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Experiment No.5

STIRLING CYCLE

To investigate the effectiveness of the Stirling cycle as a heat engine, the pressure and volume in a Stirling engine with and without a load were measured as it ran. These data were plotted, the work done by the engine was found for both cases. Using these data and various environmental measurements, the measured and theoretical engine efficiencies (η) were determined. It was found that $\eta_{experimental} = 0.012 \pm 0.083$, and that $\eta_{experimental} << \eta_{theoretical}$, likely due to friction and heat loss not present in the ideal set-up. Despite this, the Stirling cycle effectively demonstrated its application as a heat engine.

THEORY

The Stirling cycle has a distinct pattern (A→B→C→D→A...) made of two isothermal sections (A, C) alternating with two isochoric sections (B, D). Heat added to the system expands the gas, which causes the piston to move outward and the pressure to drop (A). Once fully expanded, it stops and the heat transfers to the piston (B), which then causes the piston to compress the gas fully (C). There, the gas is heated (D), which starts the cycle once more. A Stirling engine can use the work done by this process to rotate a flywheel or power an incandescent bulb. By measuring the changes in temperature, pressure, and volume, the efficiency of the engine can be determined. This experiment seeks to study the thermodynamic properties of the Stirling cycle, and determine its efficiency as a heat engine.

The theoretical efficiency of a Stirling engine is given by:

$$\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}} \tag{1}$$

where T_{hot} and T_{cold} are the maximum temperatures outputted from the hot and cold ends of the Stirling engine respectively. The efficiency of the engine can also be determined experimentally using:

$$\eta_{exp} = \frac{P_{ouput}}{P_{input}} \tag{2}$$

where P_{input} is the power inputted into the system. P_{output} is the power outputted by the engine and is found using:

$$P_{output} = \frac{W}{\mathsf{T}} \tag{3}$$

where W is the work done by the system and T is the period of the engine's cycles. The theoretical power output of the system is given by

$$P_{th} = \frac{V^2}{R} \tag{4}$$

where V is the steady state voltage, and R is the resistance of the resistor.

APPARATUS

- Stirling engine
- Computer with LoggerPro connected to pressure sensor
- Stirling engine with incandescent bulb
- Two temperature sensors
- Voltmeter
- Burner

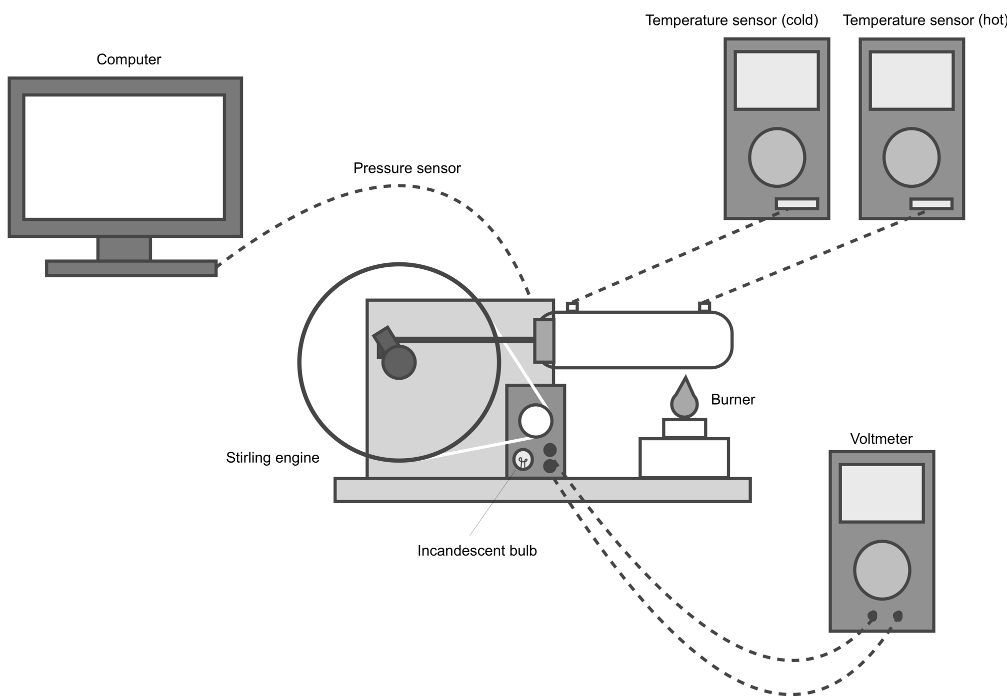


FIG. 1. Experiment set-up diagram.

