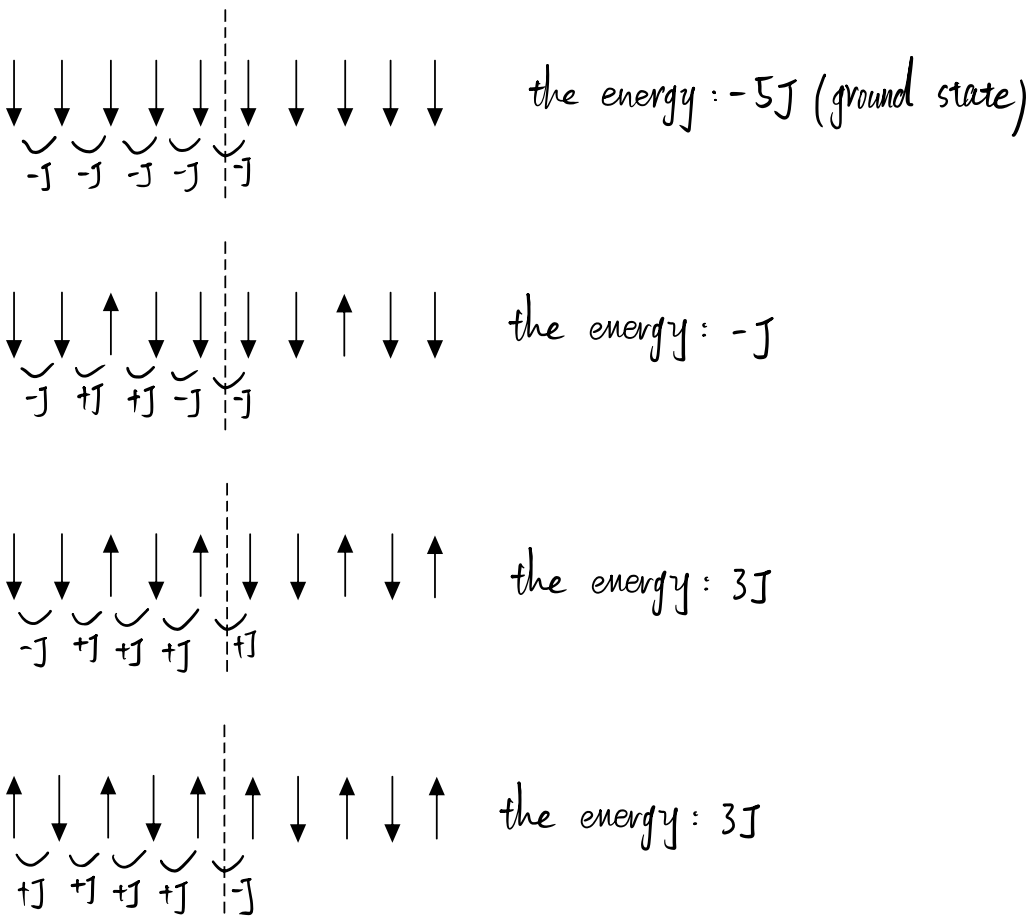
Assume there are n spins, we are going to investigate the 1D-Ising model.

For example, n=5. There are many different configurations, but some of them have the same energy:

(Using the periodic boundary conditions)



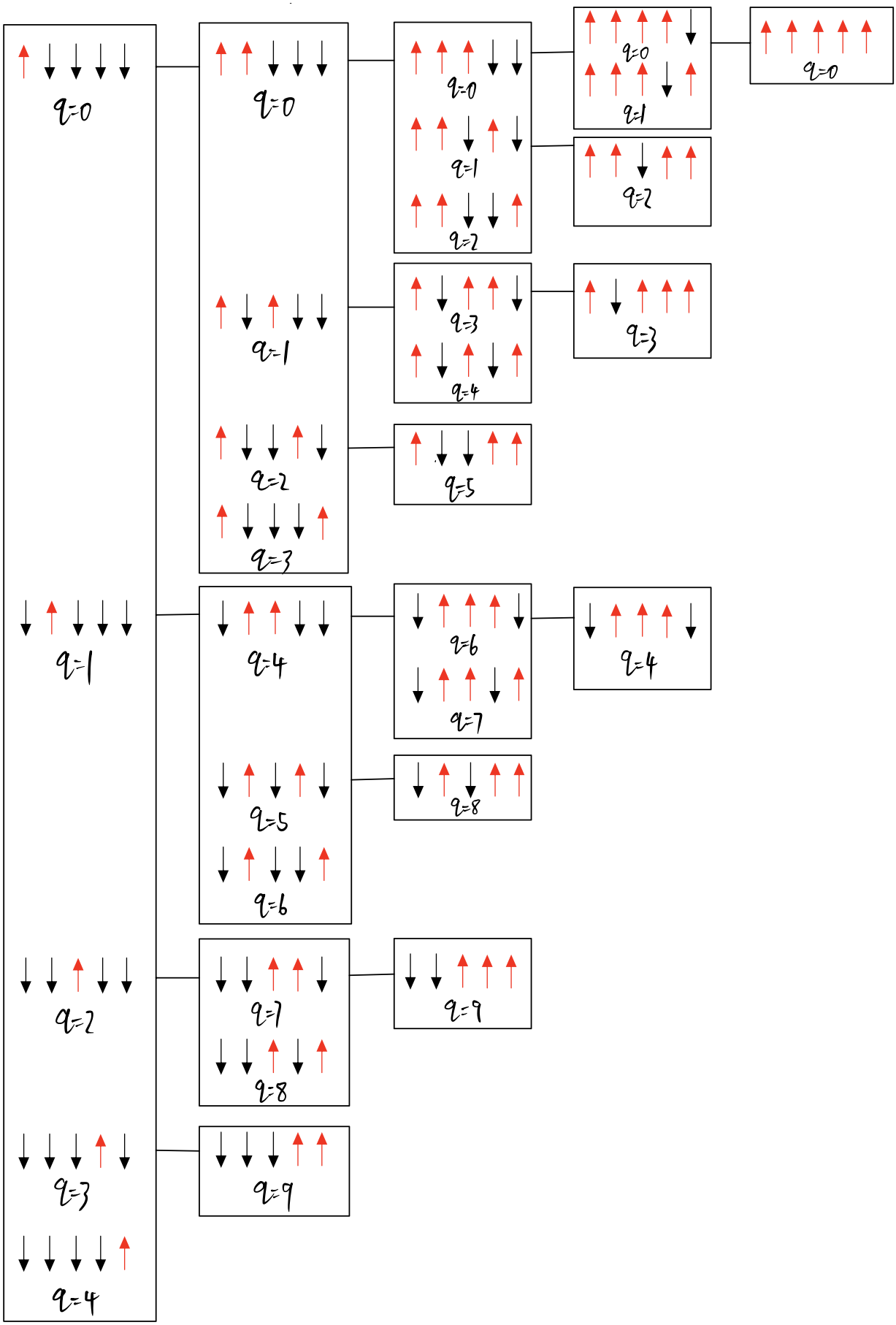
Hence, different energy level have different degeneracy.

