MATH 211: HOMEWORK 3

Section 2.3

5ab (MAT), 16 (MAT) *Note: Topic2_NM is a function for running Newton's method.*

Section 3.1

2ab, 13ab

Problem 1

Suppose we are interested in finding $3^{1/3}$. In this exercise, you will solve this using three methods: Fixed point iterations, Newton's method, and the following cubic method (See Exercise 13 in Section 2.4):

$$p_n = g(p_{n-1}) = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})} - \frac{f''(p_{n-1})}{2f'(p_{n-1})}\left[\frac{f(p_{n-1})}{f'(p_{n-1})}\right]^2.$$

a) Consider $g(x) = x - \alpha(x^3 - 3)$ on $[1, 2]$. Pick $\alpha > 0$ and initial guess p_0 properly, and prove the fixed point iterative method converges using Theorem 2.4. Use this same p_0 for Newton's method and the cubic method in part (b).

b) Write a MATLAB code that approximates $3^{1/3}$ using the three methods. Plot the absolute error on a semilogy plot, with iteration number on the x-axis and absolute error on the y-axis. Have all three plots on the same graph. For your *true* solution, use the result for the cubic method run till iteration 20.

Problem 2

Consider

$$f(x) = \Big(\cos(x) + \sin(\sqrt{2}x)\Big)e^{-x}.$$

Using MATLAB, find the first three positive zeros $(p^{(1)}, p^{(2)}, p^{(3)})$ of the function f using your choice of root-finding algorithms up to 10^{-10} tolerance in the absolute error. You may use plotting to get your initial guesses $p_0^{(1)}, p_0^{(2)}, p_0^{(3)}$ corresponding to the three zeros $p^{(1)}, p^{(2)}, p^{(3)}$, or any other method you prefer.

Date: today.

Problem 3

This is not a MATLAB exercise

Suppose we have the Lagrange polynomials $L_{nk}(x)$ for query points $\{x_i\}_{i=0}^n$ and the Lagrange polynomials $\tilde{L}_{nk}(y)$ for the query points $\{y_j\}_{j=0}^n$. Consider $f(x,y)$ defined on $\mathbb{R}^2$, and consider points $\{(x_i,y_j)\}_{i,j=0}^n$. Write a polynomial approximation $P(x,y)$ of order at most $2n$ such that $P(x_i,y_j)=f(x_i,y_j)$ for all $0\leq i,j\leq n$.

Problem 4

This is not a MATLAB exercise

Consider the sequence $p_n=\frac{3}{2}+5^{-5^n}\to\frac{3}{2}$ as $n\to\infty$. Find the order of convergence and asymptotic constant for this sequence.