Illinois Institute of Technology
Department of Computer Science

Homework 8

Due Apr. 14 2014

CS 430 Introduction to Algorithms
Spring Semester, 2014

1. **Problem 1** Suppose we wish to make change for a bill of a certain value into smaller coins of denominations $d_1, d_2...d_n$ each of weight $w_1, w_2...w_n$ units. Given only one coin of the each denomination, design a dynamic programming algorithm to determine if it is possible to make change for an input bill of value V with coins of weight at most W .

   (a) Use a recurrence to express the boolean function CoinChange(V', i, W') which is true if there is a change for a bill of value $V' \leq V$ and weight at most W' using coins of denomination $d_i...d_n$.

   (b) Use memoing to solve the recurrence for all possible choices of the parameter $V'W'$ and i where $1 \leq V' \leq V$, $1 \leq W' \leq W$ and $1 \leq i \leq n$. Note that the weighted coin-change problem has a solution if Coin Change$(V, 1, W)$ is true

   (20 pts)

2. **Problem 2** A modification of the longest common subsequence problem is to find the longest common substring problem (all the characters are contiguous). Determine an $O(mn)$ algorithm to determine the longest common substring. (20 pts)

3. **Problem 3** Suppose we have a grid of size $n \times n$ the column and rows numbered 1 through n left to right and top to bottom, respectively. A mouse gathers a bunch of cheese bits while moving from matrix square numbered $(1, 1)$ to (n, n) and at each matrix element (i, j) is able to gain cheese worth $c(w(i, j)))$ calories, where $w(i, j)$ is the weight of the cheese at the $(i, j)^{th}$ square and $c(w)$ is a function that maps weight to calories and that the mouse can compute. The mouse can only move right or down, i.e. from square (i, j) he can go to either $(i+1, j)$ or $(i, j+1)$. Find a path from $(1, 1)$ to (n, n) so that the mouse can maximize the total value of the cheese gathered. Use the following steps:

   (a) Set-up a recurrence $W(i, j)$ that represents the maximum value of cheese that the mouse can gather starting from square (i, j).

   (b) Solve it using memoing.

   (20 pts)