# TTK4255: Robotic Vision

# Homework 4: Linear algorithms

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## Part 1 Transformation between a plane and its image

### Task 1.1

Let $\mathbf{x} = (x, y)$ and $\tilde{\mathbf{x}} = (\tilde{x}, \tilde{y}, \tilde{z})$. And let $\mathbf{G} = \alpha\mathbf{H}$. Then:

$$\mathbf{G}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \alpha\mathbf{H}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \alpha\tilde{\mathbf{x}} \tag{1}$$

So dehomogenizing $\alpha\tilde{\mathbf{x}}$ gives:

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \alpha\tilde{x}/\alpha\tilde{z} \\ \alpha\tilde{y}/\alpha\tilde{z} \end{bmatrix} = \begin{bmatrix} \tilde{x}/\tilde{z} \\ \tilde{y}/\tilde{z} \end{bmatrix} \tag{2}$$

So dehomogenization with transformation $\alpha\mathbf{H}$ and $\mathbf{H}$ yields the same result.

### Task 1.2

Since the first two columns of $\mathbf{H}$ comes from a rotation matrix $\mathbf{R}$ these columns must be orthogonal. Therefore they can't be chosen independently of each other and $\mathbf{H}$ is more restricted than an arbitrary homography.

# Part 2 The direct linear transform

## Task 2.1

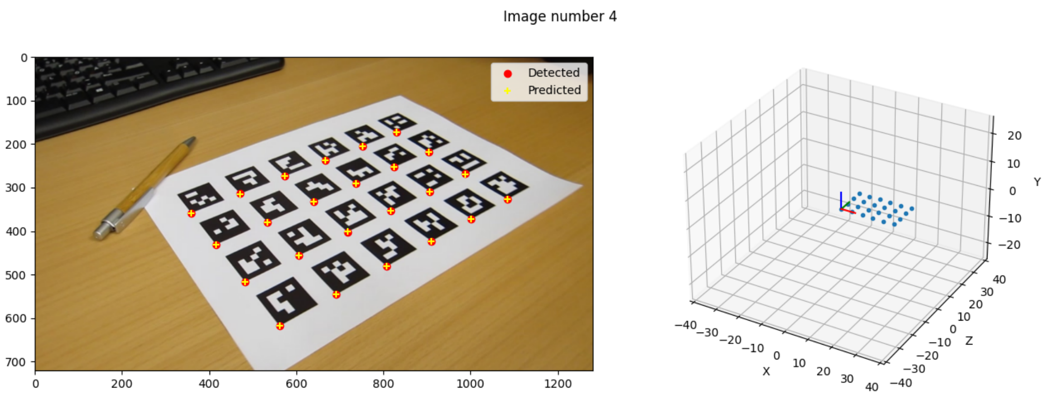


Figure 1: Image 4 with estimated marker locations

## Task 2.2

For image 4 the errors where:

