

# KERNEL METHODS FOR MOLECULAR MACHINE LEARNING

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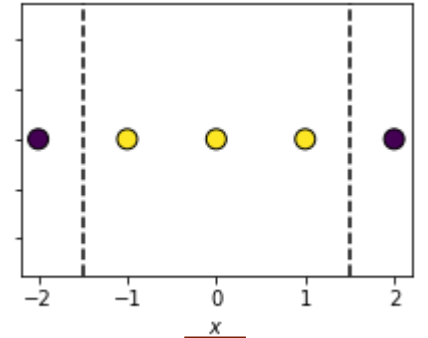
# What are kernel methods?

The “kernel trick” is to change data from **low to high dimension** space using a **pairwise similarity** “kernel” function.

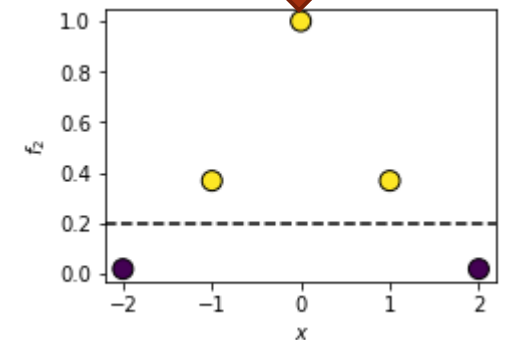
## Key terms:

- Kernel function:  $k(x_i, x_j)$
- Kernel matrix:  $K_{ij} = k(x_i, x_j)$
- Kernel Ridge Regression:  $f(x_i) = K\alpha = \sum_j \alpha_j k(x_i, x_j)$

Difficult Learning



Kernel Trick



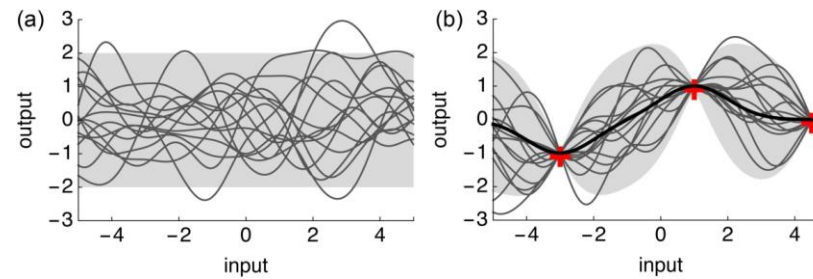
Easy Learning

**Main Concept:** “Make learning simple with a kernel function”

# Kernel Methods have great properties

- Instance based learning: Complexity grows with data

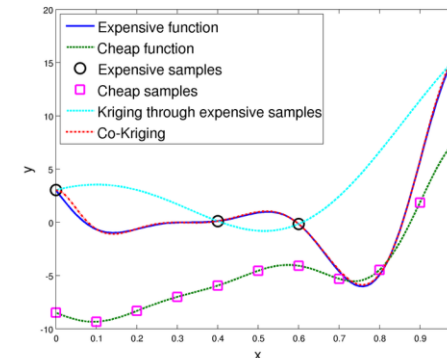
- Good uncertainty methods (GPR)



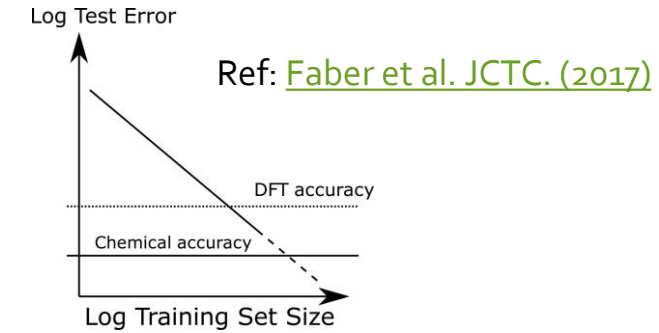
Ref: [Rupp. Int. J. Quant. Chem., \(2015\)](#)

