TTK4255: Robotic Vision

Homework 3: Geometric image formation

Jens Erik Kveen

1 Part 1 - Choosing a sensor and lens

Task 1.1

From assignment text we get:

$$Z = 50m, \quad Y = 1cm, \quad Y' = 0.01mm \tag{1}$$

som the focal legnth will be:

$$f = \frac{Z * Y}{Y'} = 50mm \tag{2}$$

Task 1.2

Image resolution of $1024x1024$ gives a distance of $1024cm$. With the speed of $50m/s$ and 5 image speed of 5 images per second gives a displacement of $1024 - 1000$ pixels. So we get:

$$\frac{1024 - 1000}{1024} = 2.34\% \tag{3}$$

2 Part 2 - Implementing the pinhole camera model

Task 2.1

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \mathbf{KX} = \begin{bmatrix} s_x f X + c_x Z \\ s_y f Y + c_y Z \\ Z \end{bmatrix} \tag{4}$$

So dividing $\tilde{u}$ and $\tilde{v}$ by $\tilde{w}$ gives:

$$u = s_x f \frac{X}{Z} + c_x \tag{5}$$

$$v = s_y f \frac{Y}{Z} + c_y \tag{6}$$

Task 2.2

3 Part 3 - Homogeneous coordinates and transformations

Task 3.1

$$\mathbf{X} = \left[\frac{\tilde{x}}{\tilde{w}}, \frac{\tilde{y}}{\tilde{w}}, \frac{\tilde{z}}{\tilde{w}} \right]^{\mathsf{T}} \tag{7}$$

From equation 4 we get:

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w}' \end{bmatrix} = \mathbf{KX} = \begin{bmatrix} s_x f \frac{\tilde{x}}{\tilde{w}} + c_x \frac{\tilde{z}}{\tilde{w}} \\ s_y f \frac{\tilde{y}}{\tilde{w}} + c_y \frac{\tilde{z}}{\tilde{w}} \\ \frac{\tilde{z}}{\tilde{w}} \end{bmatrix} \tag{8}$$

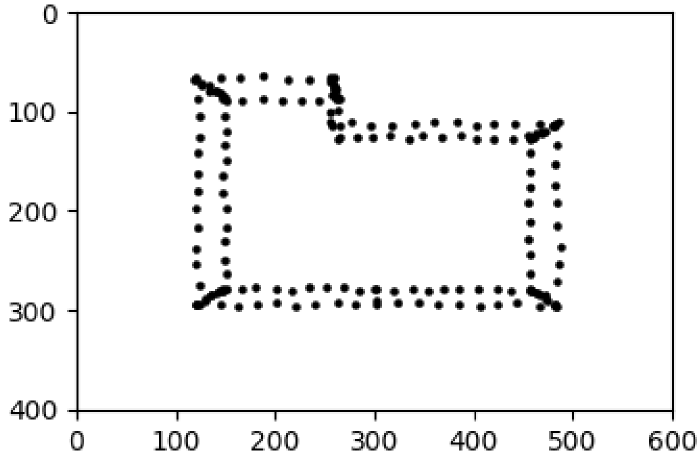

Figure 1: Dots projected into image

So by dehomogenizing the equations, ie. dividing by $\tilde{w}'$ we get:

$$u = \frac{\tilde{u}}{\tilde{w}} = s_x f \frac{\tilde{x}}{\tilde{z}} + c_x \tag{9}$$
$$v = \frac{\tilde{v}}{\tilde{w}} = s_y f \frac{\tilde{y}}{\tilde{z}} + c_y \tag{10}$$

So the dehomogenized coordinates are independent of $\tilde{w}$, therefore we can drop them.

Task 3.2

The elementary rotation matrices and the translation matrix are given by:

$$R_x(\theta) \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \qquad R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \tag{11}$$

$$R_z(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T(x, y, z) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

to get the rotation matrices to work with our coordinates with the added 1 we need to expand them with a row and column with zeros and a 1 in the last position:

$$\mathbf{R} = \begin{bmatrix} \mathbf{R} & 0 \\ 0 & 1 \end{bmatrix} \tag{13}$$

now the transformation from object to camera is simply:

$$\mathbf{T}_o^c = \mathbf{T}(0, 0, 6)\mathbf{R}_x(15°)\mathbf{R}_y(45°)\mathbf{R}_z(0) \tag{14}$$

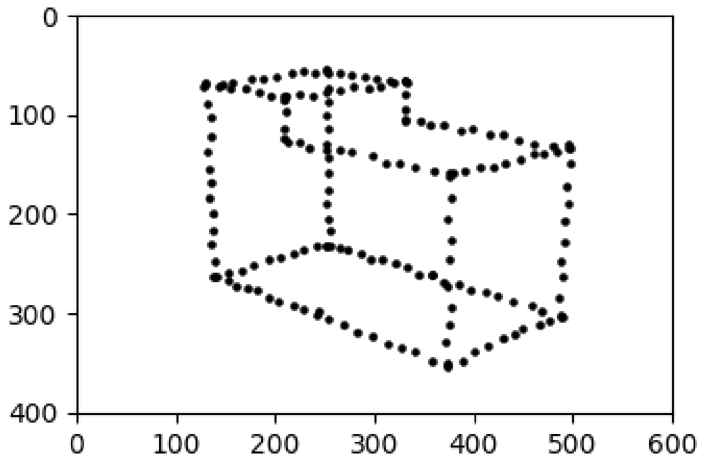

Figure 2: Dots from task 2 with applied transformations

4 Part 4 - Image formation model for the Quanser helicopter

Task 4.1

Define $L = 11.45$ cm the position of the screws in base coordinates are:

$$[0,0,0] \quad [L,0,0] \quad [L,L,0] \quad [0,L,0] \tag{15}$$

Task 4.2

By applying the camera transformation given we can plot the scre locations:

