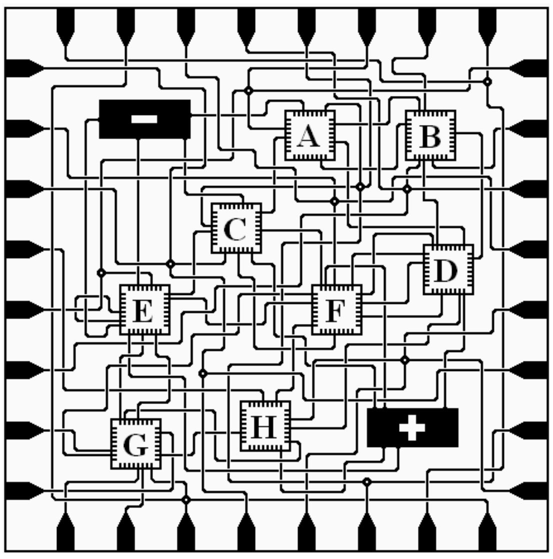
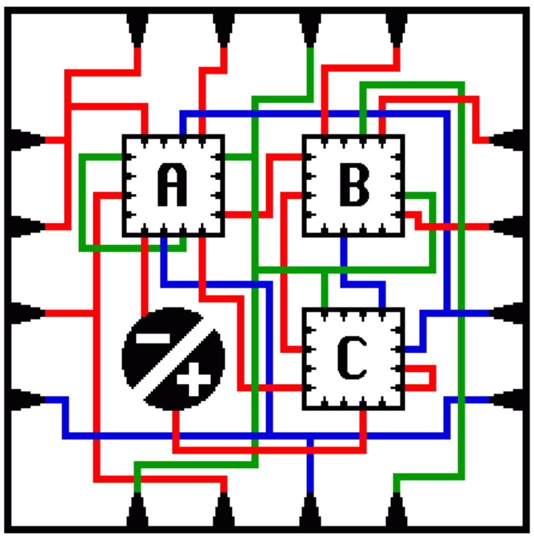
Omeometo's diary

2018 - 12 - 28

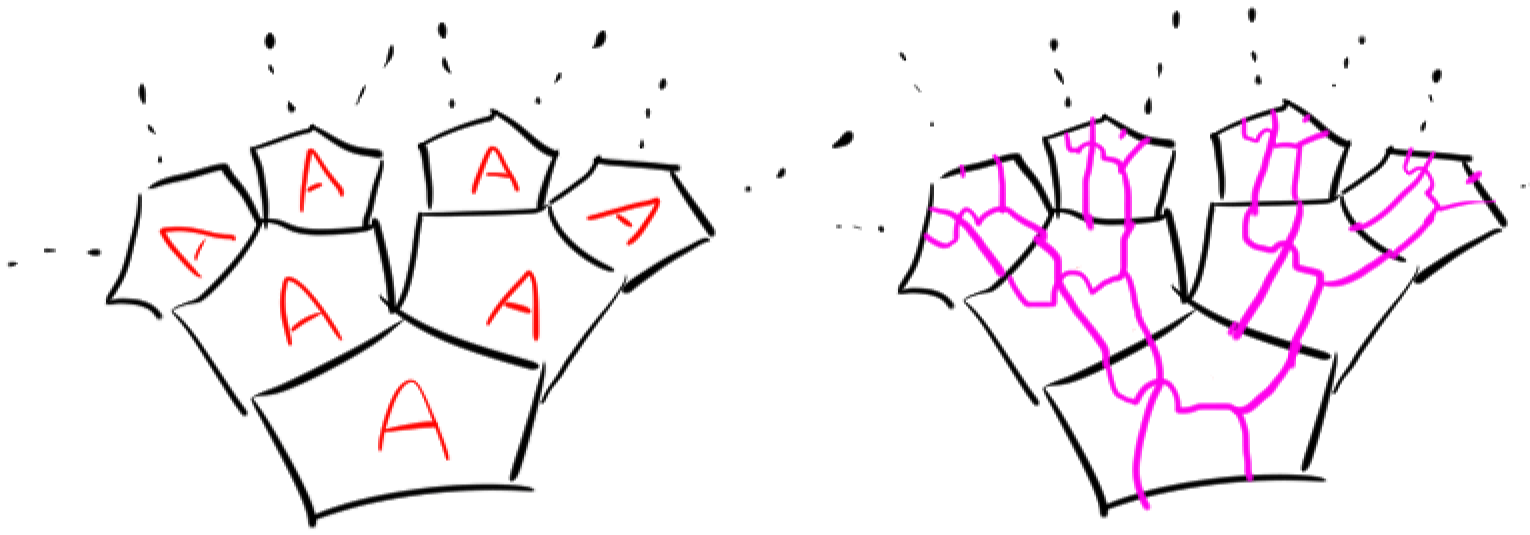
Fractal maze etc.

