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Tut 6: Machine Learning 1

1. [Probability] Assume that the probability of obtaining heads when tossing a coin is λ.
a. What is the probability of obtaining the first head at the $(k + 1)$-th toss?

b. What is the expected number of tosses needed to get the first head?

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1) a) $P(H) = \lambda$

$P(T) = 1 - \lambda$

$P(H \text{ at } k+1 \text{ th toss}) = P(T \text{ at } k \text{ toss and } H \text{ at } k+1 \text{ th toss})$

$= (1-\lambda)^k \lambda$

b) Let n be the no. of tosses required to get the first head $\Rightarrow S = E[n]$

As tosses are independent and expectation is additive

$$S = \lambda \times 1 + (1-\lambda)(S+1)$$

$$= \lambda + S + 1 - \lambda S - \lambda$$

$$\therefore S\lambda = 1$$

$$\therefore S = \frac{1}{\lambda}$$

2. [Probability] Assume X is a random variable.

a. We define the variance of X as: $\mathrm{Var}(X) = E[(X - E[X])^2]$. Prove that $\mathrm{Var}(X) = E[X^2] - E[X]^2$.

b. If $E[X] = 0$ and $E[X^2] = 1$, what is the variance of X? If $Y = a + bX$, what is the variance of Y ?

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2) $X \rightarrow$ random variable

a) Variance of X : $\text{var}(x) = E[(x - E[x])^2]$

TPT :- $\text{Var}(x) = E[x^2] - E[x]^2$

Given :- $\text{var}(x) = E[(x - E[x])^2]$

$= E[(x^2 - 2E[x]x + E[x]^2)]$

$= E[x^2] - 2E[E[x]x] + E[x]^2$

$= E[x^2] - 2E[x]^2 + E[x]^2$

$= E[x^2] - E[x]^2$

- Hence Proved.

b) $E[x] = 0$ & $E[x^2] = 1$

To find :- ① $\text{var}(x)$

② if $y = a + bx$, $\text{var}(y)$

① $\text{var}(x) = E[x^2] - E[x]^2$

$= 1 - 0^2 = \underline{1}$

② $E[Y] = E[ax + b] = E[x]b + a = 0b + a = \underline{a}$

$E[Y^2] = E[a^2x^2 + b^2 + 2axb] = \underline{a^2 + b^2}$

$\therefore \text{var}(y) = E[Y^2] - E[Y]^2 = a^2 + b^2 - a^2 = \underline{b^2}$

3. [Probability] Your friend Aku is a great predictor about winning a horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99. 2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999. 3) With probability 10^{-5}, Aku predicts that a black beauty is a winning horse. This also means with probability $1 - 10^{-5}$, Aku predicts that a black beauty will not win.

a. Given a horse, what is the probability that it wins?

b. What is the probability that Aku correctly predicts a black beauty is winning ?

3] Let A be the event that "Aku predicts that the given horse is winning horse".

$\neg A$ vice versa

Let B be the event that given horse wins & $\neg B$ vice versa.

a] $P(B) = P(B, A) + P(B, \neg A)$

$= P(B|A)\,P(A) + P(B|\neg A)\,P(\neg A)$

$= 0.99 \times 10^{-5} + (1 - 0.99999) \times (1 - 10^{-5})$

$\therefore P(B) \simeq 1.99 \times 10^{-5}$

b]

$$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(A|B)\,P(A)}{P(B)}$$

$$= \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$$

$$= 0.497$$