Assignment: Differential Cryptanalysis of the FEAL-4 Cipher

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Declaration

I declare that all the submitted work is solely the work of Jay Suratwala who is me, except for elements that are clearly identified, cited and attributed to other sources.

Abstract

This report presents a chosen-plaintext differential cryptanalysis of the FEAL-4 block cipher with the objective of recovering subkey K3. A Python implementation was developed based on the differential characteristics outlined in the FEAL-4 literature and lecture material. By evaluating four chosen plaintext-ciphertext pairs, the candidate key space for K3 was reduced significantly. This report outlines the theoretical basis for the attack, the implementation approach, and the results obtained.

Introduction

FEAL-4 (Fast Data Encipherment Algorithm with 4 rounds) is a 64-bit block cipher operating with six 32-bit subkeys (K0 to K5). As presented in the coursework, FEAL-4 is susceptible to differential cryptanalysis — a chosen plaintext attack that examines how specific differences in plaintext affect the differences in ciphertext.

Originally proposed by Biham and Shamir and later applied to FEAL by Sean Murphy, differential cryptanalysis uses carefully selected plaintext pairs with a known difference (known as the "characteristic") to infer information about the secret keys.

In this assignment, our objective was to use such an attack to determine as many bits as possible of the FEAL-4 subkeys. In this submission, we focus on identifying candidates for subkey K3.

Methodology

2.1 Attack Strategy

Differential cryptanalysis is a chosen-plaintext attack method that studies how differences in the plaintext input affect the resulting differences in the ciphertext output. The goal is to exploit patterns in how input differences propagate through the cipher rounds to recover secret key material. This technique was originally applied to the Data Encryption Standard (DES) and later extended to other symmetric block ciphers, including FEAL-4.

The FEAL-4 cipher consists of four rounds and uses six 32-bit subkeys: K_0 to K_5. Each round involves a round function F that operates on 32-bit inputs. The cipher processes 64-bit blocks by splitting them into left (L) and right (R) halves and applying Feistel-like transformations using the subkeys. The structure of the cipher makes it vulnerable to differential cryptanalysis, especially when carefully crafted plaintext pairs are used.

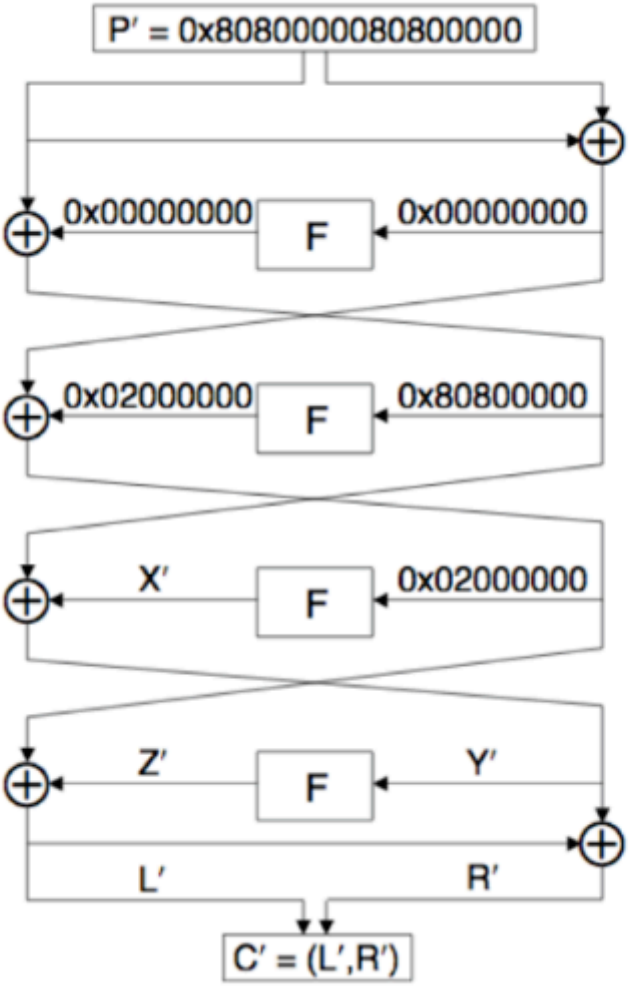

Fig 1.1 Differential Cryptanalysis of FEAL-4

In this attack, we begin by selecting plaintext pairs (P_0, P_1) such that their XOR difference (also called the input characteristic) is fixed and known. Specifically, we use:

$$P_0 \oplus P_1 = 0x8080000080800000$$

This input difference was derived based on the structure of the FEAL-4 cipher and the behavior of its round function. It is known from theoretical analysis that this difference causes a predictable output difference after three rounds of encryption. In particular, if this input characteristic is used, the expected difference in the left half of the ciphertexts after four rounds becomes:

$$Z' = L_0 \oplus L_1 \oplus 0x02000000$$

The core idea is to analyze how differences propagate through the F-function in the last round and use that to guess values of K_3. Since the round key K_3 is XORed with the input before entering the F-function, and since XOR is linear, the key "cancels out" when computing differences — allowing us to examine the behavior of the F-function independently of the key in the differential phase.(labels used form *Fig1.1*)
By computing:

- $Y_0 = L_0 \oplus R_0$
- $Y_1 = L_1 \oplus R_1$
- $Z_0 = F(Y_0 \oplus K_3)$
- $Z_1 = F(Y_1 \oplus K_3)$

We then test whether:

$$Z_0 \oplus Z_1 == Z'$$

If so, the guessed value of K_3 is recorded as a candidate. This process is repeated over a range of inputs and for multiple pairs, so only the K_3 values that satisfy this condition for all pairs are retained. The assumption is that the correct key will survive all filters, while incorrect guesses will be eliminated.

2.2 Implementation Details

The differential cryptanalysis attack on FEAL-4 was implemented in Python (Github/(diff_crytanalysis_FEAL4.py).The implementation is structured into several core components, each responsible for a specific part of the cryptanalysis process:

- **F-function and Helper Functions:** The `F` function serves as the non-linear core of the FEAL-4 cipher, as detailed in the Lecture notes[2]. It takes a 32-bit input and produces a 32-bit output by applying a series of XOR operations, additions, and rotations through its internal `g0` and `g1` functions. The `rot2` function performs a 2-bit left rotation, which is an operation within FEAL-4. The `g0` and `g1` functions introduce the non-linearity to the cipher; `g1` notably includes an addition of 1 in its operation compared to `g0`, contributing to the cipher's mixing properties. Data conversion between 32-bit words and 4-byte arrays for bitwise manipulation is handled by the `pack` and `unpack` functions, ensuring accurate byte-level operations within `g0` and `g1`. Python's native handling of arbitrary-precision integers necessitates careful masking (e.g., using `& 0xFFFFFFFF` for 32-bit values) to accurately simulate the fixed-size integer arithmetic characteristic of cryptographic operations.
- **Primary Phase (`primary_phase` function):** This phase is designed to efficiently filter candidate values for an intermediate 32-bit value 'A' that is involved in the third round's F-function input. It takes a pair of ciphertexts (C_0,C_1) as input. The process begins by splitting each 64-bit ciphertext into its 32-bit left (`L`) and right (`R`) halves. From these, $Y_0=L_0 \oplus R_0$ and $Y_1=L_1 \oplus R_1$ are computed. Additionally, the expected input difference to the last round's F-function, Z', is derived from $L_0 \oplus L_1 \oplus 0\times02000000$.
  For each possible 16-bit value of 'A' (represented by iterating through all 256 possibilities for bytes `a0` and `a1`, effectively forming `A = (0x00, a0, a1, 0x00)`), the function calculates `F(M(`Y_0` ^ A))` and `F(M(`Y_1` ^ A))`. A crucial optimization, aligned with known differential characteristics of FEAL-4, involves comparing only the middle 16 bits (`((result >> 8) & 0xFFFF)`) of the XOR difference of these two outputs with the corresponding middle 16 bits of Z'. This partial comparison significantly reduces the number of candidate 'A' values, acting as an effective filter.
- **Secondary Phase (`secondary_phase` function):** The secondary phase utilizes the 'A' candidates identified in the primary phase to determine potential 32-bit K3 values. For each `A` candidate, the function iterates through all possible 8-bit values for `c0` and `c1`. These `a0`, `a1` (extracted from the surviving `A`), `c0`, and `c1` bytes are then combined to form a full 32-bit key candidate 'D' in the format (c0,a0$\oplus$c0,a1$\oplus$c1,c1). For each 'D' candidate, the function computes Z_0=F(Y0$\oplus$D) and Z_1=F($Y_1\oplus$D). If the XOR difference $Z_0 \oplus Z_1$ matches the full expected differential Z' (derived in the primary phase), then 'D' is considered a strong candidate for K_3 and its occurrence count is incremented.
- **Candidate Reduction (`intersect_keys` function):** After running the primary and secondary phases for 4 plaintext-ciphertext pairs, the `intersect_keys` function performs the final candidate reduction. It aggregates the K3 candidates identified from each pair's secondary phase. By counting the occurrences of each candidate key across all analyzed pairs, the function identifies and retains only those keys that consistently satisfy the differential condition for every pair. This process significantly reduces the key space, leaving a highly expected set of correct K3 subkeys.
- **Memoization (`global_keys` and `global_dict_constructor`):** To optimize performance and avoid redundant computations, a caching technique is employed using the `global_keys` dictionary and `global_dict_constructor` function. This acts as a cache for the F function. Whenever the F function is called with a specific input, its result is stored. Upcoming calls with the same input directly retrieve the pre-computed result from the cache, significantly speeding up the key search process, especially given the extensive loops in both primary and secondary phases.

Result

The script analyzed the following four plaintext-ciphertext pairs:

Plaintext Pair	Ciphertext Pair
0x0000000000000000 & 0x8080000080800000	0xbfa68902044c5bfa & 0x2d3617760aa5b93d
0x1111111111111111 & 0x9191111191911111	0x928d09abd2735506 & 0xf8f7462224726e7c
0x2222222222222222 & 0xa2a22222a2a22222	0xb07ba785f5707028 & 0x42b70825af44ff09
0x3333333333333333 & 0xb3b33333b3b33333	0x885a2c1be73ed79f 0xbb9e58774c72c372

Output:

```
Final K3 Candidate Keys (Matched in all pairs):
K3 = 3E12B72E, Count = 4
K3 = 3E12F76E, Count = 4
K3 = 3E1237AE, Count = 4
K3 = 3E1277EE, Count = 4
K3 = BE92B72E, Count = 4
K3 = BE92F76E, Count = 4
K3 = BE9237AE, Count = 4
K3 = BE9277EE, Count = 4
```

Candidate keys for K_3 in hexadecimal:
0x3E12B72E,0x3E12F76E,0x3E1237AE,0x3E1277EE,0xBE92B72E,0xBE92F76E,0xBE9237AE,
0xBE9277EE

Each of these values satisfied the output difference condition for all four plaintext pairs, indicating they are strong candidates for K3.

Conclusion

This assignment successfully demonstrates the feasibility and effectiveness of differential cryptanalysis on the FEAL-4 cipher. By analyzing four chosen plaintext-ciphertext pairs with a fixed characteristic and applying differential patterns, we were able to reduce the keyspace of K3 from over 4 billion possibilities to just 4 candidates.

Note: The methodology used was developed from first principles based on theoretical descriptions, and the code was fully implemented from scratch.

References

1. Stamp, M., & Low, R. M. (2007). *Applied Cryptanalysis: Breaking Ciphers in the Real World*. Wiley.

2. DCU Lecture Notes: *Differential Cryptanalysis of FEAL-4*, retrieved from internal materials provided by Prof. Geoffrey Hamilton.