

Module 2: Crystallography

Miller Indices of Direction

- A direction in a lattice is expressed as a set of three integers with the same relationship as the components of a vector in that direction.
- The three vector components are expressed in multiples of basis vectors, and then the three integers are reduced to smallest set of integers retaining the relationship between them.
- e.g. [h k l] read as direction hkl, is the direction indicated by a vector $\vec{ha} + \vec{kb} + \vec{lc}$ from origin.

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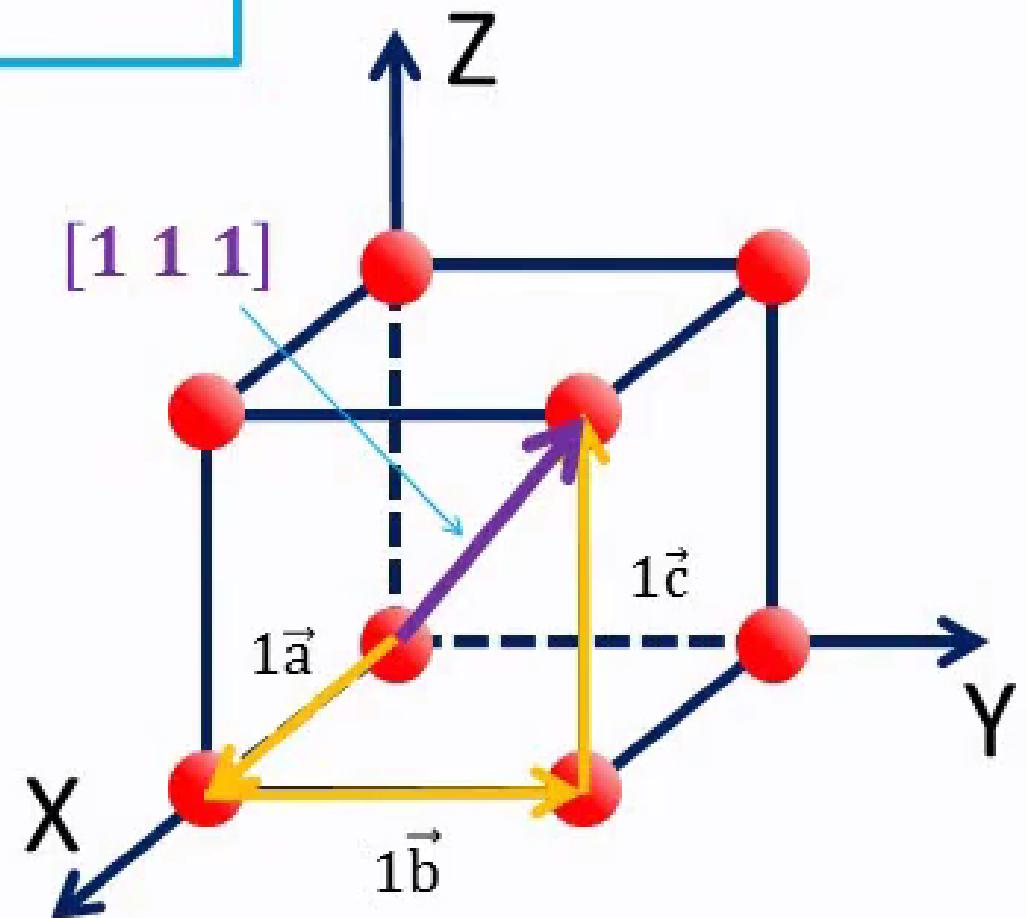
Miller Indices of Direction

- As in case of planes, many directions in a lattice are equivalent, depending only on the arbitrary choice of orientation for the axes. °
- Such equivalent direction indices are placed in angular brackets <>.
- For example, the crystals axes in a cubic lattice [100], [010] and [001] are equivalent and are called as <100> directions.
- In cubic lattice, a direction **[hkl]** is perpendicular to the plane **(hkl)**. This is convenient in analysis of cubic unit cells. However, it is not necessarily true in non-cubic systems

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Drawing the Direction in Unit Cell

$$[1 \ 1 \ 1]$$
$$1\vec{a} + 1\vec{b} + 1\vec{c}$$



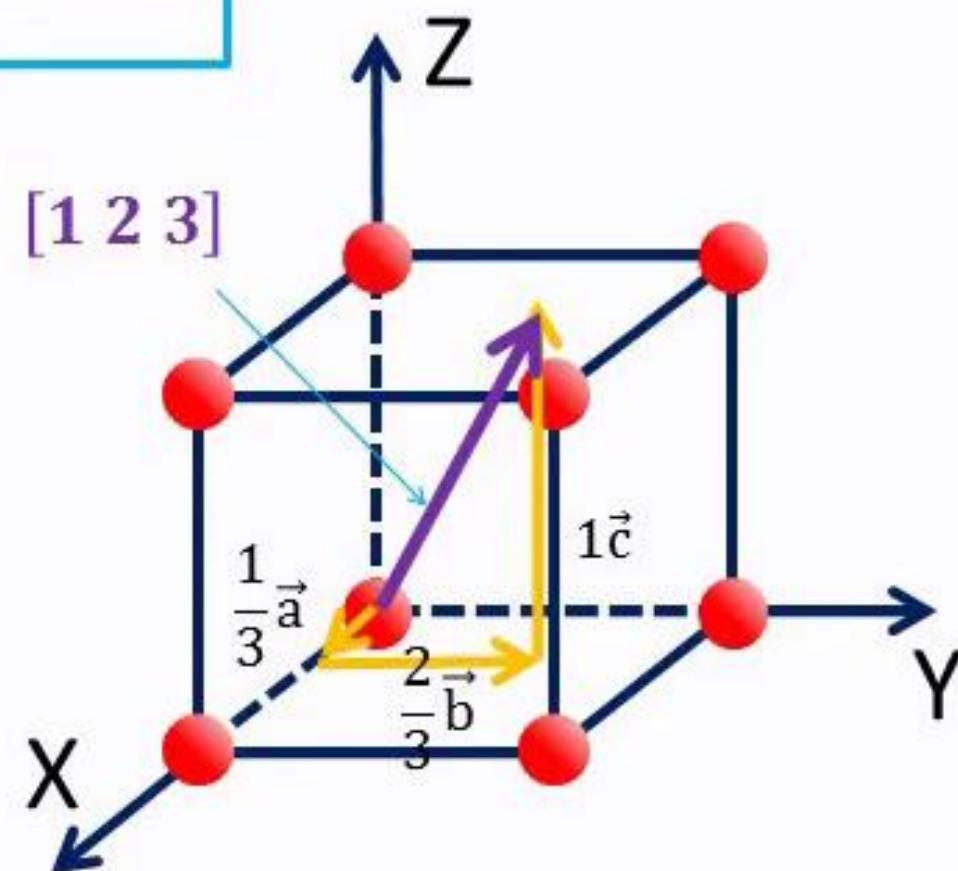
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Drawing the Direction in Unit Cell

$$[1 \ 2 \ 3]$$

$$1\vec{a} + 2\vec{b} + 3\vec{c}$$

$$\frac{1}{3}\vec{a} + \frac{2}{3}\vec{b} + 1\vec{c}$$



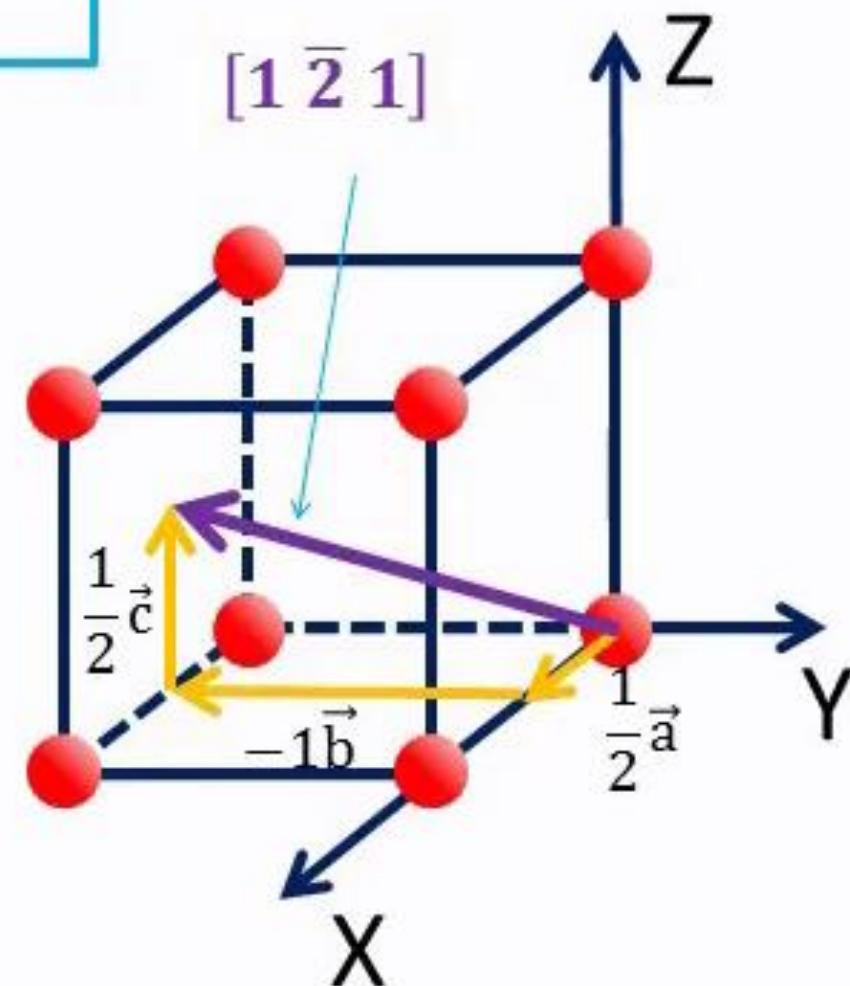
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Drawing the Direction in Unit Cell

$$[1 \bar{2} 1]$$

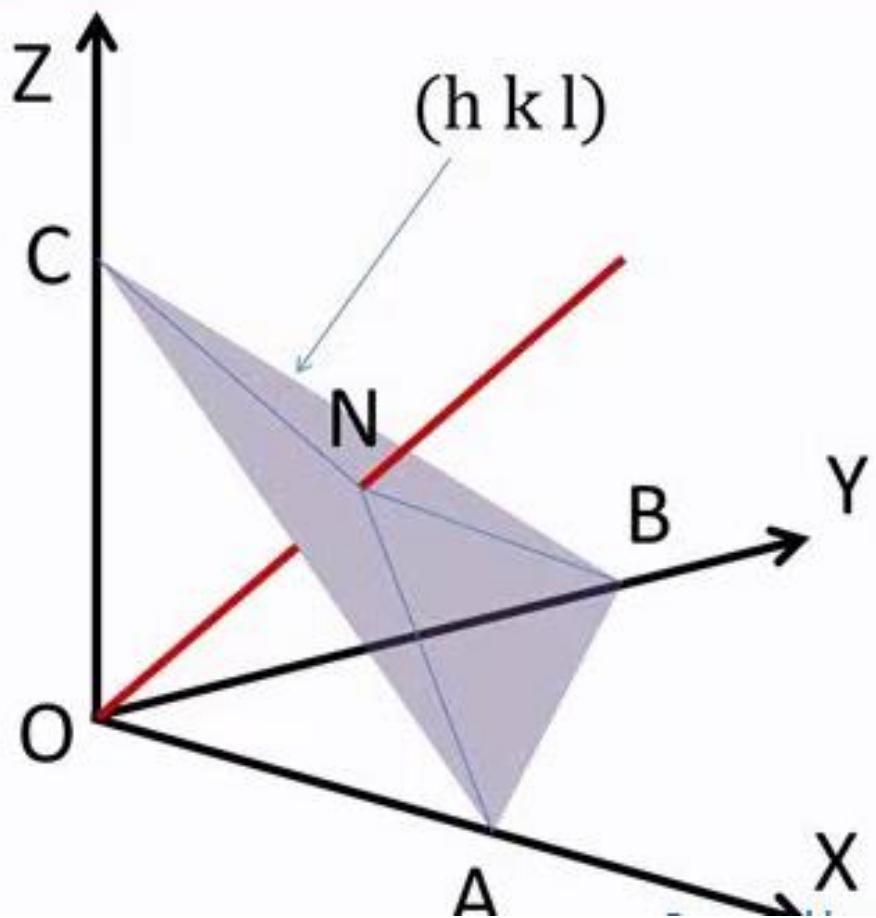
$$1\vec{a} - 2\vec{b} + 1\vec{c}$$

$$\frac{1}{2}\vec{a} - 1\vec{b} + \frac{1}{2}\vec{c}$$



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Relation between Interplanar Spacing and Cube edge in a **Cubic Crystal**



Let the plane ABC be one of the planes that has intercepts OA, OB and OC on X, Y and Z axes respectively

$$\therefore OA = \frac{a}{h}; OB = \frac{b}{k}; OC = \frac{c}{l}$$

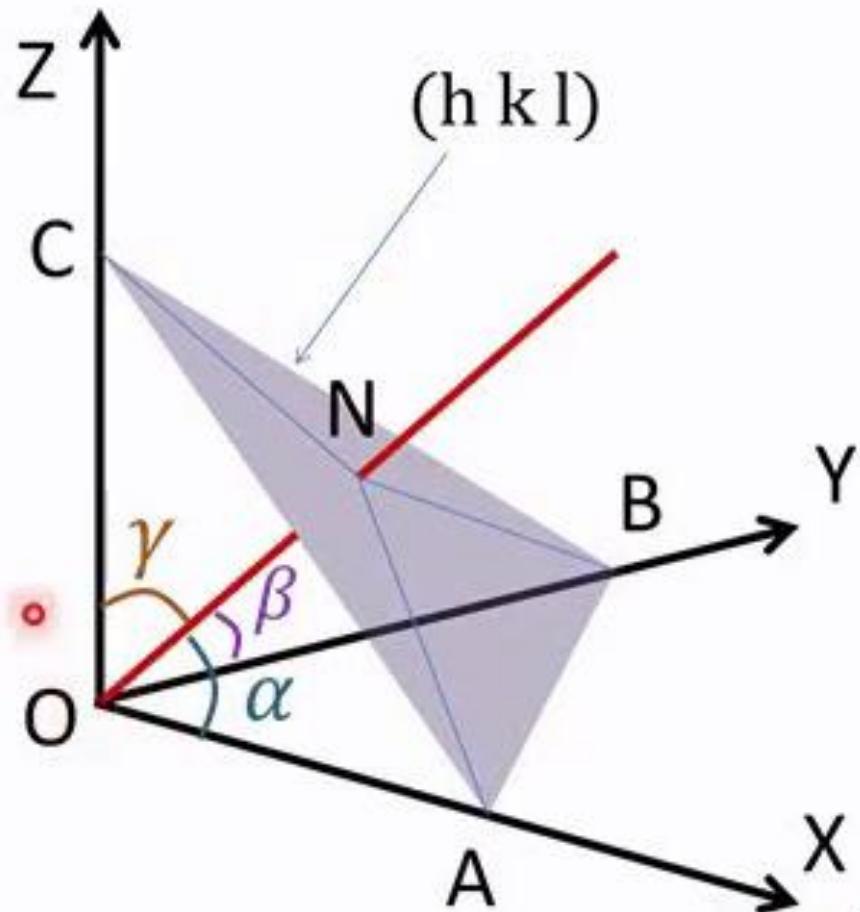
In a cubic crystal, $a = b = c$

$$\therefore OA = \frac{a}{h}; OB = \frac{a}{k}; OC = \frac{a}{l}$$

1

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Relation between Interplanar Spacing and Cube edge in a **Cubic Crystal**



$$\therefore OA = \frac{a}{h}; OB = \frac{a}{k}; OC = \frac{a}{l}$$

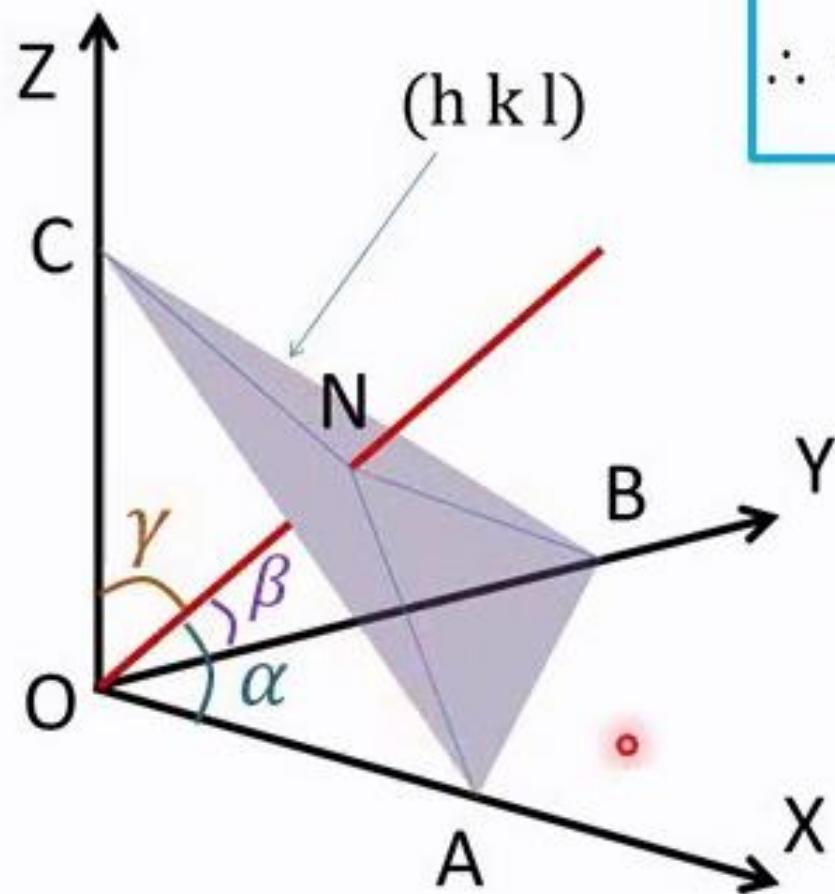
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- Let the next plane parallel to plane ABC passes through origin.
- Let ON be the perpendicular drawn on plane ABC from O.
- Hence $ON = d_{hkl}$ represents inter-planar distance.

Let α , β and γ be the angles between the coordinate axes X, Y, Z and ON respectively.

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Relation between Interplanar Spacing and Cube edge in a **Cubic Crystal**



$$\therefore OA = \frac{a}{h}; OB = \frac{a}{k}; OC = \frac{a}{l}$$

1

ON = d_{hkl}

As, $ON \perp NA$, From ΔONA , we can write

$$\cos \alpha = \frac{ON}{OA} = \frac{d}{a/h} = \frac{dh}{a}$$

2

Similarly, From ΔONB and ΔONC we can write

$$\cos \beta = \frac{ON}{OB} = \frac{d}{a/k} = \frac{dk}{a}$$

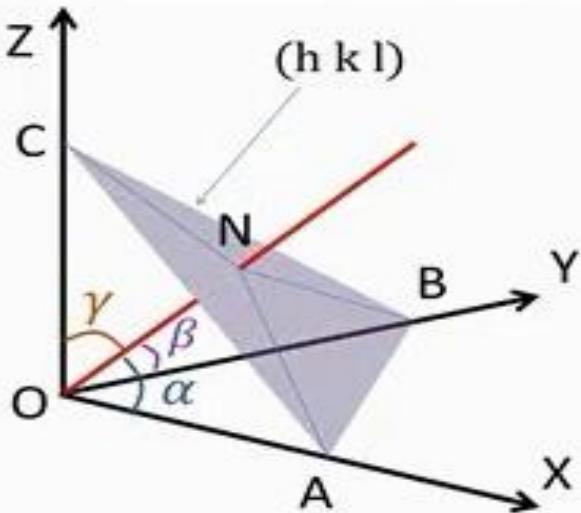
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$$\cos \gamma = \frac{ON}{OC} = \frac{d}{a/l} = \frac{dl}{a}$$

4

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Relation between Interplanar Spacing and Cube edge in a **Cubic Crystal**



$$\cos \alpha = \frac{ON}{OA} = \frac{d}{a/h} = \frac{dh}{a}$$

$$\cos \beta = \frac{ON}{OB} = \frac{d}{a/k} = \frac{dk}{a}$$

$$\cos \gamma = \frac{ON}{OC} = \frac{d}{a/l} = \frac{dl}{a}$$

We can write,

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\therefore \frac{d^2 h^2}{a^2} + \frac{d^2 k^2}{a^2} + \frac{d^2 l^2}{a^2} = 1$$

$$\therefore \frac{d^2}{a^2} (h^2 + k^2 + l^2) = 1$$

$$\therefore d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$



Silver has FCC structure and its atomic radius is 1.44 \AA^0 . Find the spacing of the (220), (200) and (111) planes.

$$\text{For FCC structure, } a = \frac{4r}{\sqrt{2}} = \frac{4 \times 1.44}{\sqrt{2}} = 4.073 \text{ \AA}^0$$

$$\text{Now, } d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$\text{For (220) plane, } d_{220} = \frac{4.073}{\sqrt{2^2 + 2^2 + 0^2}} = 1.44 \text{ \AA}^0$$

$$\text{For (200) plane, } d_{200} = \frac{4.073}{\sqrt{2^2 + 0^2 + 0^2}} = 2.036 \text{ \AA}^0$$

$$\text{For (111) plane, } d_{111} = \frac{4.073}{\sqrt{1^2 + 1^2 + 1^2}} = 2.35 \text{ \AA}^0$$

A crystal lattice plane (326) makes an intercept of 1.5 \AA^0 on X-axis in a crystal having lattice constant 1.5 \AA^0 , 2 \AA^0 and 2 \AA^0 respectively on X, Y and Z axis. Find Y and Z intercept.

Given : $(h k l) = (3 2 6)$, $a = 1.5 \text{ \AA}^0$, $b = 2 \text{ \AA}^0$, $c = 2 \text{ \AA}^0$

$$\text{Intercept on X - axis} = \frac{a}{h} = \frac{1.5 \text{ \AA}^0}{3} = 0.5 \text{ \AA}^0$$

$$\text{Intercept on Y - axis} = \frac{b}{k} = \frac{2 \text{ \AA}^0}{2} = 1 \text{ \AA}^0$$

$$\text{Intercept on Z - axis} = \frac{c}{l} = \frac{2 \text{ \AA}^0}{6} = \frac{1}{3} \text{ \AA}^0$$

$$\therefore \text{Intercept on Y - axis} = 1 \text{ \AA}^0 \times 3 \\ = 3 \text{ \AA}^0$$

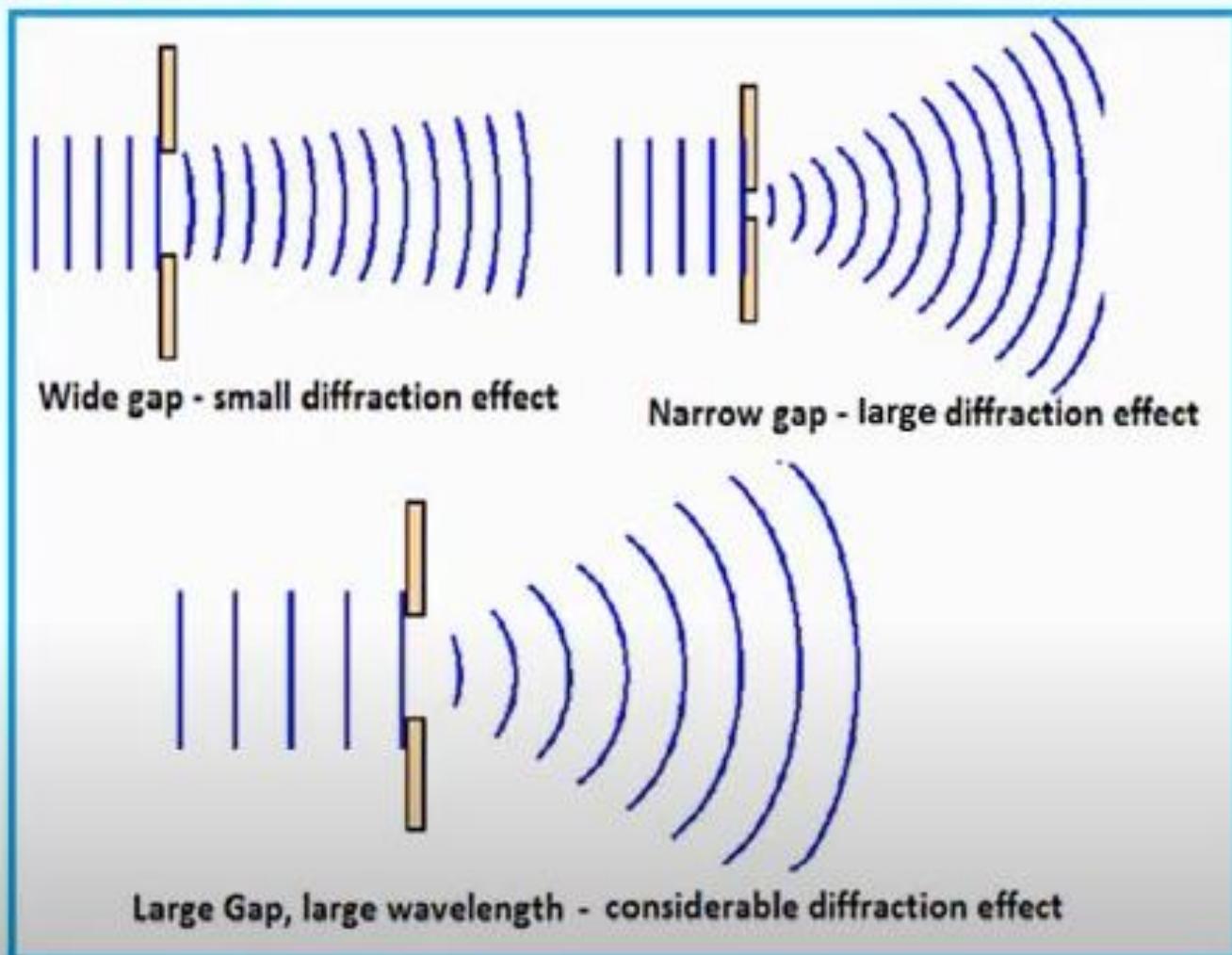
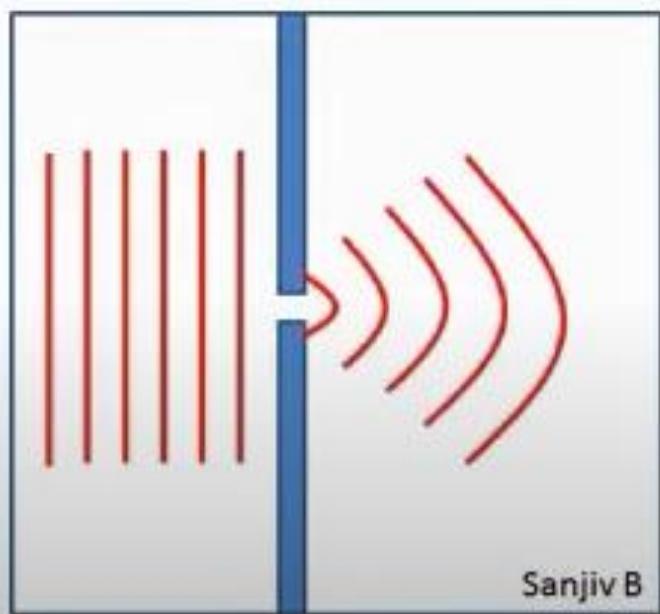
But actual intercept on X-axis is given as 1.5 \AA^0 which is 3 times of calculated X-intercept. So the plane given in the problem is parallel to the plane having intercepts 0.5 \AA^0 , 1 \AA^0 and $\frac{1}{3} \text{ \AA}^0$. Its Y and Z intercepts can be obtained by multiplying calculated Y and Z intercepts by 3.

$$\therefore \text{Intercept on Z - axis} = \frac{1}{3} \text{ \AA}^0 \times 3 \\ = 1 \text{ \AA}^0$$

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X-ray Diffraction

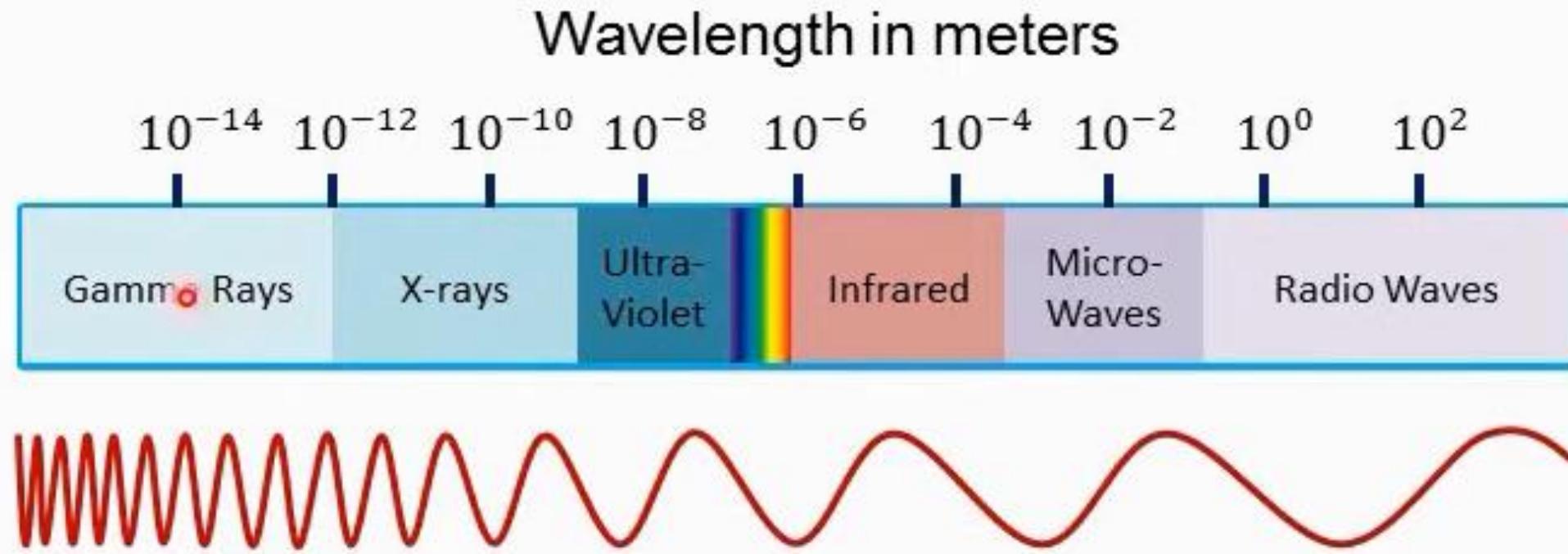
What is diffraction?



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X-ray Diffraction

What are X-rays?

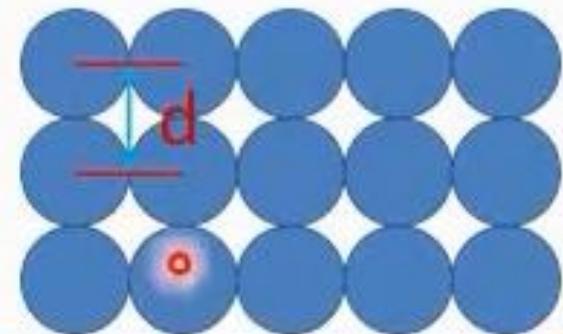


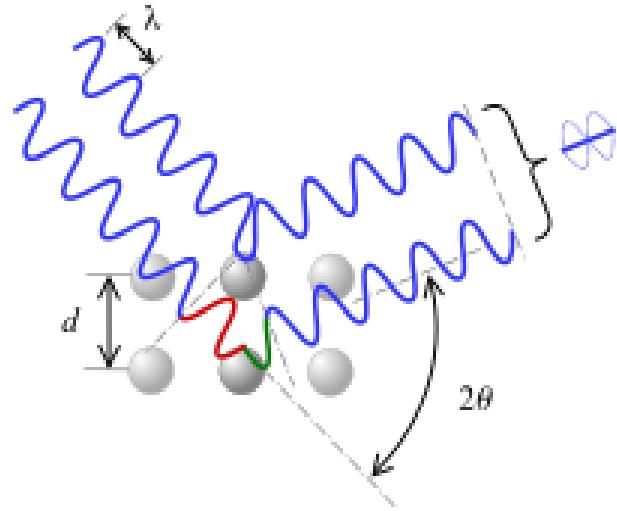
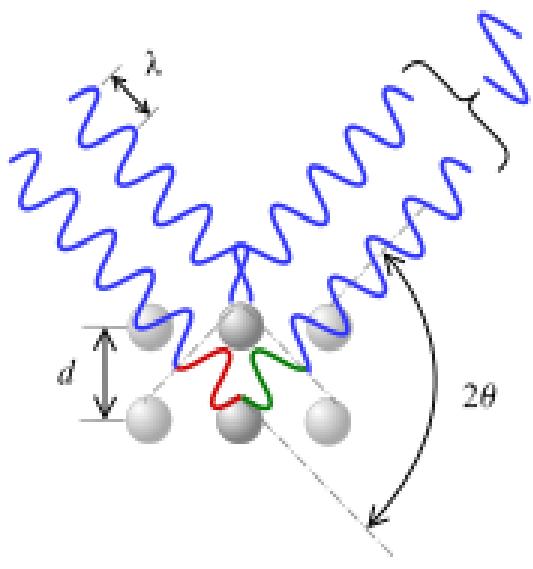
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X-ray Diffraction

Why X-rays are used in study of Crystal Physics?

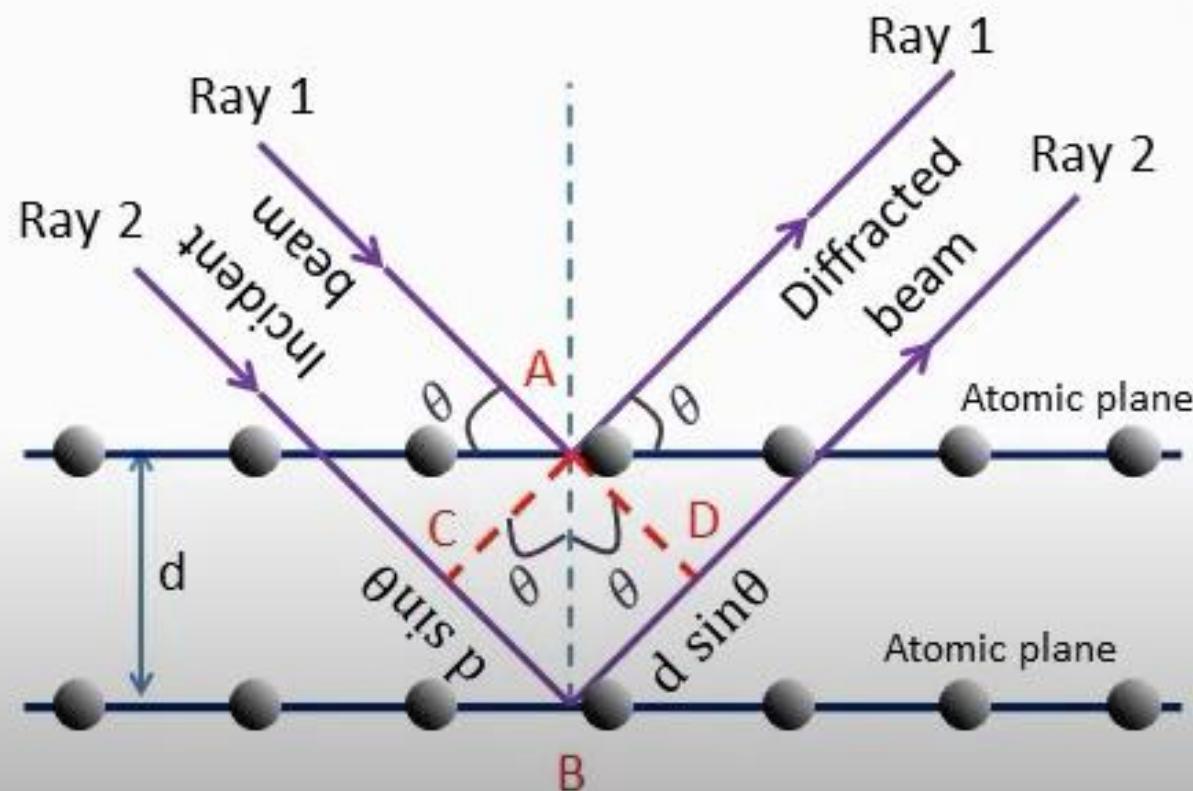
- Arrangement of atoms inside the crystal can be studied by performing diffraction experiment.
- A crystal which consists of three dimensional arrays of regularly spaced atoms can be used as a grating.
- Diffraction pattern can be obtained if the width of the slit in grating is comparable to wavelength of the wave used in the experiment.
- As the separation between the atoms (used as a slit) is of the order of 1 \AA^0 , we will get an observable pattern if we use a wave having wavelength of this order and wavelength of X-rays is of the order of few \AA^0
- Therefore X-rays are used in study of crystal physics.





