Illinois Institute of Technology
Department of Computer Science

# Solutions to Homework 8

CS 430 Introduction To Algorithms
Spring Semester, 2013

1. (a) The recursive equation for $CoinChange(V', i, W')$ is

$$CoinChange(V', i, W') = \begin{cases} CoinChange(V' - d_i, i+1, W' - w_i) & \text{if } d_i \text{ is used} \\ CoinChange(V', i+1, W') & \text{otherwise} \end{cases}$$

(b) Use memoing to solve the recurrence

**Algorithm 1:** $CoinChange(v', i, w')$

```
if memo(v', i, w') ≠ NULL then
    return memo(v',i,w');
else
    if d_i = v' and w_i < w' then
        memo(v', i, w') = True;
        return True ;
    else if i > n or v' ≤ 0 or w' ≤ 0 then
        memo(v', i, w') = False;
        return False ;
    else
        return CoinChange(v' − d_i, i + 1, w' − w_i) ∨ CoinChange(v', i + 1, w')
```

2. Let $L[i][j]$ be the length of the longest substring ending at position $i$ of the string $S_1$ with $n$ characters and position $j$ of the string $S_2$ with $m$ characters. Then we have the recursive equation

$$L[i][j] = \begin{cases} L[i-1][j-1] + 1 & \text{if } S_1[i] = S_2[j] \\ 0 & \text{otherwise} \end{cases}$$

```
for i = 1 to n do
    for j = 1 to m do
        if S_1[i] = S_2[j] then
            if i = 1 or j = 1 then
                L[i][j]= 1;
            else
                L[i][j]=L[i-1][j-1]+1;
        else
            L[i][j]=0;
```

3. (a) The recursive equation for $W(i, j)$ is

$$W(i, j) = \max\{W(i + 1, j), W(i, j + 1)\} + c(w(i, j))$$

(b) Use memoing to solve the recurrence

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$W(n, n) = c(w(n, n))$;
**for** $i = n - 1$ *to* $1$ **do**
  $W(i, n) = W(i + 1, n) + c(w(i, n))$;
**for** $j = n - 1$ *to* $1$ **do**
  $W(n, j) = W(n, j + 1) + c(w(n, j))$;
**for** $i = n - 1$ *to* $1$ **do**
  **for** $j = n - 1$ *to* $1$ **do**
    $W(i, j) = \max\{W(i + 1, j), W(i, j + 1)\} + c(w(i, j))$;