

03/03/25

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Experiment No.1
RATIO OF SPECIFIC HEAT CAPACITIES

To investigate the ratios of specific heat capacities for air, argon, and carbon dioxide, as a ball oscillated in a tube connected to an aspirator filled with each gas, the oscillations of the pressure in the aspirator and the amplitude of a ball over time were measured. Using this data and various environmental measurements, the specific heat ratios were determined to be $\gamma_{argon} = 1.65 \pm 0.09$ and $\gamma_{CO_2} = 1.35 \pm 0.08$, and $\gamma_{air} = 1.19 \pm 0.09$, the latter being the only one inconsistent with its accepted value. Given potential differences in air composition and imperfections in the experimental set-up, as well as confirmation of the insignificance of damping on the results, it was determined these observations likely still supported the validity of the equation for the ratio of the specific heat capacity.

THEORY

An adiabatic process is a process that occurs without heat transfer between the system and its environment, so that any temperature change occurs from internal changes in the system. In this type of process, changes in temperature, pressure, and volume are dependent on the specific heat ratio. The specific heat capacity of a substance is the amount of heat that needs to be added to one mass unit to cause an increase of one temperature unit. When a process occurs at constant volume this is C_v , and at constant pressure it is C_p , and their ratio is known as the specific heat ratio (γ). Consider a ball which moves up and down in a tube connected to an aspirator filled with some gas, then γ for this gas is given by

$$\gamma = \frac{C_p}{C_v} = \frac{64mV}{d^4 P_{eq} T^2} \quad (1)$$

$m = m_{ball}$ is the mass of the ball, V is the volume of the aspirator, d is the internal diameter of the tube, and T is the average period of the ball's damped oscillation. P_{eq} is the pressure in the aspirator at equilibrium and can be calculated by

$$P_{eq} = P_0 + \frac{mg}{A} \quad (2)$$

where P_0 is the barometric room pressure, and A is the cross-sectional area of the tube given by $A = \pi r^2$. However, given that the system is damped, the damping constant may have an affect the period which causes these relations to not hold true. Thus the effect of the damping constant on the period must be checked, and it is insignificant only if

$$\frac{k^2 T_1^2}{4} \ll 4\pi^2 \quad (3)$$

where k is the damping constant, and T is the period. Given an unknown damping constant, this can be found using the oscillatory nature of the system by

$$\frac{kT_1}{2} = \frac{\ln(A_n) - \ln(A_{n+\ell})}{\ell} \quad (4)$$

where A_n and $A_{n+\ell}$ are the peak amplitudes of the n th and $(n+\ell)$ th oscillations respectively. This experiment seeks to determine the ratio of the specific heats for air, carbon dioxide (CO_2) and argon.

APPARATUS

- Aspirator with pressure sensor and foam mat
- Precision glass tubing
- Steel ball
- Computer with LoggerPro
- Supplies of air, argon, and CO₂

