

$$1) a) \quad A = \begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 3 \\ 5 & 1 \end{bmatrix} \quad C = \begin{bmatrix} -6 & 0 \\ 3 & 2 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 8 & 3 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 36 & 10 \\ -5 & -1 \end{bmatrix}$$

$$A \times B \times C = \begin{bmatrix} 36 & 10 \\ -5 & -1 \end{bmatrix} \begin{bmatrix} -6 & 0 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -186 & 20 \\ 27 & -2 \end{bmatrix}$$

$$b) \quad C \times A \times B$$

$$= \begin{bmatrix} -6 & 0 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 36 & 10 \\ -5 & -1 \end{bmatrix} = \begin{bmatrix} -216 & -60 \\ 98 & 28 \end{bmatrix}$$

$$2) a) \quad E = \begin{bmatrix} -1 & 0 & 2 & 1 & 4 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$E \cdot X = 0$$

$$E: R_2 \rightarrow R_2 - R_1$$

$$E = \begin{bmatrix} -1 & 0 & 2 & 1 & 4 \\ 0 & 1 & -2 & 0 & -4 \\ 0 & 1 & 1 & 0 & 2 \end{bmatrix}$$

$$E: R_3 \rightarrow R_3 - R_2$$

$$E = \begin{bmatrix} -1 & 0 & 2 & 1 & 4 \\ 0 & 1 & -2 & 0 & -4 \\ 0 & 0 & 3 & 0 & 6 \end{bmatrix}$$

$$R_3 \rightarrow R_3/3$$

$$E = \begin{bmatrix} -1 & 0 & 2 & 1 & 4 \\ 0 & 1 & -2 & 0 & -4 \\ 0 & 0 & 1 & 0 & 2 \end{bmatrix}$$

$$R_1 \rightarrow R_1 - 2R_3$$

$$E = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 & -4 \\ 0 & 0 & 1 & 0 & 2 \end{bmatrix}$$

$$R_2 \rightarrow R_2 + 2R_3$$

$$E = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 \end{bmatrix}$$

$$\text{Eqn} \quad -x_1 + x_4 = 0$$

$$x_2 = 0$$

$$x_1 = x_4$$

$$x_3 = -2x_5$$

$$2x_5 + x_3 = 0$$

$$\underline{\text{Null space}} \Rightarrow \vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 1 \end{bmatrix} x_5 \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$3) \quad \begin{bmatrix} 2 & 0 & 3 \\ 1 & 4 & -1 \\ -5 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ -16 \end{bmatrix}$$

$$A \cdot x = B$$

$$x = A^{-1} \cdot B$$

$$\begin{aligned} |A| &= 2(4(1) - (-1)(1)) - 0(1 \times 1 - (-1)(-5)) \\ &\quad + 3(1 \times 1 - 4(-5)) \\ &= 2(4+1) + 3(1+20) \\ &= 10 + 63 \\ &= 73 \end{aligned}$$

Cofactor matrix:

$$\begin{bmatrix} 4 - (-1) & - (1 - 5) & (1 - (-20)) \\ - (0 - 3) & 2 - (-15) & - (2 - 0) \\ (0 - 12) & - (-2 - 3) & (8 - 0) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 & 21 \\ 3 & 17 & -2 \\ -12 & 5 & 8 \end{bmatrix}$$

$$A^{-1} = \text{Adj } A \times \frac{1}{\det(A)}$$

transpose of cofactor matrix  $\Rightarrow \begin{bmatrix} 5 & 3 & -12 \\ 4 & 17 & 5 \\ 21 & -2 & 8 \end{bmatrix} = \text{Adj}(A)$

$$A^{-1} \times B = \frac{1}{73} \begin{bmatrix} 5 & 3 & -12 \\ 4 & 17 & 5 \\ 21 & -2 & 8 \end{bmatrix} \begin{bmatrix} 6 \\ -1 \\ -16 \end{bmatrix}$$

$$= \frac{1}{73} \begin{bmatrix} 36 - 3 + (12 \times 16) \\ 24 - 17 - 80 \\ 126 + 2 - (8 \times 16) \end{bmatrix}$$

$$= \frac{1}{73} \begin{bmatrix} 219 \\ -73 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$

4) a)  $D = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$$\det(D) = 3(6) - 5(4) \\ = -2$$

b)  $D$  is non-singular as  $\det(D) \neq 0$  and its inverse can be calculated

$$5) \quad F = \begin{bmatrix} -3 & -2 & 1 & 3 \\ 2 & 4 & 1 & -2 \\ -1 & 2 & 2 & 1 \\ -1 & 4 & -3 & 1 \end{bmatrix}$$

$$\text{ref}(F) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Basis for column space} = \left\{ \begin{bmatrix} -3 \\ 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix} \right\}$$

pivot columns, column 1, 2, 3

6) Row space

$$\text{ref}(F) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Basis for row space} = \left\{ [-3 \ -2 \ 1 \ 3], [2 \ 4 \ 1 \ -2], [-1 \ 2 \ 2 \ 1] \right\}$$

7) Subspaces

a) Rank of matrix =  $\dim(\text{col}(\text{matrix}))$

Rank of matrix represents linearly independent columns, which is directly the basis of column space

$$\text{Rank}(F) = 3$$

b) column space of  $F = \text{col}(F)$

row space of  $F = \text{row}(F)$

$$\boxed{\dim(\text{col}(F)) = \dim(\text{row}(F))}$$

This is also called rank theorem, which states that rank of a matrix is same for row & column space.

## 8) Eigenvalues and Eigenvectors

- a) Eigenvector is a non-zero ~~matrix~~ vector  $x$  that, when multiplied by a matrix, results in a scaled version of the eigenvector  $x$ . The scaling factor is  $\lambda$  (eigenvalue)

b)  $G = \begin{bmatrix} 3 & 6 \\ 2 & 2 \end{bmatrix}$

$$\det(G - \lambda I) = 0$$

$$G - \lambda I = \begin{bmatrix} 3 & 6 \\ 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 3-\lambda & 6 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(2-\lambda) = 12$$

$$\lambda^2 - 5\lambda - 6 = 0$$

$$(\lambda - 6)(\lambda + 1) = 0$$

$$\boxed{\lambda = 6, -1}$$

$$\text{Eigenvalues (L)} = \begin{bmatrix} 6 & 0 \\ 0 & -1 \end{bmatrix}$$

For  $\lambda = 6$

$$(G - 6I)x = 0$$

$$\begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\boxed{x_1 = 2x_2}$$

$$x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

For  $\lambda = -1$

$$(G + I)x = 0$$

$$\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow x_2 = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$



$$\text{Eigenvectors} \rightarrow \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$$

$$\text{Normalized } \vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad |\vec{v}_1| = \sqrt{5}$$

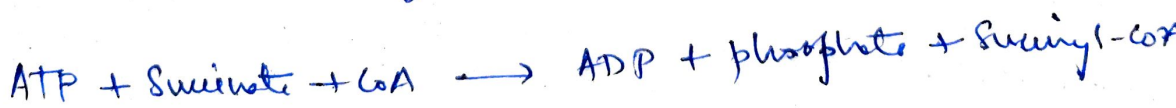
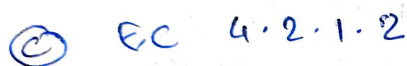
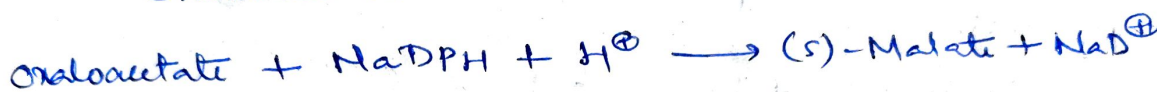
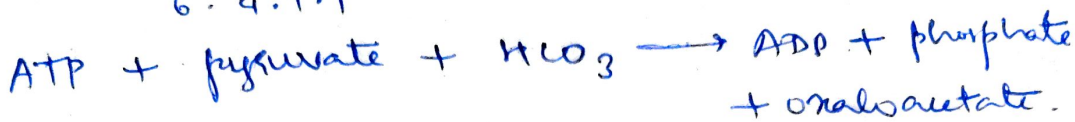
$$= \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.8944 \\ 0.4472 \end{bmatrix}$$

$$\vec{v}_2 = \begin{bmatrix} -3 \\ 2 \end{bmatrix} \quad |\vec{v}_2| = \sqrt{13}$$

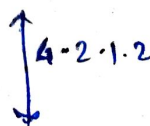
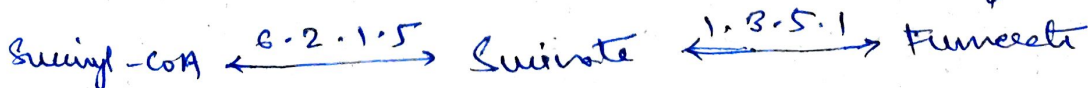
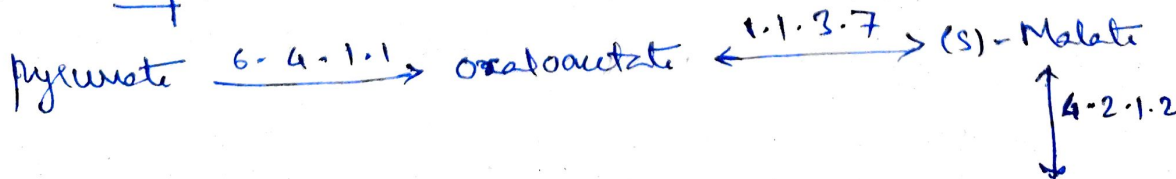
$$= \frac{1}{\sqrt{13}} \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.8321 \\ 0.5547 \end{bmatrix}$$