There is something called a fractal maze. It was probably first proposed here http://www.mathpuzzle.com/18Nov2003.html , where a copy of the entire maze is embedded recursively within the maze itself (image taken from the above site) .





Another way to look at this is that there are many identical parts lined up, as shown in the diagram below. By the way, the diagram below was drawn roughly for explanation purposes, so it probably doesn't function very well as a maze.

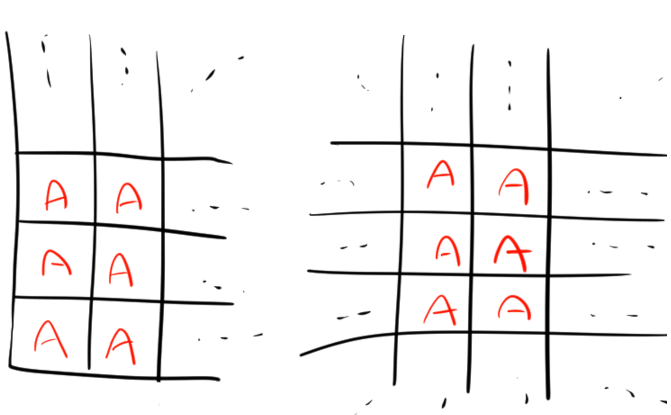


So, I've been thinking about it a lot.

One-dimensional guy

For now, the simplest way to arrange them is like this. After the appearance of the fractal maze mentioned above, things like this have also appeared on mathpuzzle.com ([http://www.mathpuzzle.com/17Dec06.html , Nov 13 2006] and [http://www.mathpuzzle.com/23Dec2010.html , Aug 9 2010]). Actually, didn't the one below, which stretches infinitely to the left and right, appear on topcoder at some point?

