MATH 211: HOMEWORK 5

Book problems

Sec 4.1: 22, 28
Sec 4.3: 19, 24

Problem 1

Write MATLAB functions that approximates f' at a point using the forward difference formula, the three-point midpoint formula, and the five-point midpoint formula. Have your functions defined as so:
function deriv = derivForwardDiff(f, h, x)
function deriv = deriv3PointMid(f, h, x)
function deriv = deriv5PointMid(f, h, x)
Your functions are thus inputing the function f, the step-size h, and the query point(s) x.

Now test your code using $f(x) = 2\sin(5x)+\cos(3x)$ and $x = .5$. For your approximation $D_h f$, let the error be $E(h) = |D_h f(x) - f'(x)|$. On the same plot, plot your three error functions as a function of h using a loglog plot. Use sample points $h = 10^{-\frac{1}{2}n}$, $n \in \{1, 2, \cdots, 20\}$.

Plot the lines corresponding to the error rate you expect (when discounting arithmetic error) for the Forward Difference scheme and the three-point midpoint formula, but don't plot the line corresponding to the five-point midpoint formula. Recall if h is a MATLAB vector of our approximation constant, then you can plot the line using the code:
loglog(h,h. ∧ a, '--') , where a is the rate you expect.

Problem 2

Consider $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. Write a MATLAB function that approximates the error function using the composite trapezoidal method (see the form from Theorem 4.5). Have the query points evenly spaced. In other words, your function should take the form: *function value = error_func(x,n)*. It returns the approximate integral using the query points $x_j = \frac{jx}{n}$, where $j = 0, 1, \cdots n$.

Plot your error function (I'll call it erf_0) on $[0, 5]$ for $n = 20$. Then on a separate plot, plot the point-wise error $E(x) = |\operatorname{erf}_0(x) - \operatorname{erf}(x)|$ on the same interval, where $\operatorname{erf}(x)$ is MATLAB's built in error function.

Date: today.

Problem 3

Suppose we have the following function where k is a very large integer:

$$f(x) = \begin{cases} \cos(kx + .1)\sin(kx), & x \in [0, \pi] \\ \cos(x + .1)\sin(x), & x \in [\pi, 2\pi]. \end{cases}$$

Suppose we are given $N = (k+1)n + 1$ query points. Propose a distribution of query points $x_0, \cdots x_N$ and integration scheme coefficients a_j such that

$$\sum_{j=0}^{N} a_j f(x_j) = \int_0^{2\pi} f(x)dx + O\left(\frac{1}{n^2}\right).$$

In particular, the error bound *does not* depend on k.

Problem 4

Suppose $f \in C^4[a, b]$, and suppose we discretize $[a, b]$ into n evenly spaced intervals. Let $h = \frac{b-a}{n}$.

(a) Propose a composite method for approximating $\int_a^b f(x)dx$ with error $O(h^\alpha)$ such that $\alpha > 0$ is as large as possible given f's smoothness criteria. What is α?

(b) Do the same, except instead of approximating $\int_a^b f(x)dx$ we approximate $\int_a^b F(x)dx$ for $F(x) = \int_a^x f(s)ds$.