Diffusion by Brownian Motion

Lab Report, PHYSICS 3P03

Jay Mehta

400285158

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Department of Physics & Astronomy, McMaster University

1 Abstract

Diffusion is an imperative property of particles that have found uses in diverse applications and the rate of flow of the diffusing substance is found to be proportional to the concentration gradient. This experiment attempts to measure the diffusion coefficient of particles undergoing Brownian motion by varying several known parameters of this motion. The methodology used in this analysis have proven to be capable of measuring the diffusion coefficient (**D**) of the utilized superparamagnetic colloids precisely to be $\mathbf{1.3 \pm 0.3\ \mu m^2/s}$. Several manifolds behind the uncertainties and effects that arise during the experiment have been concisely discussed in the following sections.

2 Introduction

Diffusion is the net movement of any substance, usually from a region of higher concentration to a region of lower concentration. There are numerous potential applications and new concepts manifested from the diffusive property of atoms, ions, molecules and particles in general. It is widely used in Physics, Chemistry, Biology, Economics, Finance, Sociology, Astronomy etc.(Dill et al., 2010) The main notion of diffusion in each of them follows a similar routine; a bunch of particles undergoing diffusion spreads out from a point at which there is a higher concentration of that substance.

Fick's law describes diffusion of Brownian particles in a medium. As illustrated in equation 1, we can see how this law relates the diffusion flux (J), also called diffusion current density, to the concentration of corresponding particles. Hence, the driving force of diffusion in Fick's law is the negative gradient of concentration.

$$J = -D\frac{d(\phi)}{dx} \tag{1}$$

where, D is the diffusion coefficient (in m^2/s), and ϕ is the concentration of the given substance.

$$\frac{\partial \phi}{\partial t} = D\frac{\partial^2 \phi}{\partial x^2} \tag{2}$$

In single dimensional case, we can demonstrate this relationship with the aforementioned equation 2, which is essentially a partial differential equation of the second degree.

One may use the Fourier Transform technique to decompose the equation 2 in reciprocal (Fourier) basis and, upon solving the differential form in real space, we get the following "**Gaussian**" form (as in equation 3) of the concentration of the diffusing substance, in one dimension, for $t > 0$:

$$\phi(x,t) = \frac{\phi_0}{(4\pi Dt)^{1/2}} \exp\left[-\frac{(x-x_0)^2}{4D(t-t_0)}\right] \tag{3}$$

As the particles diffuse from regions of higher concentration to the regions of lower concentration, the Gaussian curve will become wider and its peak gets shorter with time. Also, a standard measure of the width of a Gaussian curve is its full width at half-maximum (**FWHM**), which is given as;

$$W = 4\sqrt{\ln 2 Dt} \tag{4}$$

Where, W is the width of the Gaussian curve and t is time.

In the following image 1a, we can observe how the the superparamagnetic colloids align themselves in the presence of an external magnetic field and in the figure 1b we can witness the Brownian motion of the substance and the diffusion taking place in our experimental setup.

(a) This is the initial alignment of colloid particles in the presence of an external magnetic field

(b) This is the final alignment of colloid particles undergoing Brownian motion; long after being diffused

Additionally, A distinguishing feature of diffusion is that it depends on particle random walk. And as a consequence of that property, the Stokes-Einstein equation for a substance made of perfect sphere gives the following result;

$$D = \frac{k_B T}{6\pi R\eta} \tag{5}$$

where, k_B is the Boltzmann constant, T is the temperature, η is the viscosity, and R is the radius of the particles.

In the context of this experiment we are required to determine the diffusion coefficient by observing the diluted sample of colloids, that occurs due to the Brownian motion of the particles. This can be set up by aligning superparamagnetic colloids of about 0.2 % typical concentration by volume and applying an external magnetic field, which once removed, causes a dispersion that obeys Gaussian Distribution. By analyzing the width at half the maximum value of this distribution over time, we can calculate the diffusion coefficient of this Brownian motion.

3 Data Analysis

Example of the concentration VS distance data; one, from a frame early in the time sequence, and one much later.

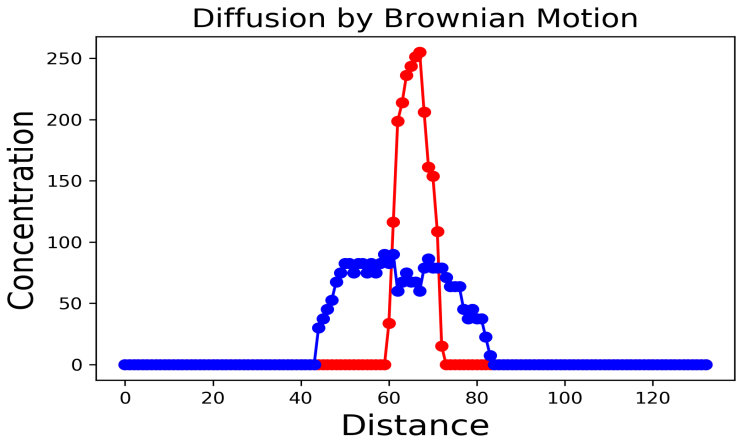


Figure 2: example of concentration vs distance data.

