D_{-1} = -0,07938 + 0,01264j$$

$$P_1 = P_0 + (D_1) + (D_{-1})$$

$$= 0,038703 + 2 \cdot (0,07938)^2 + 2 \cdot (0,01264)^2$$

$$= 0,051625 \text{ W}$$

$$P_1 / P_f = \frac{0,051625}{0,051818} = 99,63\%$$

$$B = 1 \text{ Hz}$$