Q 2.1.1) log likelihood function of x given λ

$$P(X = k \mid \lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \qquad k \in \{0, 1, 2, \ldots\}$$

$$\text{Likelihood} = \prod_{i=1}^{n} P(X = k_i \mid \lambda)$$

$$= \prod_{i=1}^{n} \frac{\lambda^{k_i}}{k_i!} e^{-\lambda}$$

- Since all observations are independent.

Log likelihood :

$$\ln\left(\prod_{i=1}^{n} e^{-\lambda} \cdot \frac{\lambda^{k_i}}{k_i!}\right)$$

$$= \sum_{i=1}^{n} \ln\left(e^{-\lambda} \cdot \frac{\lambda^{k_i}}{k_i!}\right)$$

$$= \sum_{i=1}^{n} \left(\left(\ln e^{-\lambda}\right) + \left(\ln \frac{\lambda^{k_i}}{k_i!}\right) \right)$$

$$= \sum_{i=1}^{n} \left[(\ln e^{-\lambda}) - (\ln k_i!) + (\ln \lambda^{k_i}) \right]$$

$$= \sum_{i=1}^{n} \left[(-\lambda) - (\ln k_i!) + (k_i \ln \lambda) \right]$$

$$= -n\lambda - \sum_{i=1}^{n} (\ln k_i!) + \ln \lambda \sum_{i=1}^{n} (k_i)$$

Q2.1.2) MLE for λ

$$\hat{\lambda} = \underset{\lambda}{\operatorname{argmax}} \text{ (likelihood)}$$

$$= \underset{\lambda}{\operatorname{argmax}} \text{ (loglikelihood)}$$

Differentiating log likelihood wrt λ ;

$$-n + \frac{1}{\lambda} \sum_{i=1}^{n} k_i = 0$$

$$\therefore \lambda = \frac{1}{n} \sum_{i=1}^{n} k_i$$

Q2.1.3) MLE using observed x :

$$\lambda = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$= \frac{1}{7} (4+5+3+5+6+9+3)$$

$$= \frac{1}{7} (35)$$

$$= \underline{\underline{5}}$$

Q2.2.1) $P(\lambda \mid \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} \quad \lambda > 0$

Delays ressemble Gamma ~~[illegible]~~ distribution.

$\lambda \sim \Gamma(\alpha, \beta)$

$\text{mean} = \frac{\alpha}{\beta}$

$\text{mode} = \frac{\alpha - 1}{\beta} \quad \text{for} \quad \alpha > 1$

prior distribution of $\lambda = \Gamma(\lambda \mid \alpha \beta)$

$P(X = k \mid \lambda) = \frac{\lambda^{k}}{k!} e^{-\lambda}$

$P(X=k \mid \lambda)$

$P(\lambda \mid \alpha \beta)$

$P(\lambda \mid x=k)$

$$= \frac{P(X=k \mid \lambda)\, P(\lambda)}{P(x=k)}$$

$$= P(x=k \mid \lambda)\, P(\lambda)$$

$$\lambda^{\Sigma x_e + \alpha - 1}\, e^{-\lambda(n+\beta)}$$

$$= e\, \lambda^{\Sigma x_i} \cdot \lambda^{\alpha-1} \cdot e^{-\lambda n} \cdot e^{-\lambda\beta}$$

$$= \left(\lambda^{\Sigma x_e} \cdot e^{-\lambda\beta}\right) \cdot \left(\lambda^{\alpha-1} \cdot e^{-\lambda n}\right)$$

$$= \lambda^{\Sigma x_e} \cdot e^{-\lambda n} \left(\lambda^{\alpha-1} \cdot e^{-\beta\lambda}\right)$$

$$P(x=k \mid \lambda) = \frac{\lambda^{k}}{k!}\, e^{-\lambda}$$

$$\frac{1}{\Gamma(\alpha)} \quad \lambda \quad e^{\cdots} \cdot \frac{\lambda^{k}}{k!} e^{-\lambda}$$

$$= e^{-\beta\lambda-\lambda} \cdot \lambda^{\alpha-1+k} \left(\frac{\beta^{\alpha}}{k!\,\Gamma(\alpha)}\right)$$

prior of λ is $\Gamma(\lambda|\alpha,\beta)$

$\therefore$ posterior is $\rightarrow$

$\cancel{P(\lambda|\lambda)} =$

$$P(\lambda|x) = \frac{P(x|\lambda)\,P(\lambda)}{P(x)}$$

$$\rightarrow \left(\lambda^{n} \cdot e^{-\lambda n}\right) \left(\lambda^{\alpha-1} \cdot e^{-\beta\lambda}\right)$$

$$P(\text{data}|\text{model}) = P(x=k_i|\lambda) = \frac{\lambda^{k}}{k!} e^{-\lambda}$$

$$P(\text{model}) = p(\lambda) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$

