Illinois Institute of Technology
Department of Computer Science

Homework 9

CS 430 Introduction to Algorithms
Spring Semester, 2014

1. **Problem 1** Problem 22.4-2 (Pg 614) CLRS(3rd Edition): Give a linear-time algorithm that takes as input a directed acyclic graph $G = (V, E)$ and two vertices s and t, and returns the number of simple paths from s to t in G. For example, the directed acyclic graph of Figure 22.8 contains exactly four simple paths from vertex p to vertex v: pov , $poryv$, $posryv$, and $psryv$. (Your algorithm needs only to count the simple paths, not list them.) (20 pts)

2. **Problem 2** Problem 22-1 (Pg 621) CLRS(3rd Edition):A depth-first forest classifies the edges of a graph into tree, back, forward, and cross edges. A breadth-first tree can also be used to classify the edges reachable from the source of the search into the same four categories.

   (a) Prove that in a breadth-first search of an undirected graph, the following properties hold:

      i. There are no back edges and no forward edges.

      ii. For each tree edge (u, v) , we have $v.d = u.d + 1$.

      iii. For each cross edge (u, v), we have $v.d = u.d$ or $v.d = u.d + 1$.

   (b) Prove that in a breadth-first search of a directed graph, the following properties hold:

      i. There are no forward edges.

      ii. For each tree edge (u, v) , we have $v.d = u.d + 1$.

      iii. For each cross edge (u, v), we have $v.d \leq u.d + 1$..

      iv. For each back edge (u, v), we have $0 \leq v.d \leq u.d$

   (20 pts)

3. **Problem 3** Problem 22-4 (Pg 623) CLRS(3rd Edition): Let $G = (u, v)$ be a directed graph in which each vertex $u \in V$ is labeled with a unique integer $L(u)$ from the set $\{1, 2, ..., |V|\}$. For each vertex $u \in V$, let $R(u) = \{v \in V : u \to v\}$ be the set of vertices that are reachable from u. Define $min(u)$ to be the vertex in $R(u)$ whose label is minimum, i.e., $min(u)$ is the vertex v such that $L(v) = min\{L(w) : w \in R(u)\}$. Give an $O(V + E)$-time algorithm that computes $min(u)$ for all vertices $u \in V$. (20 pts)

4. **Problem 4** Problem 23.2-5 (Pg 637) CLRS(3rd Edition):Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How fast can you make Prims algorithm run? What if the edge weights are integers in the range from 1 to W for some constant W ? (20 pts)