False Vacuum Decay in Non-Equilibrium Environment

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Motivation

Study of a metastable state decaying into the ground state is ubiquitous. False vacuum decay represents a quantum mechanical tunneling process.

- **Examples**: Possible phase transitions in the early universe, Origin of the cosmic inflation, Metastability of the Higgs vacuum, Production of baryon asymmetry and dark matter of the universe.
- Such events are preceded by nucleation of bubbles [see figure 2 (left)] which expand and fill the whole universe.

What we know!

* Quantum tunneling through a potential barrier
* Direct resemblance of false vacuum decaying in scalar field theory and in the quantum mechanics of many variables in the continuous limit [2]
* Semiclassical approximation: Euclidean bounce solution

What we don't know!

* Existence and expansion of Euclidean bubble under non-equilibrium situations
* Exploring time-dependent boundary conditions and the consequences followed
* Location of singularities on complexified time

Background

- **Existing semiclassical approach:** In weakly coupled theories, vacuum decay is semiclassical in nature. Tunneling exponent can be determined by the "bounce" solution to the classical field equations in Euclidean space-time with appropriate boundary conditions.

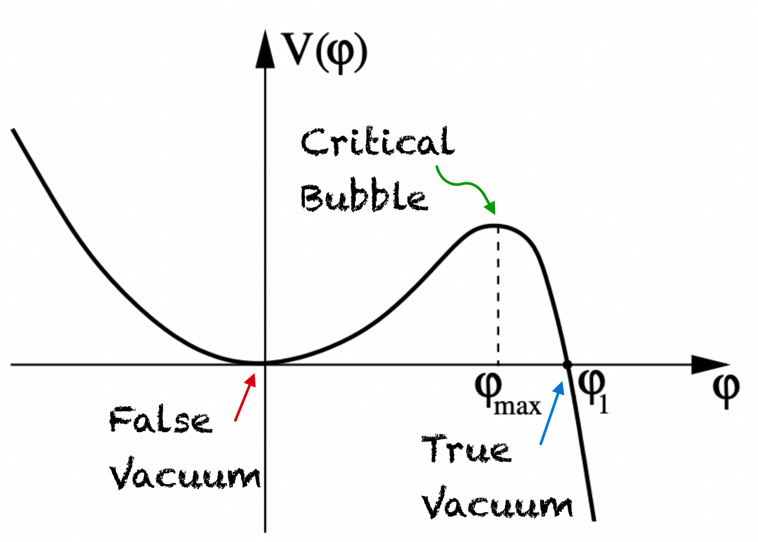

Figure 1: Inverted Liouville potential considered in this study as a schematic model.

- Tunneling solution lives in purely imaginary (Euclidean) time.
- In thermal equilibrium, classical Euclidean action on the bounce solution corresponds to the leading exponent of the tunneling probability.
- These are finite action solutions of the Wick-rotated Euclidean equations of motion.