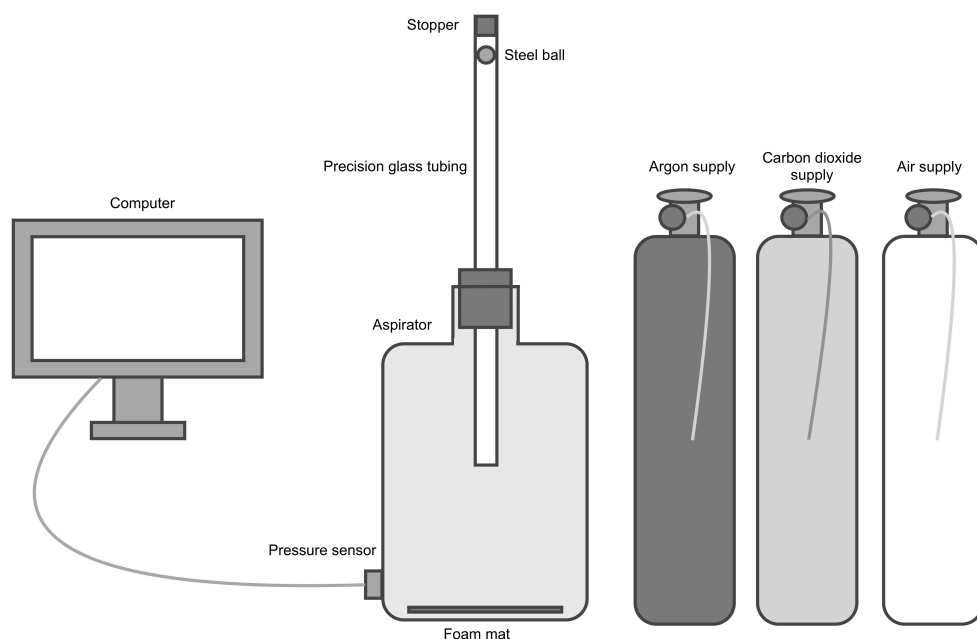


FIG. 1. Experiment set-up diagram.

PROCEDURE

Before beginning, the aspirator was filled with air for 3 minutes and LoggerPro was set up with a sampling speed of 20 samples/s and an experiment length of 30s. Once this was done, the glass tube was inverted so that the steel ball was held in place at the top by the tube's pressure, then the stopper was inserted, and the tube was snugly placed into the aspirator. The stopper was then removed, and a capture of the pressure over time was taken as the ball oscillated back and forth in the tube. This capture was repeated five times, then the aspirator was refilled with argon, and the entire process was repeated with the new gas. This was also done with carbon dioxide. Following this, the amplitudes of the ball over time, barometric pressure and temperature of the room, volume and diameter of the tube, and ball mass were recorded.

DATA

	V	P_0	m_{ball}	d_{tube}
Measured	10.75 ± 0.5 L	743.4 ± 1 mmHg	16.51 ± 0.05 g	17.78 ± 0.05 mm
SI Units	$(1.08 \pm 0.05) \times 10^{-2}$ m ³	$(9.91 \pm 0.01) \times 10^4$ Pa	$(16.51 \pm 0.05) \times 10^{-3}$ kg	$(17.78 \pm 0.05) \times 10^{-3}$ m

TABLE I. Set-up measurements of the aspirator volume (V), barometric pressure (P_0), steel ball mass (m_{ball}), and glass tubing diameter (d_{tube}) in both measured and SI units.

Measurements of the environmental and set-up conditions were recorded as in TABLE I. In addition to this, the temperature in the room was found to be approximately 295.05 ± 0.5 K.

	Amplitude [cm]
A_1	64.1
A_2	59.8
A_3	56.5
A_4	53.5
A_5	50.5
A_6	48.3
A_7	45.8
A_8	43.7
A_9	42.1
A_{10}	40.5
A_{11}	38.7

TABLE II. Peak amplitudes (± 0.5 cm) of the steel ball over time as it oscillated in an argon system.

	Peaks	t_i [s]	t_f [s]	T [s]	T_{avg} [s]
Air	8	0.20	9.30	1.14	0.98 ± 0.04
	6	0.05	5.85	0.97	
	5	1.9	6.50	0.94	
	6	1.2	6.85	0.94	
	5	0.35	4.95	0.92	
Argon	4	0.55	3.60	0.76	0.83 ± 0.02
	6	0.75	5.90	0.86	
	5	1.65	5.80	0.83	
	5	0.45	4.65	0.84	
	6	0.45	5.65	0.87	
CO ₂	5	1.15	5.95	0.96	0.92 ± 0.02
	6	1.75	7.65	0.98	
	4	0.35	3.80	0.86	
	5	0.50	5.10	0.92	
	4	0.65	4.10	0.86	

TABLE III. Measurements of the time occurrences (± 0.05 s) of two pressure crests, and the number of peaks between them. The calculated corresponding periods and average period determined in Calculation 1.