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Bragg's Law of X-ray Diffraction



From figure, we can write -

$$CB = BD = d \sin\theta$$

Path difference between Ray 1 and Ray 2 is -

$$\Delta = CB + BD$$

$$= d \sin\theta + d \sin\theta = 2d \sin\theta$$

Condition for maximum is -

path difference $\Delta = n \lambda$

where $n = 1, 2, 3, 4, \dots$

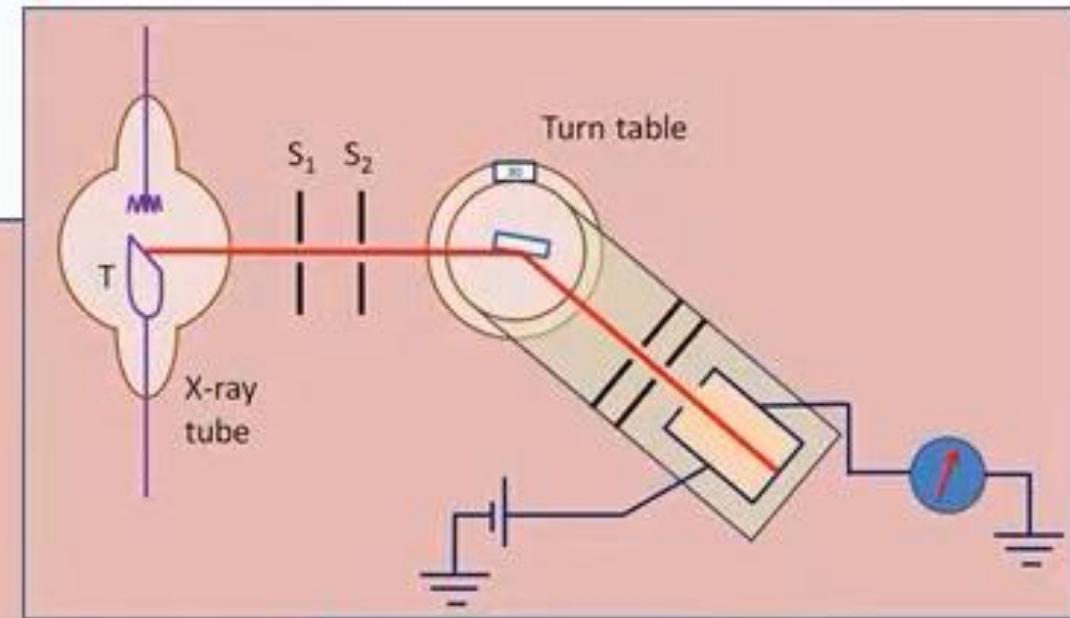
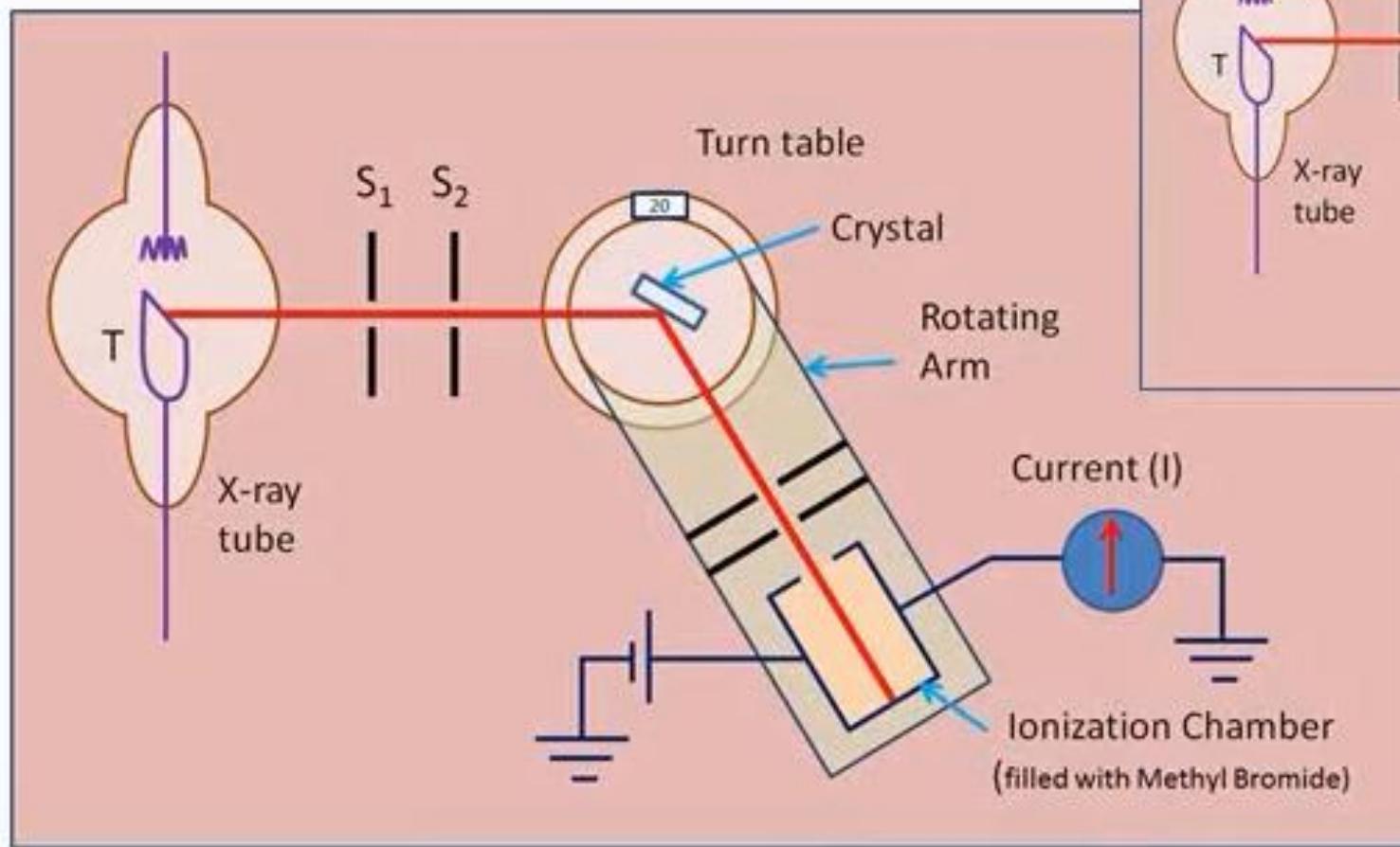
for 1st, 2nd, 3rd order maximum respectively

$$\therefore 2 d \sin\theta = n \lambda$$

This is known as
Bragg's law of X-ray diffraction

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Bragg's X-ray Spectrometer

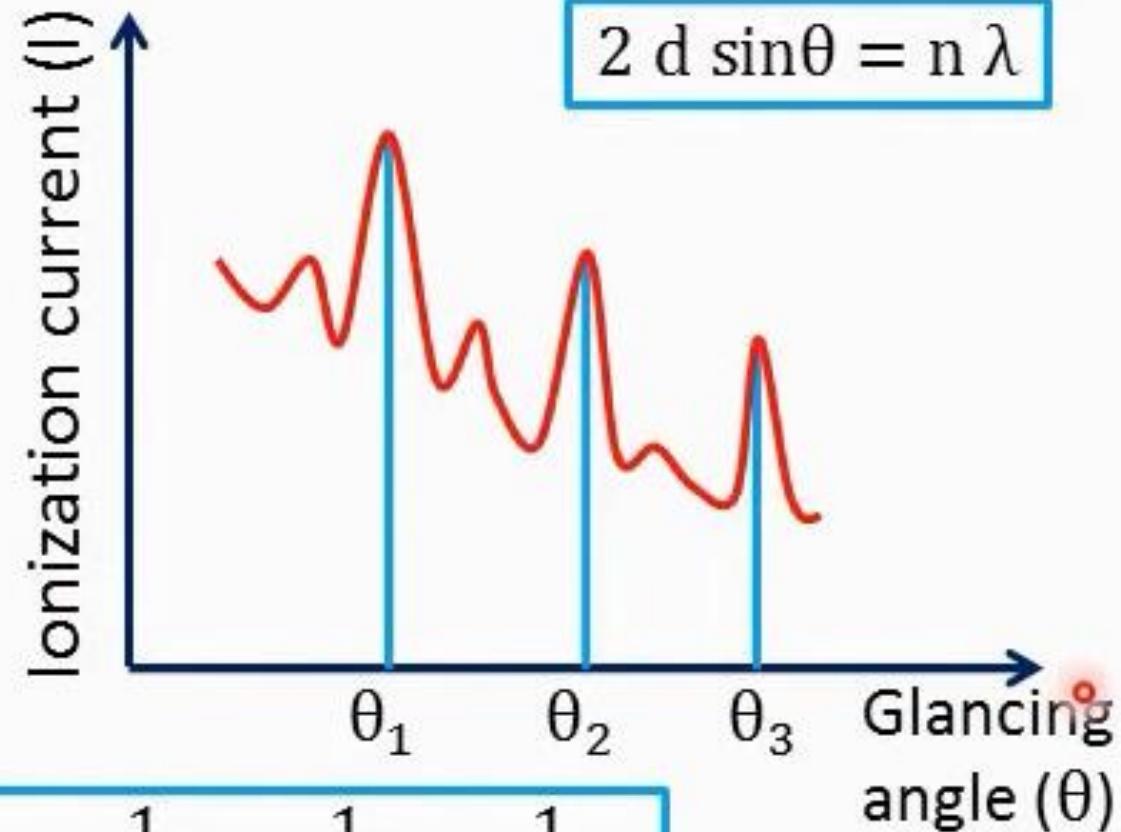
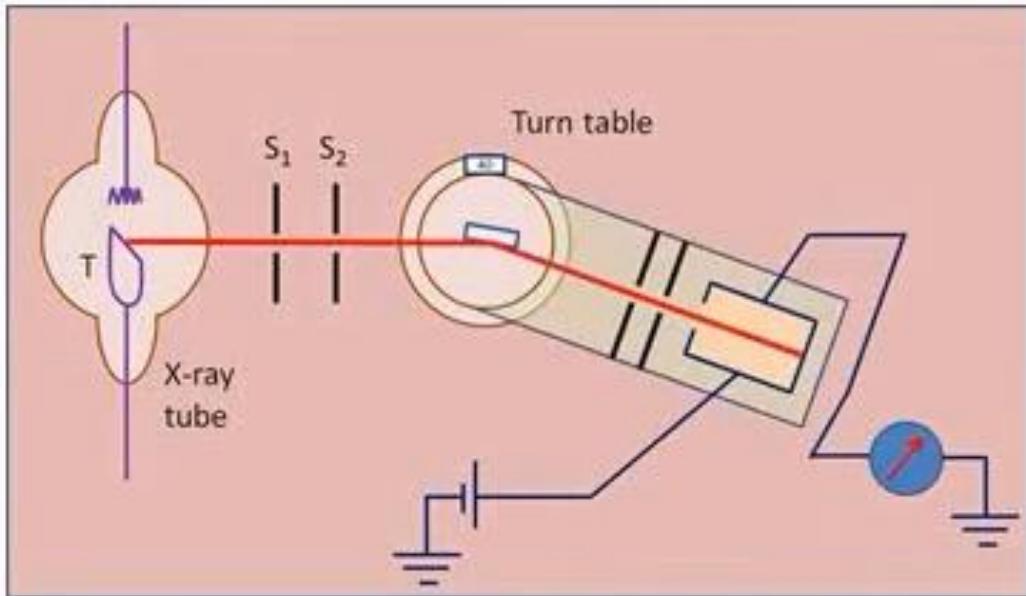


A Crystal is used as
diffraction grating

$$2 d \sin\theta = n \lambda$$

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Bragg's X-ray Spectrometer



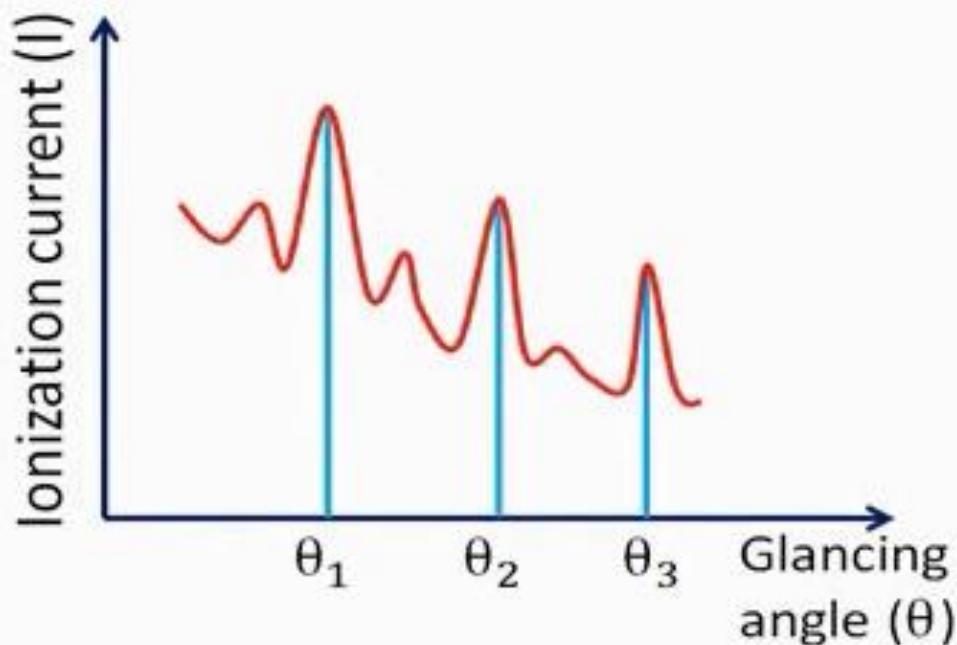
$$\therefore d \propto \frac{\lambda}{\sin\theta}$$

$$\therefore d_{100} : d_{110} : d_{111} = \frac{1}{\sin\theta_1} : \frac{1}{\sin\theta_2} : \frac{1}{\sin\theta_3}$$

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Bragg's X-ray Spectrometer

$$\therefore d_{100} : d_{110} : d_{111} = \frac{1}{\sin\theta_1} : \frac{1}{\sin\theta_2} : \frac{1}{\sin\theta_3}$$



	d ₁₀₀	d ₁₁₀	d ₁₁₁
Simple Cubic	1	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{3}}$
Body Centered Cubic	1	$\sqrt{2}$	$\frac{1}{\sqrt{3}}$
Face Centered Cubic	1	$\frac{1}{\sqrt{2}}$	$\frac{2}{\sqrt{3}}$

For NaCl, first order maxima for (100), (110) and (111) planes are obtained as 5.9° , 8.4° and 5.2° respectively. Which crystal structure is followed by NaCl?

$$d_{100}:d_{110}:d_{111} = \frac{1}{\sin\theta_1} : \frac{1}{\sin\theta_2} : \frac{1}{\sin\theta_3}$$

$$\therefore d_{100}:d_{110}:d_{111} = \frac{1}{\sin(5.9)} : \frac{1}{\sin(8.4)} : \frac{1}{\sin(5.2)}$$

$$\therefore d_{100}:d_{110}:d_{111} = 9.73 : 6.84 : 11.03$$

$$\therefore d_{100}:d_{110}:d_{111} = 1 : \frac{1}{\sqrt{2}} : \frac{2}{\sqrt{3}}$$

This is a ratio for FCC structure.
Hence NaCl follows FCC structure.

Bragg's reflection of the first order was observed at 21.7° for parallel planes of a crystal under test. If the wavelength of X-rays used is 1.54 \AA° , find the interplanar spacing for the planes in the crystal.

$$n = 1, \quad \lambda = 1.54 \text{ \AA}^{\circ} = 1.54 \times 10^{-10} \text{ m}, \quad \theta = 21.7^{\circ}$$

According to Bragg's Law of X-ray Diffraction

$$2 d \sin\theta = n \lambda$$

$$\begin{aligned}\therefore d &= \frac{n\lambda}{2 \sin\theta} = \frac{1 \times 1.54 \times 10^{-10}}{2 \times \sin 21.7} \\ &= 2.083 \times 10^{-10} \text{ m} \\ &= 2.083 \text{ \AA}^{\circ}\end{aligned}$$

The spacing between the principle planes in a crystal of NaCl is 2.82 \AA^0 . It is found that the first order Bragg's reflection occurs at 10° .

- a) What is the wavelength of X-rays?
- b) At what angle, the second order reflection occurs?
- c) What is the highest order of reflection seen?

$$d = 2.82 \text{ \AA}^0 = 2.82 \times 10^{-10} \text{ m}, \quad n = 1, \quad \theta = 10^\circ$$

a) According to Bragg's Law of X-ray Diffraction $2 d \sin\theta = n \lambda$

$$\therefore \lambda = \frac{2 d \sin\theta}{n} = \frac{2 \times 2.82 \times 10^{-10} \times \sin 10}{1} = 9.79 \times 10^{-11} \text{ m} = 0.979 \text{ \AA}^0$$

b) For $n = 2 \quad \therefore \theta = \sin^{-1} \left(\frac{n \lambda}{2 d} \right) = \sin^{-1} \left(\frac{2 \times 9.79 \times 10^{-11}}{2 \times 2.82 \times 10^{-10}} \right) = 20.31^\circ$

c) For highest order, maximum value of $\sin\theta = 1$

$$\therefore n_{\max} = \frac{2 d (\sin\theta)_{\max}}{\lambda} = \frac{2 \times 2.82 \times 10^{-10} \times 1}{9.79 \times 10^{-11}} = 5.76$$

As n is an integer, highest order than can be seen is 5

Calculate the smallest glancing angle at which k-copper line of 1.549 \AA^0 will be reflected from crystal having atomic spacing 4.255 \AA^0 .

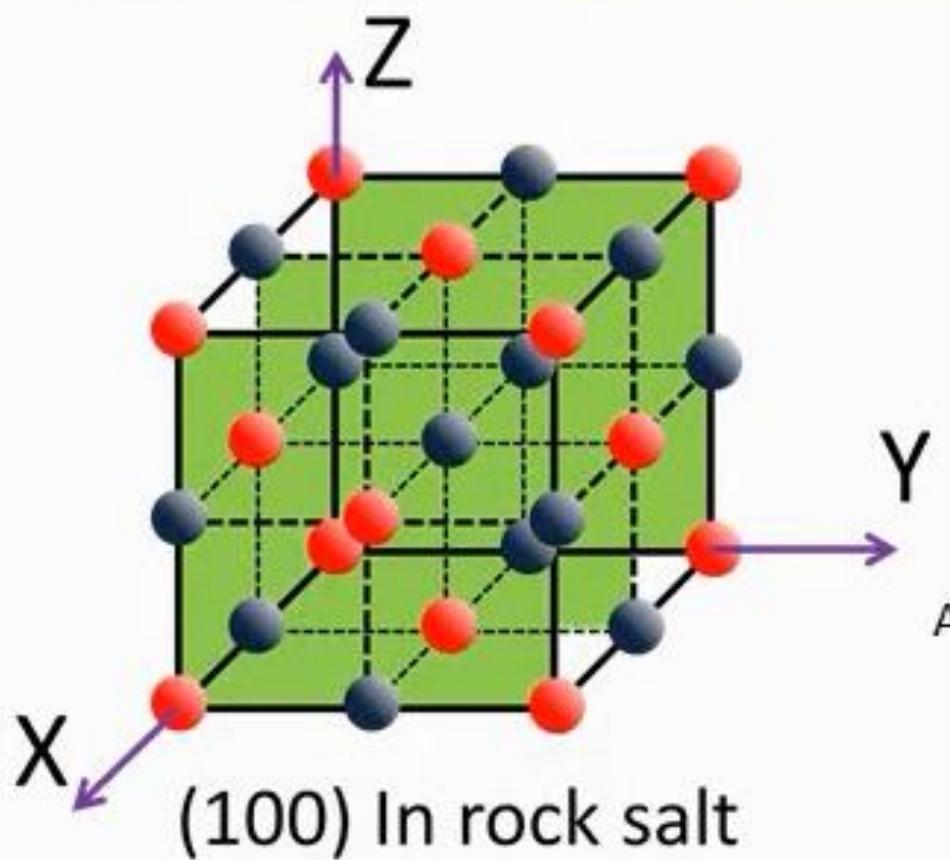
$$\lambda = 1.549 \text{ \AA}^0 = 1.549 \times 10^{-10} \text{ m}, \quad \text{smallest glancing angle means } n = 1,$$
$$d = 4.255 \text{ \AA}^0 = 4.255 \times 10^{-10} \text{ m}, \quad \theta = ?$$

According to Bragg's Law of X-ray Diffraction

$$2 d \sin \theta = n \lambda$$

$$\therefore \theta = \sin^{-1} \left(\frac{n \lambda}{2 d} \right) = \sin^{-1} \left(\frac{1 \times 1.549 \times 10^{-10}}{2 \times 4.255 \times 10^{-10}} \right) = 10.48^0$$

Calculate the glancing angle on the plane (100) of a rock salt ($a = 2.814 \text{ \AA}^0$) corresponding to 2nd order diffraction maximum for X-rays of wavelength 0.714 \AA^0 .



$$(h k l) = (1 0 0)$$

$$\text{Lattice constant } a = 2.814 \text{ \AA}^0 = 2.814 \times 10^{-10} \text{ m}$$

$$\lambda = 0.714 \text{ \AA}^0 = 0.714 \times 10^{-10} \text{ m}, \quad n = 2,$$

$$\theta = ?$$

From the unit cell of rock salt, for plane (100), inter planar spacing is -

$$d = \frac{a}{2} = \frac{2.814 \times 10^{-10}}{2} = 1.407 \times 10^{-10} \text{ m}$$

According to Bragg's Law of X-ray Diffraction

$$2 d \sin\theta = n \lambda$$

$$\therefore \theta = \sin^{-1} \left(\frac{n \lambda}{2 d} \right) = \sin^{-1} \left(\frac{2 \times 0.714 \times 10^{-10}}{2 \times 1.407 \times 10^{-10}} \right) = 30.5^\circ$$

X-rays of wavelength 1.5418 \AA^0 are diffracted by (111) planes in a crystal at an angle of 30° in the first order. Calculate the lattice constant of the cubic unit cell.

$$\lambda = 1.5418 \text{ \AA}^0 = 1.5418 \times 10^{-10} \text{ m}, \quad (h k l) = (1 1 1), \quad \theta = 30^\circ$$

$$n = 1, \quad \text{Lattice constant } a = ?$$

inter planar spacing is given by - $d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

According to Bragg's Law of X-ray Diffraction

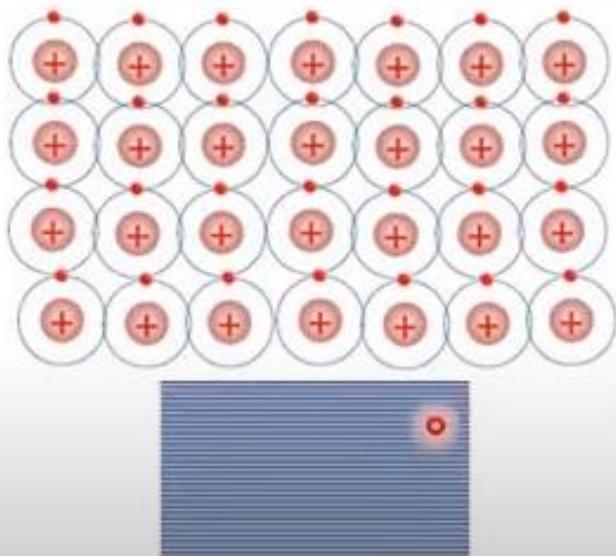
$$2 d \sin \theta = n \lambda$$

$$\therefore d = \frac{n \lambda}{2 \sin \theta} = \frac{1 \times 1.5418 \times 10^{-10}}{2 \times \sin 30^\circ} = 1.5418 \times 10^{-10} \text{ m} = 1.5418 \text{ \AA}^0$$

$$\therefore a = d_{hkl} \times \sqrt{h^2 + k^2 + l^2} = 1.5418 \times \sqrt{1^2 + 1^2 + 1^2} = 2.670 \text{ \AA}^0$$

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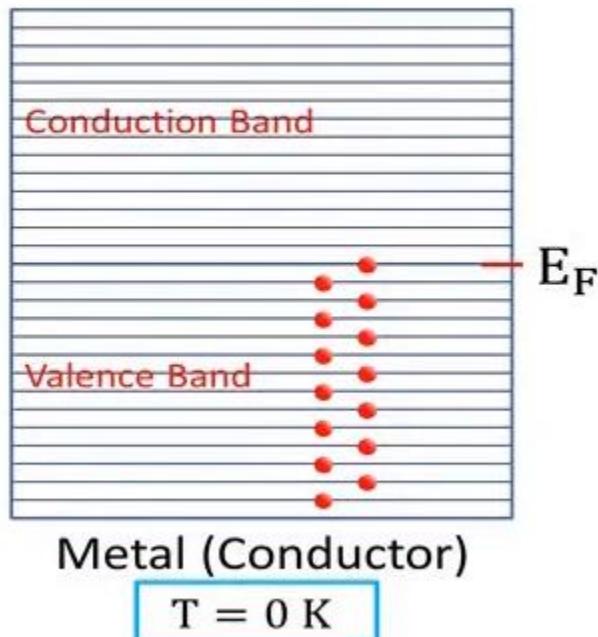
Fermi Level



- A solid is formed by bringing a large number of atoms together.
- When large number of atoms come together, the energy levels of electrons split into large number of closely packed levels and **energy bands** are formed.

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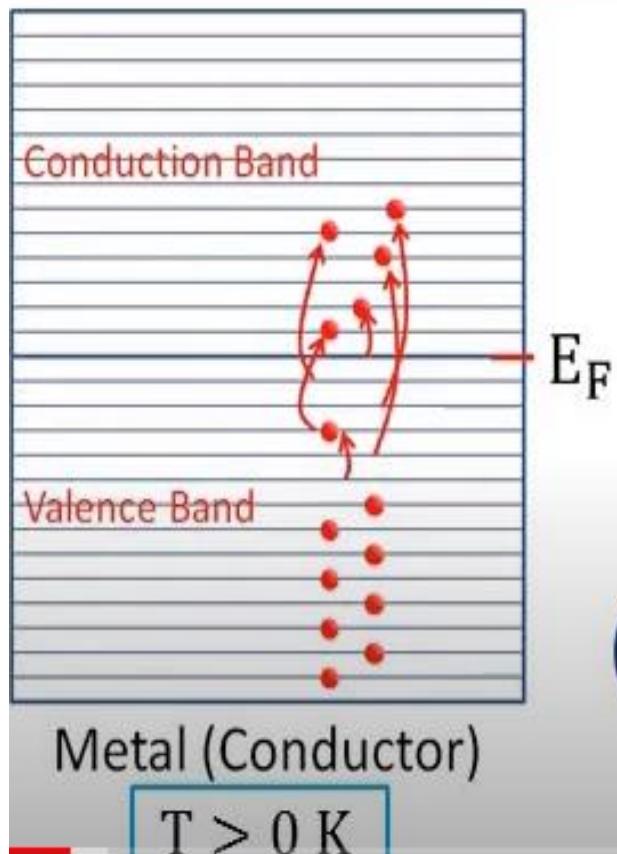
Fermi Level



- In metal, there is one partially filled band which is a result of conduction band overlapping with valence band.
- In this band lowest energy levels are filled first.
- The highest occupied energy level at absolute zero temperature (0K) is called as **Fermi level**.
- Energy corresponding to it is called as **Fermi energy denoted by E_F**

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Fermi Level and Fermi-Dirac Distribution Function



- At temperature $T > 0\text{K}$, the distribution of electrons over a range of allowed energy levels at thermal equilibrium is given by Fermi-Dirac Distribution function

$$f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

$f(E)$ is the probability of occupancy for energy level E

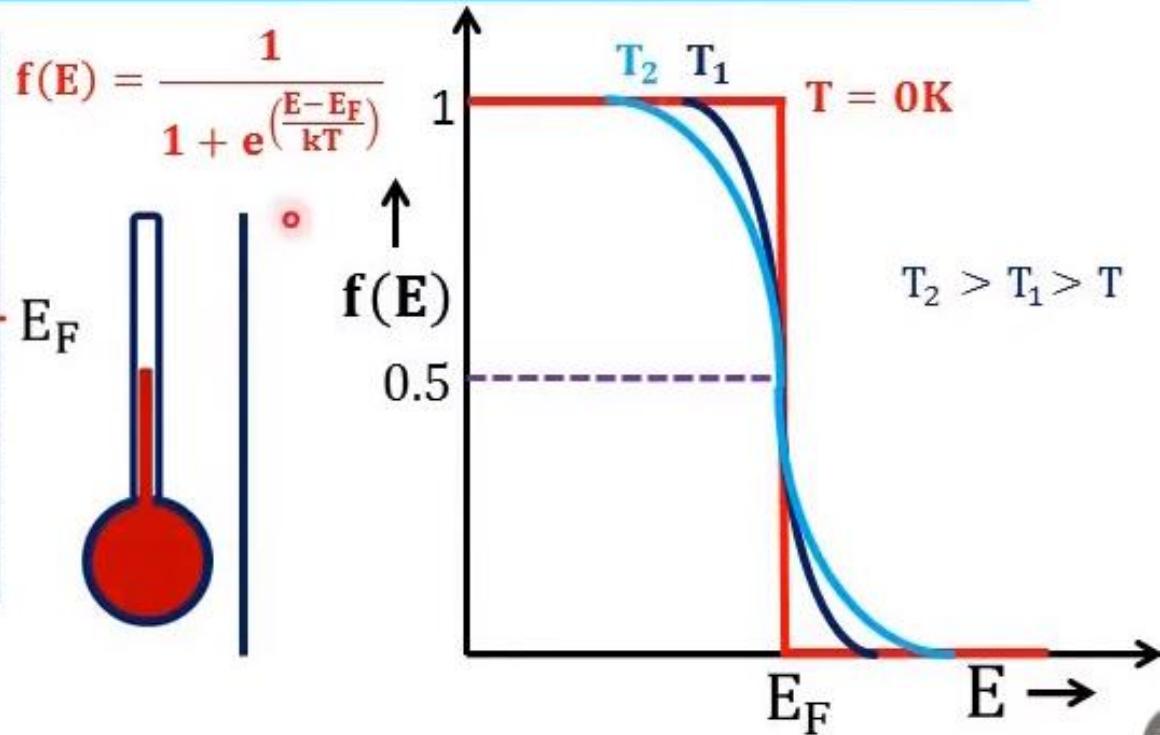
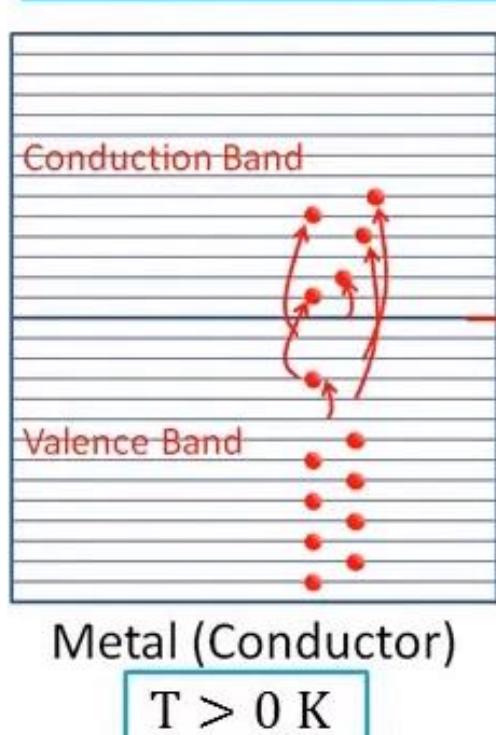
E_F is Fermi energy

T is temperature in ^0K and

$$k = 1.38 \times 10^{-23} \text{ J/K} = 8.625 \times 10^{-5} \text{ eV/K}$$

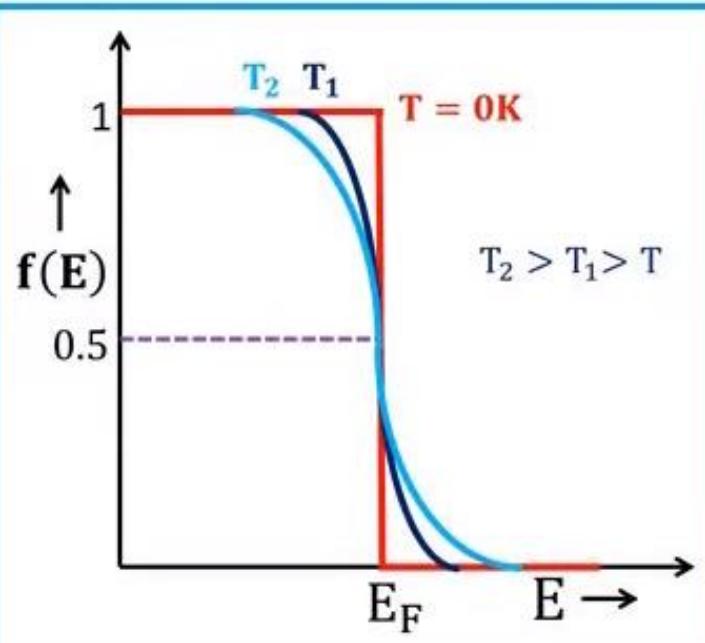
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Fermi-Dirac Distribution Function at various temperatures



Module-3 : Semiconductor Physics

Fermi-Dirac Distribution Function at various temperatures



$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

At $T = 0 K$, For $E < E_F$

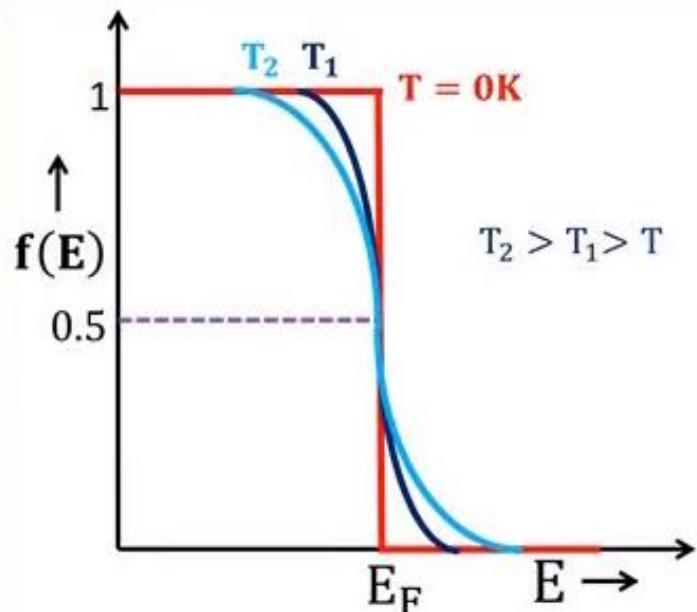
$$e^{\left(\frac{E-E_F}{kT}\right)} = e^{\frac{-ve\ number}{0}} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$\therefore f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1 + 0} = 1$$

This indicates all energy levels below E_F are completely filled at absolute zero temperature \circ

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Fermi-Dirac Distribution Function at various temperatures



$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

At $T = 0\text{ K}$, For $E > E_F$

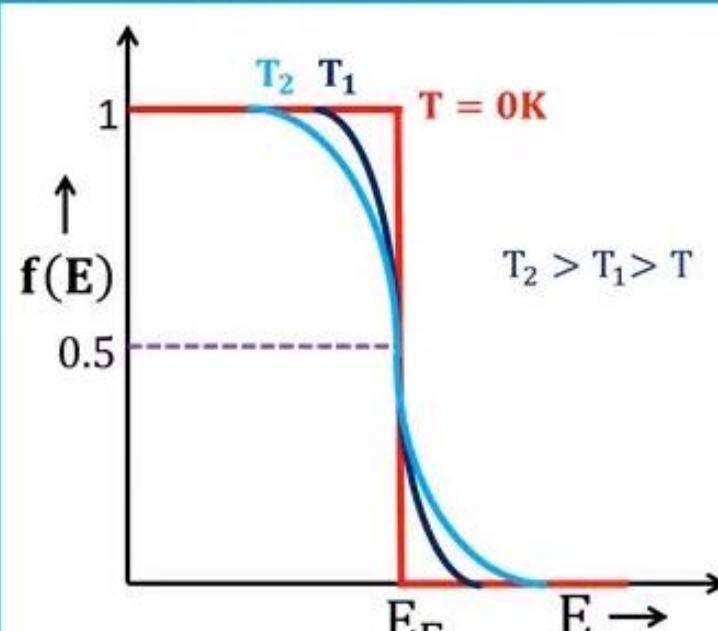
$$e^{\left(\frac{E-E_F}{kT}\right)} = e^{\frac{+ve\ number}{0}} = e^{+\infty} = \infty$$

$$\therefore f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1 + \infty} = 0$$

This indicates all energy levels above E_F are completely empty at absolute zero temperature

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Fermi-Dirac Distribution Function at various temperatures



$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

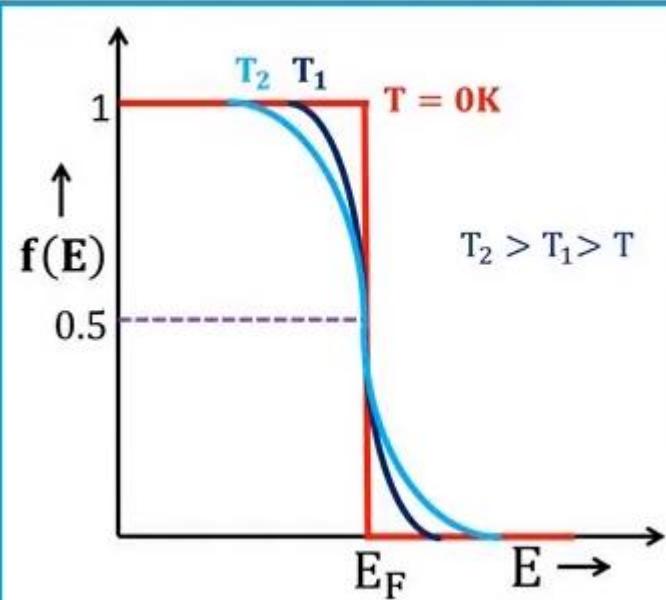
At $T = 0\text{ K}$, For $E = E_F$

$$e^{\left(\frac{E-E_F}{kT}\right)} = e^0 = \text{NOT DEFINED}$$

Fermi-Dirac Distribution function can not give us probability of occupancy of Fermi level at 0 K.