PROCEDURE

The apparatus was assembled as in Fig. 1, and Logger Pro was set with an experiment length of 20 seconds and a sampling speed of 200 samples/second. The program was calibrated so that the voltage was measured in the range of 32 to 44 cm^3. The burner was lit, and the engine was started when the hot temperature sensor reached $\sim 140°$C. Once the temperature reading stopped increasing, both hot (T_{hot}) and cold (T_{cold}) temperatures were recorded. A data capture was taken using LoggerPro where the switch was level for the first 5 seconds, and down for the last 15 seconds. The behaviour of the engine was observed when the switch was up and the incandescent bulb was on.

FIG. 2: Pressure vs Volume (Unloaded)

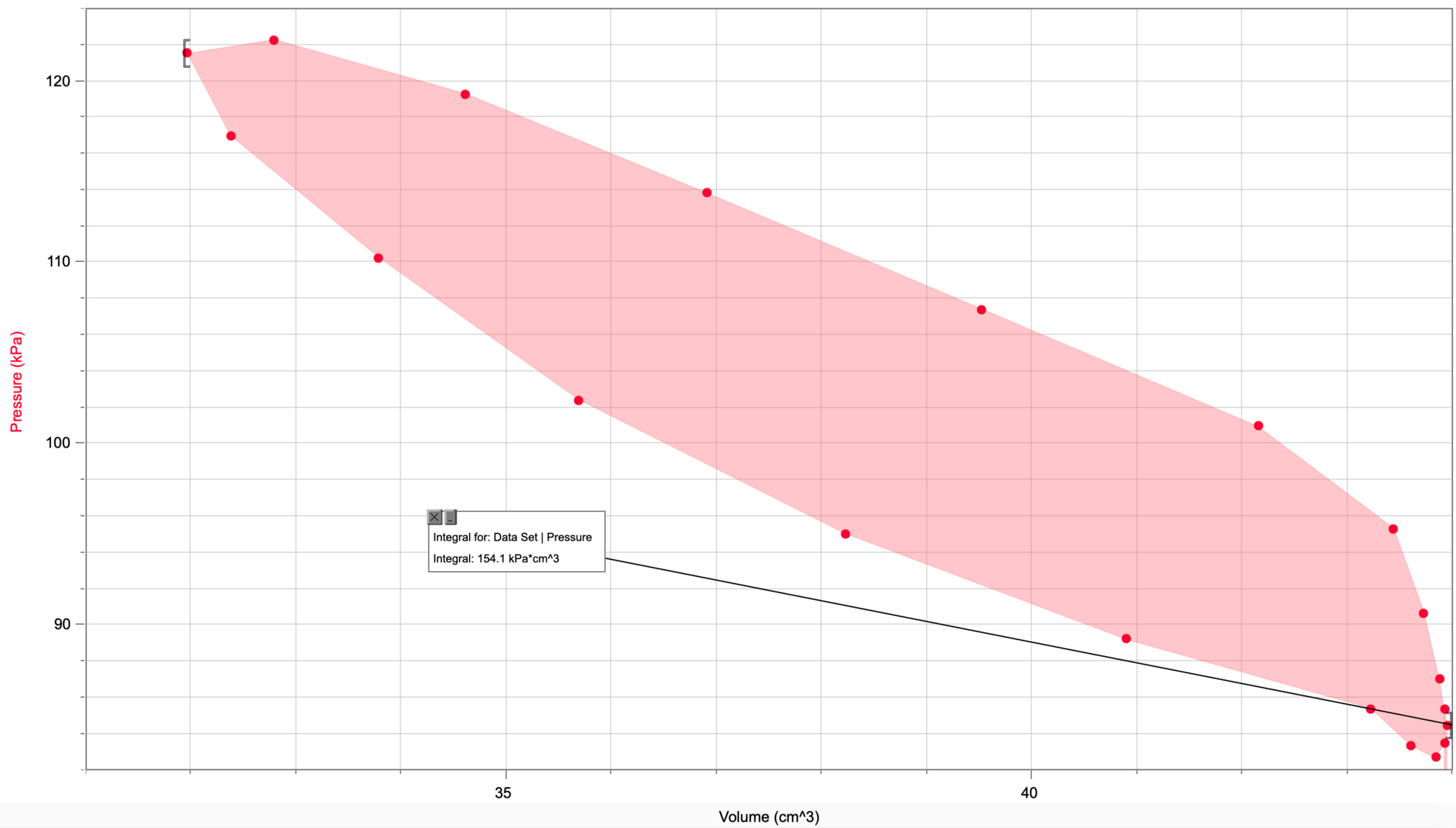
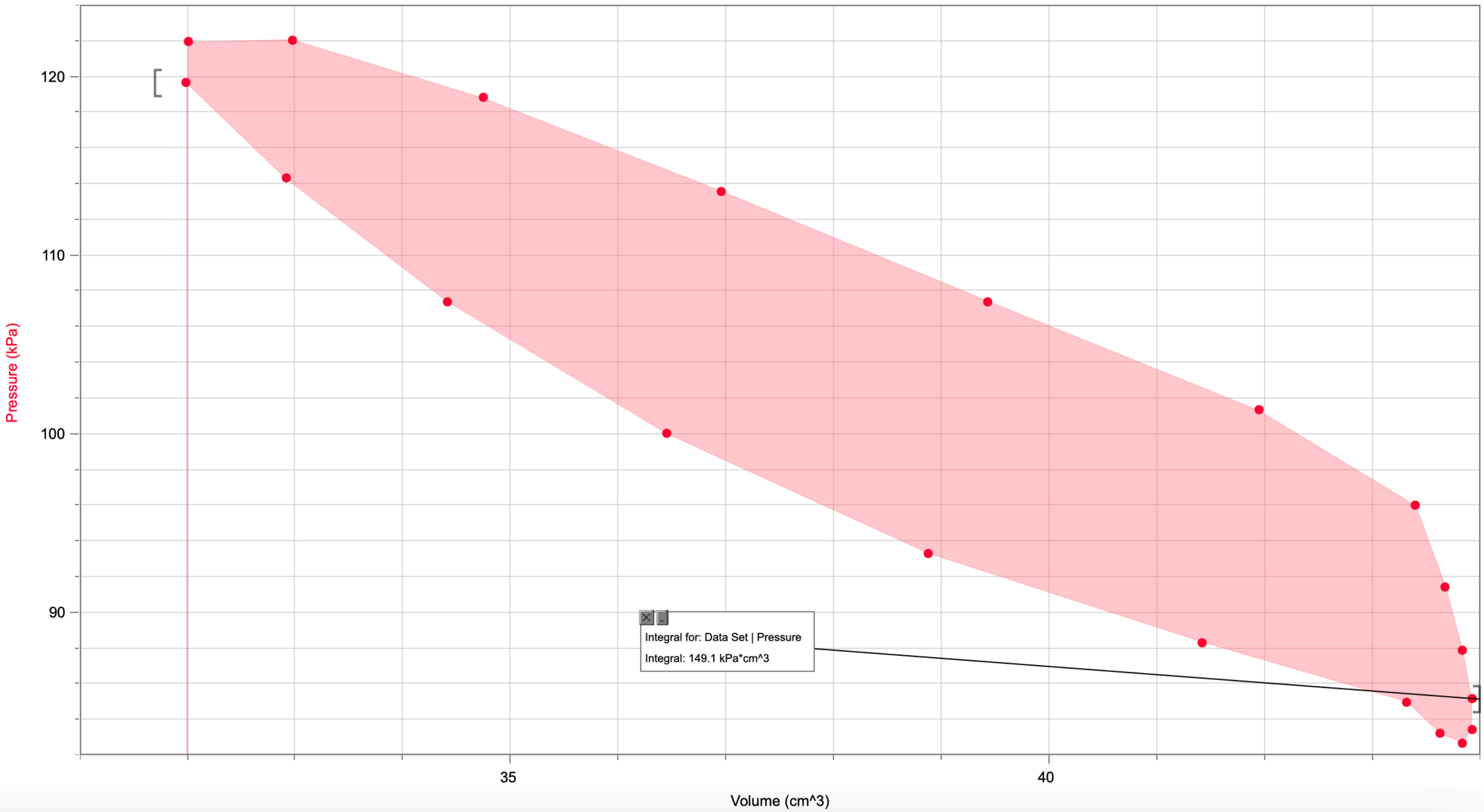