The specific heat ratios for each gas were determined to be $\gamma_{air} = 1.19 \pm 0.09$, $\gamma_{argon} = 1.65 \pm 0.09$, and $\gamma_{CO_2} = 1.35 \pm 0.08$ using the method shown in Calculation 2.

FIG 1: Pressure vs Time (Air)

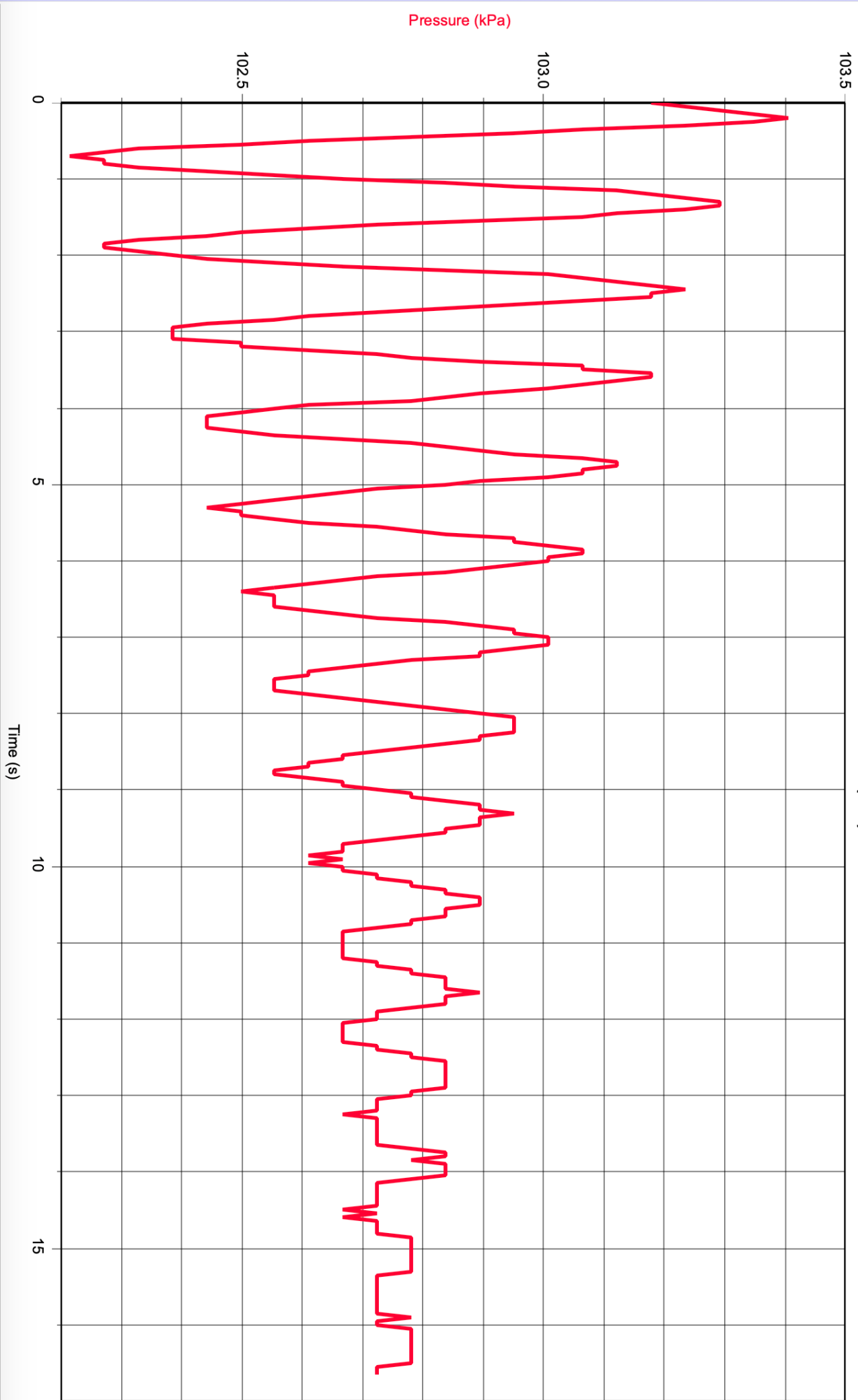


FIG 2: Pressure vs Time (Argon)