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Fermi-Dirac Distribution Function at various temperatures



$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

At $T > 0$ K, For $E = E_F$

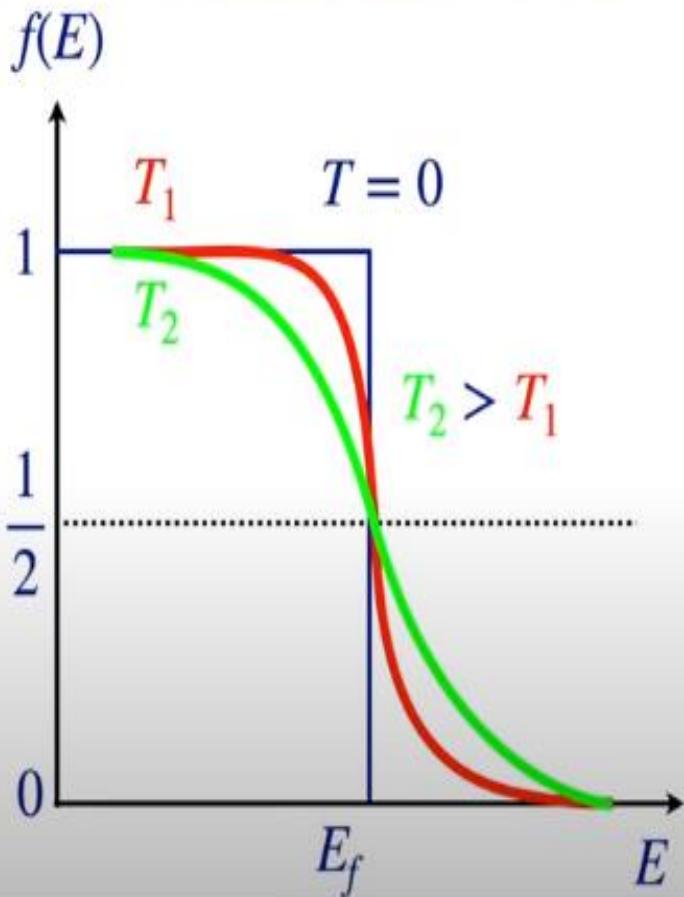
$$e^{\left(\frac{E-E_F}{kT}\right)} = e^{\frac{0}{kT}} = e^0 = 1$$

$$\therefore f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1 + 1} = \frac{1}{2}$$

At $T > 0$ K, probability of occupancy for Fermi level is always half

Fermi Function

Fig. 4: The Fermi function



Case I: At $T = 0K$, for $E < E_f$

$$f(E) = \frac{1}{1 + e^{\frac{(E - E_f)}{kT}}} = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + 0} = 1$$

Case II: At $T = 0K$, for $E > E_f$

$$f(E) = \frac{1}{1 + e^{\frac{(E - E_f)}{kT}}} = \frac{1}{1 + e^{+\infty}} = \frac{1}{1 + \infty} = 0$$

Case III: At $T = 0K$, for $E = E_f$

$$f(E) = \frac{1}{1 + e^{\frac{(E - E_f)}{kT}}} = \frac{1}{1 + e^{0/0}} = \text{Indeterminate}$$

Case IV: At $T > 0K$, for $E = E_f$

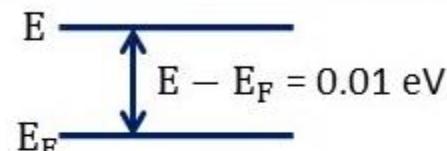
$$f(E) = \frac{1}{1 + e^{\frac{(E - E_f)}{kT}}} = \frac{1}{1 + e^0} = \frac{1}{1 + 1} = \frac{1}{2} = 0.5$$

Module-3 : Semiconductor Physics

Calculate probability of non-occupancy for the energy level which lies 0.01 eV above the Fermi energy level at 27 °C.

Given : $T = 27 \text{ } ^\circ\text{C} = 300 \text{ } ^\circ\text{K}$, $k = 8.625 \times 10^{-5} \frac{\text{eV}}{\text{°K}}$

$$\text{Probability of occupancy } f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{kT}\right)}}$$



$$\therefore \text{Probability of non-occupancy} = 1 - f(E) = 1 - \frac{1}{1 + e^{\left(\frac{E - E_F}{kT}\right)}}$$

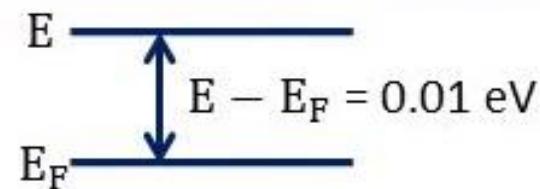
$$= 1 - \frac{1}{1 + e^{\left(\frac{0.01}{8.625 \times 10^{-5} \times 300}\right)}} \\ = 0.595$$

Module-3 : Semiconductor Physics

Calculate probability of non-occupancy for the energy level which lies 0.01 eV above the Fermi energy level at 27 °C.

Given : $T = 27 \text{ } ^\circ\text{C} = 300 \text{ } ^\circ\text{K}$, $k = 8.625 \times 10^{-5} \frac{\text{eV}}{\text{ } ^\circ\text{K}}$

$$\text{Probability of occupancy } f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{kT}\right)}}$$



$$\therefore \text{Probability of non-occupancy} = 1 - f(E) = 1 - \frac{1}{1 + e^{\left(\frac{E - E_F}{kT}\right)}}$$

Module-3 : Semiconductor Physics

Fermi level for silver is 5.5 eV. Find out the energy for which the probability of occupancy at 300 K is 0.9.

$$\text{Given : } T = 27^\circ\text{C} = 300^0\text{K}, \quad k = 8.625 \times 10^{-5} \frac{\text{eV}}{\text{K}}, \quad E_F = 5.5 \text{ eV}$$

when $f(E) = 0.9, E = ?$

$$\text{Probability of occupancy } f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} \quad \therefore E - E_F = kT \ln\left(\frac{1}{f(E)} - 1\right)$$

$$\therefore 1 + e^{\left(\frac{E-E_F}{kT}\right)} = \frac{1}{f(E)}$$

$$\therefore E = E_F + kT \ln\left(\frac{1}{f(E)} - 1\right)$$

$$\therefore e^{\left(\frac{E-E_F}{kT}\right)} = \frac{1}{f(E)} - 1$$

$$\therefore \frac{E - E_F}{kT} = \ln\left(\frac{1}{f(E)} - 1\right)$$

For $f(E) = 0.9$

$$\begin{aligned} E &= 5.5 + 8.625 \times 10^{-5} \times 300 \ln\left(\frac{1}{0.9} - 1\right) \\ &= 5.443 \text{ eV} \end{aligned}$$

Module-3 : Semiconductor Physics

Fermi level in potassium is 2.1 eV. What are the energies for which the probability of occupancy at 300 K are 0.99 and 0.01.

Given : $T = 27^\circ\text{C} = 300^\circ\text{K}$, $k = 8.625 \times 10^{-5} \frac{\text{eV}}{\text{°K}}$, $E_F = 2.1 \text{ eV}$

when $f(E) = 0.99$ and $f(E) = 0.01$, $E = ?$

$$\text{Probability of occupancy } f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{kT}\right)}} \quad \therefore E = E_F + kT \ln\left(\frac{1}{f(E)} - 1\right)$$

$$\therefore 1 + e^{\left(\frac{E - E_F}{kT}\right)} = \frac{1}{f(E)}$$

$$\therefore e^{\left(\frac{E - E_F}{kT}\right)} = \frac{1}{f(E)} - 1$$

$$\therefore \frac{E - E_F}{kT} = \ln\left(\frac{1}{f(E)} - 1\right)$$

For $f(E) = 0.99$

$$E = 2.1 + 8.625 \times 10^{-5} \times 300 \ln\left(\frac{1}{0.99} - 1\right) = 1.981 \text{ eV}$$

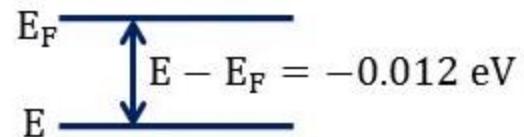
For $f(E) = 0.01$

$$E = 2.1 + 8.625 \times 10^{-5} \times 300 \ln\left(\frac{1}{0.01} - 1\right) = 2.218 \text{ eV}$$

Module-3 : Semiconductor Physics

In a solid, consider the energy level lying 0.012 eV below the Fermi energy level at 27 °C. What is the probability of this level being occupied by an electron? ○

Given : $T = 27 \text{ } ^\circ\text{C} = 300 \text{ } ^\circ\text{K}$, $k = 8.625 \times 10^{-5} \frac{\text{eV}}{\text{°K}}$



$$\text{Probability of occupancy } f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$\therefore \text{Probability of occupancy } f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1+e^{\left(\frac{-0.012}{8.625 \times 10^{-5} \times 300}\right)}}$$
$$= 0.614$$

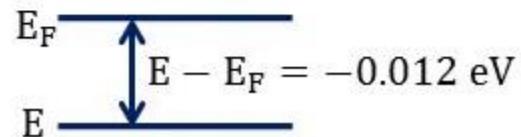
$$\therefore \frac{E - E_F}{kT} = \ln\left(\frac{1}{f(E)} - 1\right)$$

$$E = 2.1 + 8.625 \times 10^{-5} \times 300 \ln\left(\frac{1}{0.01} - 1\right) = 2.218 \text{ eV}$$

Module-3 : Semiconductor Physics

In a solid, consider the energy level lying 0.012 eV below the Fermi energy level at 27 °C. What is the probability of this level being occupied by an electron? ○

Given : $T = 27 \text{ } ^\circ\text{C} = 300 \text{ } ^\circ\text{K}$, $k = 8.625 \times 10^{-5} \frac{\text{eV}}{\text{K}}$



$$\text{Probability of occupancy } f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$\begin{aligned}\therefore \text{Probability of occupancy } f(E) &= \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1 + e^{\left(\frac{-0.012}{8.625 \times 10^{-5} \times 300}\right)}} \\ &= 0.614\end{aligned}$$

Module-3 : Semiconductor Physics

Fermi level in Semiconductor

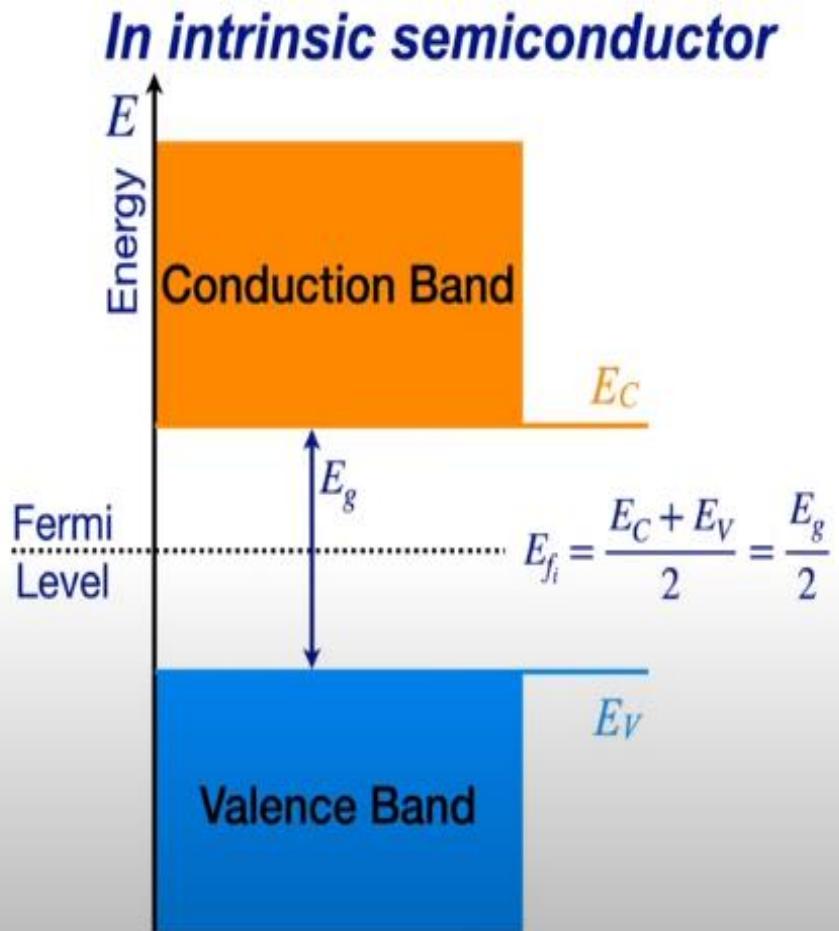
- In semiconductors, the **Fermi level** is defined as the reference level that gives the probability of occupancy of states in conduction band as well as unoccupied states in valence band.
- **Fermi level** in semiconductors may also be defined as energy level that corresponds to the centre of gravity of conduction band electrons and valence band holes weighted according to their energies.

Module-3 : Semiconductor Physics

Position of Fermi level

- Thus, Fermi level in **intrinsic semiconductors** is exactly in the middle of the **forbidden energy gap (E_g)**.

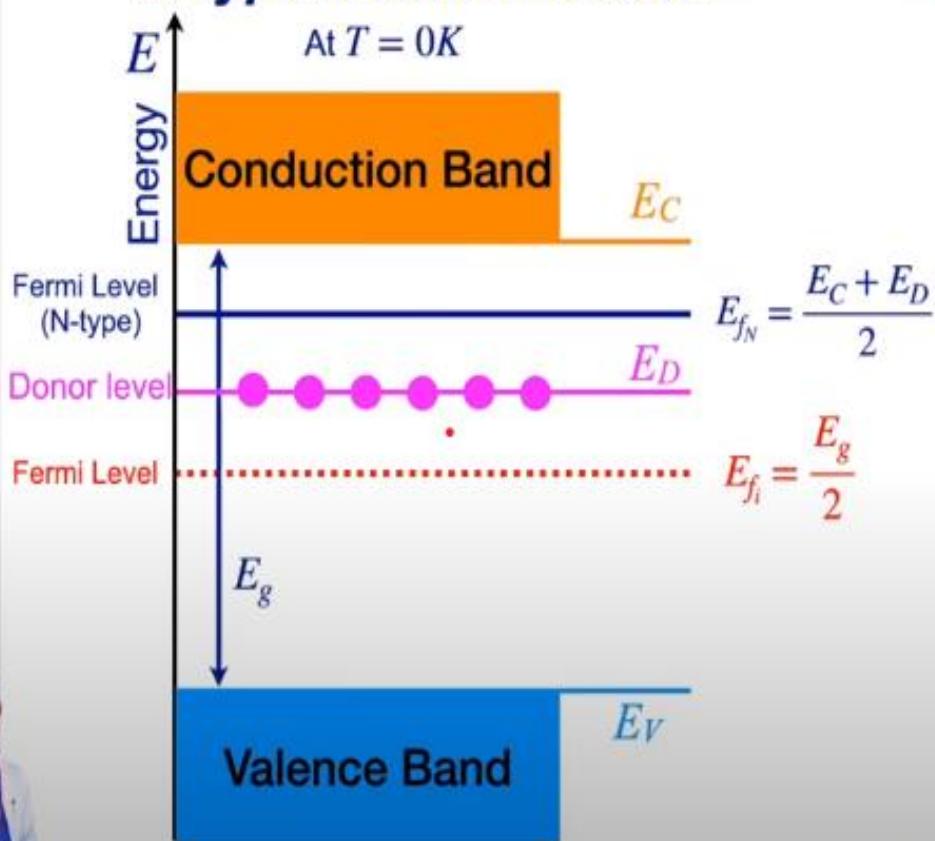
$$E_{fi} = \frac{E_C + E_V}{2}$$



Module-3 : Semiconductor Physics

Position of Fermi level

N-type semiconductor



- An intrinsic semiconductor is doped with a pentavalent donor impurity atom.

- The energy levels of donor atoms are very close to bottom of unfilled conduction band.

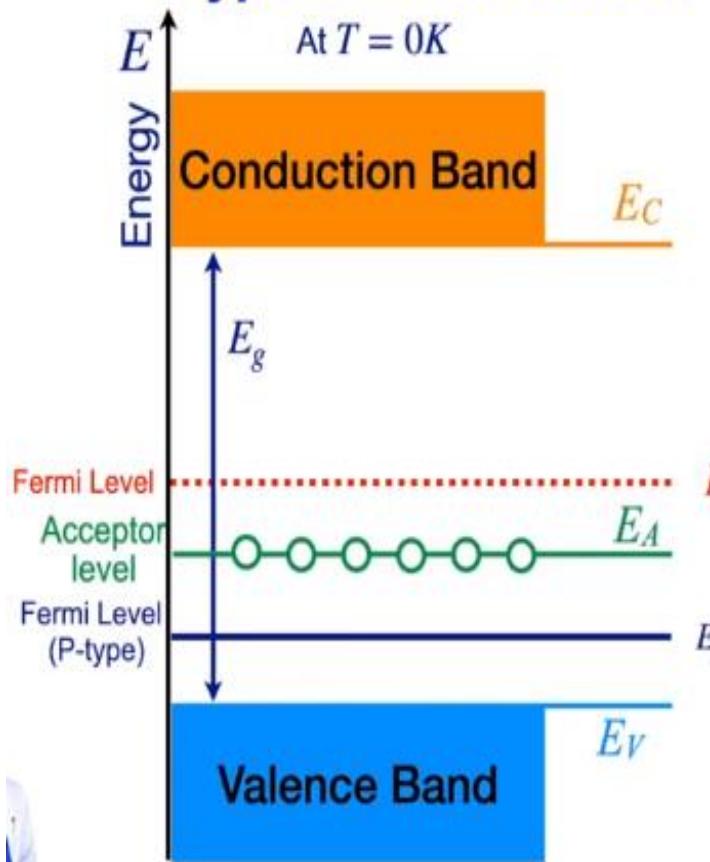
At $T = 0K$

$$E_{f_N} = \frac{E_C + E_D}{2}$$

Module-3 : Semiconductor Physics

Position of Fermi level

P-type semiconductor



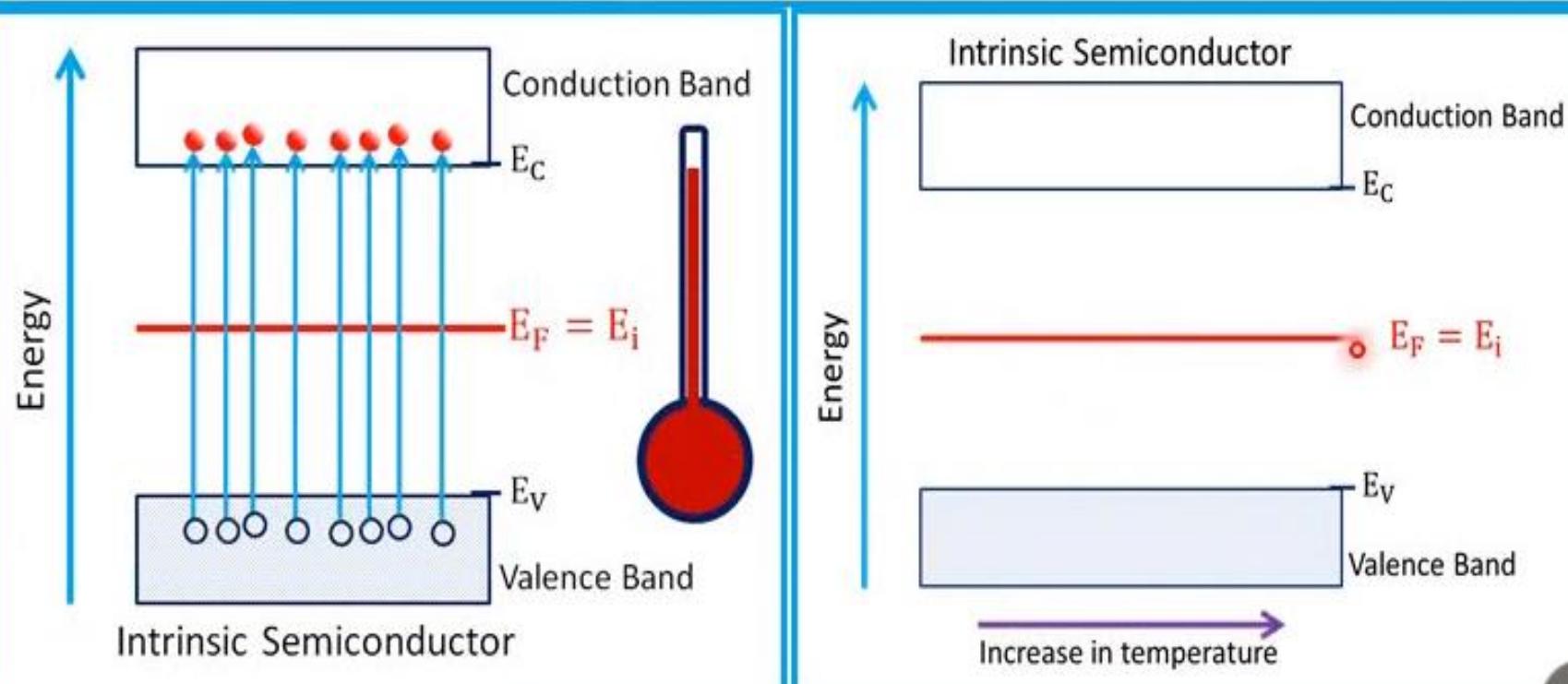
- An intrinsic semiconductor is doped with a trivalent impurity atom.
- Its energy levels are very close to the top of filled valence band.
- Fermi level lies midway between the acceptor levels and the top of the valence.

At $T = 0K$

$$E_{f_P} = \frac{E_V + E_A}{2}$$

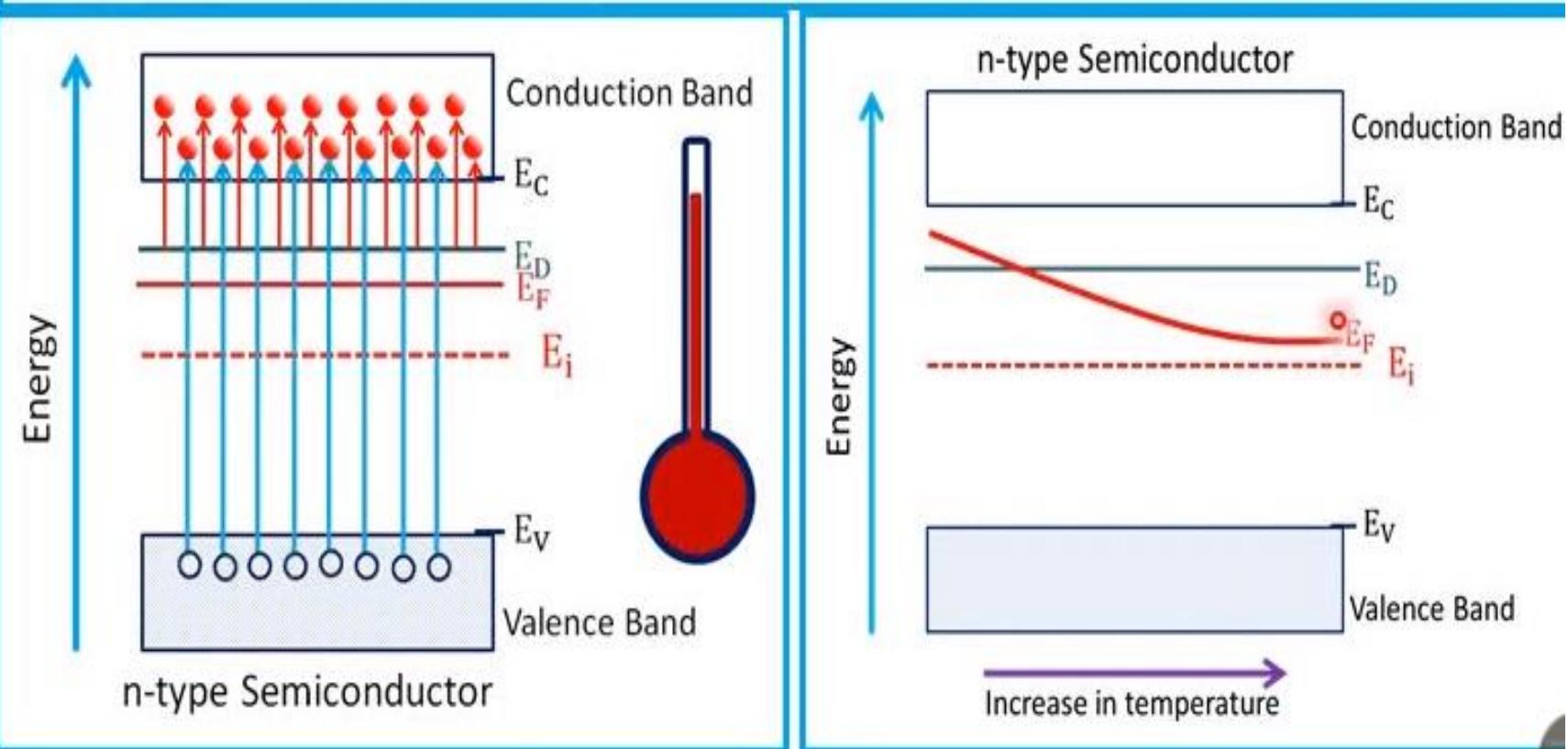
Module-3 : Semiconductor Physics

Effect of increasing temperature on Fermi level
(Intrinsic Semiconductor)



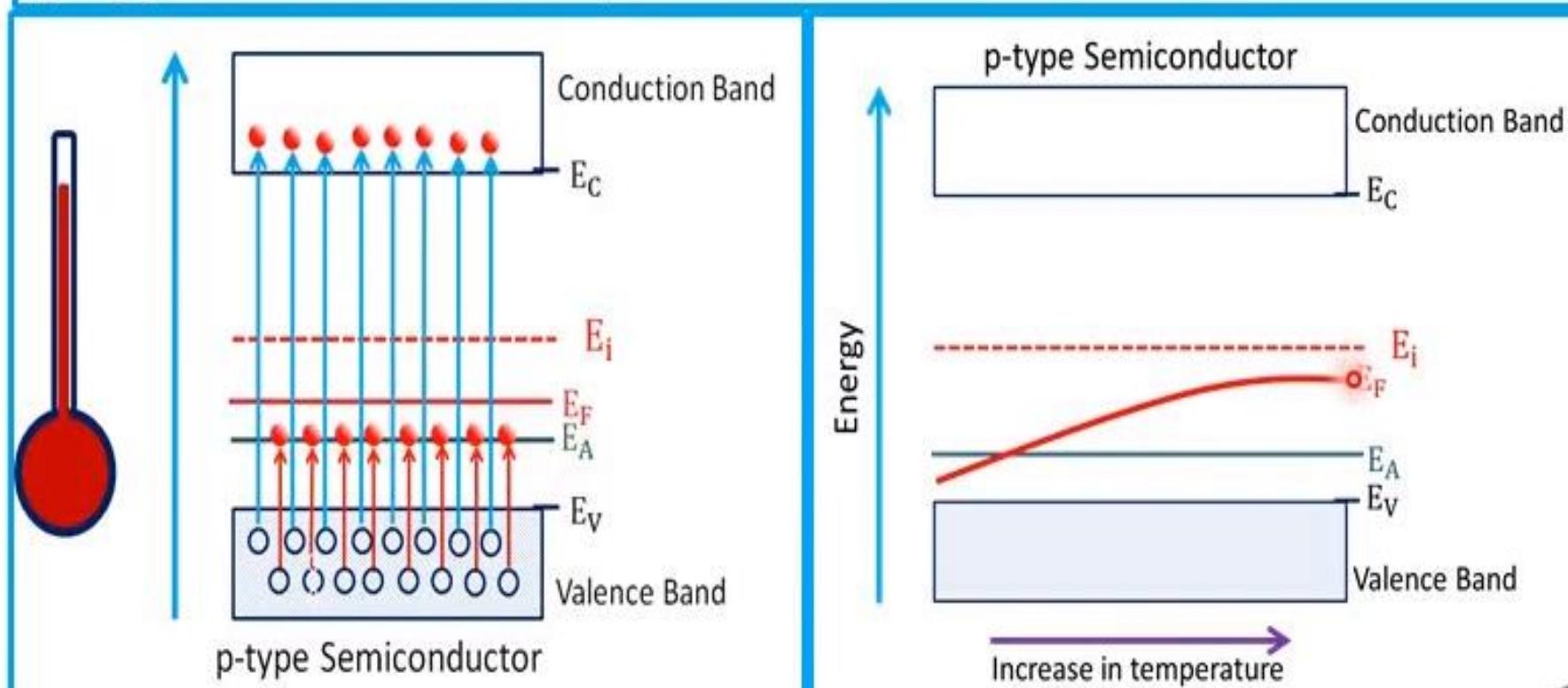
Module-3 : Semiconductor Physics

Effect of increasing temperature on Fermi level
(n-type Semiconductor)



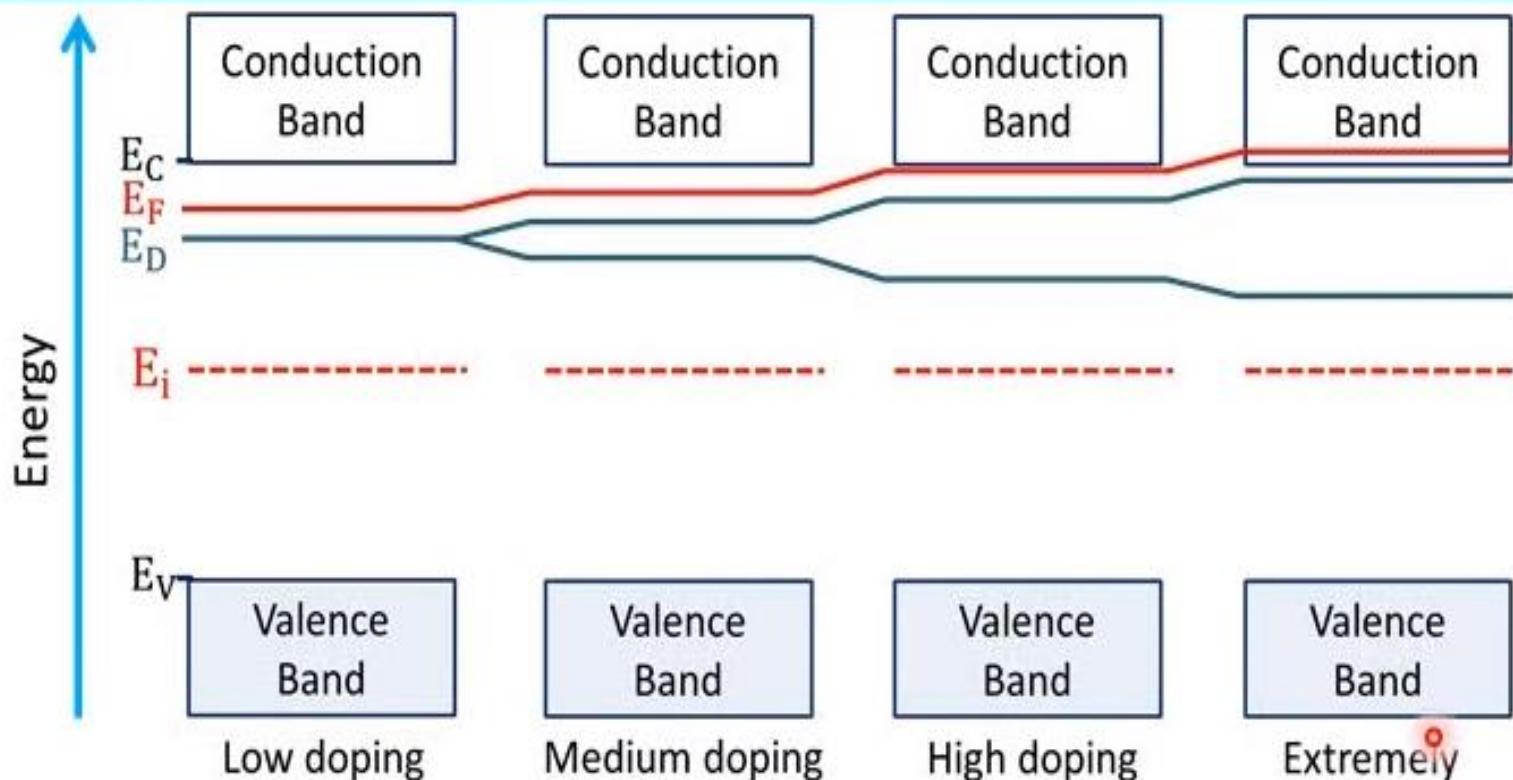
Module-3 : Semiconductor Physics

Effect of increasing temperature on Fermi level
(p-type Semiconductor)



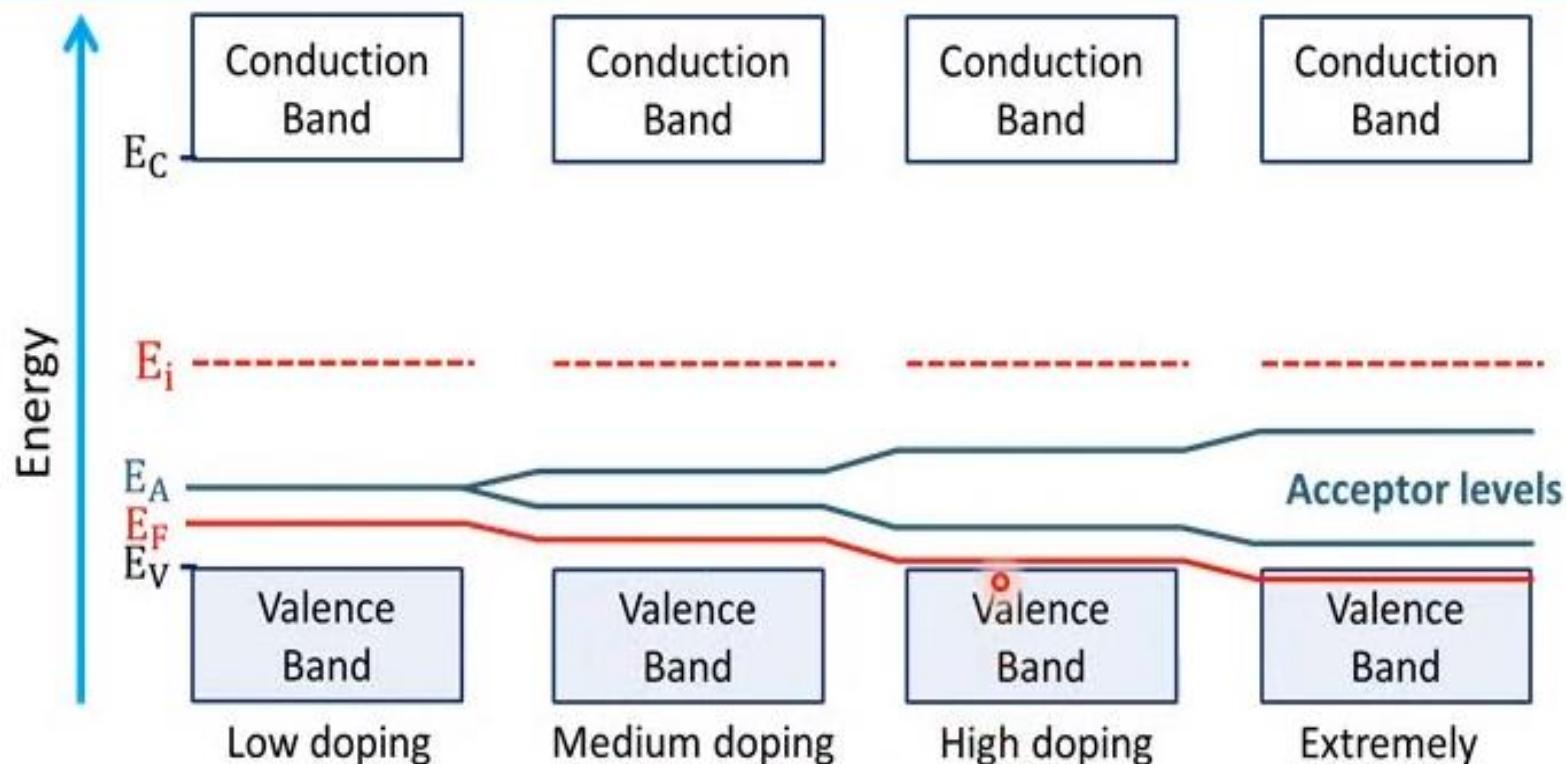
Module-3 : Semiconductor Physics

Effect of increasing doping concentration on Fermi level
(n-type Semiconductor)



Module-3 : Semiconductor Physics

Effect of increasing doping concentration on Fermi level
(p-type Semiconductor)



Module-3 : Semiconductor Physics

Carrier Concentration

The Fermi distribution function can be used to calculate the concentration of holes and electrons in the semiconductor, if the density of available states in the valence and conduction bands is known. The concentration of electrons in the conduction band is given by –

$$n = \int_{E_C}^{\infty} f(E)N(E)dE$$

1

The number of electrons per unit volume in the energy range dE is the product of the density of states $N(E)$ and the probability of occupancy $f(E)$. Thus total electron concentration is integral over the entire conduction band.