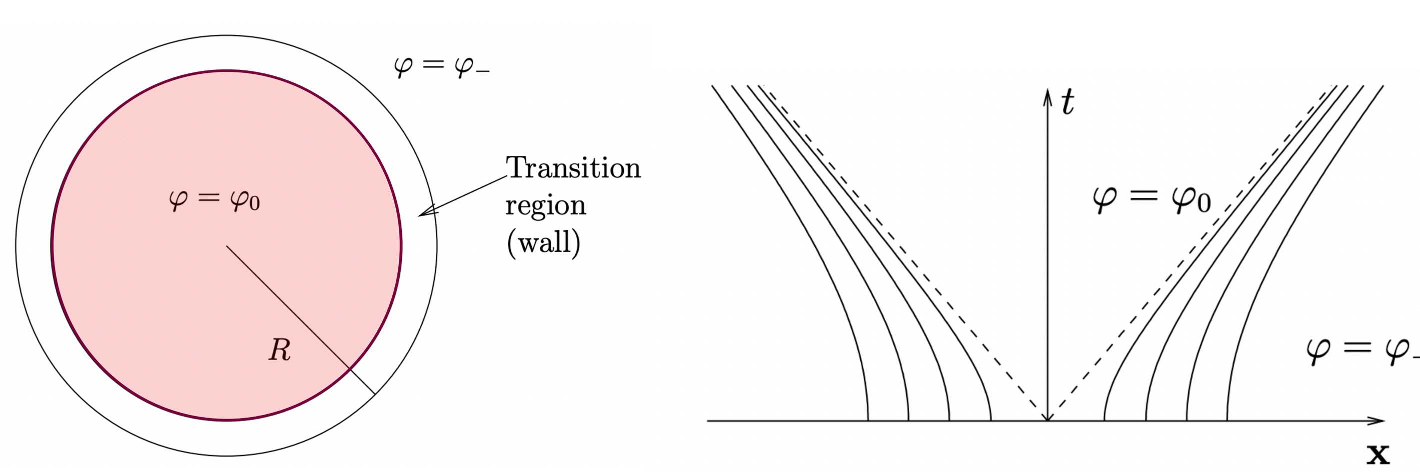

Figure 2: (left) Spherically symmetric Euclidean bubble (bounce) solution [2]; (right) Minkowski space-time diagram of the classical growth of the bubble of true vacuum after its materialization. Hyperbolas are the path traced out by the bubble wall.

Setup & Goals

- Consider a quantized massive scalar field in two dimensional flat spacetime with inverted Liouville potential: $V(\varphi) = m^2\varphi^2/2 - 2\kappa\left(e^{\varphi} - 1\right)$ [figure 1].
- Field equation: $\Box\varphi - m^2\varphi + 2\kappa e^{\varphi} = 0$.
- The motion of a reflecting boundary (a moving mirror, see figure 3) induces disturbance in the quantum state and an irreversible production of entropy would occur that leads to new field quanta. Expected to regulate the tunneling probability of the false vacuum decay.
- Parameters m and κ obey: $\ln\left(m/\sqrt{\kappa}\right) \gg 1$; ensuring overlap region exists.

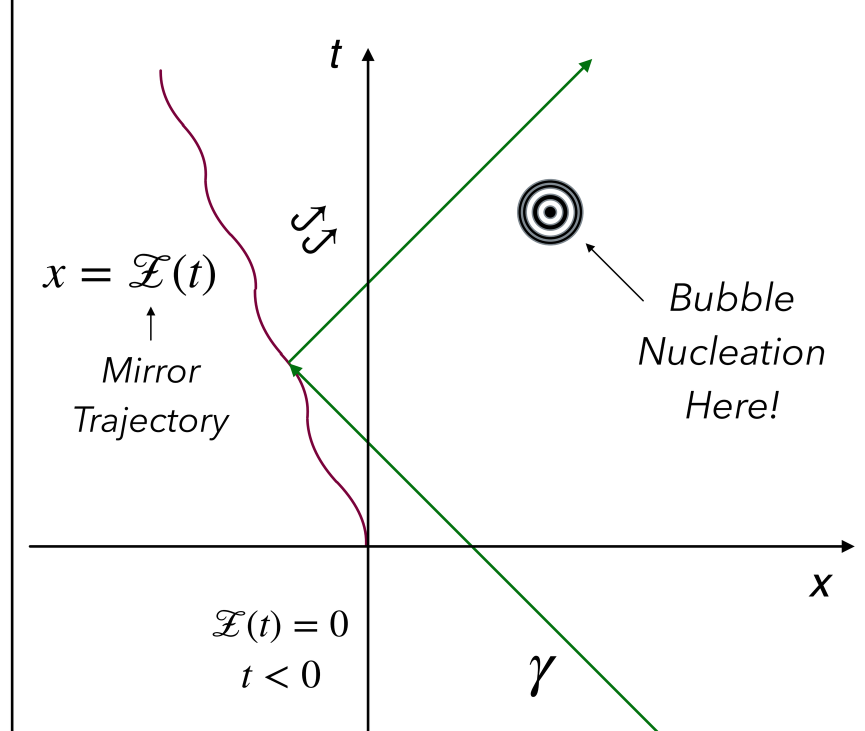

Figure 3: Null rays (γ) coming from the in-region. As a boundary condition, the scalar field quantas vanish upon mirror reflection [1].

