

Wilcoxon's W

In this document, we will explain Wilcoxon's W through a scenario of a fitness coach trying to determine if a new exercise routine is working.

Scenario:

Imagine you are a fitness coach who wants to determine if a new exercise routine has affected your clients' performance times. You have the performance times (in minutes) of 10 clients before and after trying the new routine.

Performance Times:

- **Before:** 15, 18, 20, 22, 17, 19, 16, 21, 23, 20
- **After:** 14, 19, 19, 23, 16, 21, 15, 22, 22, 21

Steps in the Wilcoxon Signed-Rank Test:

1. **Calculate Differences:** Calculate the difference between the "after" and "before" times for each client.
 - Differences: -1, +1, -1, +1, -1, +2, -1, +1, -1, +1
2. **Rank the Absolute Differences:** Ignore the signs and rank the absolute values of the differences.
 - Absolute Differences: 1, 1, 1, 1, 1, 2, 1, 1, 1, 1
 - Ranked Absolute Differences: 2.5, 2.5, 2.5, 2.5, 2.5, 10, 2.5, 2.5, 2.5, 2.5
3. **Assign Signs to Ranks:** Assign the signs of the original differences to their respective ranks.
 - Signed Ranks: -2.5, +2.5, -2.5, +2.5, -2.5, +10, -2.5, +2.5, -2.5, +2.5
4. **Calculate W Statistics:** Sum the ranks with their signs to get the W statistic.
 - Sum of positive ranks (W+): $2.5 + 2.5 + 2.5 + 10 + 2.5 + 2.5 + 2.5 = 25$
 - Sum of negative ranks (W-): $-2.5 + -2.5 + -2.5 + -2.5 + -2.5 + -2.5 = -15$

The W statistic is usually taken as the smaller of the absolute sum of positive ranks (W+) and the absolute sum of negative ranks (W-).

Interpretation:

In this example, $W^+ = 25$ and $W^- = 15$. The Wilcoxon Signed-Rank Test uses the W statistic to determine if the differences are statistically significant.

Summary:

The W statistic in this example is the sum of the signed ranks of the differences between the paired observations. Here, some differences are negative, resulting in negative ranks. The W statistic, which in this case is 15 (the smaller absolute value of W^+ and W^-), indicates the sum of negative differences. To determine significance, this W statistic would be compared to critical values in a Wilcoxon test table or used to calculate a p-value with software. If the p-value is below a certain threshold (e.g., 0.05), the null hypothesis would be rejected, concluding that the new exercise routine has significantly affected performance times.