Introduction To Algorithms
HomeWork 4 Solutions

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1.Give an example where quicksort requires $O(n^2)$ steps.

Consider a list:

$$10, 9, 8, 7, 6, 5, 4, 3, 2, 1$$

We choose the last digit in the last as the pivot.Thus time complexity is given as:

$$T(n) = T(n-1) + T(0) + \Theta(n)$$

By using substitution method, we could get:

$$T(n) = \Theta(n^2)$$

If it is $\Theta(n^2)$, it is also a $O(n^2)$.

2.Problem 4-6 (Page 110) CLRS(3rd Edition).

a. Need to prove "if and only if", thus the proof will have to separate parts

Proof of 'Only if':

If A is a Monge array, by definition, we have:

$$A[i,j] + A[k,l] < A[i,l] + A[k,j] \quad \forall i , j , k , l$$

$$where \ 1 < i < k < n , \ 1 < j < l < m$$

Let $k = i + 1, \ l = j + 1$, we will have:

$$A[i,j] + A[i+1,j+1] < A[i,j+1] + A[i+1,j] \quad \forall i , j$$

$$where \ 1 < i < i+1 < n , \ 1 < j < j+1 < m$$

$$where \ 1 < i < n-1 , \ 1 < j < m-1$$

'Only if' has been proved.

Proof of 'if':
Induction method will be used separately on rows and columns.
For rows: