Project, ESD 40.242-Mathematical Finance, Spring 2016

- **Due date:** April 18, 2016 (Monday). Respect the due date, **no late projects accepted!**

- This project is about pricing European and American vanilla options using the binomial model. The project must be done individually.

- **Programming and analysis:** any programming language can be used. C/C++ is preferred. Excel can be used for the analysis and presentation of data. Submit both a hardcopy and an electronic (word or pdf) report including your codes. Compress your documents into a single zip file. Your program should output both option price and computational time.

- **Grading criteria:** (1) Are the results complete and correct? (2) Is the report well organized? (3) Are the results well explained graphically? (4) Do you make effort on reducing computational time? (5) Is the report generally impressive?

- **Binomial model**
  1. Implement the CRR binomial model to price European and American puts and calls on a stock paying continuous dividend yield:

     $$\textit{Binomial(Option}, K, T, S, \sigma, r, q, n, \textit{Exercise)}$$

     where *Option* = C for calls and P for puts, K is the strike, T is the maturity, S is the initial stock price, σ is the volatility, r is the continuous compounding risk free interest rate, q is the continuous dividend yield, n is the number of time steps, and *Exercise* = A for American options and E for European options.

  2. Consider a 1-year European call option with strike K = 100. The current stock price is 100. Other parameters are $r = 0.05, q = 0.04, \sigma = 0.2$. Use the Black-Scholes formula to compute the price of the call option. In the binomial model, take a sequence of increasing numbers of time steps n. Verify that the binomial option prices converge to the Black-Scholes option price as n increases. Construct a table/plot to visualize the convergence.

  3. Consider an American put with K = 100, T = 1, σ = 0.2, r = 0.05. The underlying stock doesn't pay dividends. Calculate and plot the put value as a function of the initial stock price S. What is the initial stock price $S^*(12)$ for which it first becomes optimal to early exercise the put option? For $T = \frac{i}{12}, i = 0, 1, \dots, 12$, with one month interval, find the initial stock prices $S^*(i)$ for which it first becomes optimal to early exercise the put. Report and plot the resulting early exercise boundary $S^*(i)$ as a function of the option maturity $T = i/12$. How do put prices and the early exercise boundary change when the continuous dividend yield is q = 0.04? What is the intuition behind this dependence on the dividend yield?

  4. Calculate and plot American call values as a function of the initial stock price

when K = 100, T = 1, σ = 0.2, r = 0.05, q = 0.04. For $T = \frac{i}{12}, i = 0, 1, \dots, 12$, with one month interval, find the initial stock prices $S^*(i)$ for which it first becomes optimal to early exercise the call. Report and plot the resulting early exercise boundary $S^*(i)$ as a function of the option maturity $T = i/12$. How do call prices and the early exercise boundary change when the continuous dividend yield is q = 0.08? What is the intuition behind this dependence on the dividend yield?

- **Notes:** To plot the option price as a function of the initial stock price, calculate an array of option prices with reasonably small interval in S. Option prices you compute should be accurate for three digits after the decimal point. Experiment your program to find the number of time steps needed to guarantee the required accuracy. To determine $S^*(i)$, compare the option value with the option intrinsic value (the intrinsic value and the option value are considered to be equal if their difference is less than one half cent).