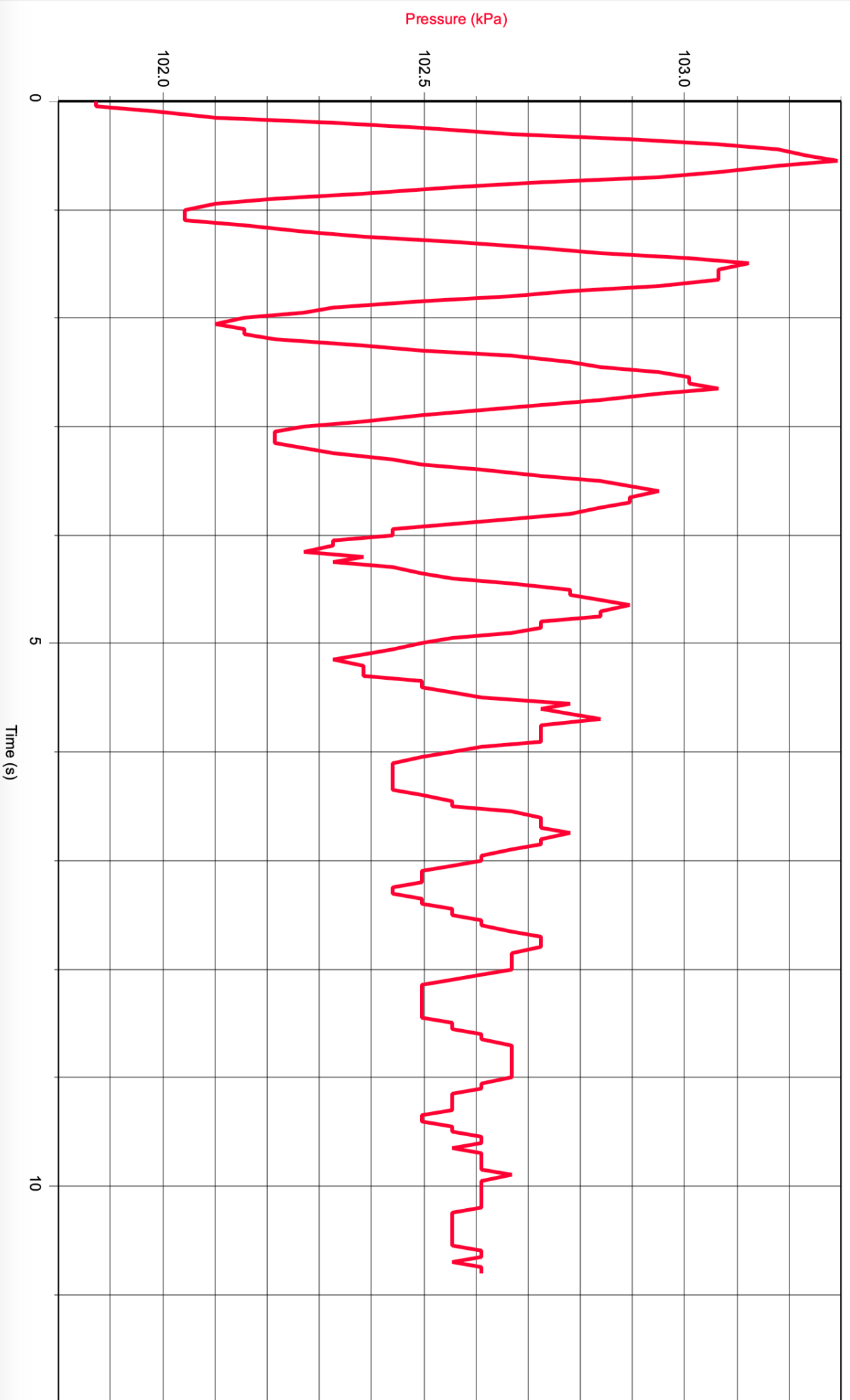