Module-3 : Semiconductor Physics

Carrier Concentration

$$n = \int_{E_C}^{\infty} f(E)N(E)dE \quad 1$$

Solving equation 1 we get the electron concentration in the conduction band as -

$$n = N_C e^{-(E_C - E_F)/kT} \quad 2$$

where, $N_C = 2 \left(\frac{2\pi m_e^* k T}{h^2} \right)^{1/2}$;

m_e^* is the effective mass of the electron

Similarly, hole concentration in the valence band can be obtained as -

$$p = N_V e^{-(E_F - E_V)/kT} \quad 3$$

where, $N_V = 2 \left(\frac{2\pi m_h^* k T}{h^2} \right)^{1/2}$;

m_h^* is the effective mass of the hole

At thermal equilibrium, electron and hole concentrations given by equations 2 and 3 are valid for intrinsic as well as extrinsic semiconductor

Module-3 : Semiconductor Physics

Carrier Concentration

$$n = \int_{E_C}^{\infty} f(E)N(E)dE \quad 1$$

$$n = N_C e^{-(E_C - E_F)/kT} \quad 2$$

$$p = N_V e^{-(E_F - E_V)/kT} \quad 3$$

Product of n and p can be written as

$$n \cdot p = N_C e^{-(E_C - E_F)/kT} \cdot N_V e^{-(E_F - E_V)/kT}$$

$$\therefore n \cdot p = N_C N_V e^{-(E_C - E_V)/kT} \quad 4$$

For intrinsic semiconductor,

$$n = n_i \text{ and } p = p_i \text{ and } E_F = E_i$$

$$n_i \cdot p_i = N_C e^{-(E_C - E_i)/kT} \cdot N_V e^{-(E_i - E_V)/kT}$$

$$\therefore n_i \cdot p_i = N_C N_V e^{-(E_C - E_V)/kT}$$

For intrinsic semiconductor, $n_i = p_i$

$$\therefore (n_i)^2 = N_C N_V e^{-(E_C - E_V)/kT} \quad 5$$

From 4 and 5

$$n \cdot p = (n_i)^2$$

Module-3 : Semiconductor Physics

Fermi level in intrinsic semiconductor lies at the middle of the forbidden gap

The electron concentration in conduction band of semiconductor at temperature T is given by -

$$n = N_C e^{-(E_C - E_F)/kT} \quad 1$$

N_C is the effective density of states in conduction band

The hole concentration in valence band of semiconductor at temperature T is given by -

$$p = N_V e^{-(E_F - E_V)/kT} \quad 2$$

N_V is the effective density of states in valence band

For intrinsic semiconductor,

$$n = p$$

$$\therefore N_C e^{-(E_C - E_F)/kT} = N_V e^{-(E_F - E_V)/kT}$$

$$\therefore \frac{e^{-(E_C - E_F)/kT}}{e^{-(E_F - E_V)/kT}} = \frac{N_C}{N_V}$$

At equilibrium, $N_C = N_V$

3

$$\therefore \frac{e^{-(E_C - E_F)/kT}}{e^{-(E_F - E_V)/kT}} = 1$$

Module-3 : Semiconductor Physics

$$n = N_C e^{-(E_C - E_F)/kT} \quad 1$$

$$p = N_V e^{-(E_F - E_V)/kT} \quad 2$$

For intrinsic semiconductor,

$$n = p$$

$$\therefore N_C e^{-(E_C - E_F)/kT} = N_V e^{-(E_F - E_V)/kT}$$

$$\text{At equilibrium, } N_C = N_V \quad 3$$

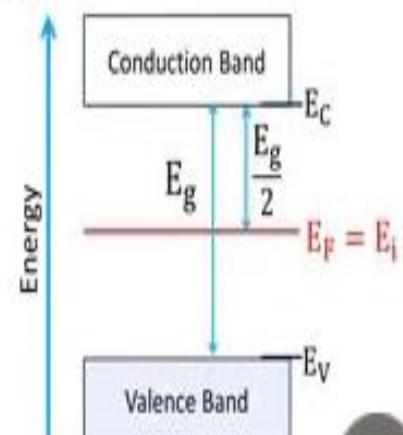
$$\therefore \frac{e^{-(E_C - E_F)/kT}}{e^{-(E_F - E_V)/kT}} = 1$$

$$\therefore e^{\frac{-(E_C - E_F - E_F + E_V)}{kT}} = 1 = e^0$$

$$\therefore \frac{E_C - E_F - E_F + E_V}{kT} = 0$$

$$\therefore E_C + E_V = 2E_F$$

$$\therefore E_F = \frac{E_C + E_V}{2}$$



Module-3 : Semiconductor Physics

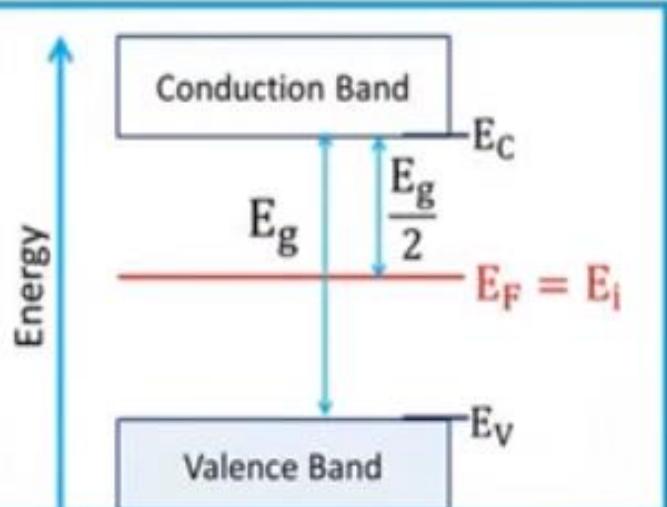
What is the probability of an electron being thermally excited to conduction band in intrinsic Si at 27 °C. The band gap energy of Si is 1.12 eV.

Given : $T = 27 \text{ } ^\circ\text{C} = 300 \text{ } ^\circ\text{K}$,

$$k = 8.625 \times 10^{-5} \frac{\text{eV}}{\text{°K}}, \quad E_g = 1.12 \text{ eV}$$

For intrinsic semiconductor $E_F = \frac{E_C + E_V}{2}$

$$\therefore E_C - E_F = \frac{E_g}{2} = \frac{1.12}{2} = 0.56 \text{ eV}$$



$$\begin{aligned}\therefore \text{Probability of occupancy } f(E_C) &= \frac{1}{1+e^{\left(\frac{E_C-E_F}{kT}\right)}} \\ &= \frac{1}{1+e^{\left(\frac{0.56}{8.625 \times 10^{-5} \times 300}\right)}} \\ &= 3.99 \times 10^{-10}\end{aligned}$$

Module-3 : Semiconductor Physics

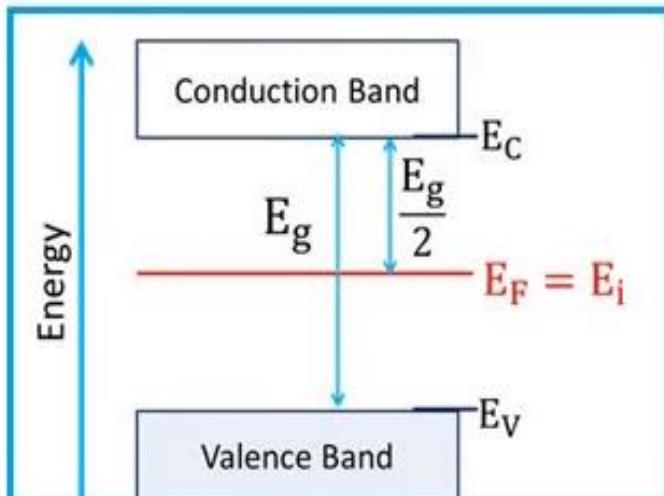
What is the probability of an electron being thermally promoted to conduction band in diamond at 27°C if band gap is 5.6 eV wide.

Given : $T = 27^{\circ}\text{C} = 300^{\circ}\text{K}$,

$$k = 8.625 \times 10^{-5} \frac{\text{eV}}{\text{°K}}, \quad E_g = 5.6 \text{ eV}$$

For diamond, $E_F = \frac{E_C + E_V}{2}$

$$\therefore E_C - E_F = \frac{E_g}{2} = \frac{5.6}{2} = 2.8 \text{ eV}$$



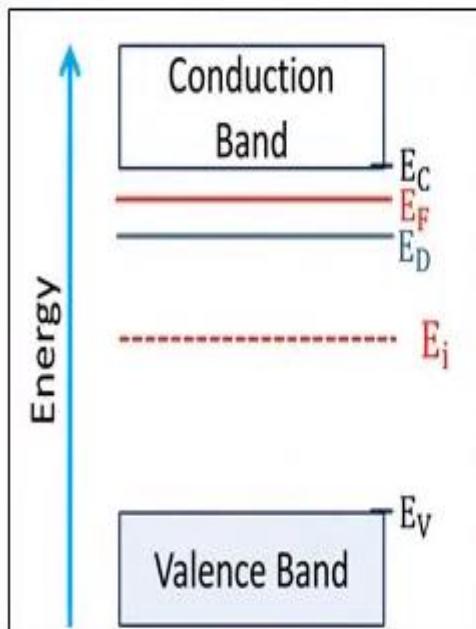
$$\begin{aligned}\therefore \text{Probability of occupancy } f(E_C) &= \frac{1}{1+e^{\left(\frac{E_C-E_F}{kT}\right)}} \\ &= \frac{1}{1+e^{\left(\frac{2.8}{8.625 \times 10^{-5} \times 300}\right)}} \\ &= 1.09 \times 10^{-47}\end{aligned}$$

Module-3 : Semiconductor Physics

In an n-type semiconductor, the Fermi level lies 0.4 eV below the conduction band. If the concentration of donor atoms is doubled, find the new position of the Fermi level w.r.t. conduction band.

Given : $E_C - E_F = 0.4 \text{ eV}$

Temperature is not mentioned.
Let us assume $T = 300 \text{ K}$



Electron concentration is given by

$$n = N_C e^{-(E_C - E_F)/kT} \quad 1$$

When donor concentration is doubled, electron concentration will also get doubled and fermi level will be shifted to E_{F2}

$$2n = N_C e^{-(E_C - E_{F2})/kT} \quad 2$$

Divide 2 by 1

$$\frac{e^{-(E_C - E_{F2})/kT}}{e^{-(E_C - E_F)/kT}} = 2$$

$$\therefore e^{\frac{-(E_C - E_{F2}) + (E_C - E_F)}{kT}} = 2$$

$$\therefore -(E_C - E_{F2}) + (E_C - E_F) = kT \ln(2)$$

$$\therefore (E_C - E_{F2}) = (E_C - E_F) - kT \ln(2)$$

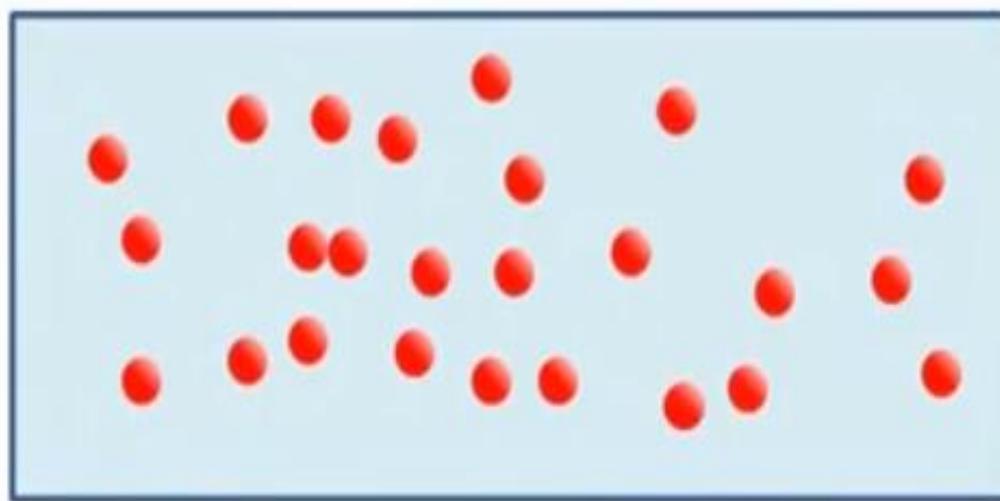
$$\therefore (E_C - E_{F2}) = 0.4 - 8.625 \times 10^{-5} \times 300 \ln(2)$$

$$\therefore (E_C - E_{F2}) = 0.382 \text{ eV}$$

So, The Fermi level shifts towards the Conduction band as donor concentration increases.

Module-3 : Semiconductor Physics

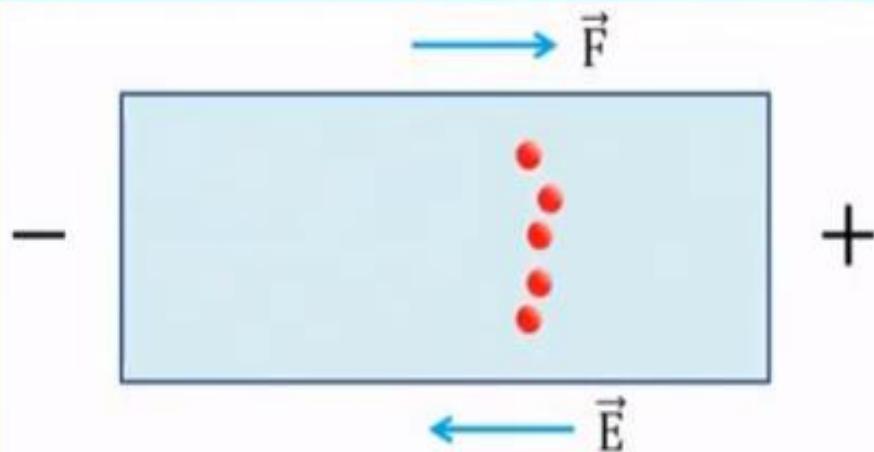
Conductivity and Mobility



At thermal equilibrium, net current due to randomly moving electrons is zero

Module-3 : Semiconductor Physics

Drift Motion and Drift Current



under the influence of electric field (E), each electron experiences a force $F = -eE$

- This results in a net motion of group of electrons in the direction opposite to the direction of electric field.
- The net motion of electrons in a particular direction under the influence of electric field is called **drift motion**.
- Current due to this motion is called as **drift current**.

Module-3 : Semiconductor Physics

Conductivity and Electric Field

Drift current density, (J), is defined as the drift current per unit area of cross section of the conductor

$$J = \frac{I}{A}$$

1

Electric field (E) is the potential drop per unit length of the conductor

$$E = -\frac{dV}{dx} = \frac{V}{L}$$

2

Resistivity (ρ) is the resistance offered by a conductor of unit length and unit cross sectional area

$$\rho = R \frac{A}{L}$$

3

Conductivity (σ) is the reciprocal of resistivity.

$$\sigma = \frac{1}{\rho} = \frac{L}{RA}$$

4

2 X 4 gives

$$\sigma E = \frac{L}{RA} \frac{V}{L} = \frac{V}{RA} = \frac{I}{A} = J$$

$$J = \sigma E$$

5

Module-3 : Semiconductor Physics

Drift Velocity and Mobility

Drift velocity (v) is defined as net displacement in electron position per unit time under the influence of electric field.

Mobility (μ) of electrons is defined as average drift velocity acquired by the electrons per unit electric field.

$$\mu = \frac{v}{E}$$

Module-3 : Semiconductor Physics

Mobility (μ) of electrons is defined as average drift velocity acquired by the electrons per unit electric field.

$$\mu = \frac{v}{E}$$

$$\text{Unit of mobility} = \frac{\text{m/s}}{\text{V/m}} = \frac{\text{m}^2}{\text{V - s}}$$

Module-3 : Semiconductor Physics

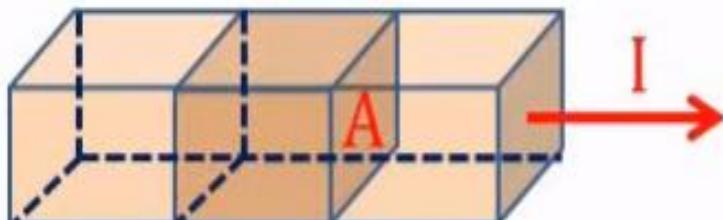
Conductivity and Mobility

Drift Current Density is given by -

$$J = \sigma E \quad 1$$

Mobility is given by -

$$\mu = \frac{v}{E} \quad 2$$



Let v is drift velocity of electrons

\therefore length traversed by the electrons
in unit time = v

\therefore Volume swept by electrons in unit time = $v A$

If 'n' is number of electrons per unit volume,
number of electrons in volume vA = $n v A$

If e is charge on electron, charge flowing per
unit time is -

$$\text{current } I = n e v A$$

\therefore Current Density,

$$J = \frac{I}{A} = \frac{n e v A}{A} = n e v$$

From 1 and 2

$$\text{Conductivity, } \sigma = \frac{J}{E} = \frac{n e v}{E} = n e \mu$$

Module-3 : Semiconductor Physics

Conductivity of Semiconductors

Conductivity of semiconductor is given by -

$$\sigma = \sigma_e + \sigma_h$$

$$\therefore \sigma = n e \mu_e + p e \mu_h$$

Where, n is electron concentration in conduction band

 p is hole concentration in valence band

 e is charge on electron

μ_e is mobility of electrons

μ_h is mobility of holes

Module-3 : Semiconductor Physics

Conductivity of Intrinsic Semiconductors

Conductivity of semiconductor is given by -

$$\therefore \sigma = n e \mu_e + p e \mu_h$$

For intrinsic semiconductor, $n = p = n_i$

\therefore Conductivity of intrinsic semiconductors is given by -

$$\therefore \sigma_i = n_i e \mu_e + n_i e \mu_h$$

$$\therefore \sigma_i = n_i e (\mu_e + \mu_h)$$

Module-3 : Semiconductor Physics

Conductivity of **Extrinsic** Semiconductors

Conductivity of semiconductor is given by -

$$\therefore \sigma = n e \mu_e + p e \mu_h$$

For n-type semiconductor, $n \gg p$

\therefore Conductivity of **n-type semiconductors** is given by -

$$\sigma = n e \mu_e = N_d e \mu_e$$

N_d is donor concentration

For p-type semiconductor, $p \gg n$

\therefore Conductivity of **p-type semiconductors** is given by -

$$\sigma = p e \mu_h = N_a e \mu_h$$

N_a is acceptor concentration

Module-3 : Semiconductor Physics

Find resistivity of Ge at 300 °K. Given density of carriers is $2.5 \times 10^{19} / \text{m}^3$. Mobility of electrons is $0.39 \text{ m}^2/\text{V}\cdot\text{sec}$, mobility of holes = $0.19 \text{ m}^2/\text{V}\cdot\text{sec}$.

Given : $n_i = 2.5 \times 10^{19} / \text{m}^3$,

$$\mu_e = 0.39 \text{ m}^2/\text{V}\cdot\text{sec}, \quad \mu_h = 0.19 \text{ m}^2/\text{V}\cdot\text{sec}$$

For intrinsic semiconductor, conductivity is given by –

$$\begin{aligned}\sigma_i &= n_i e (\mu_e + \mu_h) \\ &= 2.5 \times 10^{19} \times 1.6 \times 10^{-19} (0.39 + 0.19) \\ &= 2.32 \text{ } (\Omega \text{ -- m})^{-1}\end{aligned}$$

$$\text{Resistivity } = \rho = \frac{1}{\sigma} = \frac{1}{2.32} = 0.43 \text{ } \Omega \text{ -- m}$$

Module-3 : Semiconductor Physics

The resistivity of intrinsic InSb at room temperature is 2×10^{-4} ohm-cm. If the mobility of electron is $6 \text{ m}^2/\text{V}\cdot\text{s}$ and mobility of hole is $0.2 \text{ m}^2/\text{V}\cdot\text{s}$, calculate its intrinsic carrier density.

Given : Resistivity $\rho = 2 \times 10^{-4}$ ohm – cm = 2×10^{-6} ohm – m

$$\mu_e = 6 \frac{\text{m}^2}{\text{Vsec}}, \quad \mu_h = 0.2 \frac{\text{m}^2}{\text{Vsec}}, \quad n_i = ?$$

For intrinsic semiconductor, conductivity is given by – $\sigma = \frac{1}{\rho} = n_i e(\mu_e + \mu_h)$

$$\therefore n_i = \frac{1}{\rho e (\mu_e + \mu_h)}$$

$$\therefore n_i = \frac{1}{2 \times 10^{-6} \times 1.6 \times 10^{-19} \times (6 + 0.2)} = 5.04 \times 10^{23} / \text{m}^3$$

Module-3 : Semiconductor Physics

In an n-type semiconductor, the Fermi level lies 0.4 eV below the conduction band. If the concentration of donor atoms is doubled, find the new position of the Fermi level w.r.t. conduction band.

Given : $E_C - E_F = 0.4 \text{ eV}$

Temperature is not mentioned.

Let us assume $T = 300 \text{ K}$

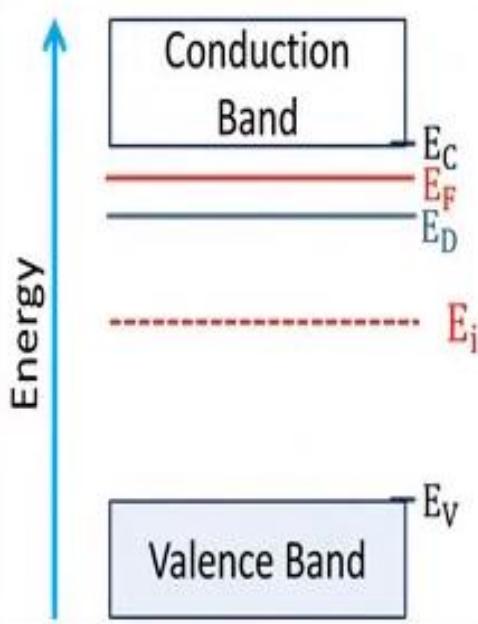
Electron concentration is given by

$$n = N_C e^{-(E_C - E_F)/kT} \quad 1$$

When donor concentration is doubled, electron concentration will also get doubled and fermi level will be shifted to E_{F2}

$$2n = N_C e^{-(E_C - E_{F2})/kT} \quad 2$$

Divide 2 by 1



$$\frac{e^{-(E_C - E_{F2})/kT}}{e^{-(E_C - E_F)/kT}} = 2$$

$$\therefore e^{\frac{-(E_C - E_{F2}) + (E_C - E_F)}{kT}} = 2$$

$$\therefore -(E_C - E_{F2}) + (E_C - E_F) = kT \ln(2)$$

$$\therefore (E_C - E_{F2}) = (E_C - E_F) - kT \ln(2)$$

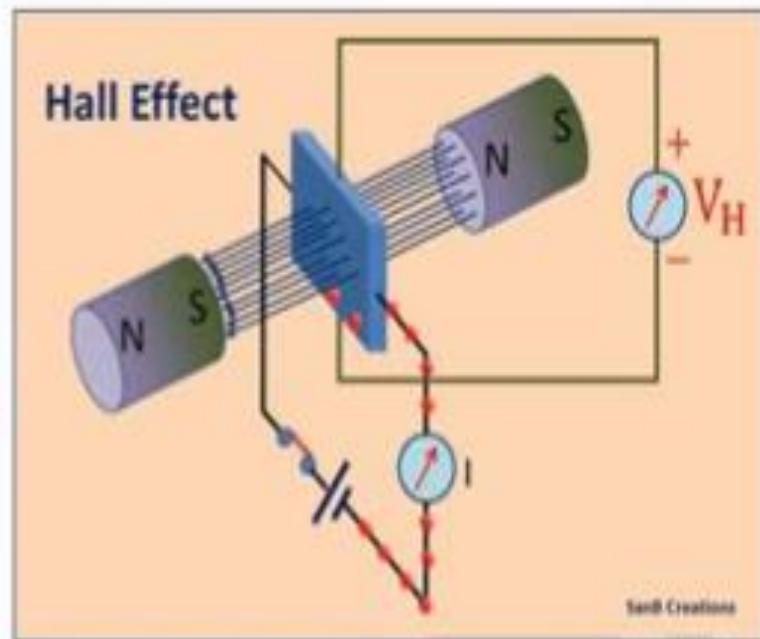
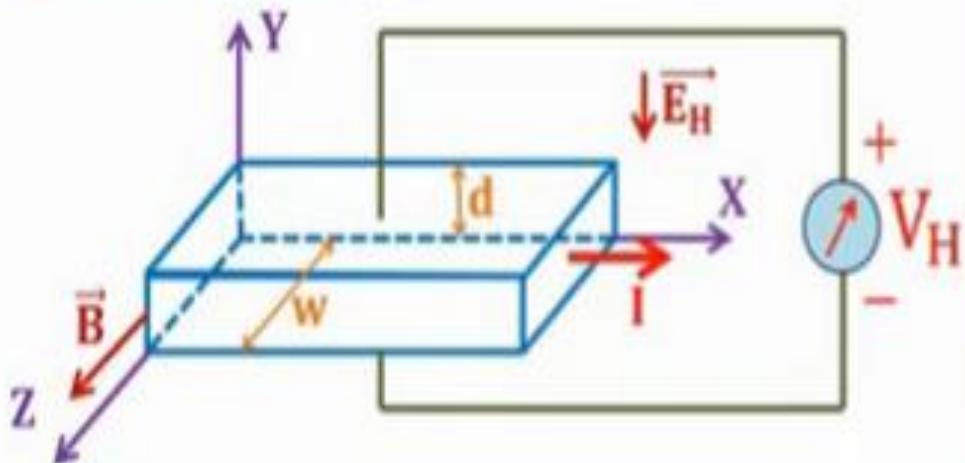
$$\therefore (E_C - E_{F2}) = 0.4 - 8.625 \times 10^{-5} \times 300 \ln(2)$$

$$\therefore (E_C - E_{F2}) = 0.382 \text{ eV}$$

So, The Fermi level shifts towards the Conduction band as donor concentration increases.

Module-3 : Semiconductor Physics

Hall Effect



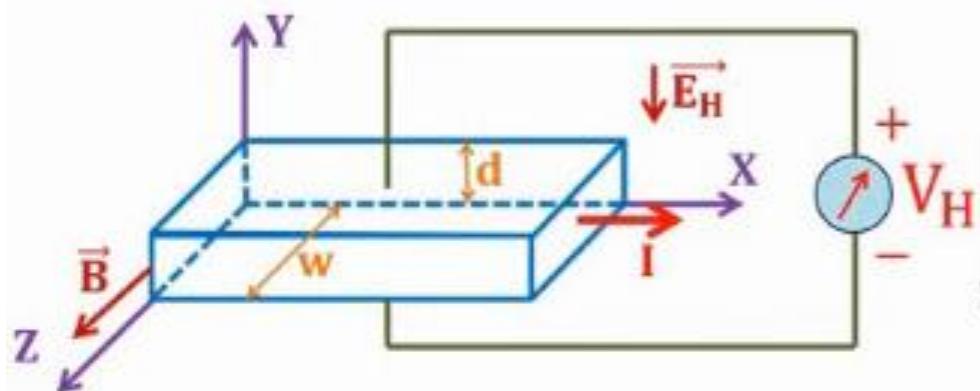
Sand Creations

If a metal or semiconductor carrying a current I is placed in a transverse magnetic field B , an electric field E is induced in the direction perpendicular to both the direction of current and magnetic field. This phenomenon is called as **Hall Effect**.

The electric field produced is called as **Hall field (E_H)**.

Module-3 : Semiconductor Physics

Hall Effect



The current through the metal semiconductor is given by -

$$I = n e v A \quad 1$$

where n is concentration of electrons,
 A is area of cross section of the plate
 e is charge on the electron,
 v is average drift velocity of electrons

At equilibrium,

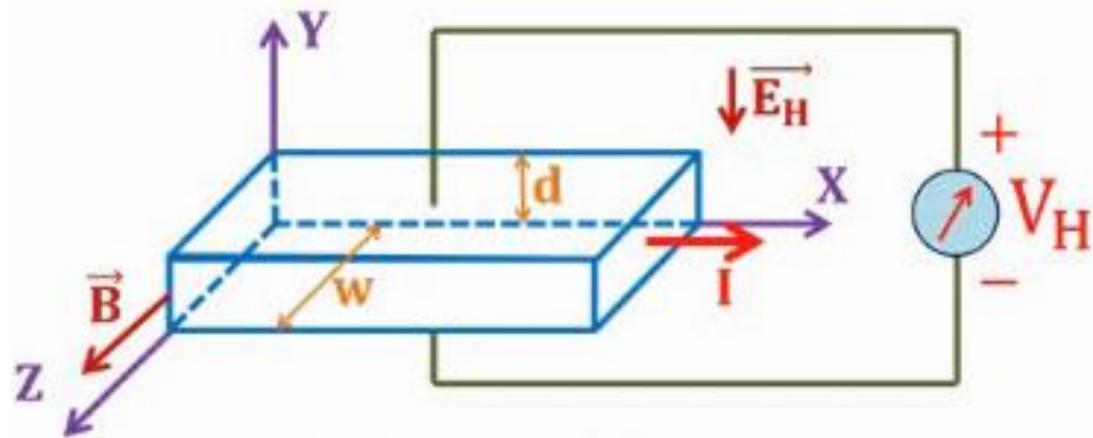
electric force on electrons = magnetic force on electrons

$$e E_H = e v B \quad 2$$

$E_H = \frac{V_H}{d}$ is the Hall field and
 V_H is Hall voltage

Module-3 : Semiconductor Physics

Hall Effect



$$I = n e v A \quad 1$$

$$e E_H = e v B \quad 2$$

$$\therefore \frac{V_H}{d} = v B$$

$$\text{As } E_H = \frac{V_H}{d}$$

$$\therefore V_H = v B d$$

$$\therefore V_H = \frac{n e v B d}{n e}$$

From 1

$$\therefore V_H = \frac{I B d}{n e A} = \frac{I B d w}{n e A w} \quad 3$$

$$\therefore V_H = \frac{B I}{n e w} \quad 3$$

Module-3 : Semiconductor Physics

Hall Effect

$$I = n e v A \quad 1$$

$$eE_H = e v B \quad 2$$

$$\therefore V_H = \frac{B I}{n e w} \quad 3$$

$$\therefore V_H = R_H \frac{B I}{w}$$

$R_H = \frac{1}{ne}$ is Hall Coefficient

$$\therefore R_H = \frac{1}{ne} = \frac{V_H w}{BI} = \frac{V_H d w}{d BI} = \frac{E_H A}{BI} = \frac{E_H}{BJ}$$

∴ Hall coefficient is defined as the Hall field per unit magnetic induction per unit current density.

Importance / Significance of Hall Effect is that it helps –

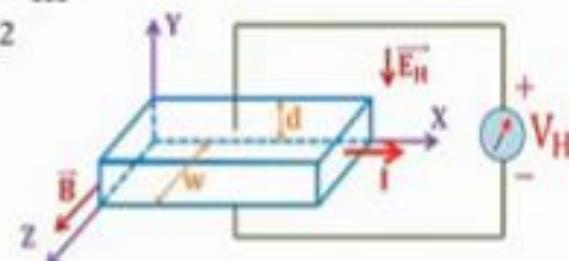
- To determine carrier concentration (n).
- To determine type of the semiconductor.
- To determine mobility μ of the charge carriers if conductivity of the semiconductor is known. $\sigma = ne\mu$

Module-3 : Semiconductor Physics

n-type Ge sample has donor concentration $10^{21}/\text{m}^3$ and thickness = 3 mm is used in a Hall effect experiment set up.

If $B = 0.5 \text{ T}$, $J = 500 \text{ A/m}^2$, Find Hall voltage.