| | |
|---|---|
| Avg: | 0.4289 |
| Max: | 0.9368 |
| Min: | 0.1239 |

# Part 3 Recover the pose

## Task 3.1

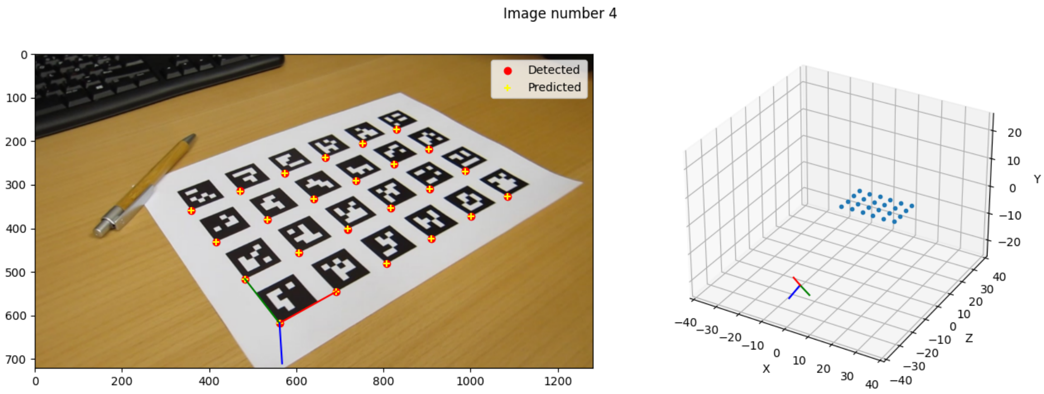


Figure 2: Pose on image 4 with with -k

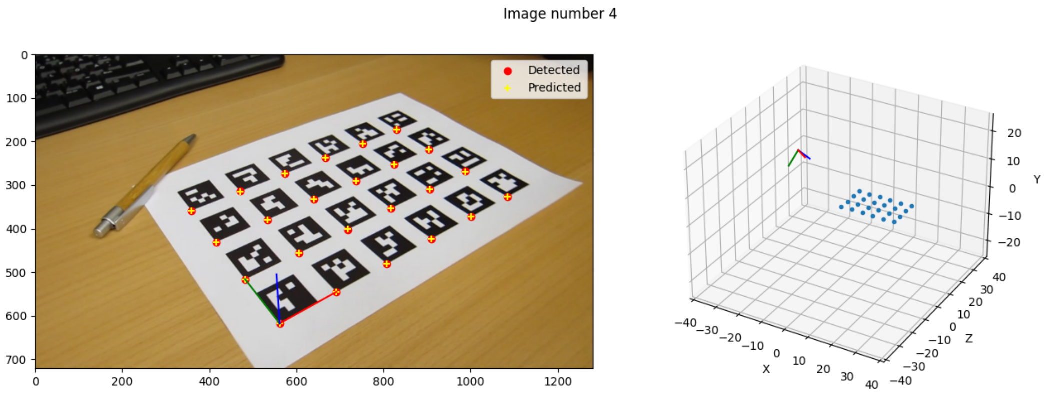


Figure 3: Pose on image 4 with +k

## Task 3.2

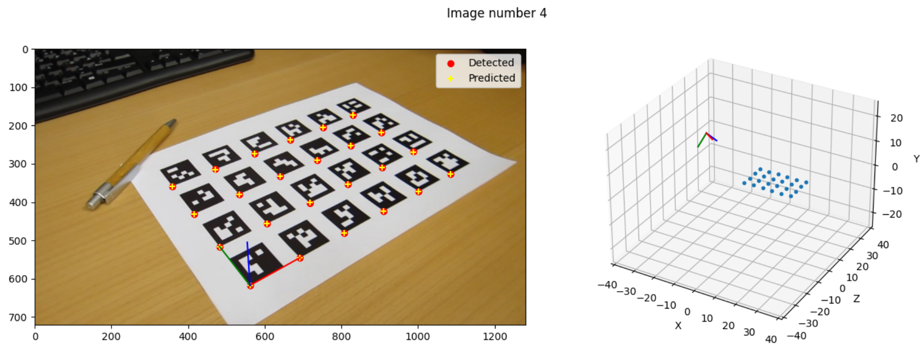


Figure 4: Image 4 with correct pose

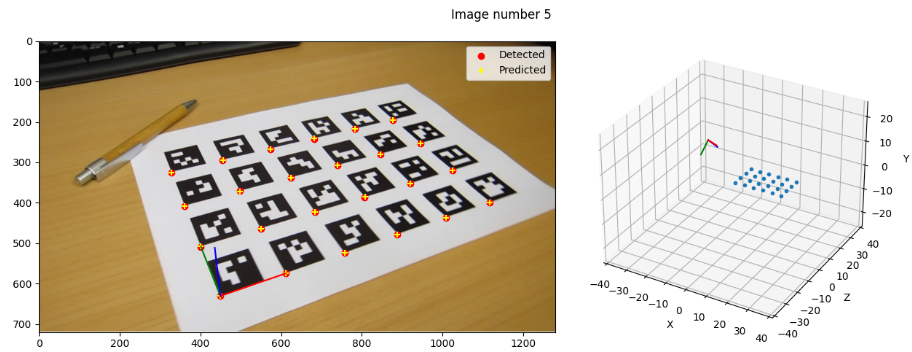


Figure 5: Image 5 with correct pose

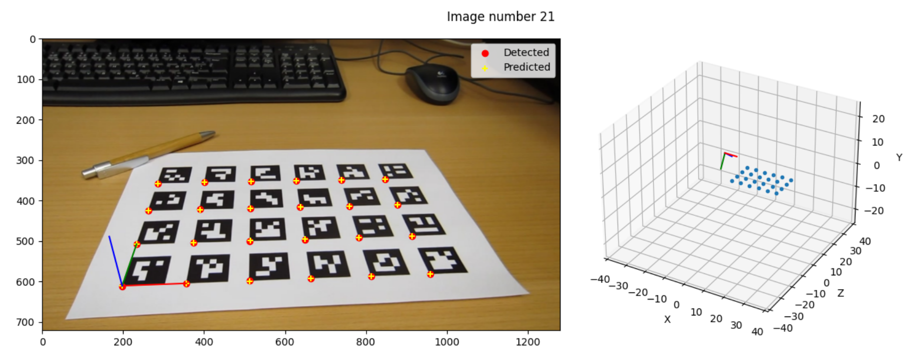


Figure 6: Image 21 with correct pose

## Task 3.3

A rotation matrix $\mathbf{R}$ has the properties: $\mathbf{R}^\mathsf{T}\mathbf{R} = \mathbf{R}\mathbf{R}^\mathsf{T} = \mathbf{I}$ and $\det(\mathbf{R}) = 1$. To get a meassure of how far off the matrix is from a rotation we can use the frobenious norm of the difference with identity, for image 4 this is:

$$\text{Before correction: } ||\mathbf{R}^\mathsf{T}\mathbf{R} - \mathbf{I}||_f = 0.00814 \tag{3}$$

$$\text{After correction: } ||\mathbf{R}^\mathsf{T}\mathbf{R} - \mathbf{I}||_f = 3.690 \times 10^{-16} \approx 0 \tag{4}$$

Also the determinant after correction is also equal to 1, but before correction it's 0.992. So after the correction we get an actual rotation matrix instead of an almost rotation matrix.

# Part 4 My own linear algorithm

## Task 4.1

Just like in the first part we have $\tilde{\mathbf{x}}_i = \mathbf{K}^{-1}\tilde{u}_i$, and the dehomogenized vector $\mathbf{x}_i = (\tilde{x}_i/\tilde{z}_i, \tilde{y}_i/\tilde{z}_i)^\mathsf{T}$. This gives together with equation (20) from the assignment text:

$$\tilde{\mathbf{x}}_i = \mathbf{R}\mathbf{X}_i + \mathbf{t} \tag{5}$$

$$x_i = \frac{\tilde{x}_i}{\tilde{z}_i} = \frac{r_{11}X_i + r_{12}Y_i + r_{13}Z_i + t_x}{r_{31}X_i + r_{32}Y_i + r_{33}Z_i + t_z} \tag{6}$$

$$y_i = \frac{\tilde{y}_i}{\tilde{z}_i} = \frac{r_{21}X_i + r_{22}Y_i + r_{23}Z_i + t_y}{r_{31}X_i + r_{32}Y_i + r_{33}Z_i + t_z} \tag{7}$$

By rearranging equation 6 and 7 we can define the following matrices to put it on linear form:

$$\mathbf{m} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{21} & r_{22} & r_{23} & r_{31} & r_{32} & r_{33} \end{bmatrix}^\mathsf{T} \tag{8}$$

$$\mathbf{b}_i = \begin{bmatrix} x_i t_z - t_x \\ y_i t_z - t_y \end{bmatrix} \tag{9}$$

$$\mathbf{A}_i = \begin{bmatrix} X_i & Y_i & Z_i & 0 & 0 & 0 & -X_i x_i & -Y_i x_i & -Z_i x_i \\ 0 & 0 & 0 & X_i & Y_i & Z_i & -X_i y_i & -Y_i y_i & -Z_i y_i \end{bmatrix} \tag{10}$$

$$\tag{11}$$

Stacking all $\mathbf{A}_i$ and $\mathbf{b}_i$ together gives dimensions of $2n \times 9$ and $2n \times 1$ respectively and yielding:

$$\mathbf{A}\mathbf{m} = \mathbf{b} \tag{12}$$

## Task 4.2

This system is inhomogenious as long as $x_i t_z - t_X \neq 0$ or $y_i t_z - t_y \neq 0$ for at least one value of $i$.

## Task 4.3

We can use the SVD to find a least squares solution to the system with an approximation of the pseudoinverse (where $D$ is a diagonal matrix of the inverse of singular values that are non-zero):

$$\mathbf{m} = \mathbf{A}^\dagger \mathbf{b} \approx \mathbf{V}\mathbf{D}^{-1}\mathbf{U}^\mathsf{T}\mathbf{b} \tag{13}$$