The red curve in the above figure 2 is the plot of data points when the magnetic field was applied and the superparamagnetic colloids were aligned in this manner, resembling the Gaussian curve pattern, initially with high peak and narrow width.

And the blue curve is the plot of data points, long time after the diffusion takes place and the Gaussian curve becomes wider and shorter with time.

Same examples with a Gaussian fit to the concentration data.

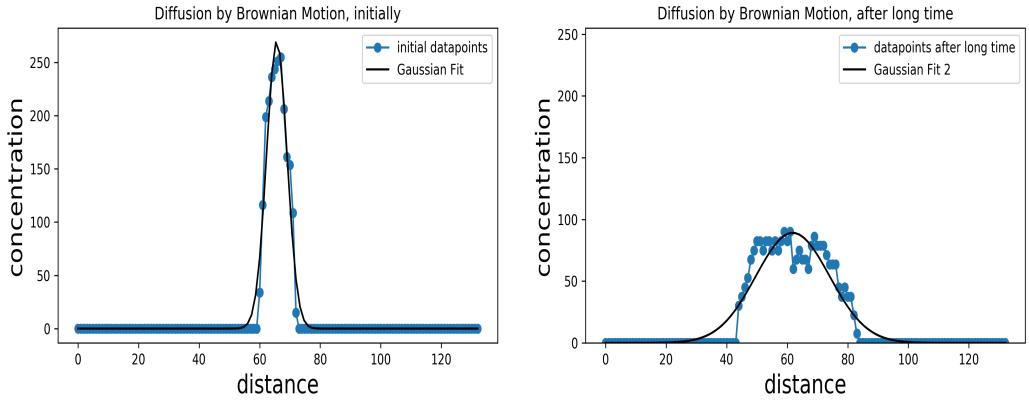


Figure 3: concentration vs distance data, with a Gaussian fit to the data points.

A plot of the width of the Gaussian VS time, to determine the diffusion coefficient.

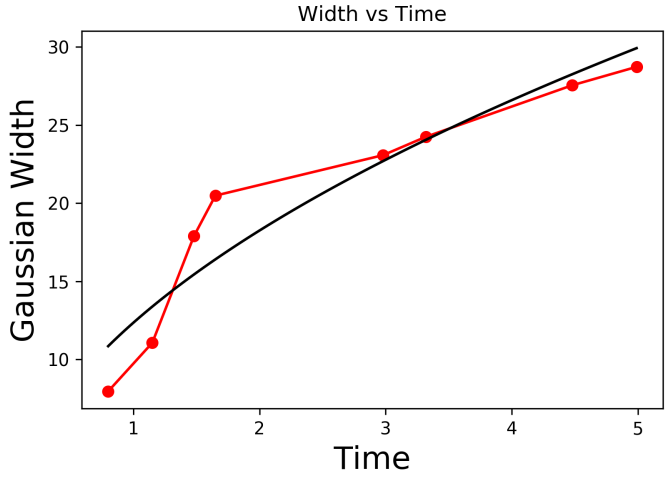


Figure 4: Graph of the full width at half-maximum (FWHM) for the Gaussian fits plotted against time to determine the diffusion coefficient.

The figures 4, 5a & 5b are essentially plotting the equation 4;

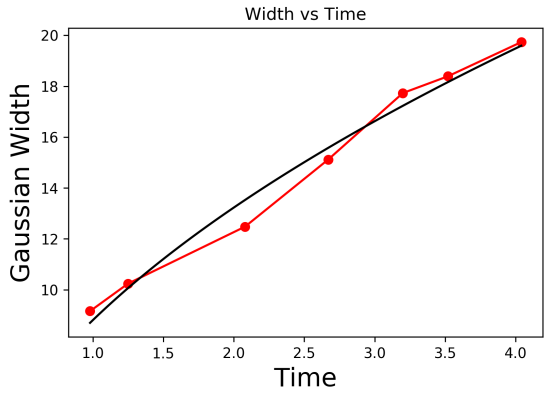
$$\mathbf{W} = 4 \times \sqrt{\ln(2) D t}$$

Where the offset is equal to the diffusion co-efficient (D).

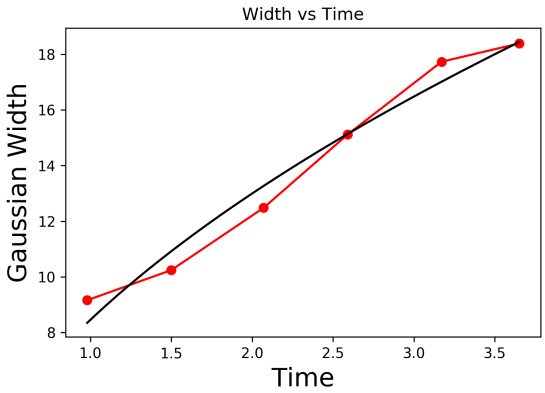
Therefore,

$$\mathbf{D = 1.8 \pm 0.5\ \mu m^2/s}$$

Determining the diffusion coefficient for several chains of colloids



(a) The value of the diffusion coefficient in this case will be, $\mathbf{D = 1.0 \pm 0.1\ \mu m^2/s}$.