Given : $n = N_d = 10^{21} / \text{m}^3$ $d = 3 \times 10^{-3} \text{ m}$
 $B = 0.5 \text{ Wb/m}^2$ $J = 500 \text{ A/m}^2$
 $V_H = ?$



$$\begin{aligned}V_H &= \frac{BI}{new} = \frac{BId}{ne wd} = \frac{BId}{ne A} = \frac{BJd}{ne} \\&= \frac{0.5 \times 500 \times 3 \times 10^{-3}}{10^{21} \times 1.6 \times 10^{-19}} = 4.688 \times 10^{-3} \text{ volts}\end{aligned}$$

Module-3 : Semiconductor Physics

The mobility of hole is $\mu_h = 0.025 \frac{m^2}{V\text{-sec}}$. What would be the resistivity of p-type silicon if the Hall coefficient of the sample is $2.25 \times 10^{-5} m^3/C$?

Given : $R_H = \frac{1}{p e} = 2.25 \times 10^{-5} m^3/C$ $\mu_h = 0.025 \frac{m^2}{V\text{-sec}}$

Resistivity = $\rho = ?$

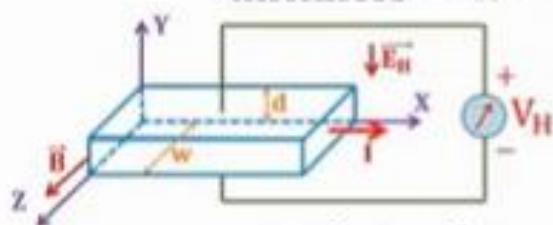
For p-type semiconductor, conductivity is given by – $\sigma = p e \mu_h$

$$\begin{aligned}\therefore \text{Resistivity } \rho &= \frac{1}{\sigma} = \frac{1}{p e \mu_h} = R_H \frac{1}{\mu_h} \\&= 2.25 \times 10^{-5} \times \frac{1}{0.025} \\&= 9 \times 10^{-4} \Omega - m\end{aligned}$$

Module-3 : Semiconductor Physics

In a Hall effect experiment, a potential difference of $4.5 \mu\text{V}$ is developed across a foil of zinc of thickness 0.02 mm , while carrying a current of 1.5 A in a direction perpendicular to applied magnetic field of 2 tesla . Calculate a) Hall coefficient for zinc b) concentration of electrons.

Given : $V_H = 4.5 \mu\text{V} = 4.5 \times 10^{-6} \text{ V}$ $I = 1.5 \text{ A}$ $B = 2 \text{ T}$
thickness = $w = 0.02 \text{ mm} = 0.02 \times 10^{-3} \text{ m}$ $R_H = ? \text{ n} = ?$



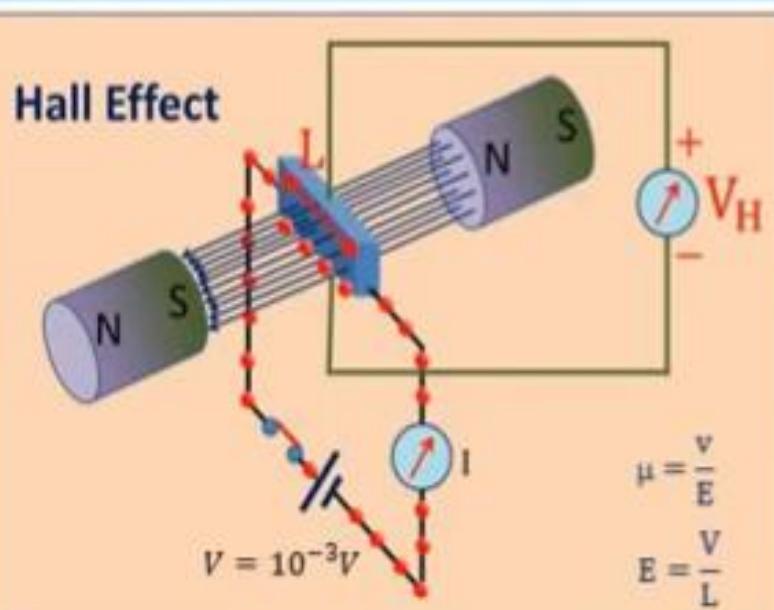
$$\text{As } V_H = \frac{B I}{n e w}$$

$$\text{Hall Coefficient} = R_H = \frac{1}{n e} = \frac{V_H w}{B I} = \frac{4.5 \times 10^{-6} \times 0.02 \times 10^{-3}}{2 \times 1.5} = 3 \times 10^{-11} \frac{\text{m}^3}{\text{C}}$$

$$\begin{aligned}\text{Concentration of electrons} &= n = \frac{B I}{V_H e w} \\ &= \frac{2 \times 1.5}{4.5 \times 10^{-6} \times 1.6 \times 10^{-19} \times 0.02 \times 10^{-3}} = 2.083 \times 10^{29} / \text{m}^3\end{aligned}$$

Module-3 : Semiconductor Physics

A bar of n-type Ge of size $0.010\text{m} \times 0.001\text{m} \times 0.001\text{m}$ is mounted in a magnetic field of $2 \times 10^{-1}\text{T}$. The electron density in the bar is $7 \times 10^{21}/\text{m}^3$. If one millivolt is applied across the long ends of the bar, determine the current through the bar and the voltage between Hall electrodes placed across the short dimensions of the bar. Assume mobility of electrons = $0.39 \frac{\text{m}^2}{\text{V-sec}}$



Given : $n = 7 \times 10^{21}/\text{m}^3$ $d = w = 0.001\text{ m}$
 $B = 0.2\text{ T}$ $V = 10^{-3}\text{ V}$
 $L = 0.010\text{ m}$ $I = ?$ $V_H = ?$

Current through the bar = $I = n e v A$

$$\begin{aligned}\therefore I &= n e \mu E d w = n e \mu \frac{V}{L} d w \\ &= 7 \times 10^{21} \times 1.6 \times 10^{-19} \times 0.39 \times \frac{10^{-3}}{0.010} \times \\ &\quad 0.001 \times 0.001 \\ \therefore I &= 43.68 \times 10^{-6}\text{ A}\end{aligned}$$

SEMICONDUCTORS

Q1.What are indirect and direct band gap semiconductors?

(M.U. Dec 2019)[3 Marks]

Direct band gap semiconductors: The semiconductors in which the bottom of the conduction band and the top of the valence band occur at the same momentum ($p = \hbar k$) as shown in *Figure 1a* are known as direct band gap semiconductors. GaAs and InP are examples of direct band gap semiconductors.

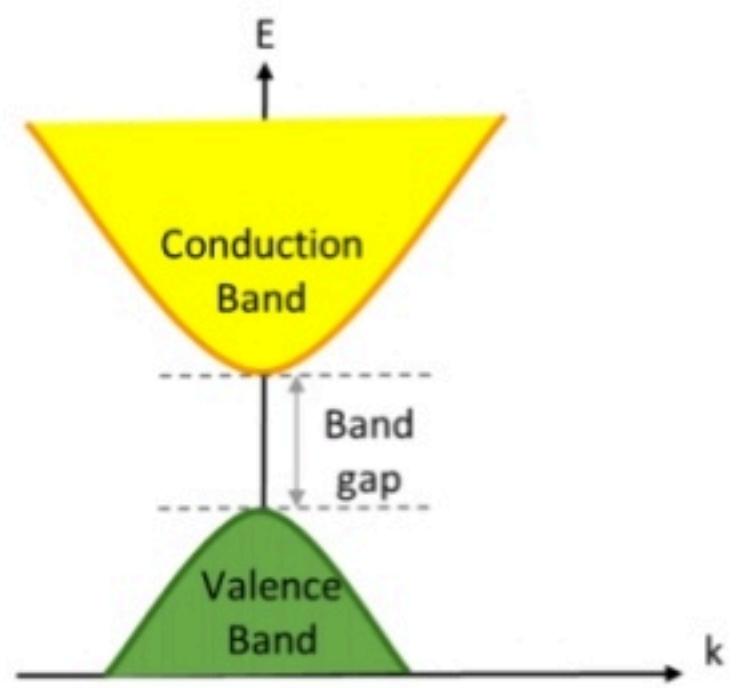


Figure 1a: Direct Band Gap

Indirect band gap semiconductors: The semiconductors in which the top of valence band and the bottom of conduction band occur at different momentums, as shown in *Figure 1b* are known as indirect band gap semiconductors. Si and Ge are examples of indirect band gap semiconductors.

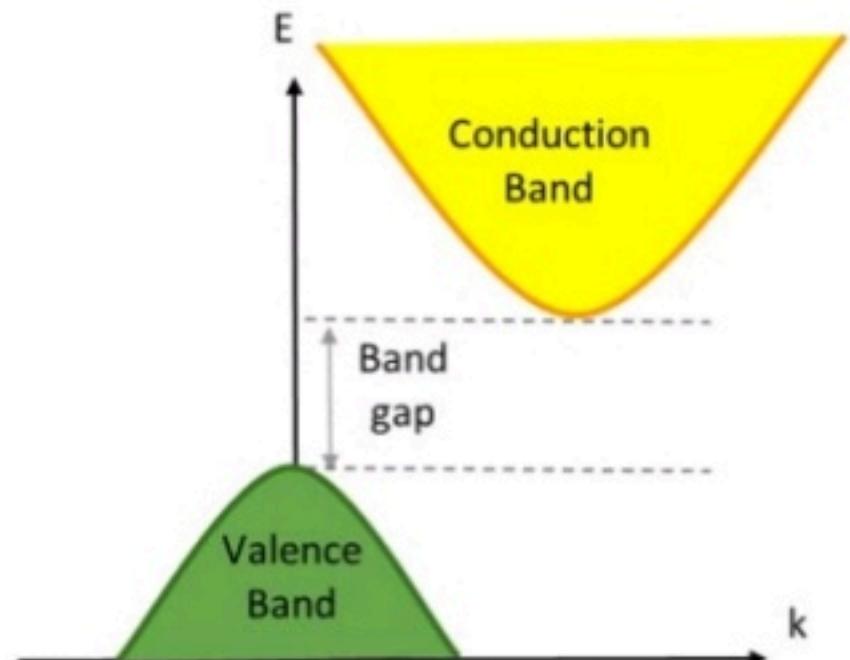


Figure 1b: Indirect Band Gap

Q2. Define drift velocity, mobility of charge, conductivity(σ), and resistivity(ρ)and state relation between them.

(M.U. May 2015, 2016; Dec 2013)[3 Marks]

Drift velocity (v_d): The net displacement of an electron's position per unit time in the steady state under the influence of electric field is called **drift velocity**.

Mobility of charge/electrons (μ): is defined as the magnitude of drift velocity obtained by the electron per unit electric field 'E'. It is given by $\mu = \frac{v_d}{E}$. Mobility represents the ease with which an electron could drift in a material.

Conductivity (σ): Conductivity is the physical property that shows the conducting ability of a material. It is given by $\sigma = ne\mu$ where, n= charge carrier concentration and e= electronic charge = 1.6×10^{-19} C.

Resistivity (ρ): Resistivity is the physical property of a material which shows the resistance 'R' of the material for given dimension(A= area of cross section, L length) of the wire. It is the inverse of conductivity. It is given by $\rho = \frac{1}{\sigma} = \frac{RA}{L}$.

Q3.Difference between intrinsic and extrinsic semiconductors.

Sr. No.	INTRINSIC SEMICONDUCTOR	EXTRINSIC SEMICONDUCTOR
1	They are pure semiconductors	They are impure or doped semiconductors
2	The charge carriers can be increased by increasing the temperature	The charge carriers can be increased by increasing the percentage of impurities or doping
3	More energy is required for electrons to cross band gap (E_g)	Less energy is required for electrons to cross (E_g)
4	Pure silicon or germanium are intrinsic semiconductors	When a semiconductor is doped by a third group impurity it forms a P-type extrinsic semiconductor and when doped by a fifth group impurity it forms a N-type extrinsic semiconductor.

Q4.What is Fermi level?

(M.U. May 2012, 2014; Dec 2013, 2015, 2019)[3-5 Marks]

Fermi level is defined as the **highest occupied energy level** of electrons at **absolute zero temperature.**

As shown in *Figure 4* ‘ E_F ’ denotes the Fermi level at 0°K . All the energy levels above Fermi level are empty (white color) and those below Fermi level are filled (Blue shaded region).

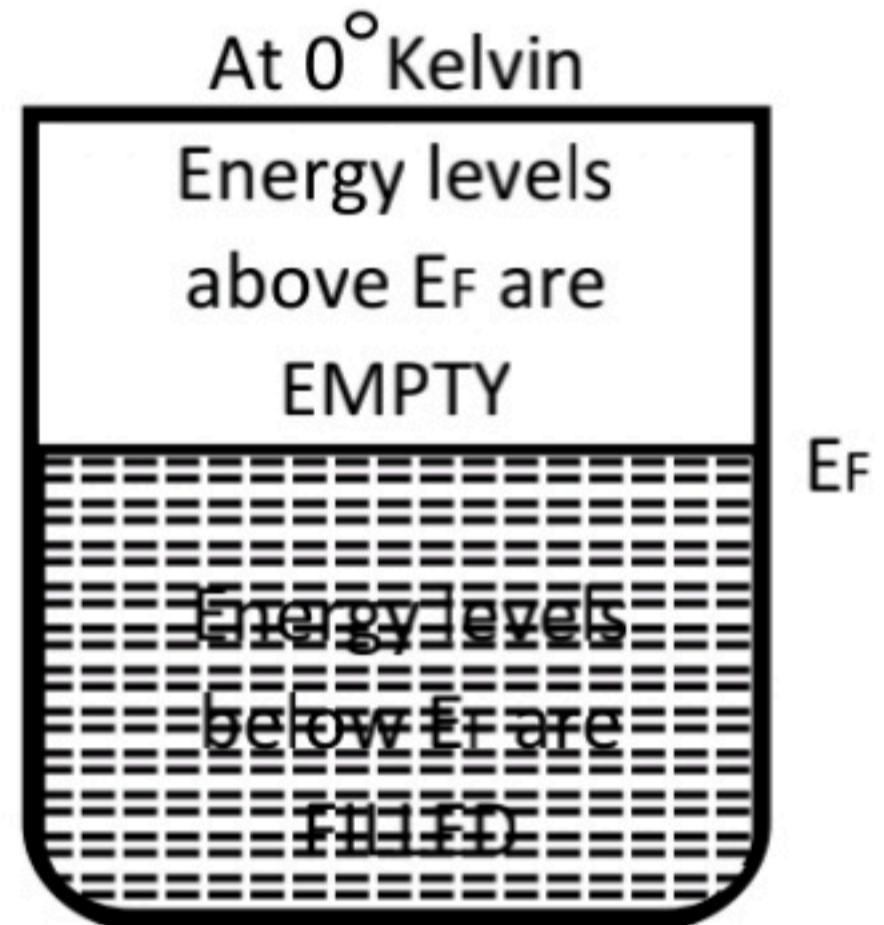


Figure 4: Fermi Level

It helps to assume that electrons in a semiconductor is like some water in a jar as shown in *Figure 4*. The level up to which the electrons (water) are (is) filled is the Fermi level. If concentration of electrons increases Fermi level rises and if concentration of electrons falls Fermi level drops.

Q5. Explain Fermi-Dirac distribution function.

Fermi-Dirac Distribution function $F(E)$:The Fermi-Dirac distribution function $F(E)$ represents the probability of an energy level with energy ‘ E ’ to be occupied by an electron. It is given by

$$F(E) = \frac{1}{1 + e^{\frac{(E-E_F)}{kT}}} \quad \dots \quad (1)$$

Where,

$F(E)$ = probability that a particular energy level 'E' is occupied

E_F = Fermi energy ,

K = Boltzmann constant and

T = temperature

In *Figure 5* the black line shows that at ' $T_0=0^0\text{K}$ ' , for $E < E_F$ the $F(E)=1$ while for $E > E_F$ the $F(E)=0$. This can be mathematically shown as:

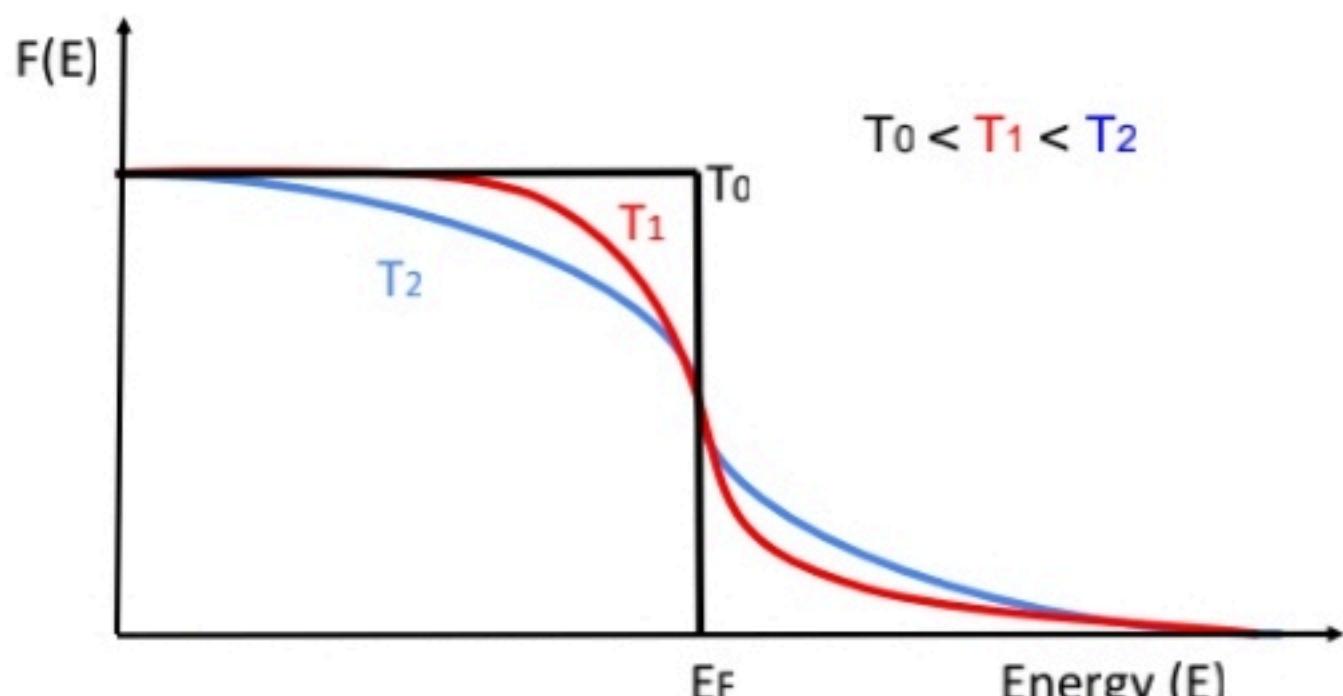


Figure 5: Fermi Dirac distribution

1. At $T = T_0=0^0\text{K}$

a. **Case A :** $F(E)$ for level at 'x' energy lower than fermi level ($E < E_F$)

$$\therefore F(E) = \frac{1}{1+\exp(\frac{-x}{T_0})} = \frac{1}{1+e^{-\infty}} = \frac{1}{1+\frac{1}{e^\infty}} = \frac{1}{1+e^\infty} = 1$$

Conclusion A: At 0^0K all energy levels below fermi level are filled

Mathematically: At 0^0K for $E < E_F$ the $F(E)=1$

b. **Case B :** $F(E)$ for level at 'x' energy higher than fermi level ($E > E_F$)

$$\therefore F(E) = \frac{1}{1+\exp(\frac{x}{T_0})} = \frac{1}{1+e^\infty} = \frac{1}{\infty} = 0$$

Conclusion B: At 0^0K all energy levels above fermi level are empty

Mathematically: At 0^0K for $E > E_F$ the $F(E)=0$

In *Figure 5* the red line indicates that as temperature increases there is a slight probability that energy levels higher than E_F are occupied and ones lower than E_F are empty. The blue one at a higher temperature than red one indicates probability of energy levels higher than E_F being occupied and ones lower than E_F being empty increases as compared to the red line. The point to note is that

for any temperature higher than 0°K at $E=E_F$ the $F(E)=1/2$. This can be mathematically shown as:

2. At $T = T_1$ or $T_2 > 0^{\circ}\text{K}$ when $E = E_F$

$$\therefore F(E) = \frac{1}{1 + \exp\left(\frac{0}{KT}\right)} = \frac{1}{1 + e^0} = \frac{1}{1+1} = \frac{1}{2}$$

Conclusion C: At $T > 0^{\circ}\text{K}$ the fermi level is half filled

Mathematically: At $T > 0^{\circ}\text{K}$ for $E = E_F$ the $F(E)=1/2$.

Q6. Show that Fermi level for an intrinsic semiconductor lies in the middle of the forbidden energy gap.

(M.U. May 2015, 2017, 2019; Dec 2014, 2015, 2017, 2019) [3-8 Marks]

Pure semiconductors are called intrinsic semiconductors. For intrinsic semiconductors the concentration of electrons (n_e) is equal to the concentration of holes (n_h) which is also called the intrinsic concentration of charge carriers (n_i).

Concentration of charge carriers 'n' in a particular band is given as

$$\therefore n = N \exp(-\Delta E/KT) \quad \text{---(1)}$$

where ,

N: maximum capacity or density of states of that band

ΔE : the difference between energy level of the band and fermi level

(KT): thermal energy (Boltzmann constant * temperature in $^{\circ}\text{K}$)

Using (1) the concentration of electrons in the conduction band can be written as,

$$n_e = N_c \exp\left(\frac{-(\Delta E_e)}{KT}\right)$$

from *Figure 6* we get ,

$$n_e = N_c \exp\left(\frac{-(E_C - E_F)}{KT}\right) \text{ ----(2)}$$

Using (1) the concentration of holes in the valence band can be written as,

$$n_h = N_v \exp\left(\frac{-(\Delta E_h)}{KT}\right)$$

from *Figure 6* we get ,

$$n_h = N_v \exp\left(\frac{-(E_F - E_V)}{KT}\right) \text{ ----(3)}$$

For an intrinsic semiconductor as the charge carrier concentrations are equal :

$$n_e = n_h \text{ ----(4)}$$

Also, assuming that the density of states in the conduction and the valence bands are nearly same we get,

$$N_c \approx N_v \text{ ----(4)}$$

Substituting equations (1), (2) and (4) in (3) we get,

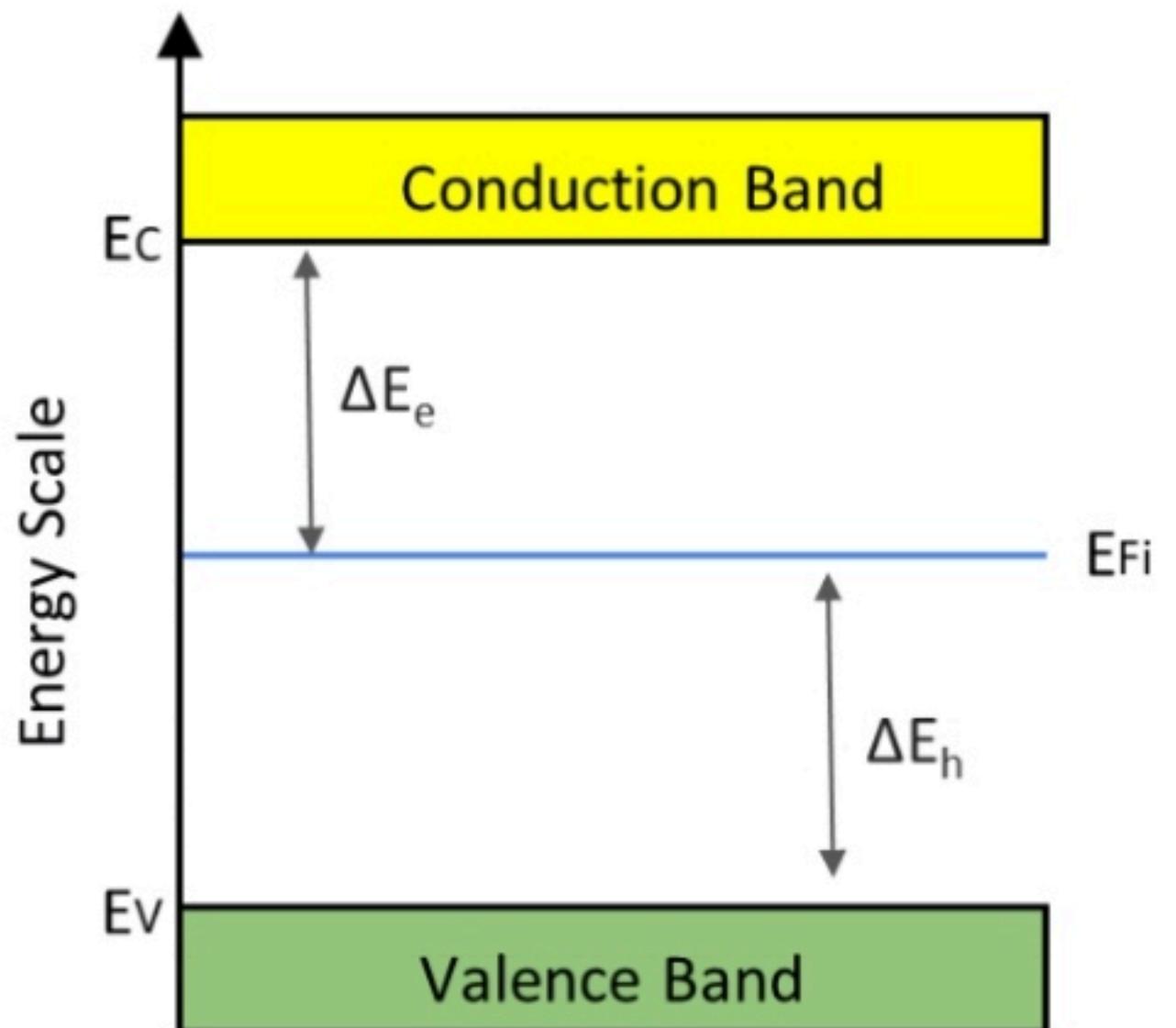


Figure 6: Energy band diagram of an intrinsic semiconductor

$$\exp\left(\frac{-E_c + E_F}{KT}\right) = \exp\left(\frac{-E_F + E_F}{KT}\right)$$

$$\therefore \left(\frac{-E_c + E_F}{KT}\right) = \left(\frac{-E_F + E_F}{KT}\right)$$

$$\therefore -E_c + E_F = -E_F + E_v$$

$$\therefore 2E_F = E_c + E_v$$

$$\therefore E_F = \frac{E_c + E_v}{2}$$

Thus, we can say that the Fermi level in an intrinsic semiconductor lies in the middle of the forbidden energy gap.

Q7. With the help of diagram, explain the variation of Fermi level with temperature in N-type and P-type semiconductors.

(M.U. May 2017, 2018; Dec 2017)[3-5 Marks]

The Fermi level for a **N-type semiconductor** (E_{Fn}) lies close to the conduction band (at 0°K $E_{Fn} = E_F$). Due to doping of the 5th group impurity donor levels (E_D) filled with electrons are created close to the conduction band just below E_F . As the temperature increases electrons from the E_D are excited to the conduction band. Thus, electrons in E_D get reduced and Fermi level (the indicator of level of electrons) falls with increase in temperature. The

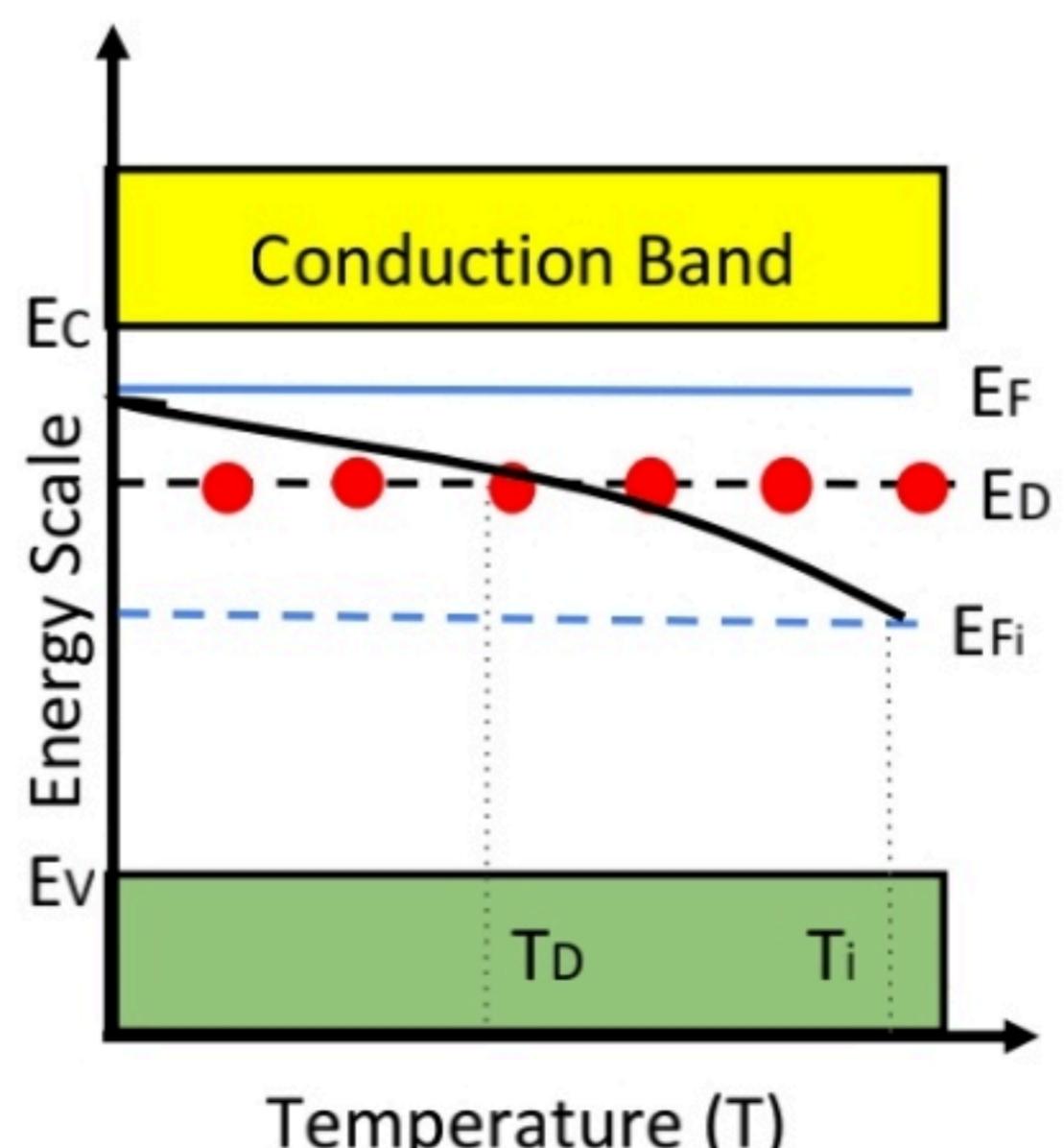


Figure 7a: Variation of E_F with T in N-type

variation of E_{Fn} is shown with a curved black line in the *Figure 7a*. While transition from energy position ' E_F ' (E_{Fn} at zero K temperature) to ' E_{Fi} ' (E_{Fn} at T_i K temperature) in the *Figure 7a* ' T_D ' shows the temperature at which the E_{Fn} coincides the E_D , while ' T_i ' is the temperature at which E_{Fn} coincides the intrinsic Fermi level (E_{Fi}).

The Fermi level for a P-type semiconductor

(E_{Fp}) lies close to the valence band (at 0^0K $E_{Fp} = E_F$). Due to doping of the 3rd group impurity Acceptor levels (E_A) filled with holes are created close to the valence band just above E_F . As the temperature increases electrons from valence band are excited to the E_A . Thus, there are electrons in E_A , hence Fermi level (the indicator of level of electrons) rises with increase in temperature. The variation of E_{Fp} is shown with a curved black line in the *Figure 7b*. While transition from energy position ' E_F ' (E_{Fp} at zero K temperature) to ' E_{Fi} ' (E_{Fp} at T_i K temperature) in the *Figure 7b* ' T_A ' shows the temperature at which the E_{Fp} coincides the E_A , while ' T_i ' is the temperature at which E_{Fp} coincides the intrinsic Fermi level (E_{Fi}).

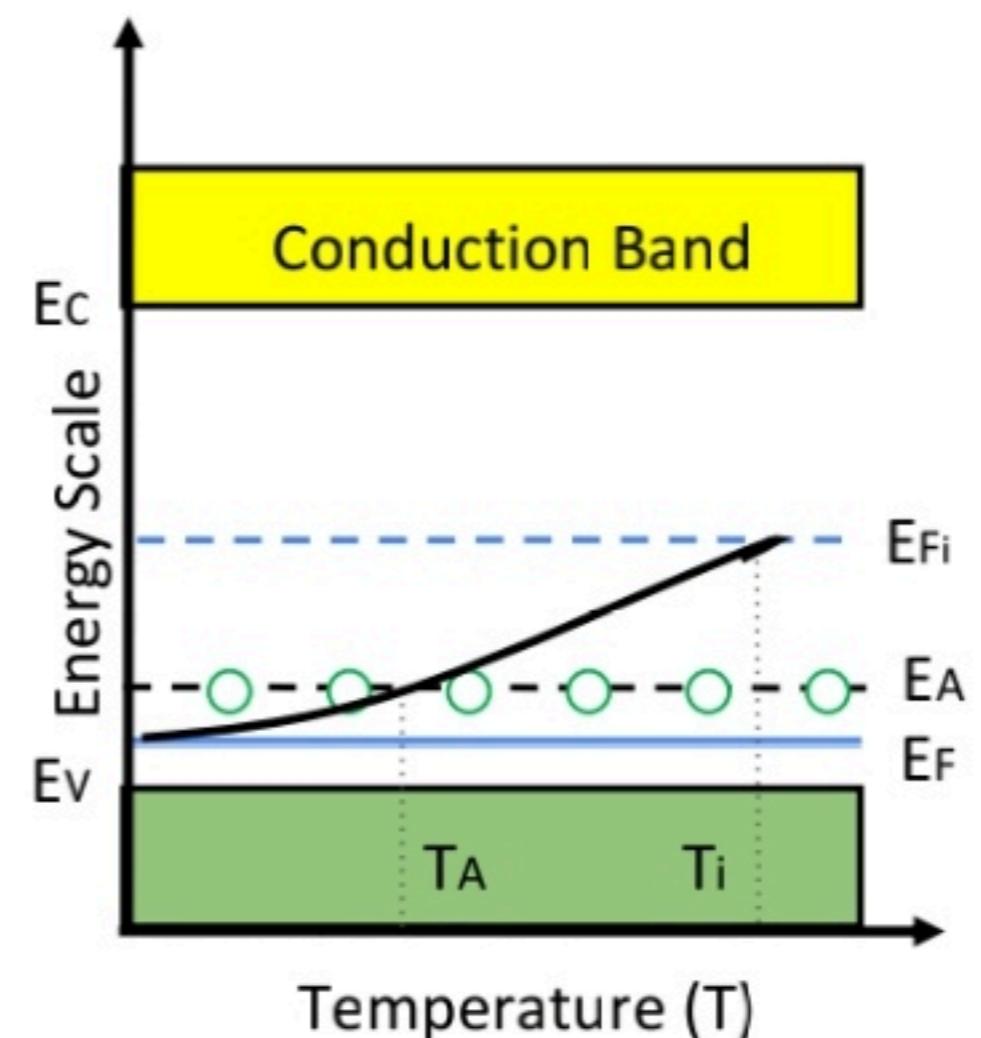


Figure 7b: Variation of E_F with T in P-type

Q8. How does the position of Fermi level change with increasing doping concentration in P-type and N-type semiconductors?

(M.U. Dec 2015, 2016) [3-5 Marks]

In an **N-type semiconductor** as the doping concentration increases more electrons get added to the donor levels. Due to increase in the concentration of electrons **Fermi level in N-type (E_{Fn}) rises** indicating this increase.