$\therefore$ $p(\text{model}\ \text{data}) =$

$$p(\lambda|x=k_i) = \frac{\frac{\lambda^{k}}{k!} e^{-\lambda}}{\int_0^{\infty}}$$

$2^3 = 8$

$2^6 = 8 \times 8$
$= 2^3 . 2^3$

Q2.3.1

$$\eta = e^{-2\lambda}$$

$$X \sim \text{Poisson}(\lambda)$$

$$P(X = k_i \mid \lambda) = \frac{\lambda^{k_i} e^{-\lambda}}{k!}$$

$$\hat{\eta} = e^{-2X}$$

$$\eta = e^{-2\lambda}$$

$$\cancel{\eta = (e^{-\lambda})^2}$$

$$e^{-2} . e^{\lambda} = \eta$$

$$e^{\lambda} = \frac{\eta}{e^{-2}}$$

$$\lambda = \ln\left(\frac{\eta}{e^{-2}}\right)$$

$\therefore \quad P(x = k_e \mid n) = \dfrac{\left(\ln\left(\frac{n}{e^{-2}}\right)\right)^{k_e}}{k!} \quad \dfrac{e^{-2}}{n} = M$

MLE for $n \rightarrow$

Taking log;

likelihood = $\underset{n}{\operatorname{argmax}}\ M$

$\therefore$ loglikelihood = $\log M$

$\rightarrow k_e \log\left(\log\left(\frac{n}{e^{-2}}\right)\right) - \log k! + \log\left(\frac{e^{-2}}{n}\right)$

$\therefore$ ignore $(-\log k!)$

$= k_e \log\left(\log \frac{n}{e^{-2}}\right) + \log e^{-2} - \log n$

$= k_e \log\left[\log n - \log e^{-2}\right] - \log n$

ignore $\log e^{-2}$

$= k_e \log\log(n) - \log(n)$

$= \cancel{k_e}\ \cancel{\log(\log n}$

$\therefore \dfrac{d}{dn} = \dfrac{k_e}{n \log n} - \dfrac{1}{n} = \dfrac{1}{n}\left(\dfrac{k_e}{\log n} - \log n\right)$

$= 0$

Q2.3.1)

$\therefore$ ~~$\log n = k_i$~~

$$n = e^{-2\lambda}$$

$$\therefore \quad n^{1/2} = e^{-\lambda}$$

$$n^{-1/2} = e^{\lambda}$$

$$\lambda = \log n^{-1/2}$$

$$= \frac{-1}{2} \log n$$

$$P(x = k_i \mid \lambda) = \frac{\lambda^{k_i}}{k_i!} e^{-\lambda}$$

$$\therefore P(x = k_i \mid n) = \frac{(-1/2 \log n)^{k_i}}{k_i!} e^{\frac{-1}{2}\log n}$$

$$= \frac{(-1/2 \log n)^{k_i}}{k_i!} n^{-1/2}$$

$$\text{likelihood} = \underset{n}{\text{argmax}} \frac{(-1/2 \log n)^{k_i}}{k_i!} n^{-1/2}$$

$$\log \text{likelihood} = \text{argmax} \log \left(\frac{-1/2 \log n}{k_i!}\right)^{k_i} + \log n$$

$$= \underset{n}{\operatorname{argmax}} \left[k_i \log\left(\frac{-1/2 \log n}{k_i!}\right) + -\frac{1}{2}\log n \right]$$

$\rightarrow 1/2 \log n = m$

$$= \underset{n}{\operatorname{argmax}} \left[k_i \log(-m) - k_i \log k_i! \; \cancel{+} \bar{m} \right]$$

$\cancel{m = \frac{1}{2}\log n}$

Differentiating wrt n;