- Flexibility in kernels: different kinds of non-linearity

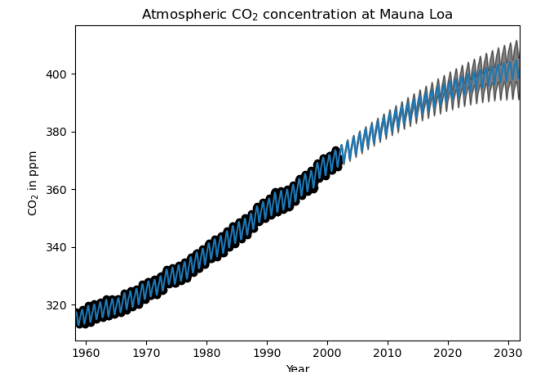
- Multi-resolution methods



Ref: [Zhang et al. AIAA 2013. \(2013\)](#)



Ref: [Faber et al. JCTC. \(2017\)](#)



Ref: [Scikit-Learn Docs](#)

# How do I make similarity functions for molecules?

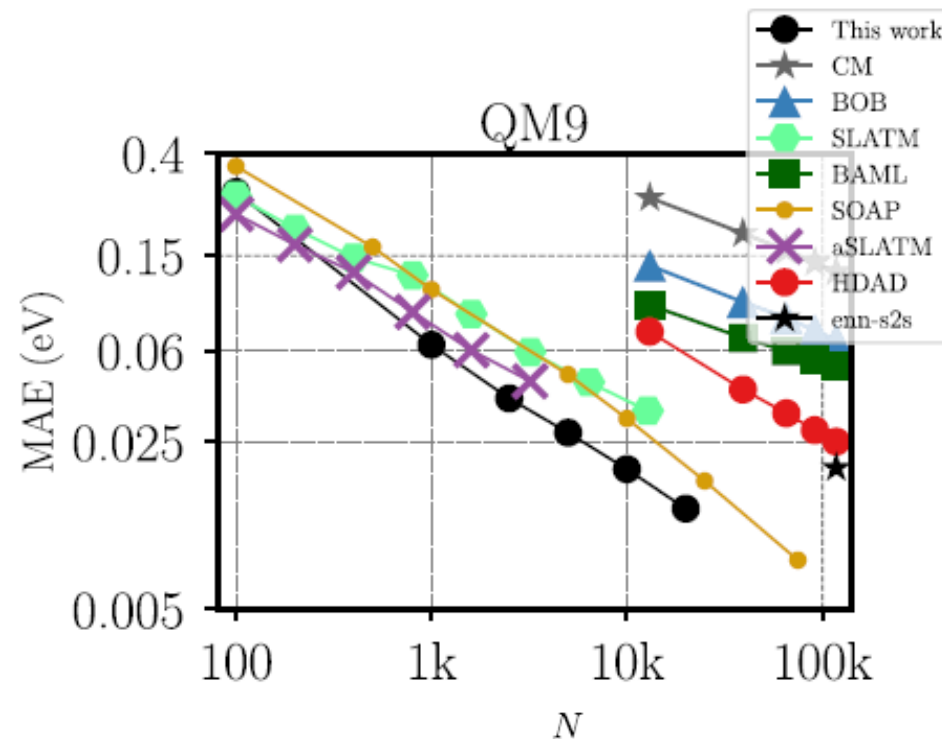
This question has been studied for 10+ years

What kind of physics do we put into the kernel

- Interpolation between changes of atom type
- Sensitivity to small changes in environments
- Invariance to rotation, translation, permutation
- ...