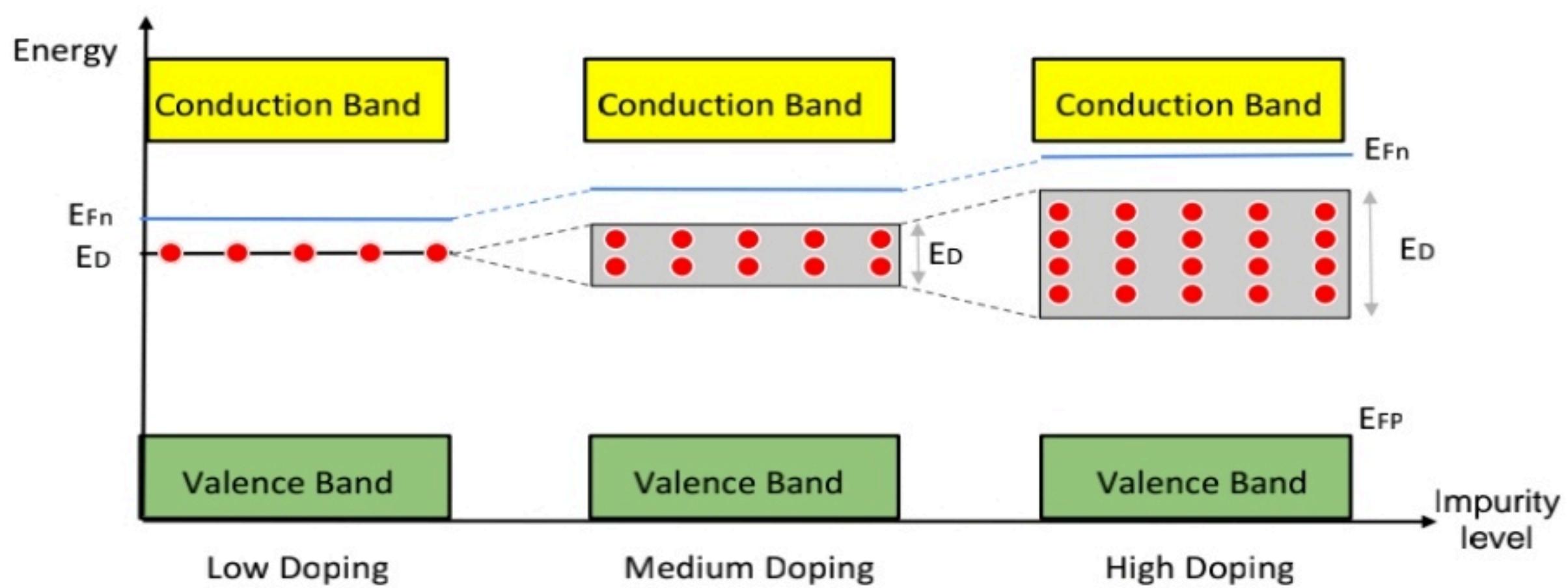


Figure 8a: Effect of Impurity on fermi level of N-type

In an **P-type semiconductor** as the doping concentration increases more holes get added to the acceptor levels. Due to increase in the concentration of holes

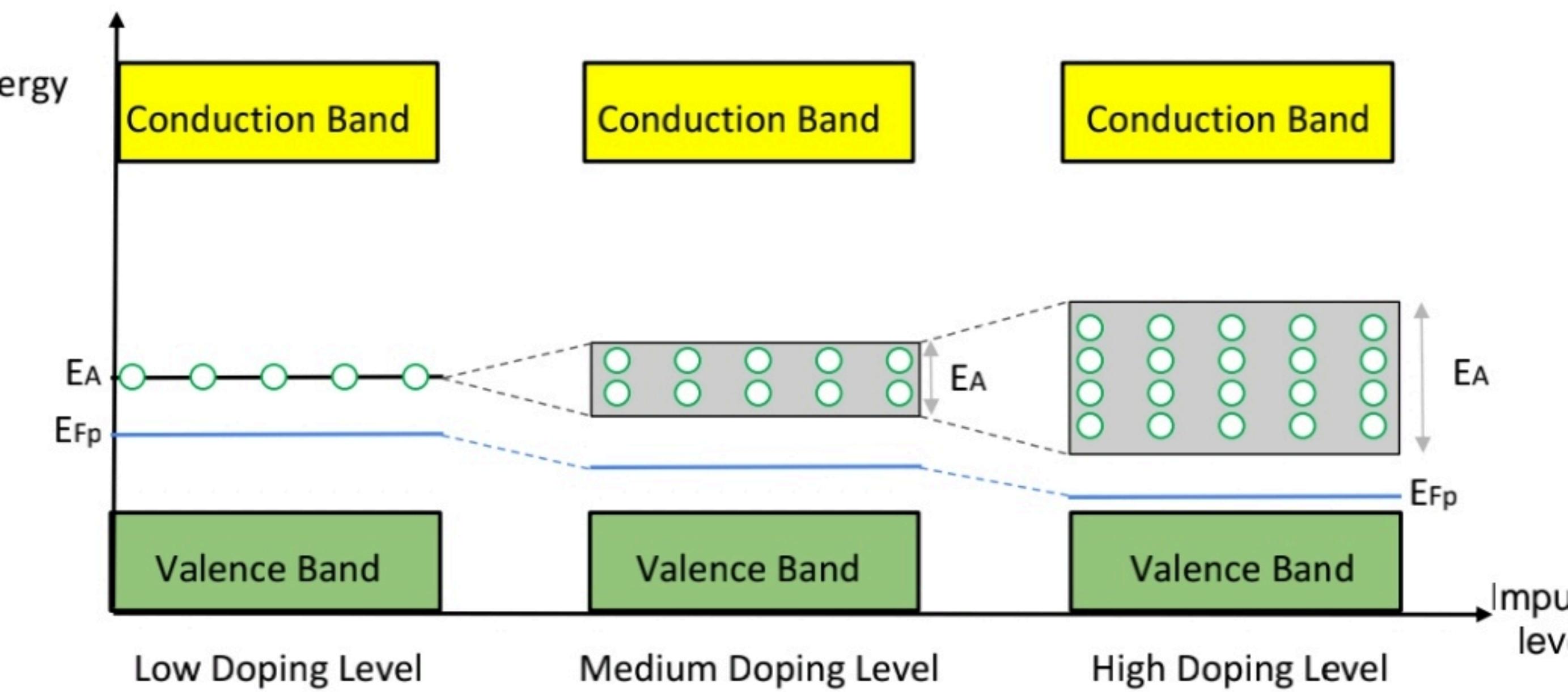


Figure 8b: Effect of Impurity on fermi level of P-type

(implies decrease in concentration of electrons comparatively) **Fermi level in P-type (E_F) falls** indicating this decrease of electrons relatively.

Q9. Define drift current, diffusion current and current density (J).

(M.U. May 2015, 2016; Dec 2013)[3 Marks]

Drift current (I_{Dr}): When there is a non-uniform distribution of electric field or potential is applied across the semiconducting material, there is a positive charge flow from the region of higher potential to the region of lower potential. Such a flow of charges is called **drift current**.

Diffusion current (I_{Df}): When there is a non-uniform concentration of charge carriers, there is a charge flow from the region of higher concentration to the region of lower concentration. Such a flow of charges is called **diffusion current**. This kind of non-uniform concentration of charge carriers can be formed by thermal or radiation excitation of a part of material or by adding carriers in the material through the surface.

Current density (J): Current density is defined as the current per unit area of cross-section of an imaginary plane held normal to the direction of current in current carrying conductor. It is given by $J = \frac{I}{A}$. Sometimes it is also expressed as $J=nev$, where n^*e is the charge density and 'v' is the velocity of charge carriers giving thereby imparting the product 'nev' dimensions of current density.

Q10. What is Hall effect? Derive an expression for Hall voltage with neat labelled diagram and give applications of Hall effect.

(M.U. May 2015, 2016, 2018, 2019; Dec 2012, 2013, 2015, 2018, 2019)

[3-8 Marks]

Hall Effect : When a metal or semiconductor carrying a current 'I' is placed in a transverse magnetic field 'B', an electric emf 'E' is induced in the direction perpendicular to both 'I' and 'B' as shown in *Figure 10*. This phenomenon is known as Hall effect and the voltage induced is Hall voltage (V_H).

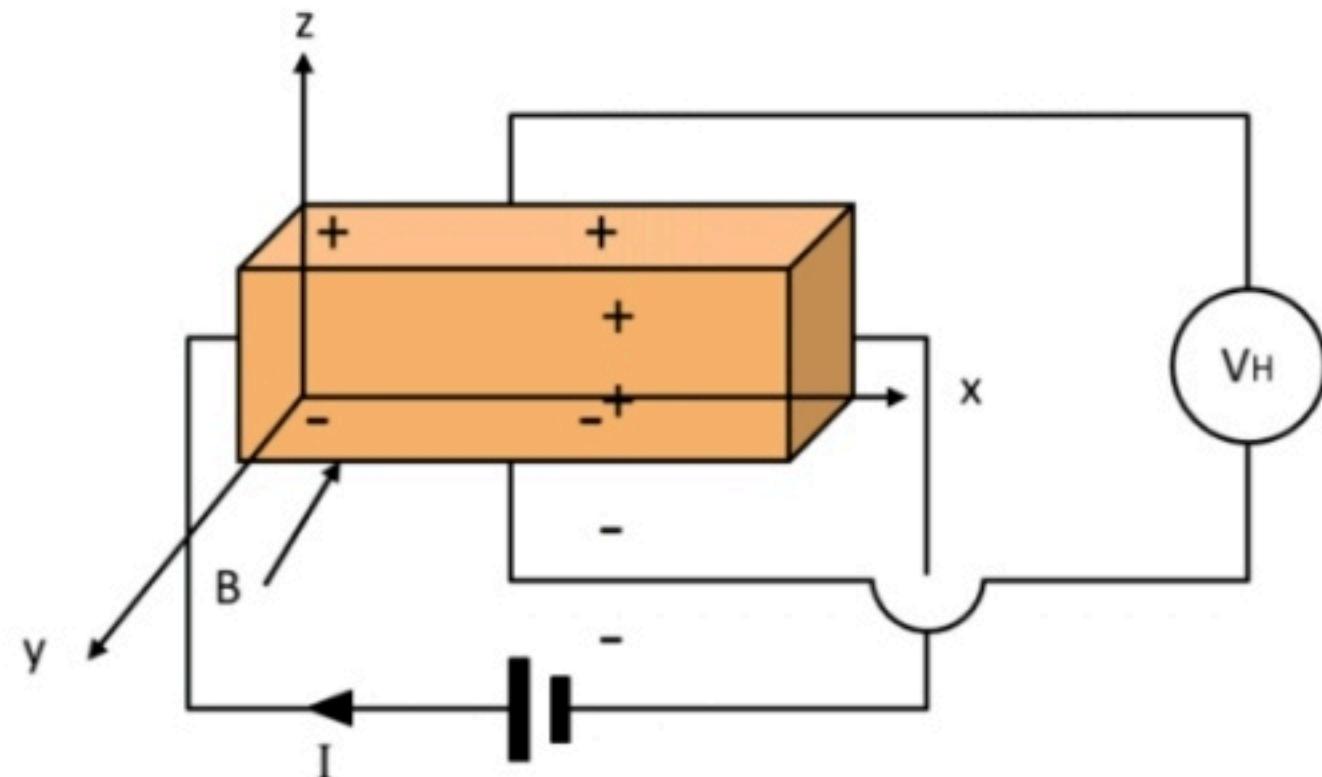


Figure 10: Hall's Effect

Derivation for expression of Hall voltage (V_H):

At equilibrium condition in the semiconductor we should have :

Electric force = magnetic force i.e. $F_E = F_M$

$\therefore eE = ev B$ (here, 'v' is the velocity of charge carriers and 'e' is their charge)

$$\therefore E = v B \quad \dots\dots(1)$$

Current density by definition is current per unit cross sectional area and it is also expressed as ' nev ' so we can write that,

$$J = \frac{I}{wd} = nev \quad \dots\dots(2)$$

The relationship between potential and electric field between two surfaces which are at a 'd' distance from each other is

$$V_H = Ed \quad \dots \dots (3)$$

Substituting (1) in (3) we get,

$$V_H = Bvd \quad \dots \dots (4)$$

Substituting 'v' from (2) in (4) we get,

$$V_H = Bd \frac{I}{newd}$$

$$V_H = \frac{BI}{new} \quad \dots \dots (5)$$

This is the expression for the hall voltage and we find that it is directly proportional to the applied magnetic field and current flowing through the semiconductor and inversely proportional to the charge density and the width of the semiconducting wafer used for hall effect. (Sometimes hall voltage is also expressed in terms of hall coefficient $R_H = \frac{1}{ne}$ hence it is then $V_H = R_H \frac{BI}{w}$)

Applications of Hall Effect:

1. Used in magnetic field meter (gauss meter).
2. Charge carrier concentration can be determined.
3. Mobility of charge can be determined.
4. Nature (N-type/P-type) of semiconductor can be determined.

Q11. Write a brief note on a normal diode. Or Draw the energy band diagram of an unbiased p-n junction and mark the barrier potential and depletion region. Or Draw the energy band diagram of p-n junction diode in forward and reverse bias condition.

(M.U. May 2013, 2014; Dec 2013, 2014, 2015, 2016)[3-5 Marks]

A diode is a two-terminal device. The circuit symbol for a diode is a small triangle with a line at the end. The line represents the cathode and the other side of the triangle represents the anode. The most commonly used ordinary diode is a small black cylinder that has a silver-grey ruling at one edge indicating that side to be the cathode.

Figure 11a and *Figure 11b* shows the circuit symbol of a diode and the cartoon representation of a real diode.

Construction:

1. The diode consists of a N-type substrate on which a P-Type layer is grown as shown in *Figure 11c*.
2. Metal contacts are provided for the P-side and N-side as anode and cathode connectors.
3. The doping is covered by a thin layer of silicon oxide for insulation.



Figure 11a: circuit symbol of diode



Figure 11b: circuit symbol of diode

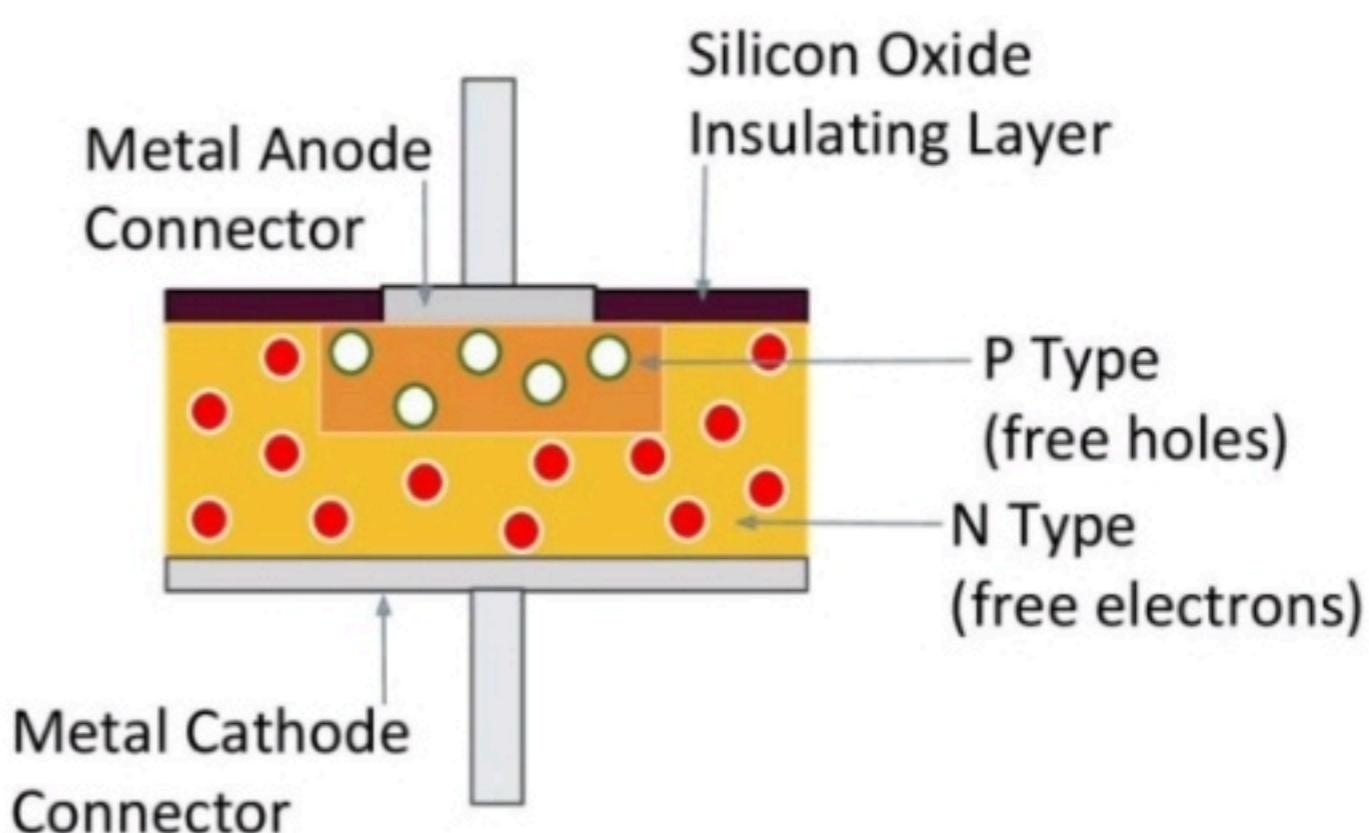


Figure 11c: construction of diode

4. A common boundary between the P-type and N-type semiconductors is called a PN junction.

ZERO bias PN junction, *Figure 11d:*

1. As the PN junction comes to existence, the electrons from N-side diffuse to P-side and recombine with holes.
2. Electron hole pair recombination at the junction creates a region devoid of charge carriers left with immobile ions only is called the depletion region or the space charge region.
3. The immobile ions in the depletion region create an electric field which further opposes the diffusion of electrons to P-side. Thus, halting the electron hole pair recombination and thereby restricting the growth of depletion region width.
4. The width of the depletion region depends on the doping concentration. Higher the doping concentration, stronger is the field created in the space charge regions and hence thinner is the depletion region.

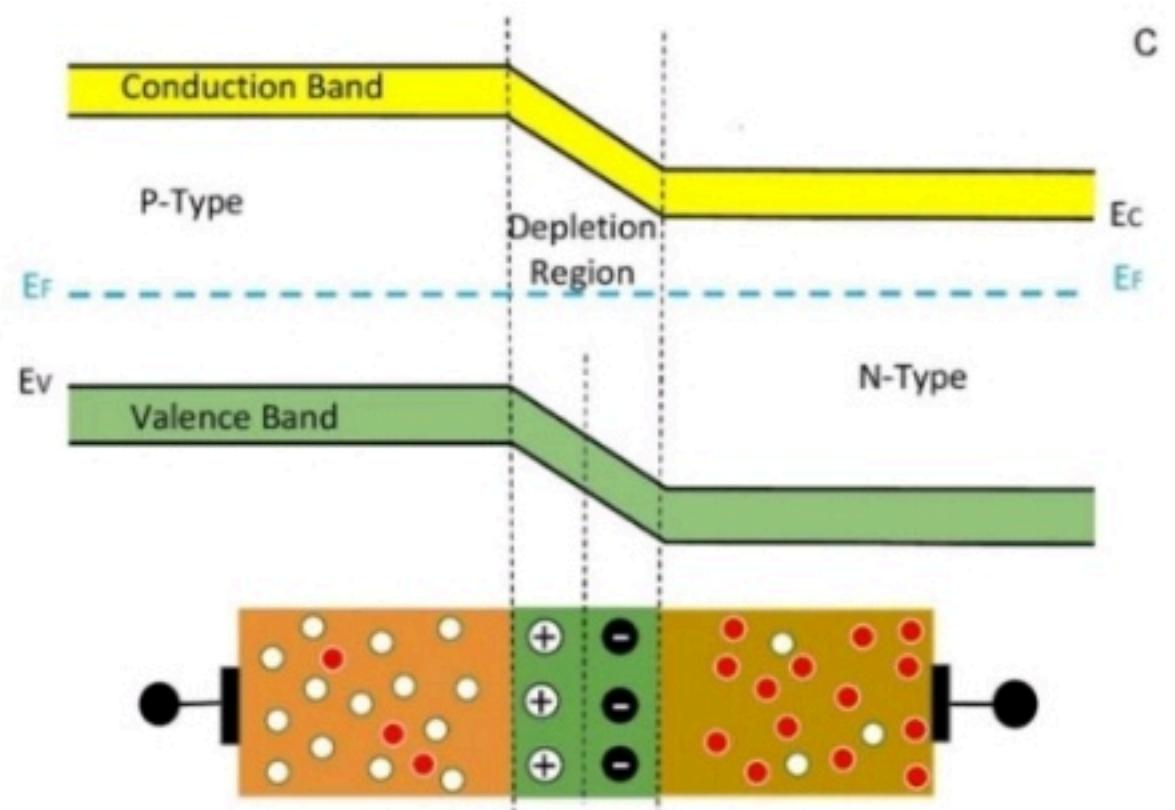
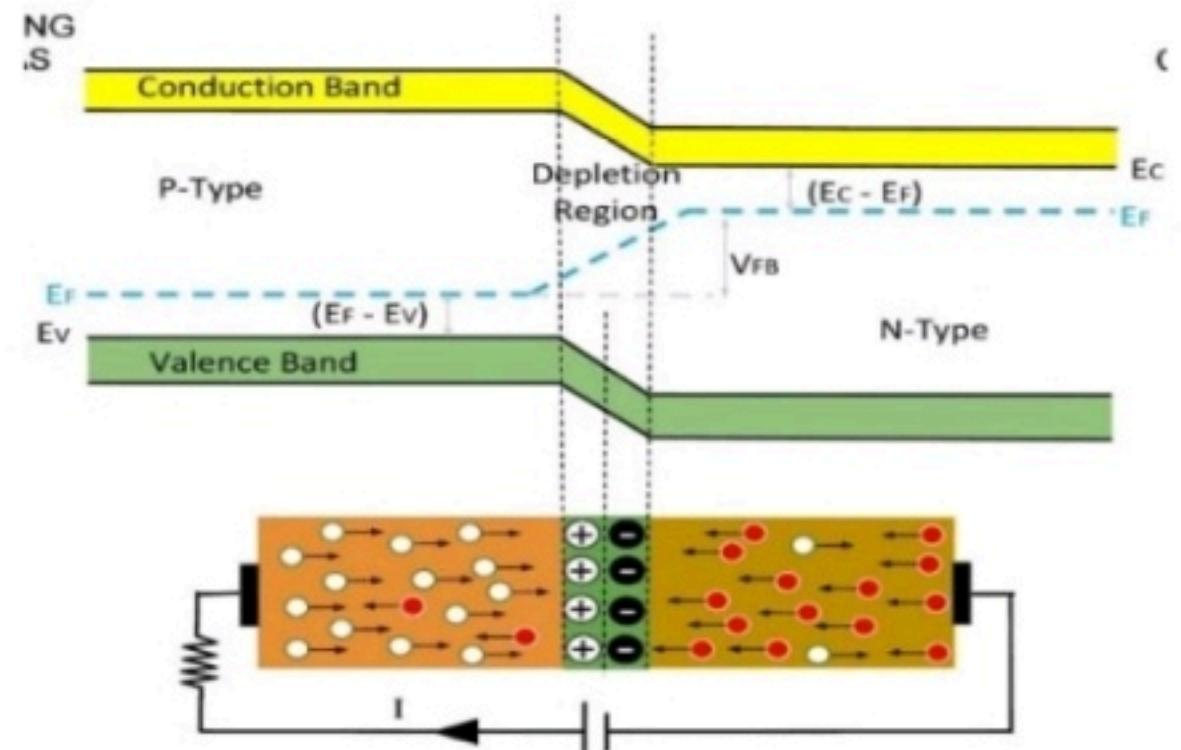


Figure 11d: Diode working in zero bias

Forward bias PN junction, [Figure 11e](#):

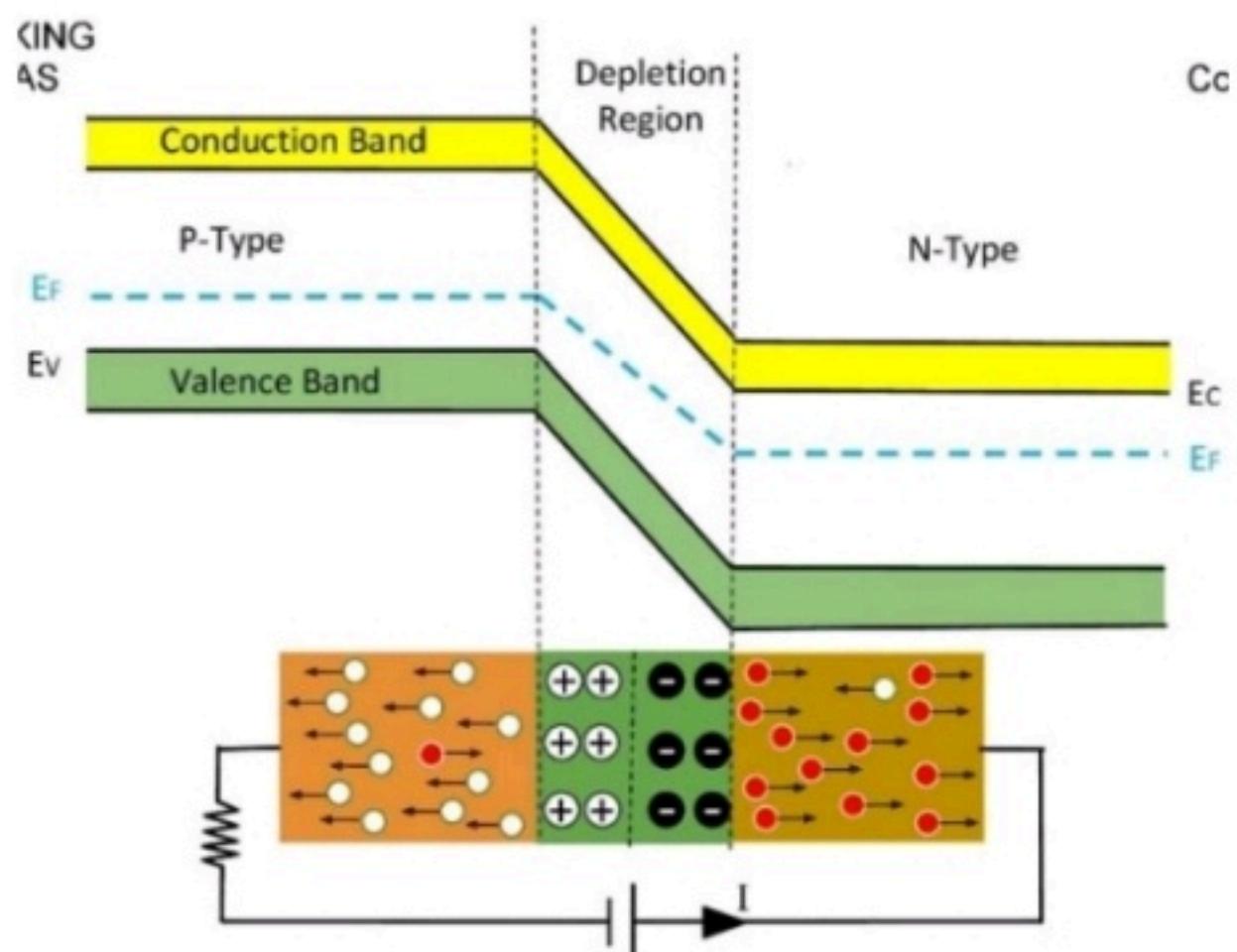
1. The PN junction is forward biased when the P-side is connected to positive of the battery and the N-side is connected to the negative of the battery.
2. Electrons from the battery pour into the N-side raising the fermi level there wherein the electrons are drained from the P-side dropping the fermi level on this side. This is indicated by the blue dotted line depicting fermi level for forward bias PN junction in [Figure 11e](#).



[Figure 11e: Forward bias](#)

Reverse bias PN junction, [Figure 11f](#):

1. The PN junction is reverse biased when the N-side is connected to positive of the battery and the P-side is connected to the negative of the battery.
2. Electrons from the battery pour into the P-side raising the fermi level there wherein the electrons are drained from the N-side dropping the fermi level on this side. This is indicated by the blue dotted line depicting fermi level for Reverse bias PN junction in Figure 11f.



[Figure 11f: Reverse bias](#)

IV characteristics of diode, *Figure 11g*:

1. **Forward bias characteristics:** As the forward bias applied goes on increasing, initially no current is observed. Once the bias applied becomes greater than the barrier potential in the depletion region (also called knee voltage) the current starts increasing exponentially shown by the red curve.

2. **Reverse bias characteristics:** As the reverse bias applied goes on increasing, initially very little almost negligible current due to minority charge carriers is observed. As the bias reaches the breakdown voltage avalanche breakdown occurs and current increases without increasing the reverse voltage. After this stage the diode breaks down and is of no use further. Reverse characteristics is shown by the blue curve.

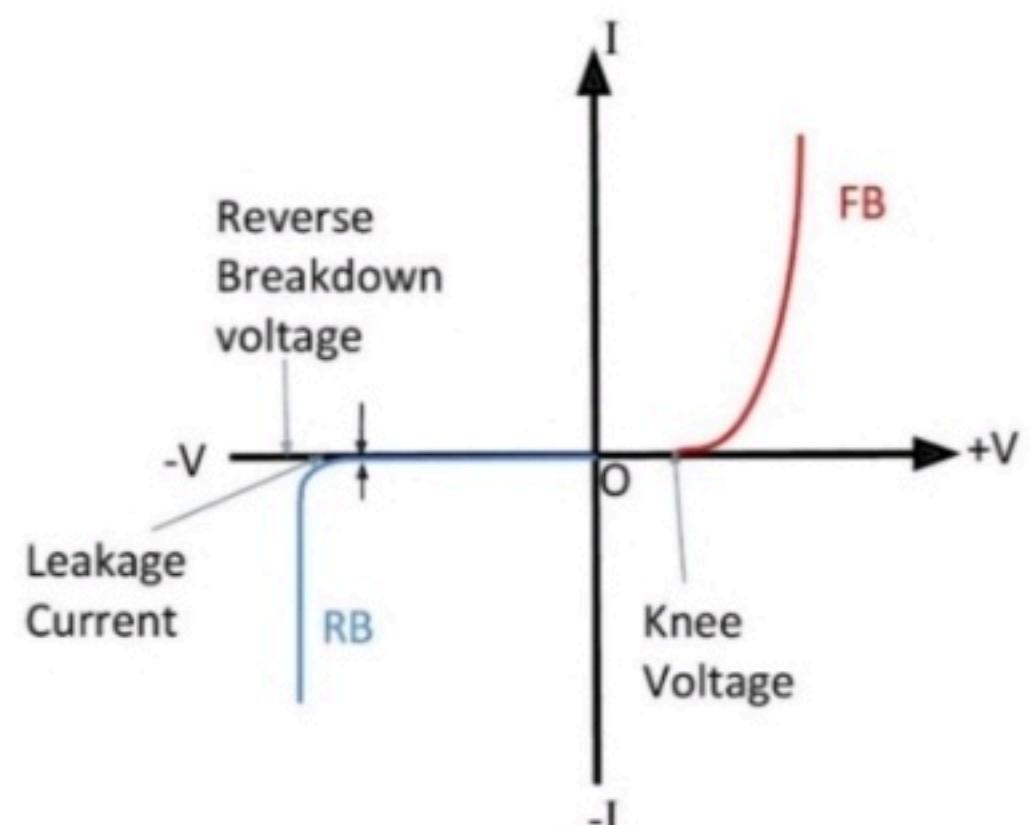


Figure 11g: IV characteristics of a diode

Q12. Explain the principle, construction and working of a LED.

(M.U. Dec 2015, 2018)[5 Marks]

LED or Light Emitting Diode is a two terminal device which emits light when supplied with electric potential. The circuit symbol of LED is shown in *Figure 12a* is just like the ordinary diode with additional two arrows pointing outward indicating emission of light as its main function. The real LED component looks like

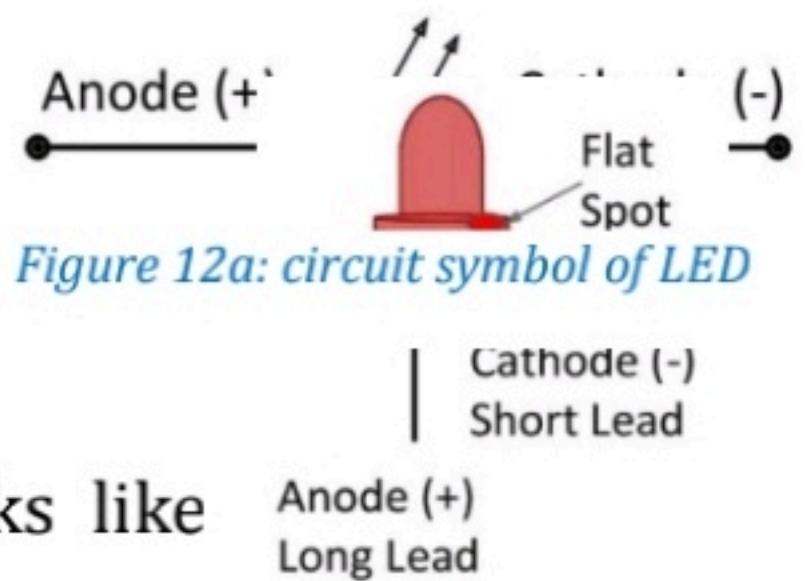


Figure 12a: circuit symbol of LED

Figure 12b where the longer leg indicated the anode and the flat spot on the top capsule marks the cathode side.

An LED is a PN junction diode which functions only in forward bias mode.

Figure 12b: real LED

Construction:

1. The most common method of constructing an LED is to stack two semiconducting layers on the substrate.
2. Indirect band gap semiconductors like silicon and germanium are used to make ordinary diodes whereas direct band gap materials like gallium arsenide (GaAs), gallium phosphide (GaP), gallium arsenide phosphide (GaAsP) etc are used in LEDs.
3. The active region is the depletion region created between the p-type and n-type semiconductors as shown in *Figure 12c*.
4. The positive terminal (anode) is connected on top of the p region, while the negative terminal (cathode) is connected below the n region via metal contacts.
5. Light emerges from the active region when recombination takes place. Since recombination takes place when the electrons move into the holes, the p-layer containing holes is kept above the n-layer containing electrons.