(b) The value of the diffusion coefficient in this case will be, $\mathbf{D = 1.1 \pm 0.2\ \mu m^2/s}$.

The following table 1 is

Trials	Diffusion Coefficient with the uncertainty
1	$\mathbf{D_1 = 1.8 \pm 0.5\ \mu m^2/s}$
2	$\mathbf{D_2 = 1.0 \pm 0.1\ \mu m^2/s}$
3	$\mathbf{D_3 = 1.1 \pm 0.2\ \mu m^2/s}$

Table 1: Values of the diffusion coefficients (with their uncertainties) obtained from the Gaussian width values of various chains of colloids plotted against time as shown above in the figures 4, 5a & 5b.

Therefore, the average value of the diffusion coefficient becomes,

$$D_{avg} = \frac{1.8 + 1.0 + 1.1}{3}$$
$$D_{avg} = \mathbf{1.3\ \mu m^2/s}$$

And the average of the individual uncertainties is,

$$\sigma_{avg} = \mathbf{0.3\ \mu m^2/s}$$

Also, the variance as well as the standard deviation of these several measurements of the diffusion coefficient are as follows,

$$\sigma_{var}^2 = 0.1296$$
$$\sigma_{std.dev} = \mathbf{0.36\ \mu m^2/s}$$

Upon calculation, it's seen that the standard deviation of the measurements and the average value of the uncertainties in each case, are quite comparable; which is also reasonable because of the nature of the experiment.

4 Discussion

The expected diffusion coefficient from equation 5 (i.e. Stokes-Einstein Relation)

The Stokes-Einstein equation for a substance made of perfect sphere is;

$$D = \frac{k_B T}{6\pi R\eta}$$

where, k_B is the Boltzmann constant, T is the temperature, η is the viscosity, and R is the radius of the particles. In our solution of superparamagnetic colloids, the mean value of inverse radius of the particles $\langle \frac{1}{R} \rangle = 1.7 \times 10^6\ m^{-1}$ and the average viscosity of the solution, $\eta = 1\ mPa \cdot s$ (Darras et al., 2017)

Substituting the aforementioned values in the Stokes-Einstein relation (equation 5) yields the expected value of the diffusion coefficient,

$$D_{theoretical} = \mathbf{0.56\ \mu m^2/s}$$

As we can witness that the theoretical value of the diffusion coefficient is almost half the experimental value we obtained by conducting the experiment. The reason lies within the margin of errors taking place throughout the experimental process. There can be numerous possible sources of errors that can infiltrate in our measurements.

Potentially, there could be three prime factors that may influence the experimental outcome significantly and lead to erroneous result; Firstly, the particles undergoing Brownian motion

in the superparamagnetic colloids possess varying radius, as opposed to the assumption that they simulate perfect spheres and the viscosity of the fluid would also be fairly inconsistent across different regions of the slide we used for microscopic observations. Secondly, the image analysis that we performed with our observed phenomenon of diffusion was quite analytical and orchestrated in such a way that it does not take account of the spread along the vertical alignment of colloids after decomposing from the chain structure. And lastly, it is quite obvious that the plot of the width of Gaussian curve against the timestamps was objectively inaccurate since the values of sigma in each Gaussian curve were handpicked from the array of values available to us.

5 Conclusion

To conclude, this experiment achieves the goal of calculating the diffusion coefficient. The objective was accomplished by preparing a colloidal solution with appropriate amount of superparamagnetic particles being present in the substance. Upon applying external magnetic field, these particles associate together to form a chain; and with the removal of the induced field, they undergo diffusion by Brownian motion. We were successfully able to observe the behaviour of these colloids on our microscopic slide as expected theoretically and according to the Fick's law, the concentration of corresponding particles were certainly forming a Gaussian curve, when plotted. Also, the width of the curve increases with time and square of the width (standard measure) seems to be directly proportional with increasing time. Several sources of error that affect our results and measurements were also discussed in detail. The experiment registers the calculated value of the diffusion coefficient to be equal to $\mathbf{1.3 \pm 0.3\ \mu m^2/s}$. From the Stokes-Einstein relation, the theoretical value of the diffusion coefficient in our setup is, $D_{theoretical} = \mathbf{0.56\ \mu m^2/s}$.

The results of this experimental data certainly agree with our expectations up to a certain extent and the justification of those results are concisely explained in this report.

References

Darras A., Fiscina J., Vandewalle N., and Lumay G. (2017). "Relating Brownian motion to diffusion with superparamagnetic colloids". *American Journal of Physics* 85.4, pp. 265–270.

Dill K. A., Bromberg S., and Stigter D. (2010). *Molecular driving forces: statistical thermodynamics in biology, chemistry, physics, and nanoscience*. Garland Science.