MATH 211: HOMEWORK 2

Section 2.1

6ac (MAT), 13 (MAT), 15 (MAT & written)

Section 2.2

1, 7 (MAT & written), 16

Problem 1

Consider $f \in C^4[a,b]$.

$$D_h^2 f(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Using computer arithmetic for each operation, we denote

$$D_h^{\text{MAT},2} f(x) = \Big(f(x \oplus h) \ominus 2 \otimes f(x) \oplus f(x \ominus h) \Big) \oslash (h \otimes h).$$

(a) Show the following convergence rate of absolute error: $|D_h^2 f(x) - f''(x)| = O(h^2)$. *Hint: Taylor expand the first and third terms centered at x and perturbed by $\pm h$.*

(b) Show $|D_h^2 f(x) - D_h^{\text{MAT},2} f(x)| = O(\delta h^{-2})$ where $\delta \approx 10^{-16}$ is the machine precision error. Assume f is not *terrible*, in other words if δ is on the order of machine error, $f(x+\delta) = f(x) + \delta'$ for some δ' on the order of machine error.

(c) We can conclude error should start growing as h gets smaller near $h \sim \delta^\alpha$. Find α.

(d) Numerically plot h vs *relative error* of the algorithm $D_h^{MAT,2} f(x)$ using a logarithmic plot with respect to both axes for h values of $10^{-(0:.1:15)}$. Assume $f(x) = \cos(x)e^{-x}$ and evaluate the algorithm at $x = .1$. *Hint: You can modify the code of Topic1_ComputerArithmetic, Part 5. You will need to find the true solution by finding $f''(x)$ to use for comparison.*

Date: today.