$$\vec{V} = \begin{bmatrix} 0.8944 & -0.8321 \\ 0.4472 & 0.5547 \end{bmatrix}$$

## "Part - II"



2) map:



### 3) S-Matrix

<u>Metabolite</u>	<u>R(a)</u>	<u>R(b)</u>	<u>R(c)</u>	<u>R(d)</u>	<u>R(e)</u>
ATP	-1	0	0	0	-1
ADP	+1	0	0	0	+1
phosphate	+1	0	0	0	+1
HCO <sub>3</sub> <sup>-</sup>	-1	0	0	0	0
pyruvate	-1	0	0	0	0
oxaloacetate	+1	-1	0	0	0
(s)-Malate	0	+1	-1	0	0
NAD <sup>+</sup>	0	+1	0	0	0
NADH	0	-1	0	0	0
H <sup>+</sup>	0	-1	0	0	0
Fumarate	0	0	+1	-1	0
H <sub>2</sub> O	0	0	+1	0	0
Succinate	0	0	0	+1	-1
Quinone	0	0	0	+1	0
quinol	0	0	0	-1	0
CoA	0	0	0	0	-1
Succinyl-CoA	0	0	0	0	+1



# 4) E - Matrix

<u>Metabolite</u>	<u>C</u>	<u>H</u>	<u>O</u>	<u>N</u>	<u>P</u>	<u>S</u>
ATP	10	16	13	5	3	0
ADP	10	15	10	5	2	0
phosphate	0	3	4	0	1	0
HCO <sub>3</sub> <sup>-</sup>	1	1	3	0	0	0
pyruvate	3	4	3	0	0	0
oxaloacetate	4	4	5	0	0	0
(S)-Malate	4	6	5	0	0	0
NAD <sup>+</sup>	21	27	14	7	2	0
NADH	21	28	14	7	2	0
H <sup>+</sup>	0	1	0	0	0	0
fumarate	4	4	4	0	0	0
H <sub>2</sub> O	0	2	1	0	0	0
Succinate	4	6	4	0	0	0
Quinone	6	4	2	0	0	0
Quinol	6	6	2	0	0	0
CoA	21	36	16	7	3	1
Succinyl - CoA	25	40	19	7	3	1

5) on doing  $E \times S$

as  $E_{17 \times 6}$  &  $S_{17 \times 5}$  to make  $E \times S$  feasible

let's take  $E^T \times S$

$$[6 \times 17 * 17 \times 5 = 6 \times 5]$$

on doing  $E^T \times S$  we have

6 rows

5 columns

$$6) \quad E^T \times S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

element (2,1)  
that means  $E^T$  second row &  $S$  first column  
is unbalance

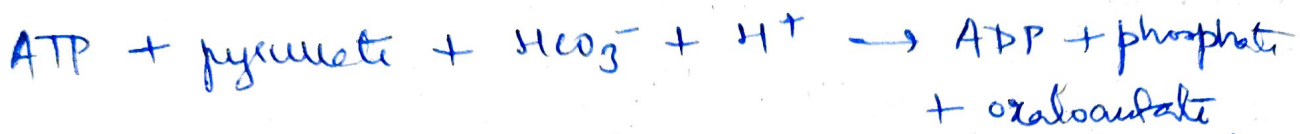
$E^T \times S$  signifies in Row 1 H is unbalanced on  
product side (one more  $H^+$  extra)



adding one  $H^+$  in reactant will make it balanced

Balanced Rxn (a)

EC : 6.4.1.1



7) EC ~~≠~~ 6.2.1.5

6. ligases

6.2 forming carbon sulfur bonds

6.2.1 Acid-thiol ligases (GDP-forming)

6.2.1.5 succinate-CoA ligase (ADP forming)

8) other pathway (in S. cerevisiae) which the reaction represented by EC- 6.2.1.5 participate in are,

- TCA cycle (citrate cycle)

- propanoate metabolism

- Exosome pathways

Gene encoding EC 6.2.1.5 → YGR244G

NCBI gene Id → 853159

NCBI protein Id → NP-011760

uniprot ; P53312