FIG. 3: Pressure vs Volume (Loaded)



DATA

From FIG. 3 the work done by the system during a loaded cycle is 149.1 kPa·cm^3 ± 10% and from FIG. 2 the work done by the system during a unloaded cycle is 154.1 kPa·cm^3 ± 10%. In addition, the input power from the alcohol burner was given to be (140 ± 20)W.

V_{loaded} [cm^3]	P_{loaded} [kPa]	$V_{unloaded}$ [cm^3]	$P_{unloaded}$ [kPa]
32.01	121.9	43.93	85.36
32.97	122.0	43.93	83.43
34.75	118.8	43.84	82.69
36.96	113.6	43.61	83.32
39.44	107.4	43.23	85.36
41.95	101.3	40.90	89.22
43.40	95.97	38.23	95.01
43.68	91.43	35.69	102.4
43.84	87.86	33.79	110.2
43.93	85.13	32.38	117.0
43.93	83.41	31.98	121.5
43.84	82.69	32.80	122.2
43.64	83.20	34.62	119.2
43.32	84.96	36.92	113.8
41.42	88.31	39.53	107.4
38.88	93.30	42.16	101.0
36.46	100.0	43.44	95.29
34.41	107.4	43.73	90.64
32.92	114.3	43.88	87.00
31.99	119.6	43.95	84.45

TABLE I. Measured volumes (±0.5 cm^3) and pressures (0.5 kPa) used for the loaded (see FIG. 3) and unloaded (see FIG. 2) graphs.