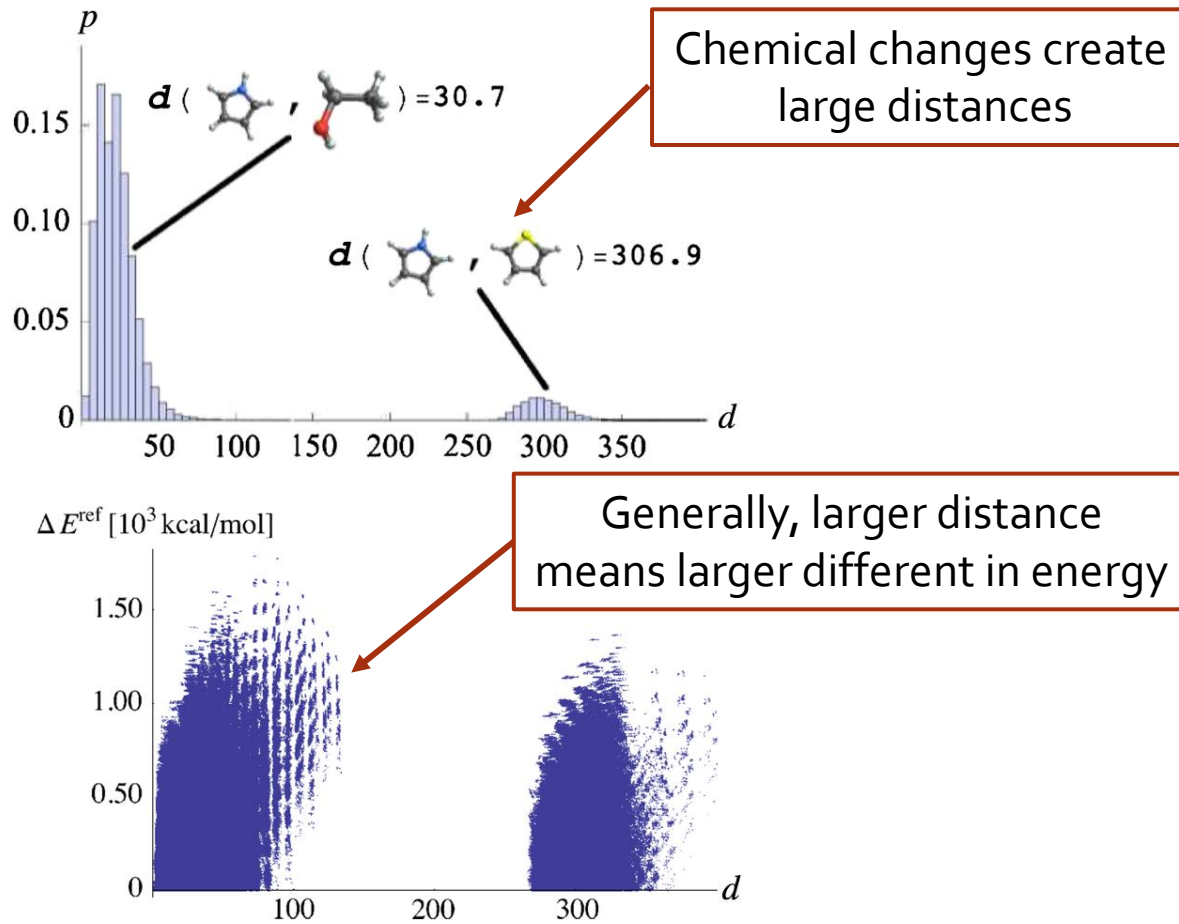
There are a half dozen (or more\*) ways to do this

\*Follow [Anatole von Lilienfeld's group](#)



Ref: [Faber et al. JCP. \(2018\)](#)

# Case Study: Coulomb Matrix



Simple formula:

$$M_{ij} = \begin{cases} 0.5Z_i^{2.4} & i = j \\ \frac{Z_i Z_j}{\|R_i - R_j\|_2} & i \neq j \end{cases}$$