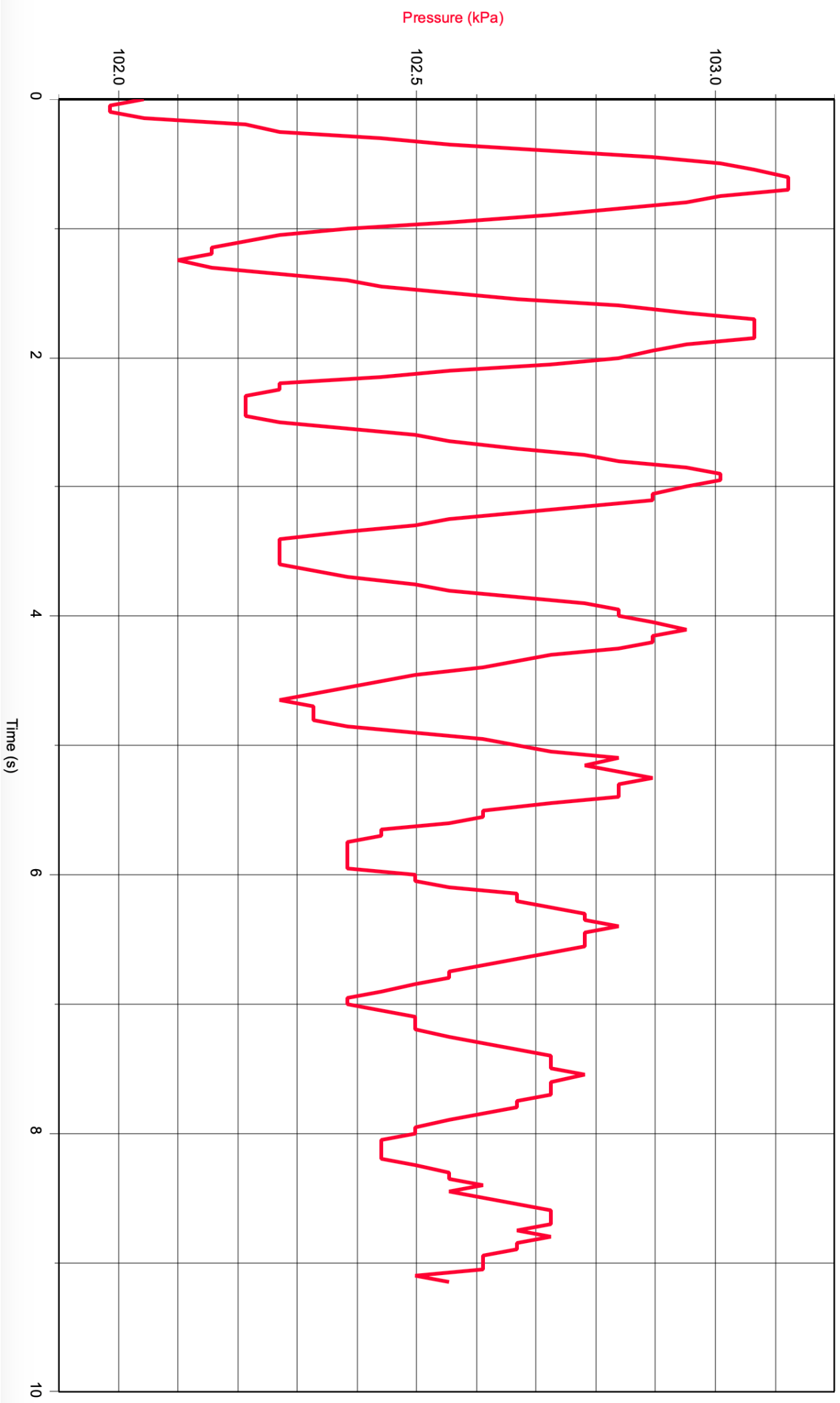