Research Goals

- Exploring a specific example of a scalar field theory with potential [in figure 1] will allow us to identify key features of the false vacuum decay phenomena in non-equilibrium cases.
- Time-dependent boundary conditions are set by time-ordered Green's function.
- Formulate Green's function and match it with non-linear solution to get the nucleated bubble.

Methods & Formalism

- **Alternative approach:** Tunneling suppression computed by evaluating path integral along $\mathscr{C}$. Developing time-ordered Feynman Green's function of the massive field.
- Method of asymptotic expansion and matching. Coincides with the bounce solution in theories with unbounded scalar potential, as in our case.
- Starting with an initial state $|i\rangle$ close to the false vacuum and a final state $|f\rangle$ in the basin of attraction of the true vacuum is given by the following path integral. The corresponding transition probability $\mathscr{P}_{decay}$ is given by:

$$\mathscr{P}_{decay} = \sum_{f\in true} \langle i|f\rangle\langle f|i\rangle = \int_{\mathscr{C}} D[\varphi_i]D[\varphi_i']D[\varphi_{\mathscr{C}}]\langle i|\varphi_i'\rangle e^{iS[\varphi_{\mathscr{C}}]}\langle\varphi_i|i\rangle$$

For large $\varphi \geq \varphi_{max}$ one can neglect the mass term and the equation reduces to the Liouville equation which has a general solution:

$$\varphi = \ln\left[\frac{4F'(-u)G'(v)}{\left(1+\kappa F(-u)G(v)\right)^2}\right]$$

Here u, v are the advanced and retarded coordinates; $F(-u)$, $G(v)$ are arbitrary functions.

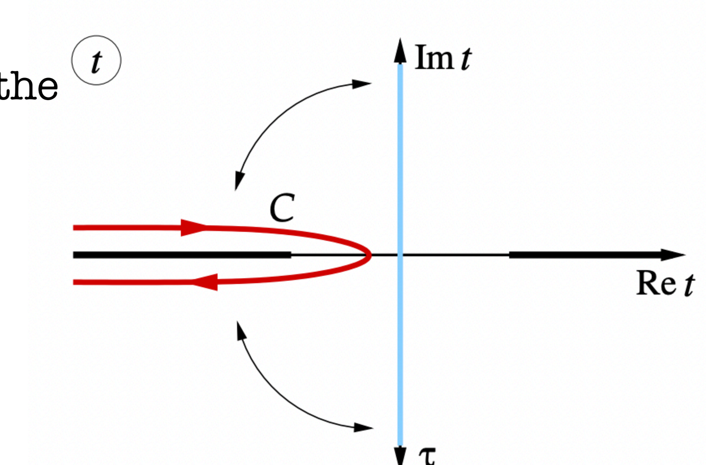

Figure 4: Analytic continuation of contour C in complex time plane [3]

- Structure of Minkowski bounce in the complex time plane in figure 4.
- The standard Euclidean bounce is defined on the imaginary time (blue). Analytic continuation to the contour C (red) satisfies Feynman boundary conditions at $\Re(t) \to -\infty$.

Results & Outlook

* Developed a thorough understanding of quantized scalar field theory to study and devise the properties of bounce solution in both the Minkowski & the Euclidean spacetime.
* Successfully formulated Feynman Green's function of the massive (quantized) scalar field given by $iG_F(t,x;t',x') = \langle 0,in|\mathscr{T}(\varphi(t,x)\varphi(t',x'))|0,in\rangle$, tunneling through the inverted Liouville potential by imposing single reflecting mirror moving on an arbitrary trajectory (as a time dependent boundary condition, $x = \mathscr{X}(t)$; $\mathscr{X}(t) = 0$ for $t < 0$).

- Insight into non-perturbative quantum field theory in curved spacetime with nontrivial causal structure.
- Connections between the probability of false vacuum decay and Black hole entropy.
- Semiclassical quantum gravity. [6]

References

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