Captures atomic positions and types

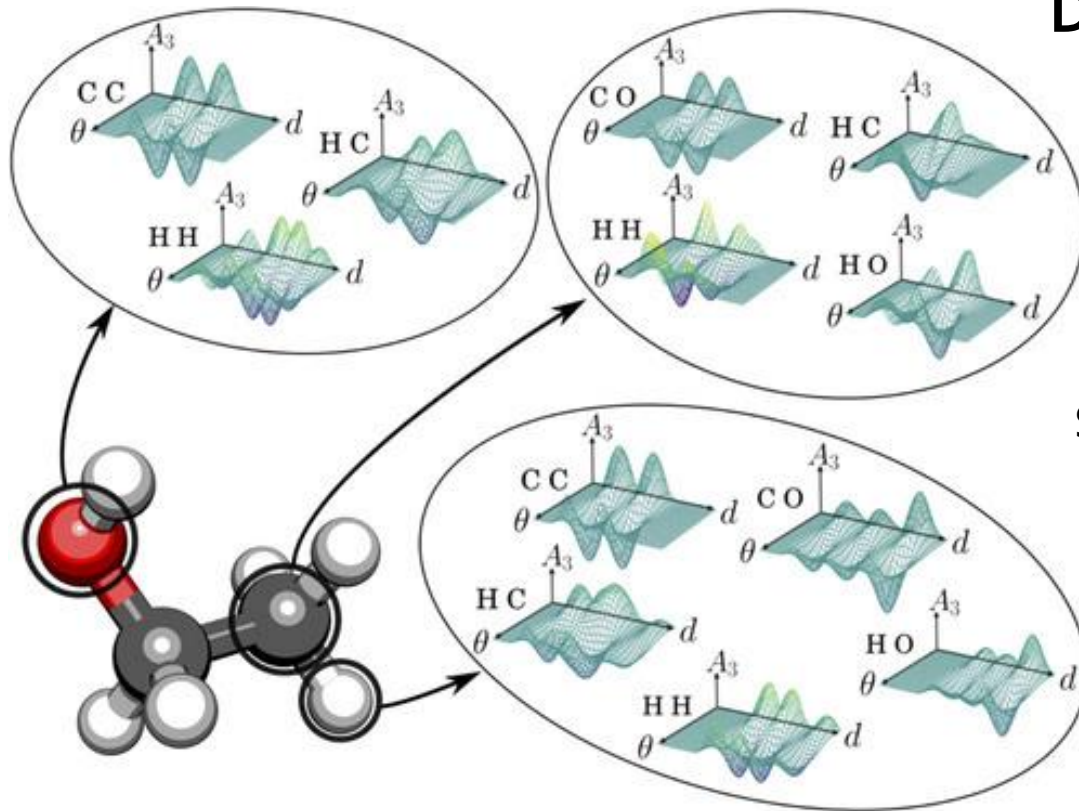
**Problem:**  $M$  not permutation invariant

**Solution:** Use eigenvalues of  $M$  ( $\epsilon$ )

$$d_{ij} = \|\epsilon_i - \epsilon_j\|_2$$

$$k(x_i, x_j) = e^{\frac{-d_{ij}}{\sigma}}$$

# Case Study: FCHL



Describes atoms using...

- “alchemical”** - difference based on period, group
- “many body”** - Capture bond distances and angles
- “distributions”** - As gaussian functions

Similarity between atoms are computed with overlap integrals

$$A_1(x, y; I) = e^{-\frac{(P_I - x)^2}{2\sigma_P^2} - \frac{(G_I - y)^2}{2\sigma_G^2}}$$

$$\Delta(A_1(I), A_1(J)) = \iint (A_1(I) - A_1(J))^2 dx dy$$

$$= \frac{1}{2} \exp \left( -\frac{(P_I - P_J)^2}{4\sigma_P^2} - \frac{(G_I - G_J)^2}{4\sigma_G^2} \right)$$

Ref: [Faber et al. JCP \(2018\)](#)

