

Introduction To Algorithms
CS430

Spring 2013
HomeWork 8
Due 3rd April

1. **Problem 1: Weighted coin-change:** Suppose we wish to make change for a bill of a certain value into smaller coins of denominations $d_1, d_2 \dots d_n$ each of weight $w_1, w_2 \dots w_n$ units. Given only one coin of the each denomination, design a dynamic programming algorithm to determine if it is possible to make change for an input bill of value V with coins of weight at most W .

(1) Use a recurrence to express the boolean function $CoinChange(V', i, W')$ which is true if there is a change for a bill of value $V' \leq V$ and weight at most W' using coins of denomination $d_i \dots d_n$.

(ii) Use memoing to solve the recurrence for all possible choices of the parameter $V'W'$ and i where $1 \leq V' \leq V$, $1 \leq W' \leq W$ and $1 \leq i \leq n$

Note that the weighted coin-change problem has a solution if $Coin_Change(V, 1, W)$ is true.

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2. **Problem 2:** A modification of the longest common subsequence problem is to find the longest common substring problem (all the characters are contiguous). Determine an $O(mn)$ algorithm to determine the longest common substring.

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3. **Problem 3:** Suppose we have a grid of size $n \times n$ the column and rows numbered 1 through n left to right and top to bottom, respectively. A mouse gathers a bunch of cheese bits while moving from matrix square numbered $(1, 1)$ to (n, n) and at each matrix element (i, j) is able to gain cheese worth $c(w(i, j))$ calories, where $w(i, j)$ is the weight of the cheese at the (i, j) th square and $c(w)$ is a function that maps weight to calories and that the mouse can compute. The mouse can only move right or down, i.e. from square (i, j) he can go to either $(i + 1, j)$ or $(i, j + 1)$. Find a path from $(1, 1)$ to (n, n) so that the mouse can maximize the total value of the cheese gathered.

Use the following steps:

(i) Set-up a recurrence $W(i, j)$ that represents the maximum value of cheese that the mouse can gather starting from square (i, j) .

(ii) Solve it using memoing.

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