Two-dimensional guy



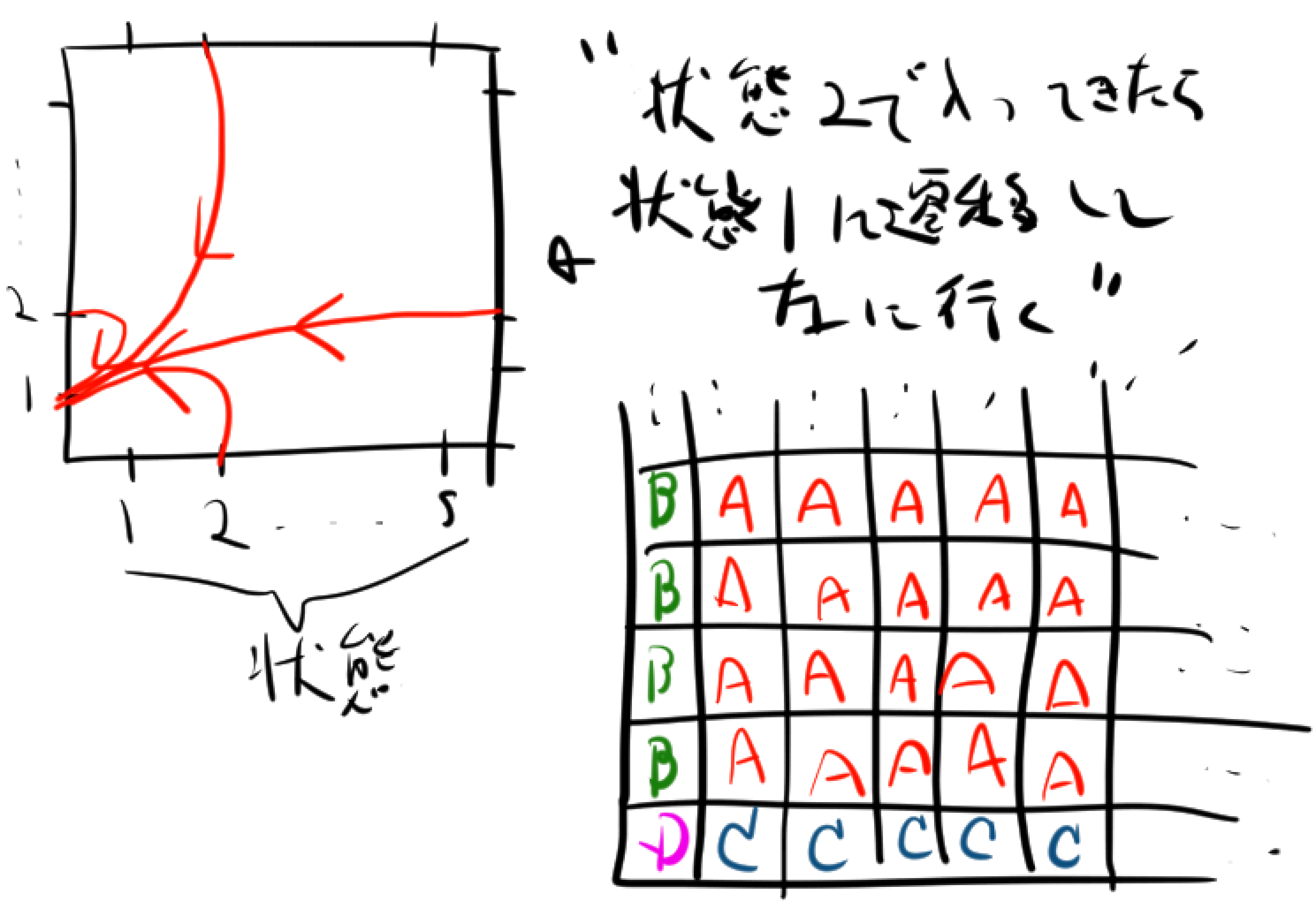
I thought it might be possible to arrange them in a two-dimensional grid.

By the way, I learned the following story the other day.

Given a machine with two registers each capable of holding nonnegative integers , and memory capable of a finite number of states, we run the machine from a given initial state by repeatedly applying the following rules:
If the current memory state is s and the register value is (x, y), the memory transitions to state s' and the register transitions to state (x+a, y+b), where a,b∈{±1, 0} and s' are determined by only three pieces of information: (s, "whether x is 0", "whether y is 0").
In this case, it is impossible to determine whether the machine will reach state s by moving it from a given initial state, or whether it will reach state (s, (x, y)).

The proof is here https://www.jstor.org/stable/1970290 https://www.jstor.org/stable/2269811 , and as usual, we simulate or emulate a Turing machine . We put the tape information and head information in the form of the exponent of the prime factorization on x, and use y to perform simple multiplication and division on x to fudge it.

This means that if you make a "near-periodic" maze (although it's a single road) using a "one-way road" like the one below, it will be impossible to determine whether you can reach the goal . Seriously? I'm starting to lose confidence, so I'd appreciate any comments from experts .



I don't know what would happen without one-way streets, or how effective "almost" periodicity would be, but when I see something like this, it doesn't seem strange at all to me that a maze with identical parts arranged periodically in two dimensions could be undecidable.