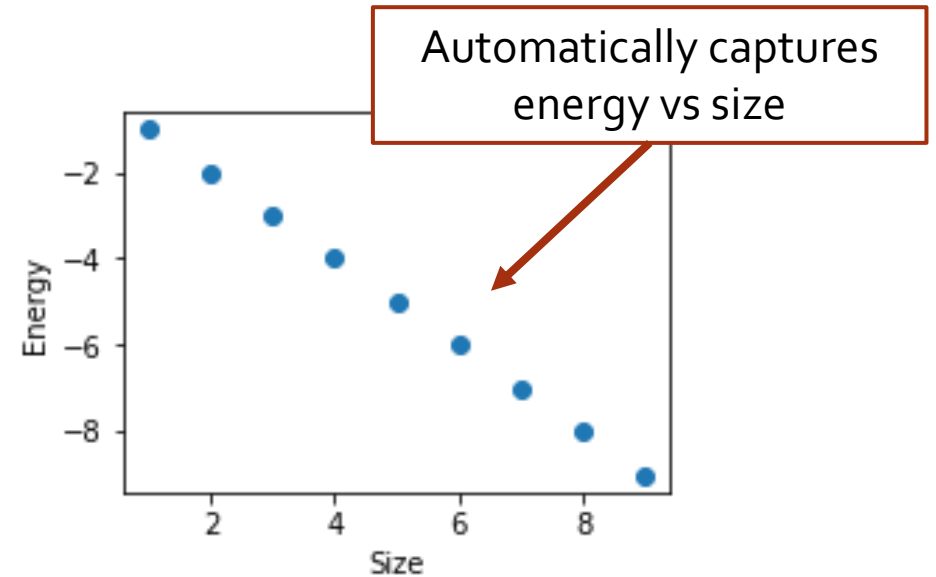
# Scalable Kernels: What and Why

**Problem:** FCHL has atomic similarity but we want molecular properties

**Solution:** Make a “scalable” kernel that encodes atomic -> molecular relationship

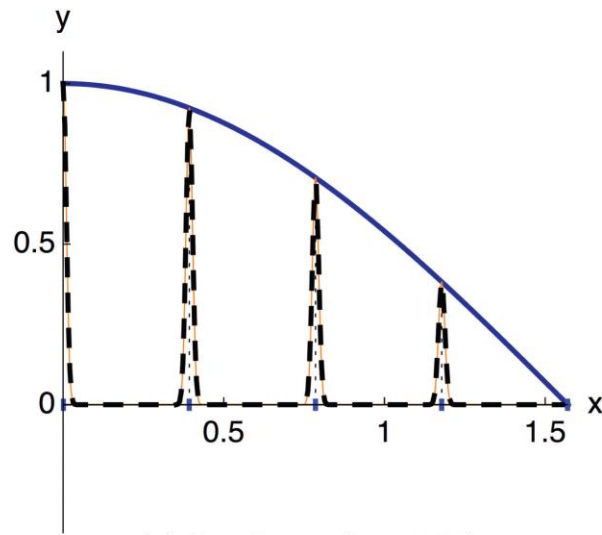
**Example:** Energy is often a sum over atoms.

$$K_{mol}(x, y) = \sum_i \sum_j k_{atom}(x_i, y_j)$$

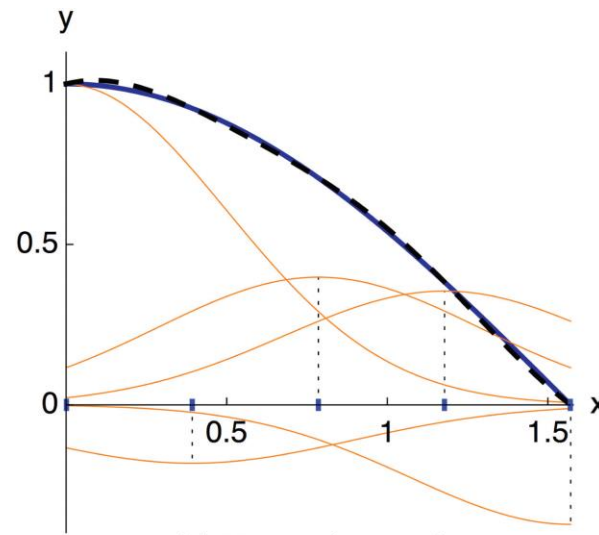


Full explanation: [“FCHL in one notebook”](#)