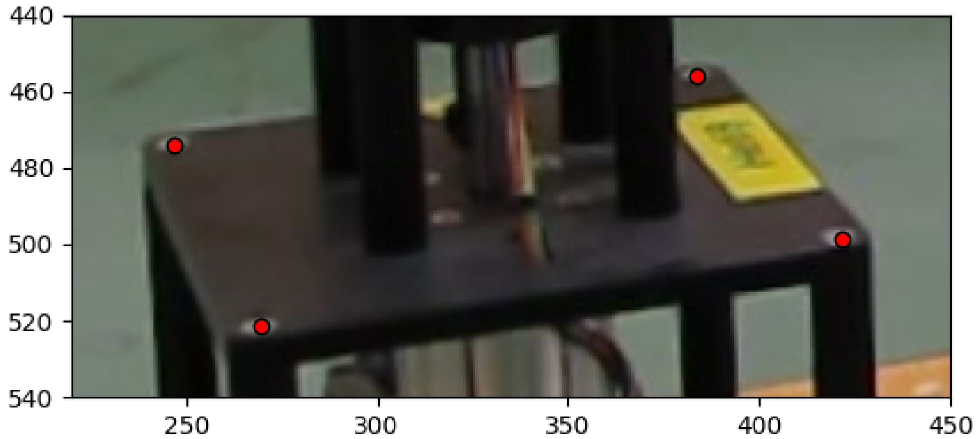

Figure 3: Plot of screw position on platform

Task 4.3

This is given as a translation to the center of the platform as well as a rotation around ψ or the z-axis:

$$\mathbf{T}_{base}^{platform} = \mathbf{T}(L/2, L/2, 0)\mathbf{R}_z(\psi) \tag{16}$$

Task 4.4

This is a translation up from the base, and a rotation around y-axis.

$$\mathbf{T}_{hinge}^{base} = \mathbf{T}(0,0,0.325)\mathbf{R}_y(\theta) \tag{17}$$

Task 4.5

Since these two frames have the same alignment this transformation is only a translation:

$$\mathbf{T}_{arm}^{hinge} = \mathbf{T}(0, 0, -0.05) \tag{18}$$

Task 4.6

Finally the last transformation is a translation down to the end of the arm and down the 3cm to the rotors, nad also a rotation in ϕ or the x-axis.

$$\mathbf{T}_{rotors}^{arm} = \mathbf{T}(0.65, 0, 0.03)\mathbf{R}_x(\phi) \tag{19}$$

Task 4.7

By multiplying the former transformations together we can get the transformation matrix from every frame the camera.This lets us plot all the frames as well as the marker locations on the helicopter:

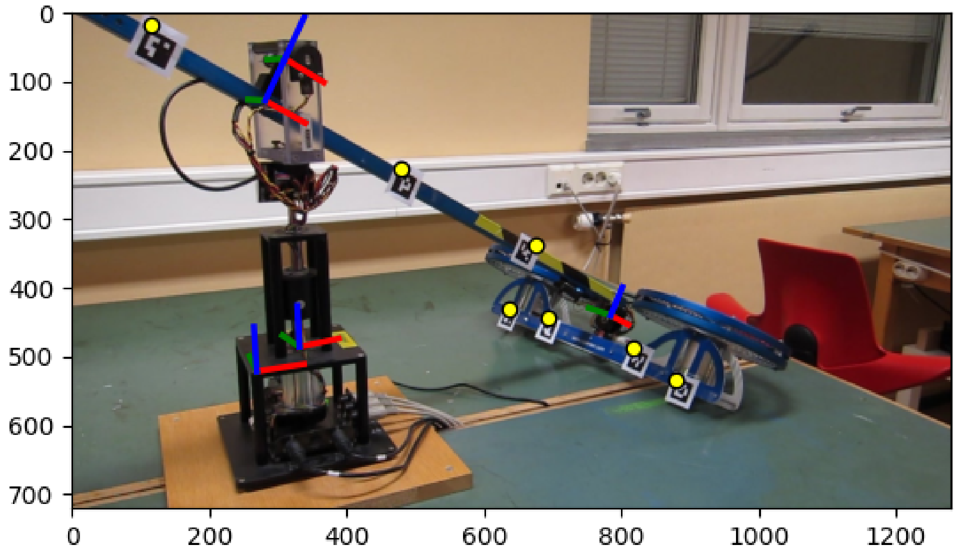

Figure 4: All coordinate frames as well as markers on helicopter