FIG 3: Pressure vs Time (CO2)



CALCULATIONS

Calculation 1: Average period

$$\begin{aligned}
 T &= \frac{t_f - t_i}{\text{Peaks}} = \frac{(6.85 \pm 0.05)s - (1.2 \pm 0.05)s}{6} = \frac{5.65 \pm 0.40s}{6} = 0.94 \pm 0.07s \\
 T_{avg} &= (1.14 + 0.97 + 0.94 + 0.94 + 0.92)/5 = 0.98 \\
 \text{stdev of mean} &= \sqrt{\frac{(1.14-0.98)^2 + (0.97-0.98)^2 + (0.94-0.98)^2 + (0.94-0.98)^2 + (0.92-0.98)^2}{4}} / \sqrt{5} = 0.04s \\
 T_{avg} &= 0.98 \pm 0.04 s
 \end{aligned}$$

Calculation 2: Equilibrium pressure

$$\begin{aligned}
 A &= \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{(17.78 \pm 0.05) \times 10^{-3} m}{2}\right)^2 = (24.83 \pm 0.10) \times 10^{-5} m^2 \\
 P_{eq} &= P_0 + \frac{mg}{A} = (9.91 \pm 0.01) \times 10^4 Pa + \frac{(9.81 m/s^2) \cdot (16.51 \pm 0.05) \times 10^{-3} kg}{(24.83 \pm 0.10) \times 10^{-5} m^2} \\
 &= (9.91 \pm 0.01) \times 10^4 Pa + (652.32 \pm 3.26) Pa = (99736.90 \pm 133.36) Pa = 99.76 \pm 0.13 \text{ kPa}
 \end{aligned}$$

Calculation 3: Specific heat ratio

$$\begin{aligned}
 \gamma_{air} &= \frac{64mV}{d^4 P_{eq} T^2} = \frac{64 \cdot [(16.51 \pm 0.05) \times 10^{-3} kg] \cdot [(1.08 \pm 0.05) \times 10^{-2} m^3]}{[(17.78 \pm 0.05) \times 10^{-3} m]^4 \cdot [(99736.90 \pm 133.36) Pa] \cdot [0.98 \pm 0.04 s]^2} \\
 &= \frac{(1.14 \pm 0.05) \times 10^{-2} m^3 kg}{[(99.94 \pm 0.56) \times 10^{-9} m^4] \cdot [(99736.90 \pm 133.36) Pa] \cdot [0.95 \pm 0.06 s^2]} = \frac{(1.14 \pm 0.05) \times 10^{-2} Pa}{(9.52 \pm 0.56) \times 10^{-3} Pa} = 1.19 \pm 0.09
 \end{aligned}$$

Calculation 4: Consistency check

According the 97th edition of the Handbook of Chemistry and Physics,^[1] the accepted values are γ at a temperature of 300K and a pressure of 0.1MPa are as follows:

- $\gamma_{air} = \frac{C_p}{C_v} = \frac{1.007 kJ kg^{-1} K^{-1}}{0.7181 kJ kg^{-1} K^{-1}} = 1.40$
- $\gamma_{argon} = \frac{C_p}{C_v} = \frac{0.5215 kJ kg^{-1} K^{-1}}{0.3124 kJ kg^{-1} K^{-1}} = 1.67$
- $\gamma_{CO_2} = \frac{C_p}{C_v} = \frac{0.8525 kJ kg^{-1} K^{-1}}{0.6593 kJ kg^{-1} K^{-1}} = 1.29$