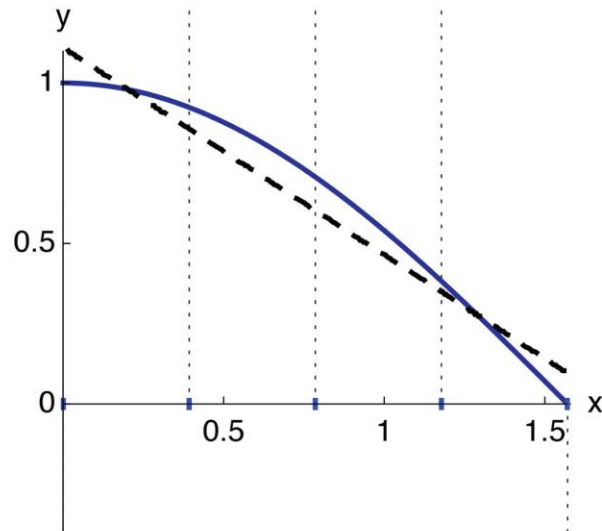
# Key issue: Adjusting hyperparameter is *very* important



(a) Overfitting ( $\sigma = 0.01$ )



(b) Fitting ( $\sigma = 0.5$ )



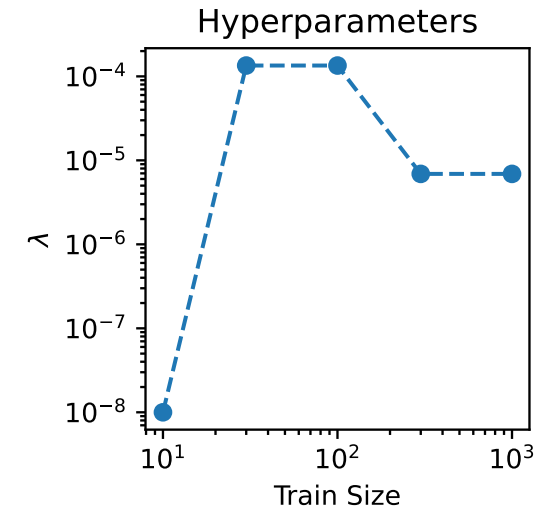
(c) Underfitting ( $\sigma = 10^4$ )

$$\mathbf{x} = \{0, \frac{1}{8}\pi, \frac{2}{8}\pi, \frac{3}{8}\pi, \frac{4}{8}\pi\}$$

	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$
(a)	1.000	0.924	0.707	0.383	0.000
(b)	0.998	-0.182	0.398	0.355	-0.372
(c)	$-10^{13}$	$10^{12}$	$10^{13}$	$10^{12}$	$-10^{12}$

(d) Training points  $\mathbf{x}$  and weights  $\alpha$

You cannot be sure if a kernel method works until after investing significant time