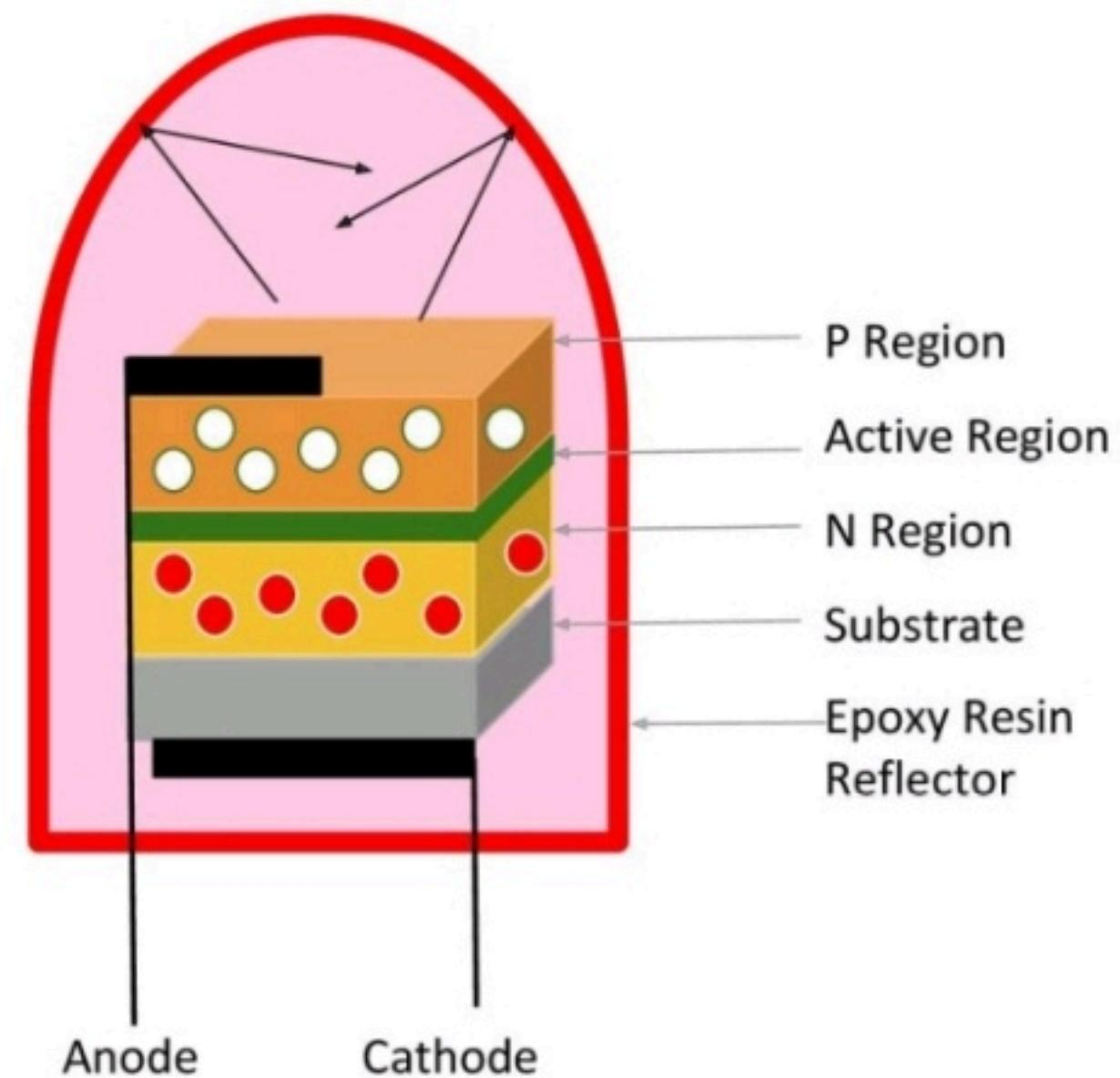


Figure 12c: LED construction

- Light thus emitted by an LED is spread in all directions. In order to prevent spreading, the structure is placed inside a small reflective hemispherical cup made from a transparent plastic or epoxy resin.
- The unique shape of the cup helps focus all light in one direction (the top) through reflection because of its unique shape. This makes the device more efficient.

Principle of LED:

When LED is forward biased the electrons in the n-region cross the junction and recombine with the holes in the p-region releasing energy in the form of light.

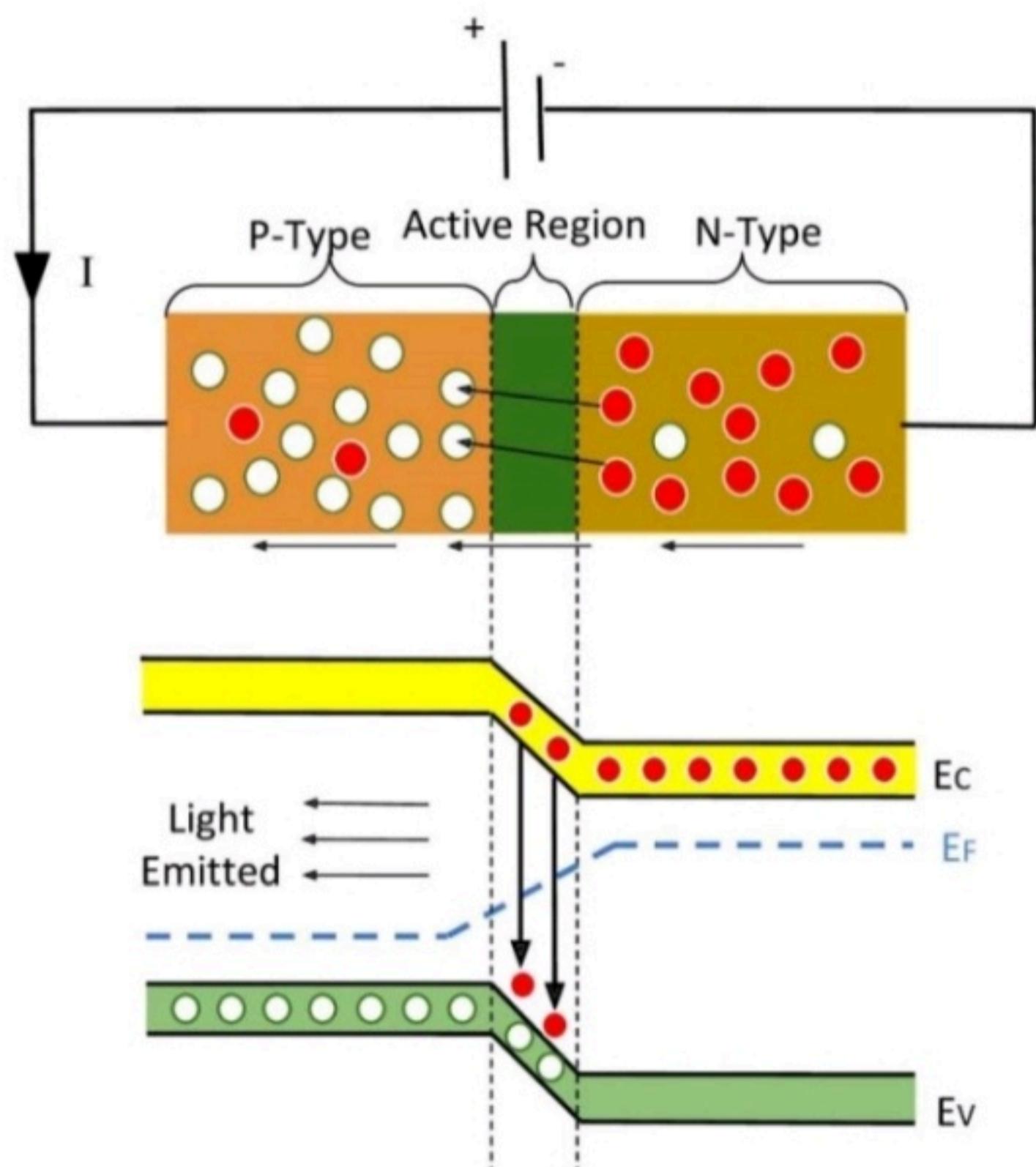


Figure 12d: LED working

- When a forward bias is applied to the LED, the energy levels of the p and n region become aligned thus allowing the electrons and holes to cross the energy gap as shown in *Figure 12d*.
- This happens because the electrons are repulsed by the battery, which causes them to move from the conduction band into the valence band to recombine with the holes.

- During this process, as the conduction band has a higher energy than valence band, the electrons move from a higher energy level, to a lower energy level to attain stability.
- This causes them to emit some energy in the form of light. This is caused by the process of recombination.
- This is how the energy emitted from LEDs is in the form of light.

This represents IV characteristics of an LED through a spectrum of light. As the wavelength of light decreases, the values of forward voltage increase for the same amount of current.

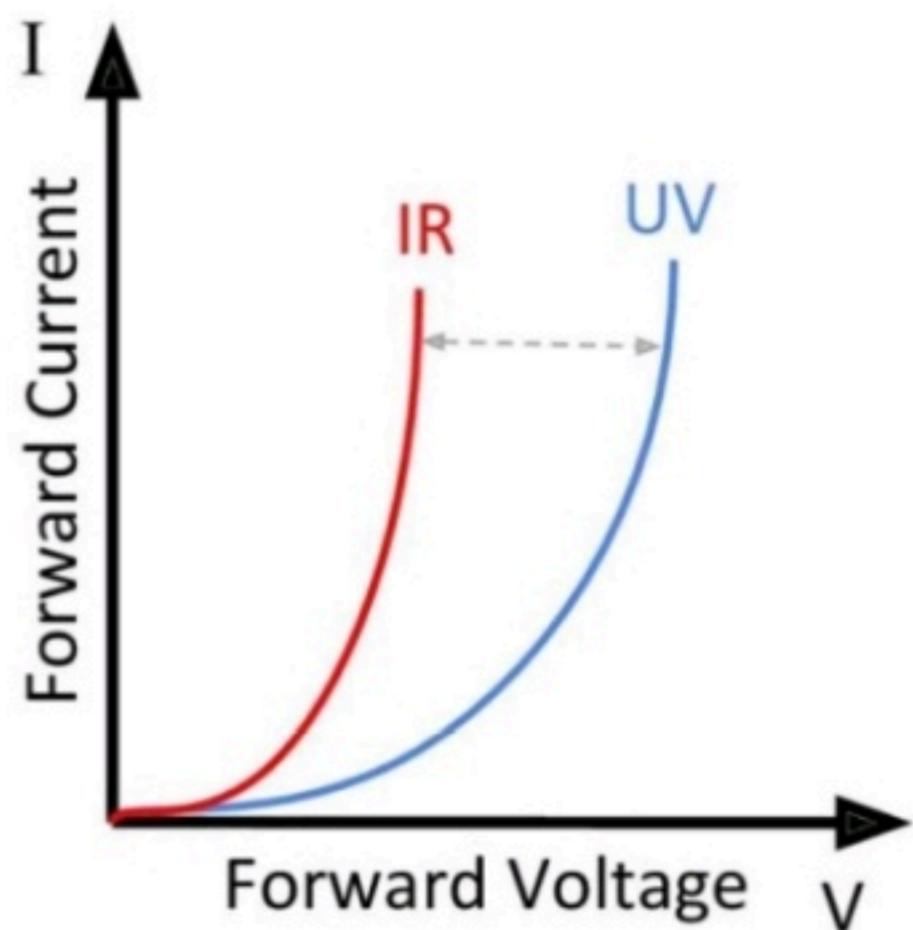


Figure 12e: IV Characteristic of LED

Q13. Explain Zener diode

It is a unique type of diode as shown in *Figure 13a* and *Figure 13b* which works in the breakdown region of the reverse bias mode. It is usually used for voltage regulation.



Figure 13a: circuit symbol



Figure 13b: symbol

Construction:

1. A Zener diode is, in essence, a silicon semiconductor which has been metalized to facilitate connections to the anode and cathode as shown in *Figure 13c*.
2. It consists of a heavily doped N-type and P-type substrate, such that the junction between the two is extremely thin.
3. It is insulated with a layer of silicon dioxide to stop the contamination of the junction.

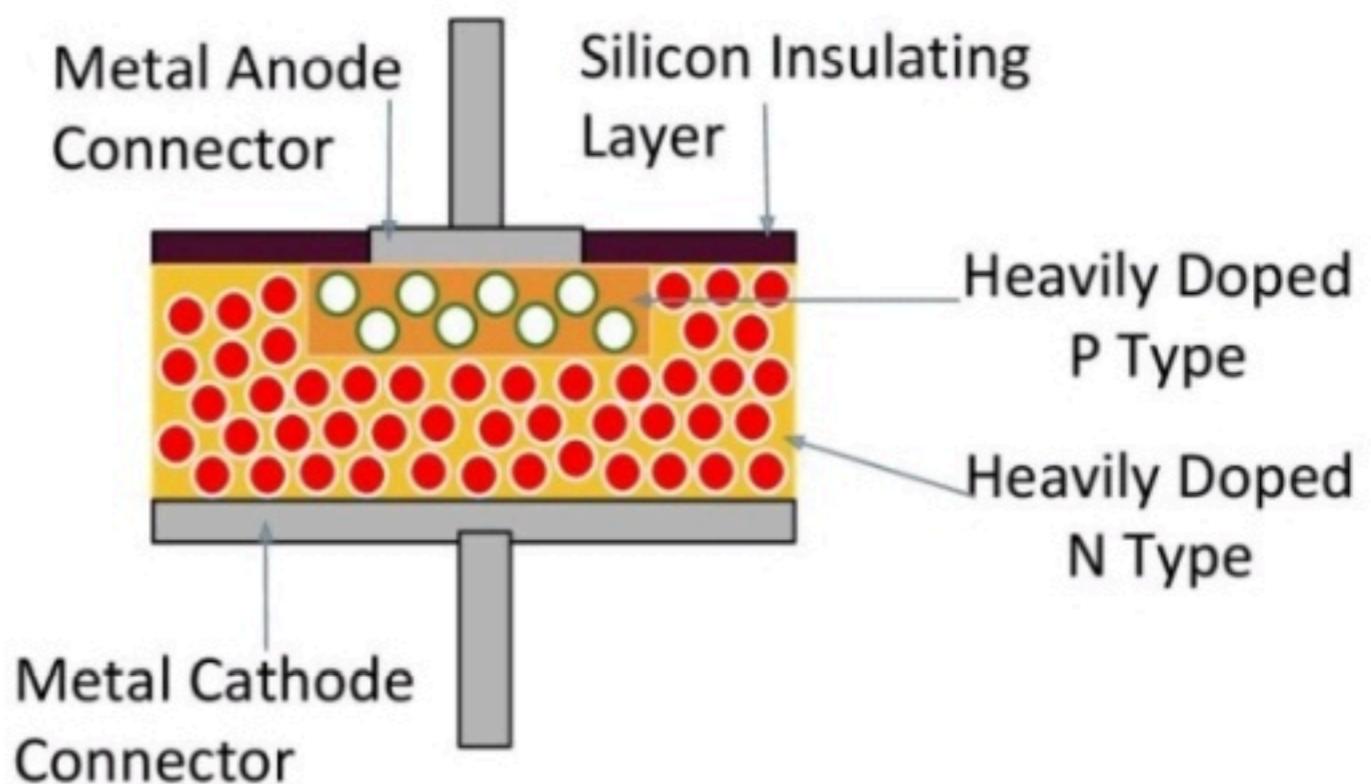
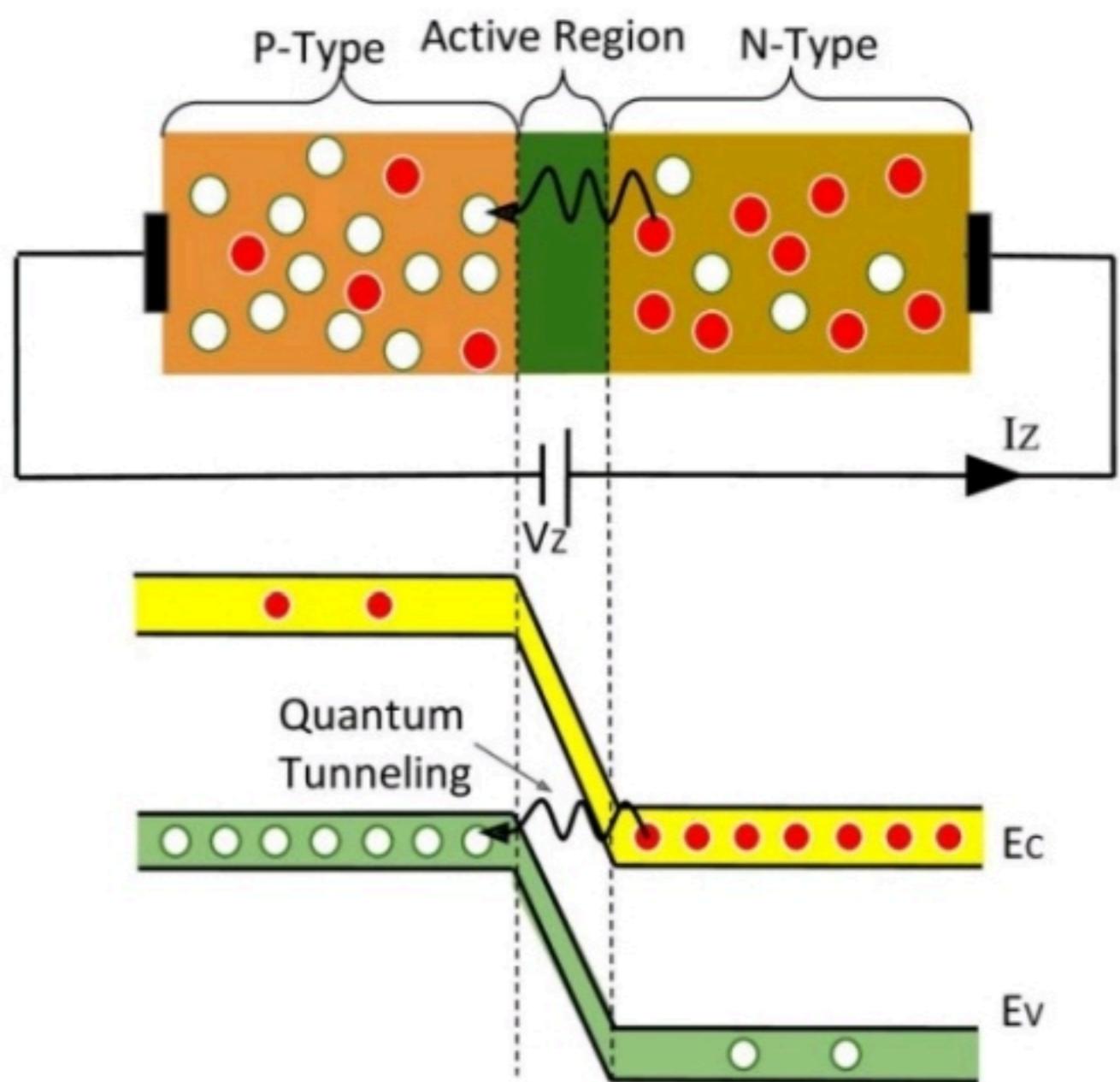


Figure 13c: Zener construction

Working:

1. A zener diode is heavily doped. Due to this the depletion region becomes extremely thin and hence breakdown can occur at a relatively low (reverse-biased) voltage.



4. In a normal diode, breakdown occurs through the avalanche effect. This effect takes place when the depletion region

Figure 13d: Zener working

becomes wider due to reverse biased modeas shown in *Figure 13d*.

2. However in a zener diode, breakdown takes place because of an effect called quantum tunneling.
3. As the width of the depletion region in the zener diode is very narrow, when a reverse biased voltage is applied to it, there is a generation of a strong, electric field.
4. When the voltage reaches zener voltage, the energy of the p type conductor increases in such a manner that the energy level of the valence band of the P-type conductor is almost equal to the energy level of the conduction band of the N-type conductor.
5. Hence, as shown in the figure, it becomes difficult for the electrons to climb up to the conduction band on the P-type conductor, but because of the narrow width, they are simply able to “tunnel” through over to the valence band of the P-type conductor. This is the quantum tunnelling effect.
6. During this process, because of the movement of free electrons across the junction, a sharp rise in current is obtained. This is Zener Breakdown.

This represents the I/V characteristics of a Zener diode. In reverse biased mode, on increasing the voltage, a sharp increase in the current is seen as shown in *Figure 13e*.

1. A Zener diode in forward bias acts exactly like a normal diode.
2. The left half side of the curve is reverse biased.
3. In reverse biased mode, once it hits the breakdown voltage, the current drastically increases. This current which spikes drastically is the breakdown current.
4. The current remains constant after the breakdown voltage.

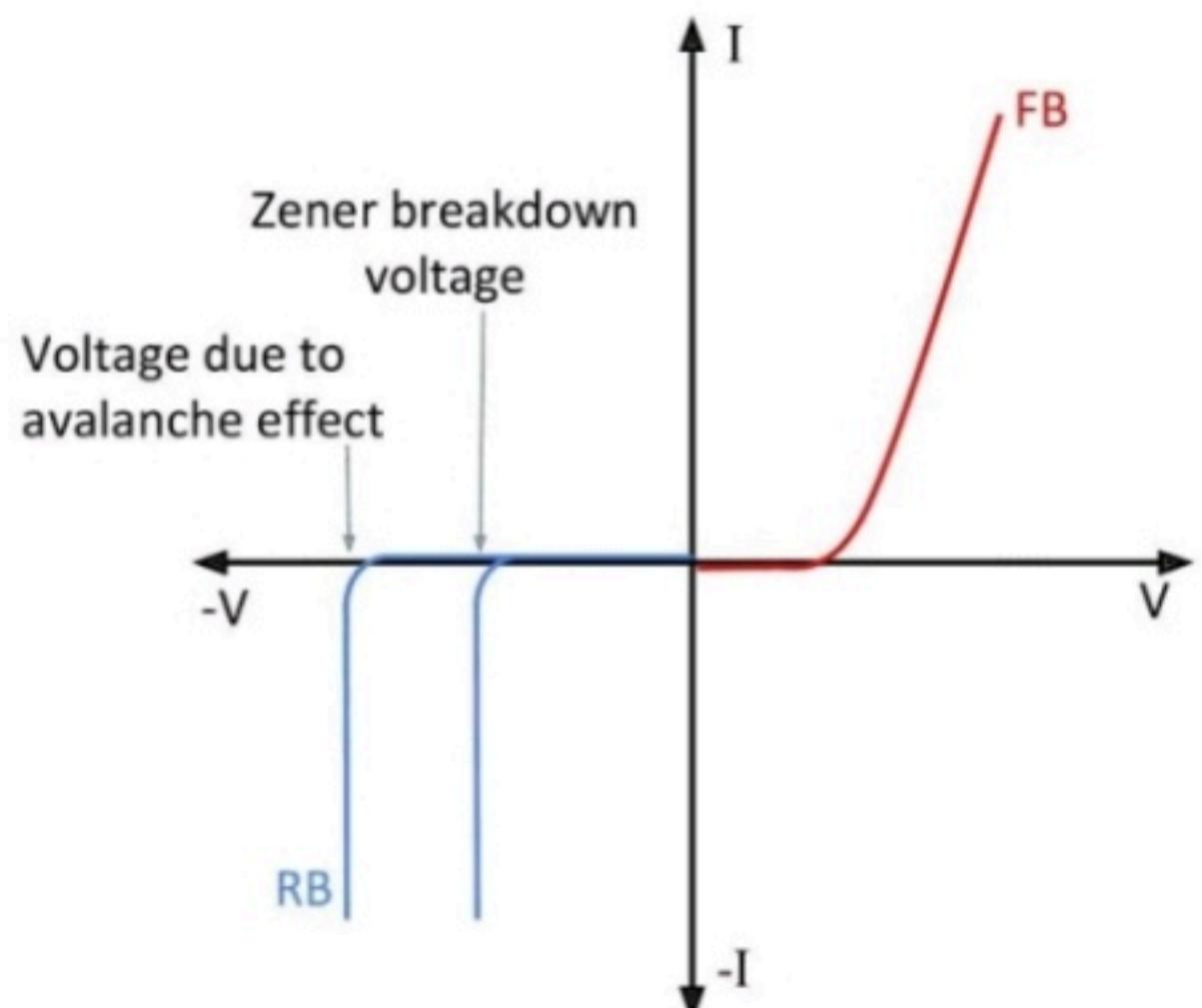


Figure 13e: Zener IV characteristics

Q14. What is the photovoltaic effect? Explain the principle and working of solar cells.

(M.U. May 2013, 2014, 2015, 2017, 2018; Dec 2012, 2013 2014, 2017)[5 Marks]

Photovoltaic effect –

1. When light energy falls on a p-n junction, a potential difference is formed across it.
2. This potential difference is capable of conducting a current through an external circuit.
3. This phenomenon is known as the photovoltaic effect.

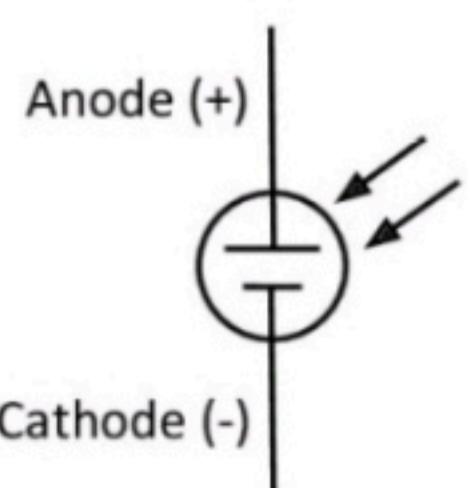


Figure 14a: Solar cell symbol

A solar cell as shown in *Figure 14a* and *Figure 14b* is a device which works on the principle of photovoltaic effect i.e. it converts incoming solar energy into electrical energy. A solar cell is made of semiconducting material. Mostly silicon, cadmium sulphide, gallium arsenide, etc. are used in making solar cells.

Construction –

1. A solar cell is basically a p-n junction connected to an external circuit as shown in *Figure 14c*.
2. The p side of the cell is made thin as compared to the n side. This is to allow the solar energy ,incident on the p side, to reach the junction.
3. The top of the solar cell is covered with glass to prevent heat (energy) loss.
4. Metallic grid contacts are fitted on the cell to connect the electrodes.
5. These solar cells are designed such that they are durable and weather resistant.

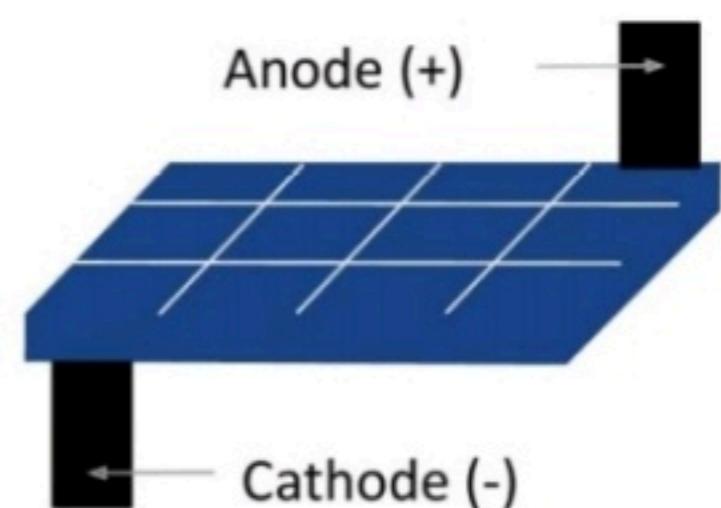


Figure 14b: symbol

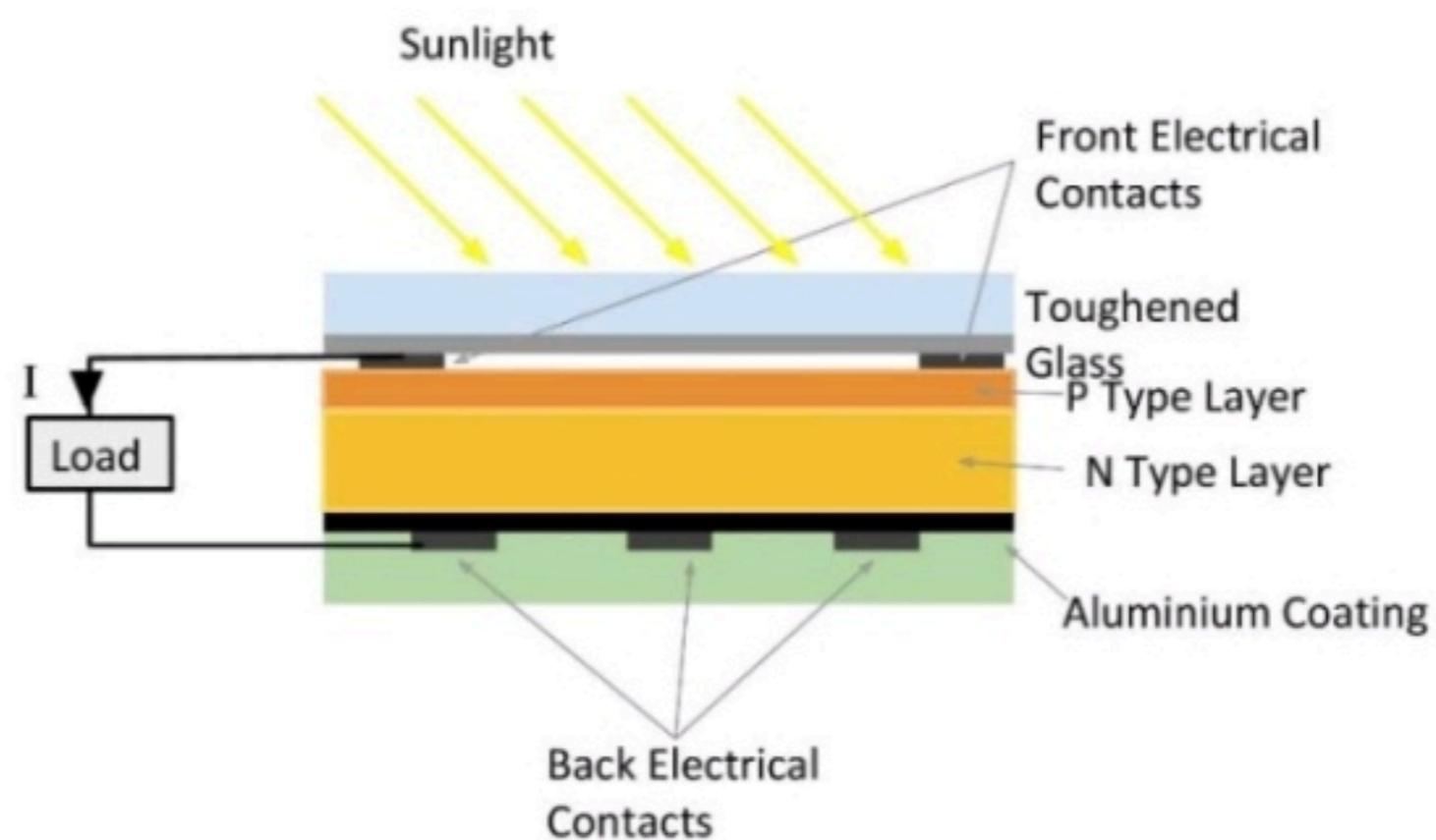


Figure 14c: Construction

Working -

1. In the equilibrium state the Fermi level lies near the conduction band of the n region and the valence band of the p region.
2. When energy in the form of solar radiation falls on the p-n junction, the electrons from the valence band get excited and reach the conduction band as shown in *Figure 14d*.
3. Subsequently, holes are formed in the valence band.
4. This process is known as electron-hole pair generation.
5. These electrons on the P side of conduction band, freely move to the N side while holes from the N side of the valence band, freely move to the P side.
6. The flow of these charge carriers results in the formation of current in solar cells.
7. This current produced is found to be directly proportional to the intensity of incident radiation.
8. With the solar cell open-circuited, the current will be at its minimum (zero) and the voltage across the cell is at its maximum, known as the solar cells open circuit voltage, or V_{oc} .
9. At the other extreme, when the solar cell is short circuited, the voltage across the cell is at its minimum (zero), known as the solar cells short circuit current, or I_{sc} .

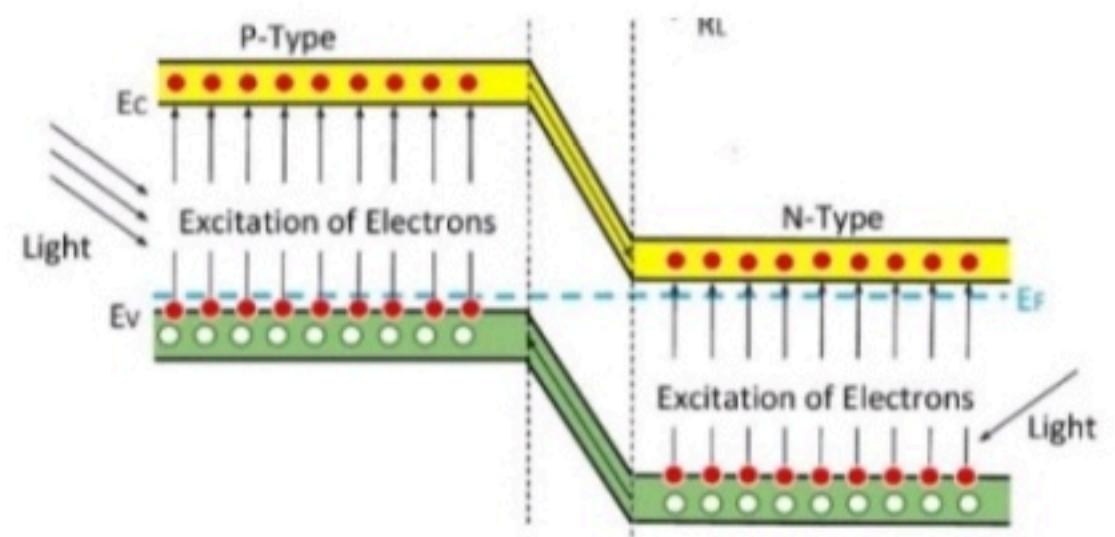


Figure 14d: Working

The span of the solar cell I-V characteristics curve as shown in *Figure 14e* ranges from the short circuit current (I_{sc}) at zero output volts, to zero current at the full open circuit voltage (V_{oc}).

There is one particular combination of current and voltage for which the power reaches its maximum value, at I_{mp} and V_{mp} . The point at which the cell generates maximum electrical power is called the “maximum power point” or MPP.

Advantages-

1. It is pollution free.
2. Solar energy is abundant in nature.
3. They are used in areas where electrical transmission is difficult.