By the way, if it is undecidable , it means that there exists a problem where the minimum number of steps to solve the problem is "unthinkably" large compared to the "appearance" of the problem, which makes it a good puzzle. Can someone make something interesting?

By the way, I remembered that something like this had appeared in a JOI-related contest some time ago, so I looked it up and it was the JOI Open Contest 2012 (JOI ninja contest). I'm sure it was presented as a solvable problem there, but I wonder what made it different. Is it the presence of a one-way road that's important, or is it the fact that the wall only has one "terminal" (→ the above configuration means it can't have a state) that's important? Or is it the fact that it extends infinitely in both directions that's important?

181230 Update: I think that if exactly identical parts are lined up as I wrote at the beginning of the section, it probably won't be impossible to calculate, or rather, I think that "making special movements when you reach the edge" seems important in doing something that resembles a calculation (→ bringing it into the halting problem).

Also, if you're only concerned with reachability, there aren't many patterns for the case where there's only one terminal each. I was asleep.

190105 Update: I think it will be impossible to calculate even without one-way streets . I think it will be possible to drop it from generalized collatz (the version with the initial value specified) (or FRACTRAN), but I thought there might be a problem with being able to reverse the calculation process, but if you make it so that you can't go anywhere after the calculation is finished, then there won't be a problem, right?

Fractal maze, another version

> I came up with the idea of a fractal game. Two players alternate between placing 1x1 square pieces (small pieces) or 2x2 square pieces (large pieces) on an NxN grid board. Each piece represents a contraction map (a map that reduces the board to pieces). The winner is the one who connects "invariant self-similar sets of contraction maps on the board".
> — Mizusumashi Morita (@nosiika) October 18, 2018

みよしじゅんいち
@nosiika · Follow

これ、一旦連結になったあと、連結じゃなくなったりすることもある。んー。ゲームになるか、ちょっと分からなくなってきました。

6:40 PM · Oct 19, 2018

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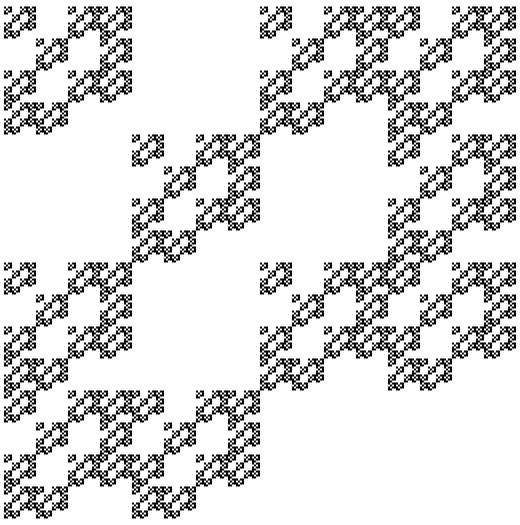
After reading this, I was thinking of different victory conditions, and I thought that a victory condition like "You lose if the top and bottom of the board are separated by an immutable set" would be nice and crazy (and have a fractal maze feel to it). For simplicity, let's limit the contraction maps to those that reduce NxN to 1x1.

For example, in the image below, try entering from the bottom edge and exiting to the top edge. The left and right are dead ends. Please think of the parts that are distorted as having infinitely fine structures.



By the way, I hit a dead end when I tried to solve this programmatically. The fractal maze introduced at the beginning of the article, where terminals are connected to each other, can be solved by "increasing the depth of the diagram one level at a time while maintaining how the outer terminals of the entire maze are connected (when the connection status stops being updated, there is no need to search any deeper)" (sorry if I misunderstood here in the first place), but if you try to do this with a maze like the one above, naively, the number of openings whose connection relationships must be maintained will increase every time you increase the depth, and it seems like you can't stop searching by saying "when the status stops being updated, there is no need to search any deeper" (→ you will continue searching endlessly if there is no solution).

How can we solve this? Or is it undecidable like the previous one? I look forward to hearing from you, algorithm experts.

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