$\cancel{\frac{dm}{dn} = \frac{1}{2}\frac{1}{n}}$

$$\frac{d}{dn}\left(k_i \log(-1/2\log m) \cancel{+} -1/2\log n\right)$$

$$= \frac{k_i}{-1/2\log n} \cdot \frac{-1}{2}\frac{1}{n} \cancel{+} -\frac{1}{2n}$$

$$= \frac{1}{2n}\left(\cancel{k_i} -1 - \frac{k_i}{-1/2\log n}\right)$$

$$\therefore \quad 1/2 \log n = -k_i = x$$

$$\log n = \cancel{\quad} -2x$$

$$n = e^{-2x}$$

Q3.1.1) $\epsilon_i \sim \mathcal{N}(0, \sigma_i^2)$

$\epsilon_1 \ldots \epsilon_n$ are independent Gaussians.

$y_i = w^T x_i + \epsilon_i$

Now; log likelihood calculation :

$$\sum_{i=1}^{N} \log \left[\left(\frac{1}{2\pi\sigma_i^2} \right)^{1/2} \exp\left(\frac{-1}{2\sigma_i^2} (y_i - \omega^T x_i)^2 \right) \right]$$

$$= \sum_{i=1}^{N} \left[\log \left(\frac{1}{2\pi\sigma_i^2} \right)^{1/2} - \frac{1}{2\sigma_i^2} (y_i - \omega^T x_i)^2 \right]$$

$$= \sum_{i=1}^{N} \left[\underbrace{\text{some } \sigma_i \text{ term}}_{\text{constant}} \; \frac{-1}{2\sigma_i^2} (y_i - \omega^T x_i)^2 \right]$$

$$\text{minimize} \sum_{i=1}^{N} \frac{-1}{2\sigma_i^2} (y_i - \omega^T x_i)^2$$

$$s_i = \frac{-1}{2\sigma_i}$$

$$\frac{\partial}{\partial w} \sum s_i (y_i - w x_i)^2 =$$

$$2s_i \sum -x_i(y_i - w x_i)$$

$$\Rightarrow \sum 2 s_i x_i (y_i - w x_i) = 0$$

$$\sum 2 s_i x_i y_i - \sum 2 s_i x_i^2 w \cancel{x} = 0$$

$$\therefore \sum x_i y_i = \sum w x_i^2$$

$$w = \frac{\sum x_i y_i}{\sum x_i^2}$$

§3.2)

$$p(\varepsilon_i) = \frac{1}{2b}\exp\left(-\frac{|\varepsilon_i|}{b}\right) = \frac{1}{2b} e^{-\frac{|\varepsilon_i|}{b}}$$

$$y(x) = w^T x + \varepsilon$$

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}} \log p(D \mid \theta)$$

$\Rightarrow$ loglikelihood :

$$\sum_{i=1}^{N} \log\left[\frac{1}{2b} e^{-\frac{|\varepsilon_i|}{b}}\right]$$

$$\Rightarrow \sum_{i=1}^{N} \log\left[\frac{1}{2b} e^{-\frac{|y_i - wx_i|}{b}}\right]$$

$$= \sum_{i=1}^{n} \left[\log(1/2b) \cancel{+} -|y_i - wx_i|\right]$$

$$= \frac{1}{b}\sum_{i=1}^{n} |x_i w - y_i| + \underbrace{n \log 1/2b}_{\text{constant}}$$

maximize loglikelihood

~~= maximize~~ $\cancel{\frac{1}{b}\sum_{i=1}^{n}(x_i w - y_i)}$

$$= \text{maximize} \sum_{i=1}^{n} |x_i w - y|$$

1271

Q4.1.1 $r_i = y_i - (w^T x_i + b)$

$\|x\|_0$ = no. of nonzero x entries

$R = Y - W^T X - B$

Time complexity = $O(\|x\|_0)$

Q4.1.2

$B_{new} = \frac{R + B_{OLD}}{N}$

Time complexity = $O(n) \rightarrow O(\|X\|_0)$
where $B \Rightarrow (N \times 1)$