$\gamma_{air} : 1.40 - 1.19 = 0.21 > 0.09$	INCONSISTENT
$\gamma_{argon} : 1.67 - 1.65 = 0.02 < 0.09$	CONSISTENT
$\gamma_{CO_2} : 1.29 - 1.35 = 0.06 < 0.08$	CONSISTENT

Calculation 5: Verify the assumption of (3)

$$\begin{aligned}
 \frac{kT_1}{2} &= \frac{\ln(A_1) - \ln(A_7)}{7} = \frac{\ln((64.1 \pm 0.5) \times 10^{-2} m) - \ln((45.8 \pm 0.5) \times 10^{-2} m)}{7} = (5.60 \times 10^{-2} \pm \sqrt{(0.5/64.1)^2 + (0.5/45.8)^2} / 7) \\
 &= (5.60 \pm 0.22) \times 10^{-2} \\
 ([(5.60 \pm 0.22) \times 10^{-2}]^2 &= (3.14 \pm 0.18) \times 10^{-3}) \ll (39.48 = 4\pi^2) \\
 \text{Thus, the effect of the damping on the period will be insignificant.}
 \end{aligned}$$

DISCUSSION

The temperature of the accepted values from the Handbook of Chemistry and Physics (HCP) were provided for $T_{th} = 300\text{K}$ and $P_{th} = 0.1\text{MPa}$, which are inexact approximations of the observed values of $T_{obs} = (295.05 \pm 0.5)\text{K}$ and $P_{eq} \approx (0.099 \pm 0.001)\text{MPa}$, likely giving a larger uncertainty to the values that was not considered. Additionally, while 3 minutes should be sufficient to flush the aspirator, its possible traces of other gases remained or were added to the system each time the glass tube was removed. This could make the gas impure, and give it a slightly different composition than that of the HCP. It should also be noted that the process observed in this experiment approximates to an adiabatic process, but cannot be purely adiabatic. This is due to unavoidable imperfections in the apparatus—such as improper insulation or a weak seal—which differ the experimental set-up from that of the ideal. This means there may have been heat transfer between the experiment and its environment, which could affect the specific heat capacity. The only specific heat ratio not consistent with its specific value was γ_{air} , which is potentially due to a slightly different composition of air in the location of this experiment compared to that of the HCP. Thus, this cannot disprove the equation for ratio of the specific heat capacity, and the consistency of the other results supports its validity. In addition, since Calculation 5 shows that $(\frac{kT_1}{2})^2 = (3.14 \pm 0.18) \times 10^{-3} < 4\pi^2$, this verifies the assumption of (3) and shows that the damping of the period is insignificant for this experiment.

CONCLUSIONS

The specific heat ratios were determined to be $\gamma_{argon} = 1.65 \pm 0.09$ and $\gamma_{CO_2} = 1.35 \pm 0.08$, which were consistent with their accepted values, and $\gamma_{air} = 1.19 \pm 0.09$ which was inconsistent. This supports the validity of the equation for the ratio of the specific heat capacity.

QUESTIONS

1. Each of the gases should have different values for the ratio of specific heat because they have different structures, and thus different degrees of freedom in their motion. The amount a particle vibrates dictates the amount of energy it can store, so the ability of a gas to gain heat is dependent on the freedom of the molecules in that gas to vibrate. Higher ratios of specific heat capacity mean that more heat is needed to increase the temperature of the system. Therefore, we could expect gases with more degrees of freedom, like CO_2 , to have lower heat capacity ratios than those with less, such as argon, because they have more energy stored in their internal vibrations.

BIBLIOGRAPHY

- [1] Haynes, William M., editor. CRC Handbook of Chemistry and Physics. 97th ed., CRC Press, 2020.