T_{hot} [K]	T_{cold} [K]	V_{steady} [V]	R [Ω]	$\mathsf{T}_{unloaded}$ [s]	T_{loaded} [s]
433.15 ± 0.50	327.15 ± 0.50	3.35 ± 0.05	100 ± 0.5	0.093 ± 0.005	0.100 ± 0.005

TABLE II. Additional data measured during the experiment.

When the incandescent bulb was switched on, it was observed that the temperatures and measured voltage remained the same, but the speed of the pinwheel's rotation noticeably slowed.

CALCULATIONS

Calculation 1: Measured powers

$W_{loaded} = [149.1(kPa \cdot cm^3)] \pm 10\% = (0.149 \pm 0.015)J$
$P_{loaded} = \frac{W_{loaded}}{\mathsf{T}_{loaded}} = \frac{(0.149 \pm 0.015)J}{(0.100 \pm 0.005)s} = (1.49 \pm 0.17)W$

$W_{unloaded} = [154.1(kPa \cdot cm^3)] \pm 10\% = (0.154 \pm 0.015)J$
$P_{unloaded} = \frac{W_{unloaded}}{\mathsf{T}_{unloaded}} = \frac{(0.154 \pm 0.008)J}{(0.093 \pm 0.005)s} = (1.66 \pm 0.19)W$

Difference: $(1.66 \pm 0.17)W - (1.49 \pm 0.19)W = (0.17 \pm 0.03)W$

We can observe that the power when the engine is unloaded is larger than when the engine is loaded, and the difference is given by $(0.17 \pm 0.03)W$.

Calculation 2: Theoretical power

$$P_{theory} = \frac{V^2}{R} = \frac{[(3.35\pm0.05)V]^2}{(100\pm0.5)\Omega} = \frac{(11.22\pm0.24)V^2}{(100\pm0.5)\Omega} = (0.112 \pm 0.002)W$$

$$|0.112 - 0.17| = 0.058 > 0.032 = (0.03 + 0.002) \qquad \text{INCONSISTENT}$$

Calculation 3: Experimental and theoretical efficiencies

$$\eta_{exp} = \frac{P_{ouput}}{P_{input}} = \frac{(1.66\pm0.17)W}{(140\pm20)W} = 0.012 \pm 0.083$$

$$\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{(327.15\pm0.50)K}{(433.15\pm0.50)K} = 1 - (0.755 \pm (0.002\%)) = 0.245 \pm 0.001$$

We can observe that $\eta_{exp} << \eta_{th}$.

DISCUSSION

The power output of the engine is larger for the unloaded cycle because the load added during the loaded cycle caused that cycle to require more power to run. This is likely also the reason why the flywheel slowed when the incandescent bulb was turned on. However, it is not easy to calculate the power dissipated in the bulb because the resistance of the incandescent filament increases as it heats up.

We observed that the theoretical power of the system and the difference in the power between the loaded and unloaded power were inconsistently different from each other. The theoretical Stirling cycle has four distinct stages with harsh transitions between motions resulting in a P-V graph similar to a parallelogram. In reality, we observe that the Stirling engine has smooth continuous motions resulting in a P-V graph similar to an ellipse existing within these bounds. Since the area of the ellipse would be smaller than that of the parallelogram, it follows that the work, and thus the power, would be smaller for the measured value. This and the fact that $\eta_{exp} << \eta_{th}$ are likely due to friction of the piston or heat loss to the walls of the engine or surrounding air. It should also be noted that, due to the data sample rate, in the P-V graphs (see FIG. 2 and FIG. 3) the starting point and ending point were not exactly the same. This resulted in a thin strip of area added to the integral that may have affected the accuracy of the results, and as such, a large uncertainty was given to the determined work.

CONCLUSION

The measured efficiency of the Stirling engine was determined to be $\eta_{exp} = 0.012\pm0.083$ and was found to be much less effective than its theoretical efficiency. Despite this, the Stirling cycle was capable of powering multiple loads and effectively demonstrated its application as a heat engine.

QUESTIONS

1. The area of within the P-V diagram has units of $kPa \cdot cm^3$ and the work done by the engine has units of J. We want to show that $kPa \cdot cm^3 \propto J$:

   $kPa \cdot cm^3 = (10^3 Pa) \cdot (10^{-6} m^3) = (10^{-3})(\frac{N}{m^2} \cdot m^3) = (10^{-3})(N \cdot m) = 10^{-3} J$ ✓