Q4.1.3 $R_{new} = R_{OLD} - B_{new} + B_{OLD}$
$O(n)$

Q4.1.4 $C_k = 2[R + W_k X_k] X_k$

vector X_k

kth element of W

Q4.1.5

121

Q4.1.3

$$R_{New} = R_{OLD} - B_{New} + B_{OLD}$$

$$O(n)$$

Q4.1.4 $$C_K = 2\left[R + W_K X_K\right] X_K$$

$$O(Z_K)$$

Q4.1.5 $$R_{NEW} = R_{OLD} + {W_K}_{OLD}^{T} X_K - {W_K}_{NEW}^{T} X_K$$

$$O(Z_K)$$

Q4.1.6
1. $O\|X\|_0$
2. $O(n)$ → $(n \times d)$ matrix considered
3. $O(n)$
4. $(d)(Z_K)$
5. $(d)(1)$
6. $(d)(Z_K)$

$\therefore$ Total time $= O\|X\|_0 + O(n) + O(d) + O(dZ_K)$

Q4.3 The λ to precision - recall relation:

As shown in graphs;

precision increases as λ increases

Recall decreases as λ increases

Changing σ to 10 →

Precision decreased;

recall remains same

→ To achieve better precision & recall;

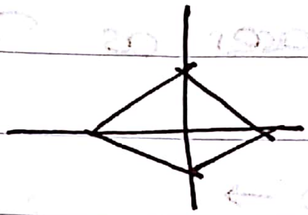
As seen through results; the λ should be increased to increase precision.

There will be a point after which recall starts falling. Stop increasing λ at this point...

Q 3-3] Laplace prefered over Gaussian -

1) laplace has thinner tail

2) laplace - solutions intersect the



curve at

edges; which helps eliminate unnecess features

3) Sparser solutions ..

~~4) The error is~~

Q1. X_1 $[0, 1]$

X_2 $[0, 1]$

$X \rightarrow \max(X_1, 2X_2)$

$$f_{X_1} = \begin{cases} 1 & 0 < x_1 < 1 \\ 0 & \text{else} \end{cases}$$

$$f_{X_2} = \begin{cases} 1 & 0 < x_2 < 1 \\ 0 & \text{else} \end{cases}$$

$X_3 = 2X_2$

$$f_{X_3} = \begin{cases} 1/2 & 0 < x_3 < 2 \\ 0 & \text{else} \end{cases}$$

$$F_{x_1} = \begin{cases} 0 & x_1 < 0 \\ x & 0 < x_1 < 1 \\ 1 & x_1 \geq 1 \end{cases}$$

$$F_{X_3} = \begin{cases} 0 & x < 0 \\ x/2 & 0 < x_3 < 2 \\ 1 & x_3 \geq 2 \end{cases}$$

$$f_X = f_{X_1} F_{X_3} + f_{X_3} F_{X_1}$$

$$f_x = \begin{cases} 0 & x < 0 \\ 1/2 & 1 < x < 2 \\ \cancel{3/4}\, x & 0 < x < 1 \end{cases}$$

$$E(x) = \int_0^1 \frac{3}{4} x^2 \, dx + \int_1^2 \frac{x}{2} \, dx$$

$$= \left[\frac{x^3}{3} \right]_0^1 + \left[\frac{x^2}{4} \right]_1^2$$

$$= \frac{1}{3} + \frac{3}{4}$$

$$= \frac{13}{12}$$

$$(13/12)^2$$

$$\text{Var}(x) = E(x^2) - (E(x))^2$$

$$= \int_0^1 x^3 \, dx + \int_1^2 \frac{x^2}{2} \, dx$$

$$= \left[\frac{x^4}{4} \right]_0^1 + \left[\frac{x^3}{6} \right]_1^2$$

$$= \frac{1}{4} + \frac{8}{6} - \frac{1}{6} = \frac{1}{4} + \frac{7}{6}$$

$$= \frac{6 + 28}{24}$$

$$= \frac{34}{24}$$

$$= \frac{17}{12}$$

- $\text{Cov}(X\,Y) = E(X\,Y) - E(X)\,E(Y)$

$\rightarrow$ $\cancel{\text{Cov}(X, X_1) = E(X X_1) - E(X)\,E(X_1)}$

$\cancel{13/12}$ $\cancel{17/12}$

$\cancel{E(X X_1)} = \cancel{\iint X X_1}$

Q4.3 The λ to precision - recall relation :

As shown in graphs ;

precision increases as λ increases

Recall decreases as λ increases

Changing σ to 10 →

Precision decreased ;

recall remains same

→ To achieve better precision & recall;

As seen through results ; the λ should be increased to increase precision.

There will be a point after which recall starts falling. Stop increasing λ at this point...

→ Smaller λ takes more iterations to converge -

- Priors added to end of PDF