Our goal is to find them, and achieve visualization.

(Please see the python script)

1. We use number "-1" and "1" to denote "↓" and "↑".
2. -1 * -1 = 1. means one spin flips from "↓" to "↑".
3. we start from only one spin to be flipped. then flip two spins based on the previous one, then flip three spins from the previous one.

num_for = 0 num_for = 1 num_for = 2 num_for = 3 num_for = 4


q=0
q=0
q=0
q=1
q=2
q=0
q=0
q=1
q=2
q=1
q=3
q=4
q=3
q=2
q=5
q=3
q=1
q=4
q=6
q=7
q=4
q=5
q=8
q=6
q=2
q=7
q=9
q=8
q=3
q=9
q=4

After finding the first 22 spins and their configurations. energy level and degeneracy. we use induction method to predict others

① when n is an even number:

2 : $\overset{+4}{\frown}$ -2 . 2 (2)

4 : -4 $\overset{+4}{\frown}$ 0 $\overset{+4}{\frown}$ 4 (3)

6 : -6 $\overset{+4}{\frown}$ -2 $\overset{+4}{\frown}$ 2 $\overset{+4}{\frown}$ 6 (4)

⋮

12 : -12 $\overset{+4}{\frown}$ -8 $\overset{+4}{\frown}$ -4 $\overset{+4}{\frown}$ 0 $\overset{+4}{\frown}$ 4 $\overset{+4}{\frown}$ 8 $\overset{+4}{\frown}$ 12 (7)

⋮

22 : -22 . -18 . -14 . -10 . -6 . -2 . 2 . 6 . 10 . 14 . 18 . 22 (12)

⋮

② when n is an odd number:

n=3 : -3 $\overset{+4}{\frown}$ 1 (2)

n=5 : -5 $\overset{+4}{\frown}$ -1 $\overset{+4}{\frown}$ 3 (3)

⋮

n=11 : -11 $\overset{+4}{\frown}$ -7 $\overset{+4}{\frown}$ -3 $\overset{+4}{\frown}$ 1 $\overset{+4}{\frown}$ 5 $\overset{+4}{\frown}$ 9 (6)

python script:

```
prelist = [ ]
prelist.append(-n)
if n%2 == 0 :
    m = int(n/2)+1
    a = -n
    for i in range(m-1) :
        a = a+4
        prelist.append(a)
```

The degeneracy of each energy level:

$n=2$: energy level: $(-2, 2)$ degeneracy: $(2, 2)$ $C_2^0 \times 2$, $C_2^2 \times 2$

$n=3$: energy level: $(-3, 1)$ degeneracy: $(2, 6)$ $C_3^0 \times 2$, $C_3^2 \times 2$

$n=4$: energy level: $(-4, 0, 4)$ degeneracy: $(2, 12, 2)$ $C_4^0 \times 2$ $C_4^2 \times 2$ $C_4^4 \times 2$

$n=5$: energy level: $(-5, -1, 3)$ degeneracy: $(2, 20, 10)$ $C_5^0 \times 2$ $C_5^2 \times 2$ $C_5^4 \times 2$

⋮

$n=9$: energy level: $(-9, -5, -1, 3, 7)$ degeneracy: $(2, 72, 252, 168, 18)$

$C_9^0 \times 2$ $C_9^2 \times 2$ $C_9^4 \times 2$ $C_9^6 \times 2$ $C_9^8 \times 2$

$n=10$: energy level: $(-10, -6, -2, 2, 6, 10)$ degeneracy: $(2, 90, 420, 420, 90, 2)$

$C_{10}^0 \times 2$ $C_{10}^2 \times 2$ $C_{10}^4 \times 2$ $C_{10}^6 \times 2$ $C_{10}^8 \times 2$ $C_{10}^{10} \times 2$

Python script

```
prelist2 = [ ]
prelist2.append(2)
q = 2
if q <= n:
    prelist2.append(C_n^q × 2)
    q = q + 2
```