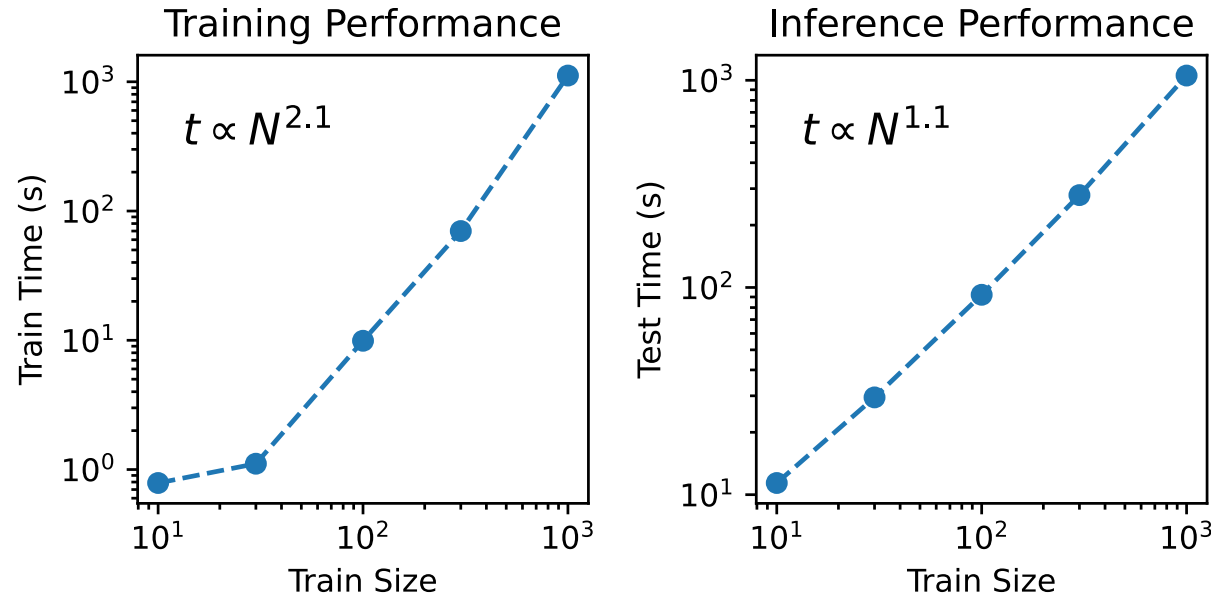


Effort must be replicated as training set changes



# Dark side of kernel methods: Scaling

## Fitting KRR models is expensive...



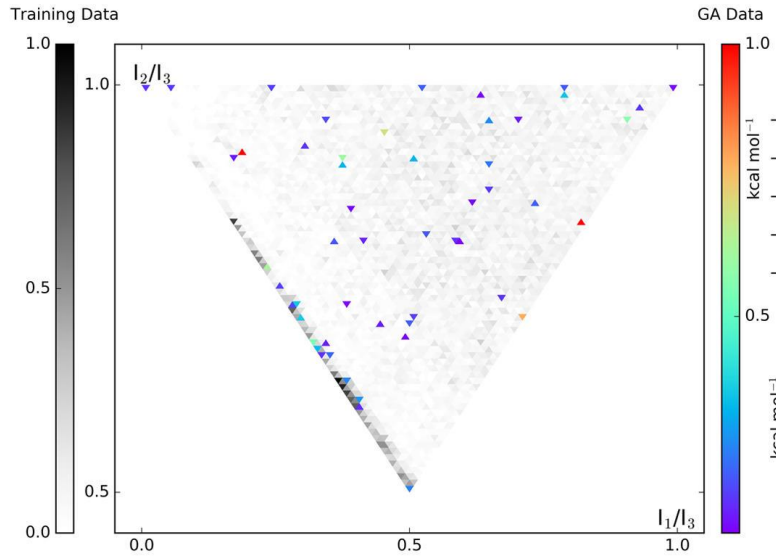
Computing  $K$  is  $O(N^2)$   
Solving KRR is  $O(N^3)$ !

New features with  
each training point =  $O(N)$

**Reframing:** *"Dial in performance vs accuracy tradeoff"*

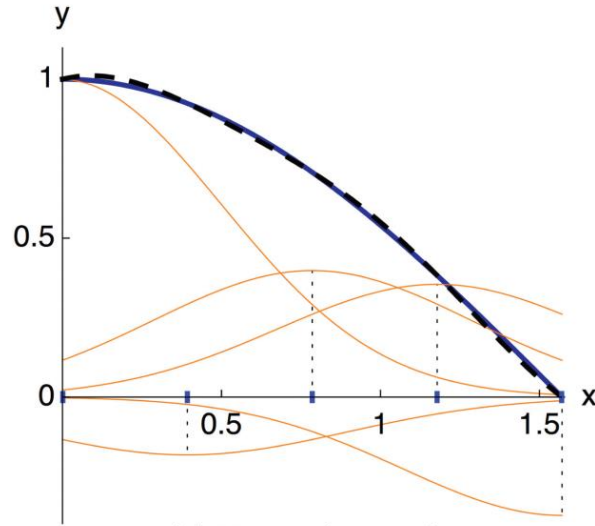
# Some routes for addressing scaling issue

## Pick “high value points”



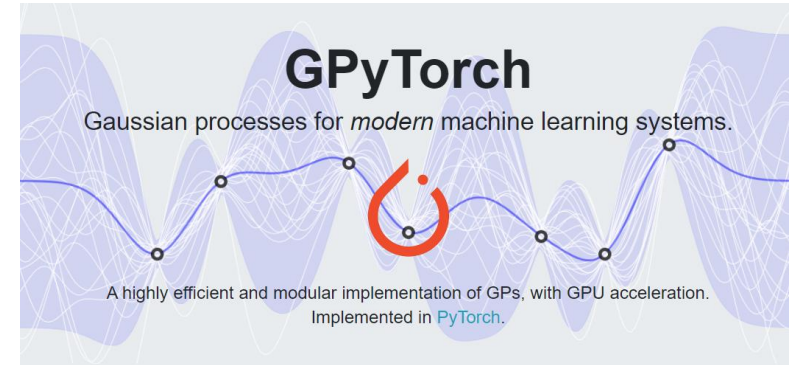
Ref: [Browning et al. JPCL. \(2017\)](#)

## Adjust basis points



Ref: [Snelson, Ghahramani. NuerIPS \(2005\)](#)

## Deploy on GPUs



You can use GPR with large datasets, its just more work

A good reference: [J. Emmanuel Johnson's Research Journal](#)

# Codes

## DScribe



0.4.x

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### Dscribe

build passing coverage 88%

Dscribe is a Python package for transforming atomic structures into fixed-size numerical fingerprints. These fingerprints are often called "descriptors" and they can be used in various tasks, including machine learning, visualization, similarity analysis, etc. To get started you can check the [basic tutorial](#).

#### Note

Version 0.4.0 includes a fix to the layout of the SOAP feature vector. See issue [#48](#) for more details. These changes break the backwards compatibility of the SOAP feature vectors. It is thus encouraged to start using version 0.4.0 or above if working with SOAP.

### Capabilities at a Glance

Dscribe currently includes the following descriptors:

- Coulomb matrix
- Sine matrix
- Ewald sum matrix
- Atom-centered Symmetry Functions (ACSF)
- Smooth Overlap of Atomic Positions (SOAP)
- Many-body Tensor Representation (MBTR)
- Local Many-body Tensor Representation (LMBTR)

Check the tutorials for more information.

## QML

» QML

Search docs

#### GETTING STARTED:

Installing QML  
Citing use of QML  
QML Tutorial  
Examples

#### SOURCE DOCUMENTATION:

Python API documentation

Docs » QML: A Python Toolkit for Quantum Machine Learning

[View page source](#)

build passing pypi package 0.4.0.27 DOI 10.5281/zenodo.817332 BETA VERSION!

### QML: A Python Toolkit for Quantum Machine Learning

QML is a Python2/3-compatible toolkit for representation learning of properties of molecules and solids. QML is not a high-level framework where you can do `model.train()`, but supplies the building blocks to carry out efficient and accurate machine learning on chemical compounds. As such, the goal is to provide usable and efficient implementations of concepts such as representations and kernels.

#### Current list of contributors:

- Anders S. Christensen (University of Basel)
- Lars A. Bratholm (University of Bristol)
- Jimmy C. Kromann (University of Basel)

# Conclusions and Outlook

## Key bits to understand:

1. Kernel methods simplify learning through *similarity functions*
2. There are many made for molecules
3. Learning curve to using them and other tradeoffs (e.g., scaling)

## Not just “historical methods”

- Extremely good predictive accuracy
- Active area of research