Disadvantages -

1. Initial cost is high.
2. It has low efficiency of energy conversion.
3. Operation at night is not possible.

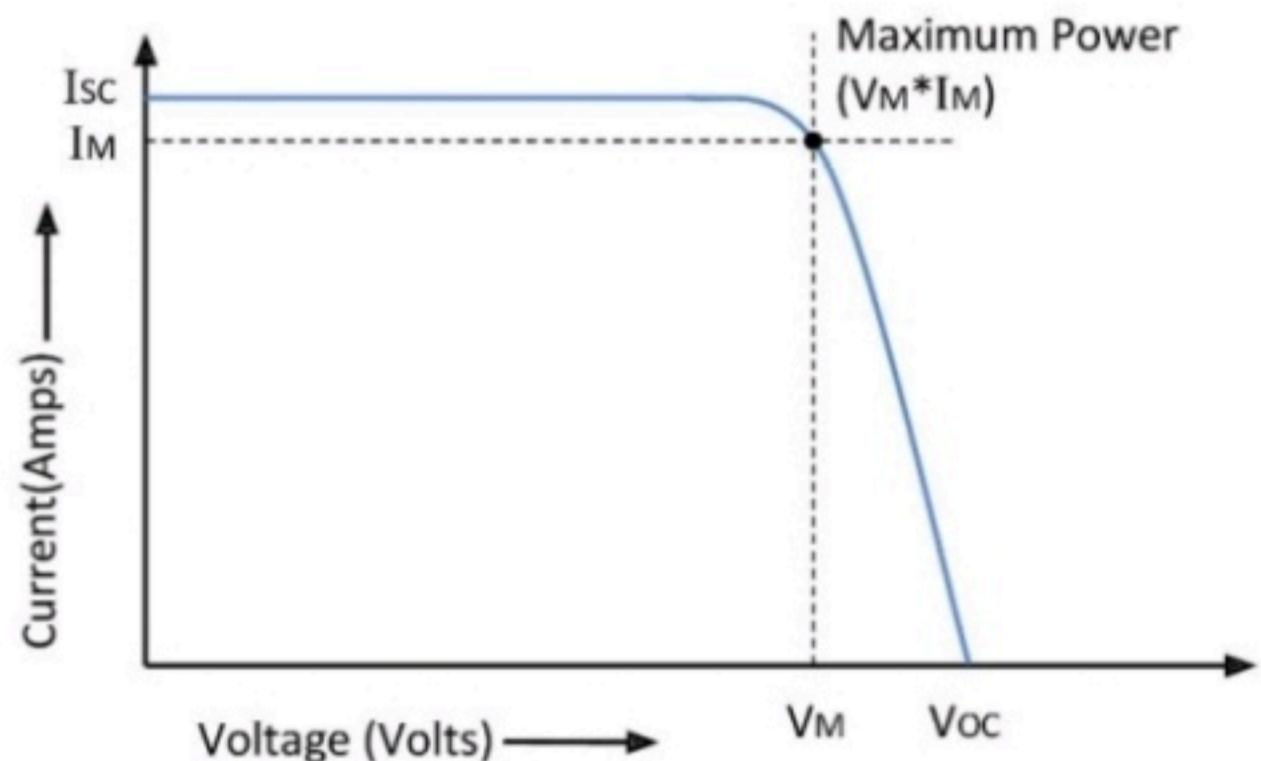


Figure 14e: IV characteristics of solar cell

FERMI-LEVEL FOR INTRINSIC SEMICONDUCTOR.

$$n = N \exp\left(\frac{-\Delta E}{KT}\right)$$

Using the concentration of electrons:

$$n_e = N_c \exp\left(\frac{-(\Delta E_e)}{KT}\right)$$

$$n_e = N_c \exp\left(\frac{-(E_C - E_F)}{KT}\right)$$

Using the concentration of holes:

$$n_e = N_c \exp\left(\frac{-(\Delta E_h)}{KT}\right)$$

$$n_h = N_V \exp\left(\frac{-(E_F - E_V)}{KT}\right) \longrightarrow n_e = n_h$$

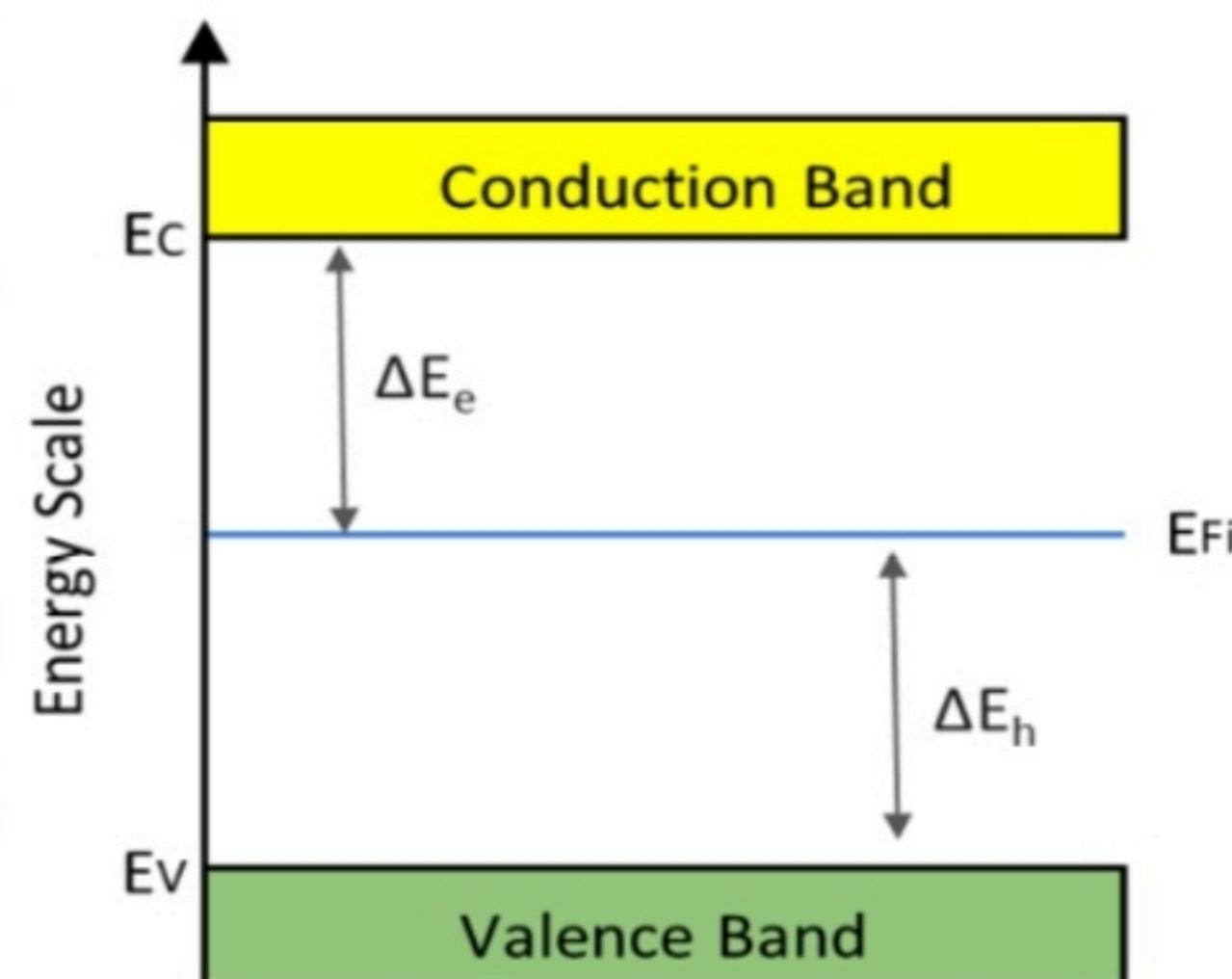
$$\exp\left(\frac{-E_C + E_F}{KT}\right) = \exp\left(\frac{-E_F + E_V}{KT}\right) \quad N_C \cong N_V$$

$$\left(\frac{-E_C + E_F}{KT}\right) = \left(\frac{-E_F + E_V}{KT}\right)$$

$$-E_C + E_F = -E_F + E_V$$

$$2E_F = E_C + E_V$$

$$E_F = \frac{E_C + E_V}{2}$$



SEMICONDUCTORS

Q1. Mobilities of electrons and holes in a sample of intrinsic Ge at room temperature are $3600 \text{ cm}^2/\text{V}\cdot\text{sec}$. and $1700 \text{ cm}^2/\text{V}\cdot\text{sec}$. respectively. If the electron and hole densities are each equal to $2.5 \times 10^{19}/\text{m}^3$, calculate the conductivity.

Given: - $\mu_e = 3600 \text{ cm}^2/\text{V}\cdot\text{sec} = 0.36 \text{ m}^2/\text{V}\cdot\text{sec}$
 $\mu_h = 1700 \text{ cm}^2/\text{V}\cdot\text{sec} = 0.17 \text{ m}^2/\text{V}\cdot\text{sec}$
 $n_i = 2.5 \times 10^{19}/\text{m}^3$

Formula: - $\sigma_{int} = n_i(\mu_e + \mu_h) \cdot e$

Solution: - $\sigma_{int} = n_i(\mu_e + \mu_h) \cdot e$
 $= 2.5 \times 10^{19} (0.37 + 0.17) \times 1.6 \times 10^{-19}$
 $= 2.16 \text{ mho/meter}$

Ans: - conductivity is **2.16 mho/meter.**

Q2. Find the resistivity of intrinsic Ge at 300° K . Given, density of carrier is $2.5 \times 10^{19}/\text{m}^3$. Mobility of electrons is $0.39 \text{ m}^2/\text{V}\cdot\text{sec}$. and mobility of holes is $0.19 \text{ m}^2/\text{V}\cdot\text{sec}$.

Charge of electron is $1.6 \times 10^{-19} \text{ C}$.

Given: - $\mu_e = 0.39 \text{ m}^2/\text{V}\cdot\text{sec}$
 $\mu_h = 0.19 \text{ m}^2/\text{V}\cdot\text{sec}$
 $n_i = 2.5 \times 10^{19}/\text{m}^3$

Formula: - $\sigma_{int} = n_i(\mu_e + \mu_h) \cdot e$

$$\rho = \frac{1}{\sigma}$$

Solution: - $\sigma_{int} = n_i(\mu_e + \mu_h) \cdot e$
 $= 2.5 \times 10^{19} \times 1.6 \times 10^{-19} \times (0.39 + 0.19)$
 $= 2.32 \text{ mho/m}$
 $\rho = \frac{1}{\sigma} = 0.431 \text{ ohm-m}$

Ans: - resistivity is 0.431 ohm-m

Q3. An impurity of 0.01 ppm (particle per million) is added to Si. The semiconductor has a resistivity of 0.25 ohm-m at 300° K. Calculate the hole concentration and its mobility. Also comment on the results. Given, atomic weight of Si 28.1 and density of Si $2.4 \times 10^3 \text{ kg/m}^3$.

Given : - impurity level=0.01ppm, $\rho = 0.25 \text{ ohm-m}$, $T=300^\circ \text{ K}$

$$M= 28.1, n=2.4 \times 10^3 \text{ kg/m}^3.$$

Formula: - $\rho = \frac{1}{en_h\mu_h}$

Solution: - no of Si atoms/unit volume

$$\begin{aligned} &= \frac{\text{avogadros no} \times \text{density}}{\text{atomic wt}} \\ &= \frac{6.023 \times 10^{23} \times 2.4 \times 10^3}{28.1} = 5.144 \times 10^{25} \text{ atoms/m}^3 \end{aligned}$$

Here,

$$1 \text{ ppm} = 1 \text{ impurity atom}/10^6 \text{ Si atoms}$$

$$1.01 \text{ ppm} = 1 \text{ impurity atom}/10^8 \text{ Si atoms}$$

\therefore no of impurity atoms

$$= \frac{5.144 \times 10^{25}}{10^8} = 5.144 \times 10^{17} \text{ atoms/m}^3$$

Each impurity contributes one hole

$\therefore 5.144 \times 10^{17}$ impurity atoms/m³ introduces the hole concentration of

$$n_h = 5.144 \times 10^{17} \text{ holes/m}^3$$

mobility , $\mu_h = \frac{1}{\rho n_h}$

$$= \frac{1}{0.25 \times 1.6 \times 10^{-19} \times 5.144 \times 10^{17}} = 48.6003 \text{ m}^2/\text{volt-sec}$$

$$\mu_h = 48.6003 \text{ m}^2/\text{volt-sec}$$

Ans: - hole concentration is 5.144×10^{17} holes/m³ and

Mobility of holes is 48.6003 m²/volt-sec.

Comment: - with the addition of impurity atoms, the hole concentration increases though its mobility remains the same.

Q4.The resistivity of Cu is 1.72×10^8 ohm-m. Calculate the mobility of electrons in Cu. Given, the number of electrons per unit volume is $10.41 \times 10^{28}/\text{m}^3$.

Given: - $\rho = 1.72 \times 10^8$ ohm-m
 $n_e = 10.41 \times 10^{28}/\text{m}^3$.

Formula: - $\mu = \frac{\sigma}{n_e \cdot e}$ and $\sigma = \frac{1}{\rho}$

Solution: - $\sigma = \frac{1}{\rho} = \frac{1}{1.72 \times 10^{-8}} \text{ mho/m}$
 $= 0.58 \times 10^8 \text{ mho/m}$

$$\mu = \frac{\sigma}{n_e \cdot e} = \frac{0.58 \times 10^8}{10.41 \times 10^{28} \times 1.6 \times 10^{-19}} = 3.482 \times 10^{-3} \text{ m}^2/\text{V-sec}$$

Ans: - hence, mobility is 3.482×10^{-3} m²/V-sec

Q5.What is the probability of an electron being thermally excited to the conduction band at Si at 27°C . the band gap is 1.12 eV.

Given : - $T = 27^\circ \text{C} = 300 \text{K}$, $E_g = 1.12 \text{eV}$

$$K = 1.38 \times 10^{-23} \text{ J/K} = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} = 86.25 \times 10^{-6} \text{ eV/K}$$

Formula: - $f(E_C) = \frac{1}{1 + e^{(E_C - E_f)/kT}}$

Solution: - Si is an intrinsic semiconductor. hence

$$E_c - E_f = \frac{E_g}{2} = 0.56 \text{eV}$$

$$f(E_C) = \frac{1}{1 + e^{\frac{0.56}{86.25 \times 10^{-6} \times 300}}} = 3.9882 \times 10^{-10}$$

Ans: - probability is 3.9882×10^{-10}

Q6. In a solid there is an energy level lying 0.012eV below Fermi level. What is the probability of this level not being occupied by electron at 27° C?

Given: -

$$E_F - E = 0.012 \text{ eV}, \quad T = 27^\circ \text{C} = 300 \text{ K}$$

$$K = 1.38 \times 10^{-23} \text{ J/K} = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} = 86.25 \times 10^{-6} \text{ eV/K}$$

Formula: - $f(E_c) = \frac{1}{1 + e^{(E_c - E_f)/kT}}$

Solution: -

Total probability=1

Probability of occupying an energy + probability of not occupying the energy state=1

$f(E) + \text{probability of not occupying the energy state} = 1$

probability of not occupying the energy state=1-f(E)

$$\text{here, } f(E_c) = \frac{1}{1 + e^{(E_c - E_f)/kT}} = \frac{1}{1 + e^{\frac{0.012}{86.25 \times 10^{-6} \times 300}}} = 0.386$$

$$\text{hence, } 1 - f(E) = 1 - 0.386 = 0.614$$

Ans : - probability of not occupying is 0.614 or 61.4%

Q7. Fermi level in K is 2.1 eV. What are the energies for which the probabilities of occupancy at 300K are 0.99 and 0.01.

Given: - $E_p = 2.1 \text{ eV}$, $f(E_1) = 0.99$, $f(E_2) = 0.01$, $T = 300 \text{ K}$

$$K = 1.38 \times 10^{-23} \frac{\text{J}}{\text{K}} = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} = 86.25 \times 10^{-6} \text{ eV/K}$$

Formula: - $f(E_c) = \frac{1}{1 + e^{(E_c - E_p)/kT}}$

Solution: -

$$f(E_1) = \frac{1}{1 + e^{(E_1 - E_p)/kT}}$$

$$0.99 = \frac{1}{1 + e^{(E_1 - E_p)/kT}}$$

$$1 + e^{(E_1 - E_p)/kT} = 1.01$$

$$e^{(E_1 - E_p)/kT} = 0.01$$

$$E_1 - E_p = -0.1192 \text{ eV}$$

$$E_1 = 1.9808 \text{ eV}$$

$$\text{And, } f(E_2) = \frac{1}{1+e^{(E_2-E_p)/kT}}$$

$$1.01 = \frac{1}{1+e^{(E_2-E_p)/kT}}$$

$$E_2 - E_p = 0.1189 \text{ eV}$$

$$E_2 = 2.2189 \text{ eV}$$

Ans :- The energies are 1.9808 eV and 2.2189 eV respectively

Q8. In n type semiconductor, the Fermi level lies 0.4eV below the conduction band .If the concentration of the donor atom is doubled, find the new position of the level w.r.t. the conduction band.

Given:-

$$E_c - E_f = 0.4 \text{ eV},$$

$$n_d' = 2n_d$$

$$T=300 \text{ K}$$

Formula:- $n_d = N_c e^{(E_F - E_C)/kT}$

Solution:-

$$n_d' = N_c e^{(E'_F - E_C)/kT}$$

$$\frac{n_d'}{n_d} = e^{[(E'_F - E_C) - (E_F - E_C)]/kT}$$

$$\begin{aligned} E_C - E'_F &= (E_C - E_F) - kT \ln 2 \\ &= 0.4 - 0.018 \end{aligned}$$

$$E_C - E'_F = 0.3821 \text{ eV}$$

Ans:- The new Fermi level will be 0.3821eV below conduction band. Hence, the Fermi level will be shifted towards the conduction band by an amount (0.4-0.3821) eV=0.0179eV.

Q9. In a Hall effect set up, a n-type Ge sample with donor concentration $2.5 \times 10^{21} / m^3$ is used. If the magnetic field is $0.5 \text{ wb} / m^2$, the current density is $500 \text{ A} / m^3$ and the thickness of the sample is 4mm, find the Hall voltage.

Given: - $n_e = 2.5 \times 10^{21}$, $B = 0.5 \text{ wb} / m^2$, $J = 500 \text{ A} / m^3$

$$w = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$$

Formula: -

$$V_H = \frac{BJd}{n_e e}$$

Solution: -

$$V_H = \frac{0.5 \times 500 \times 4 \times 10^{-3}}{2.5 \times 10^{21} \times 1.6 \times 10^{-19}}$$

$$= 2.5 \text{ mV}$$

Ans: - The Hall voltage is 2.5mV.

Q10. Calculate the charge carrier density and electron mobility of n type Si specimen of which the conductivity and Hall coefficients are given as 950 mho/m and $1.5 \times 10^{-4} \text{ m}^3 / \text{C}$.

Given: -

$$\sigma = 950 \text{ mho/m}, \quad R_H = 1.5 \times 10^{-4} \text{ m}^3 / \text{C}.$$

Formula : -

$$R_H = \frac{1}{n_e e}, \quad \mu_e = \sigma R_H$$

Solution: -

$$n_e = \frac{1}{R_H e} = \frac{1}{1.6 \times 10^{-19} \times 1.5} = 4.1667 \times 10^{18} / m^3$$

$$\mu_e = \sigma R_H = 950 \times 1.5 \times 10^{-4} = 0.1425 \text{ m}^2 / \text{V-sec}$$

Ans: - The charge carrier density is $4.1667 \times 10^{18} / m^3$ and the Electron mobility is $0.1425 \text{ m}^2 / \text{V-sec}$

Q11. Determine the current density of a n type semiconductor sample with Hall electric field of 150 V/m . If its Hall coefficient is $-0.0145 \text{ m}^3/\text{C}$ and the electron mobility is $0.39 \text{ m}^2/\text{V-sec}$, determine the Hall current density

Given: - $E_H = 150$, $R_H = 0.0145 \text{ m}^3/\text{C}$ $\mu_e = 0.39 \text{ m}^2/\text{V-sec}$

Formula: -

$$J = \sigma E_H, \quad \mu_e = s R_H$$

Solutions: -

$$\sigma = \frac{\mu_e}{R_H} = \frac{0.39}{0.0145} = 26.8966 \text{ mho/m}$$

$$J = 26.8966 \times 150 = 4034.49 \text{ Amp/m}^2$$

Ans: - The current density is 4034.49 Amp/m^2

Q12. The mobility of holes is $0.025 \text{ m}^2/\text{V-sec}$. What would be resistivity of p type if the Hall coefficient of sample is $2.25 \times 10^{-5} \text{ m}^3/\text{C}$.

Given: - $\mu_h = 0.025 \text{ m}^2/\text{V-sec}$, $R_H = 2.25 \times 10^{-5} \text{ m}^3/\text{C}$.

Formula: -

$$\rho = \frac{1}{\sigma}, \quad \mu_h = \sigma R_H$$

Solution: -

$$\sigma = \frac{\mu_h}{R_H} = \frac{0.025}{2.25 \times 10^{-5}} = 1111.11 \text{ mho/m}$$

$$\rho = \frac{1}{1111.11} = 9 \times 10^{-4} \Omega \cdot \text{m}$$

Ans: - The resistivity is $9 \times 10^{-4} \Omega \cdot \text{m}$

Q13. Find the Hall coefficient of sodium assuming BCC structure for Na of lattice constant 4.28 \AA .

Given: - $a=4.28\text{ \AA}^0$, Number of atoms/unit cell=2 for BCC

Formula: - $R_H = \frac{1}{n_e e}$

Solutions :-

Volume of 1 unit cell= a^3

Number of atoms/unit volume= $2/a^3$

Assuming that each atom contributes one donor electron

$$n_e = \frac{2}{a^3} = \frac{2}{(4.28 \times 10^{-10})^3} = 2.5509 \times 10^{28} / \text{m}^3$$

$$R_H = \frac{1}{2.5509 \times 10^{28} \times 1.6 \times 10^{-19}} = 2.4501 \times 10^{-10} \text{ m}^3 / \text{C}$$

Ans: - The Hall coefficient is $2.4501 \times 10^{-10} \text{ m}^3 / \text{C}$

* Newton's Rings :-

Newton's Rings are the name given to interference pattern formed between plano convex lens & plane glass.

The air gap formed b/w PCL & glass is small & has thickness which is comparable to λ , it produces interference, when it is illuminated by light.

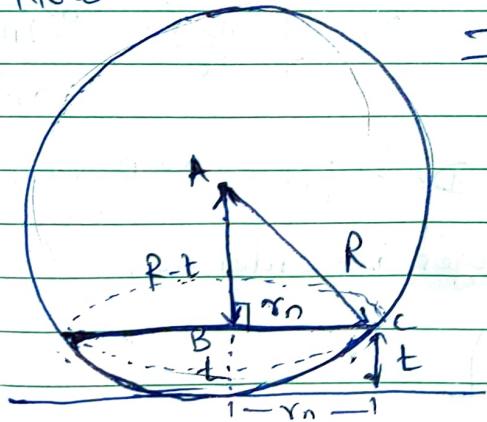
- Considering this air gap as wedge shaped air film, the path difference betⁿ reflected rays from upper & lower surface of PCL & glass (→) as,

$$\Delta = 2lt \cos(\pi + \theta)$$

- If we consider normal incidence of light $\theta=0$ & for very large value of radius of curvature (R), $\theta=0$

$$\underline{\Delta = 2lt} \quad \text{--- (1)} \quad \text{as } \cos(\pi + \theta) = \cos 0 = 1$$

Now



In fig., R → Radius of curvature of PCL
 r_n → Radius of Ring (Dark/Bright) formed at thickness t of wedge shaped film.

$\triangle ABC$ is right angle Δ $\angle B = 90^\circ$

By Pythagoras theorem

$$R^2 = (R-t)^2 + r_n^2$$

$$R^2 = (R^2 - 2Rt + t^2) + r_n^2$$

$$-R^2 = -2Rt + t^2$$

$$r_n^2 = 2Rt - t^2$$

$$r_n^2 = 2Rt$$

$$\therefore 2t = \frac{r_n^2}{R} \quad \text{--- (2)}$$

$t \ll$ small if R is large.

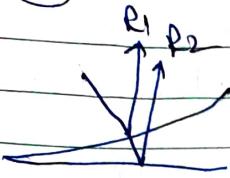
$\therefore t^2$ neglected.

Substituting (2) in (1)

$$\Delta = 2lt$$

$$\boxed{\Delta = \frac{r_n^2}{R} l} \quad \text{--- (3)}$$

(A) In Reflected system of Rays.



R₂ reflection is from denser boundary.

- It will suffer a phase shift of 180° or
- path length of γ_2 will be added in that.
- equation (3) now will become

$$\textcircled{0} \Delta = \frac{\gamma_n^2}{R} u + \gamma_2$$

For Dark Ring,

$$\Delta = (2n+1) \gamma_2$$

$$\frac{\gamma_n^2}{R} u + \gamma_2 = (2n+1) \gamma_2$$

$$\frac{\gamma_n^2}{R} u + \gamma_2 = \frac{2n\lambda}{2} + \gamma_2$$

$$\frac{\gamma_n^2}{R} u = n\lambda$$

$$\textcircled{0} \gamma_n^2 = \frac{nR\lambda}{u} \Rightarrow \boxed{\gamma_n = \sqrt{\frac{nR\lambda}{u}}}$$

$$\textcircled{0} D_n^2 = \frac{4nR\lambda}{u}$$

$$D_n \propto \sqrt{\frac{4nR\lambda}{u}}$$

$$\textcircled{0} r_n = \frac{D_n}{2}$$

$$\textcircled{0} u=1$$

$$\boxed{D_n^2 = 4nR\lambda}$$

$$D_n \propto \sqrt{n}$$

$$\textcircled{0} R, \lambda \& 4$$

are const.

$\textcircled{0}$ n here is an integer i.e. order no.

$$n=1, 2, 3 \dots$$

$$\boxed{D_n \propto \text{natural no.}}$$

for Bright Rings

$$\Delta = n\lambda$$

$$\frac{\gamma_n^2}{R} u \pm \gamma_2 = n\lambda$$

$$\frac{\gamma_n^2}{R} u = n\lambda \pm \gamma_2$$

$$\frac{\gamma_n^2}{R} u = (2n+1) \gamma_2$$

$$\gamma_n^2 = \frac{R\lambda(2n+1)}{2u}$$

$$\textcircled{0} D_n^2 = \frac{4R\lambda}{u}(2n+1)$$

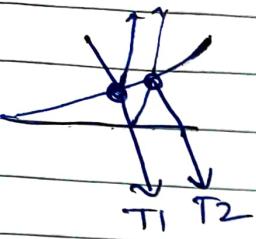
$$\textcircled{0} \gamma_n^2 = \frac{D_n^2}{4}$$

$$\textcircled{0} D_n^2 = \frac{2R\lambda}{u}(2n+1)$$

$$D_n \propto \sqrt{2n+1}$$

$\textcircled{0}$ D_n ~~add~~ natural no

(B) In transmitted system of rays



T_1 & T_2 both are reflected from denser boundary. \therefore both will suffer a phase shift of 180° on path length of $\lambda/2$.

$$\therefore \Delta = 2R \cos(\theta)$$

$$\therefore \Delta = \frac{\pi n^2}{R} u \pm \lambda/2 \pm \lambda/2$$

$$\Delta = \frac{\pi n^2}{R} u$$

For Bright Rings

$$\Delta = n\lambda$$

$$\frac{\pi n^2}{R} u = n\lambda$$

$$\therefore r_n^2 = \frac{Rn\lambda}{\pi}$$

$$r_n = \sqrt{\frac{Rn\lambda}{\pi}}$$

$$D_n^2 = \frac{4Rn\lambda}{\pi} \quad \therefore u=1 \quad D_n^2 = 4Rn\lambda$$

$$D_n = \sqrt{4Rn\lambda}$$

for Bright Rings

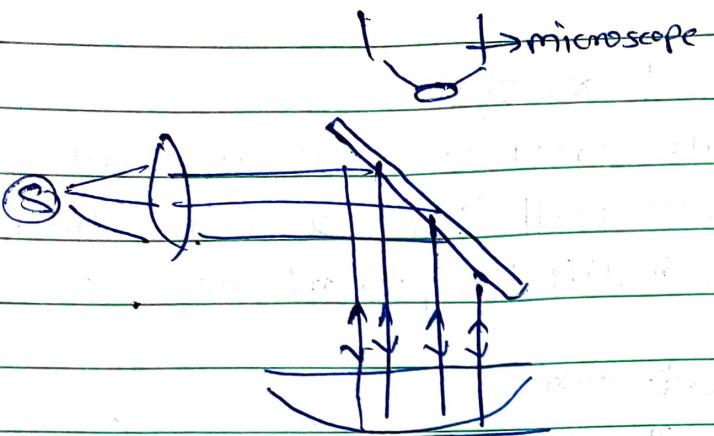
$$\Delta = (2n+1)\lambda/2$$

$$\frac{\pi n^2}{R} u = (2n+1)\lambda/2$$

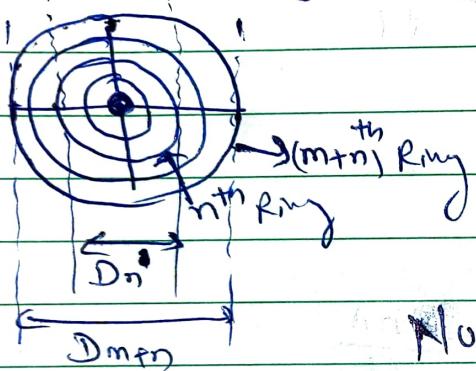
$$r_n^2 = \frac{R\lambda}{2\pi} (2n+1)$$

$$D_n^2 = \frac{4R\lambda}{2\pi} (2n+1) \Rightarrow D_n^2 = \frac{2R\lambda}{\pi} (2n+1)$$

Appn of Newton's rings \rightarrow ① Determination of ~~RP~~ Wavelength & Radius of curvature



Exp. arrangement to get Newton's Rings



Let Dia of n^{th} Dark ring is

$$D_n^2 = 4Rn\lambda \quad \text{--- (1)} \quad \text{so, } R = ?$$

Dia of $(m+n)^{\text{th}}$ Dark ring is

$$D_{m+n}^2 = 4R(m+n)\lambda \quad \text{--- (2)} \quad \text{so, } R = ?$$

$$\text{eq } (2) - \text{eq } (1)$$

$$\begin{aligned} D_{m+n}^2 - D_n^2 &= 4R(m+n)\lambda - 4Rn\lambda \\ &= 4R\lambda(m+n-n) \end{aligned}$$

$$D_{m+n}^2 - D_n^2 = 4Rm\lambda \quad \text{--- (3)}$$

Now from eq (3)

$$\boxed{\lambda = \frac{D_{m+n}^2 - D_n^2}{4Rm}}$$

With this eq, if we know D_{m+n} , D_n , R & m
we can calculate λ (wavelength) of unknown source.

Again from eq (3)

$$\boxed{R = \frac{D_{m+n}^2 - D_n^2}{4\lambda m}}$$

With this eq, if we know λ , D_{m+n} , D_n , m
we can find out Radius of curvature of PCL.

Module 1. QUANTUM PHYSICS

Session 5: Schrodinger's Time Independent and Time Dependent Equation

Session 6 : Particle trapped in an Infinite Potential Well

Session 7 : Applications of Quantum Physics

THE WAVE FUNCTION ψ :

Schrödinger assumed that a quantity ψ represents a De Broglie wave. It is called as a wave function. This wave function mathematically describes the motion of particle. It is not possible to locate the particle precisely at point (x,y,z) . This wave function gives a probability of finding the particle at (x,y,z) at time t . It is a function of space and time coordinates.

The wave function ψ introduced in Schrödinger's wave mechanics is an abstract mathematical quantity. It is this wave function, which takes care of the wave nature or wavelike behaviour of particles. It may be complex in nature

$$\psi(x,t) = A(x,t) + iB(x,t)$$

where, A is real part and B is the imaginary part.

Its complex conjugate is $\psi^*(x,t) = A(x,t) - iB(X,t)$

If $dV = dx dy dz$ is an infinitesimally small volume element surrounding point (x,y,z) the probability of finding the particle in that volume element is given by -

$$\psi^* \psi = A^2 - i^2 B^2 = A^2 + B^2 = |\psi|^2$$

And the product $\psi^* \psi = A^2 - i^2 B^2 = A^2 + B^2 = |\psi|^2$ gives the probability of finding the particle at (x,y,z) at time (t) .

If particle exists, probability of finding the particle somewhere in the space must be unity.

$$\iint \int_{-\infty}^{\infty} \Psi^* \Psi dx dy dz = 1$$

This condition is called as normalization condition. The wavefunction is normalized. It means, it satisfies this condition.

The wave function is required to fulfill the following conditions –

1. ψ must be finite, continuous and single valued everywhere.
2. Its derivative, $\frac{\partial \psi}{\partial x}$ or $\frac{\partial \psi}{\partial t}$ must also be finite, continuous and single valued everywhere.
3. ψ must have atleast some physically acceptable solutions.
4. ψ must obey the principle of linear superposition i.e. ψ can anytime be expressed as a linear combination of two wavefunctions say

$$\psi(x,t) = A\phi_1(x,t) + B\phi_2(x,t)$$

However, except condition 4, a particular condition may not be satisfied for a certain application.

SCHRODINGER'S EQUATION

It is an equation which describes the behaviour of the wave function associated with microscopic particles.

In 1926, Schrodinger developed a rigorous mathematical theory using De Broglie's idea of matter waves. This theory is known as 'Wave Mechanics'. The essential feature of this theory is that it incorporates the expression for De Broglie wavelength into the general classical equation derived for a moving particle is known as Schrodinger's wave equation.

Derivation of Schrodinger's Time Inependent Wave Equation (STIE):

According to the De Broglie theory, a particle of mass 'm' moving with velocity 'v' is associated with a wave of wavelength $\lambda = \frac{h}{mv}$. Though, we do not know, what it is that vibrates, this vibration is represented by ' ψ ', the periodic changes which are responsible for

$$\lambda = \frac{h}{mv} \quad \dots \quad (1)$$

The wave equation of stationary wave associated with the particle in terms of Cartesian coordinate system at any instant is given by –

$$\psi = \psi_0 \sin \omega t$$

$$\psi = \psi_0 \sin 2\pi v t \quad \dots \quad (2)$$

Where ψ_0 is the amplitude at the point under consideration which is a function of x, y, z and v (v is frequency of vibration).

The classical differential equation of wave motion can be written as –

$$\begin{aligned} \frac{\partial^2 \psi}{\partial t^2} &= v^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) \\ \frac{\partial^2 \psi}{\partial t^2} &= v^2 \nabla^2 \psi \quad \dots \quad (3) \end{aligned}$$

Where $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$ is called as Laplacian operator.

In one dimensional case we can write –

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2} \quad \dots \quad (4)$$

Differentiating equation (2)

$$\frac{\partial \psi}{\partial t} = \psi_0 2\pi v \cos 2\pi vt \quad \dots \quad (5)$$

Differentiating it further –

$$\frac{\partial^2 \psi}{\partial t^2} = -\psi_0 4\pi^2 v^2 \sin 2\pi vt$$

$$\frac{\partial^2 \psi}{\partial t^2} = -4\pi^2 v^2 \psi \quad \dots \quad (6)$$

$$\text{Also, frequency } (\nu) = \frac{\text{velocity } (v)}{\text{wavelength } (\lambda)}$$

∴ Equation (6) becomes –

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{4\pi^2 v^2}{\lambda^2} \psi \quad \dots \quad (7)$$

From equation (4) and (7) we get –

$$\begin{aligned} v^2 \frac{\partial^2 \psi}{\partial x^2} &= -\frac{4\pi^2 v^2}{\lambda^2} \psi \\ \therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2}{\lambda^2} \psi &= 0 \end{aligned} \quad \dots \quad (8)$$

Now, we will introduce wave mechanical concept by replacing λ by $\frac{h}{mv}$ from De Broglie theory. Thus equation (8) becomes –

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \quad \dots \quad (9)$$

The total energy of electron E is given by the sum of its kinetic energy and potential energy.

$$\text{i.e. } E = \text{K.E.} + \text{P.E.}$$

$$\therefore E = \frac{1}{2} mv^2 + V$$

$$\therefore mv^2 = 2(E - V)$$

$$\therefore m^2 v^2 = 2m(E - V) \quad \dots \quad (10)$$

Substituting (10) in (9) we get –

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$

If $\hbar = \frac{h}{2\pi}$ it becomes –

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad \dots \quad (11)$$

This equation (11) is known as one dimensional Schrodinger's Time Independent Equation. This is also termed as Schrodinger's fundamental wave equation with respect to space.

This equation is independent of time and gives a steady value. It is particularly useful when the energy of the particle is very small as compared to its rest energy. In most atomic problems, energy of the particle is very small when compared to rest energy.

Derivation Of Schrodinger's Time Dependent Wave Equation (STDE)

Let us consider a free particle of mass 'm' moving with velocity 'v' in one dimension. Let 'p' be the momentum and 'E' be the energy of the particle. By the term free particle, it means that no forces are acting on it and its total energy E is entirely kinetic energy.

$$\therefore E = \text{Kinetic energy} = \frac{1}{2} m v^2 = \frac{1}{2m} m^2 v^2 = \frac{p^2}{2m} \quad \dots \quad (1)$$

This moving particle is associated with De Broglie waves which have wavelength λ and frequency ν . These are related as –

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \frac{2\pi}{\lambda} = \hbar k \quad \dots \quad (2a)$$

$$E = h\nu = \frac{h}{2\pi} 2\pi \nu = \hbar \omega \quad \dots \quad (2b)$$

Where $k = \frac{2\pi}{\lambda}$ represents the propagation constant and $\omega = 2\pi\nu$ represents angular frequency of the waves.

From equation (1) and (2) we get –

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} \quad \dots \quad (3)$$

Now, a wave equation is needed which will describe these traveling waves. This wave equation will also involve equation (3) in it. Further the function should be harmonic one, because such functions can be superimposed, thereby giving a wave packet which will represent a particle. It means that it must be sine, cosine or exponential function of $(kx - wt)$. Let Such function is $\psi = A e^{i(kx - wt)}$.

Now,

$$\begin{aligned} i\hbar \frac{\partial \psi}{\partial t} &= i\hbar \frac{\partial [A e^{i(kx - wt)}]}{\partial t} \\ i\hbar \frac{\partial \psi}{\partial t} &= i\hbar (-i\omega) A e^{i(kx - wt)} \\ i\hbar \frac{\partial \psi}{\partial t} &= \hbar\omega \psi = E\psi \quad \dots \quad (4) \end{aligned}$$

and

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = \frac{-\hbar^2}{2m} (ik)^2 A e^{i(kx-wt)}$$

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = \frac{\hbar^2 k^2}{2m} \psi = \frac{p^2}{2m} \psi \quad \dots \quad (5)$$

From (3), (4) and (5), we can write –

$$\begin{aligned} \hbar \omega \psi &= \frac{\hbar^2 k^2}{2m} \psi \\ \text{i.e. } i\hbar \frac{\partial \psi}{\partial t} &= \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \end{aligned} \quad \dots \quad (6)$$

If we include the effect of forces acting on the particle i.e. when the particle is not free, the potential energy of the particle is V .

Now, total energy of the particle is

$$E = \frac{p^2}{2m} + V$$

If we represent these as operators

$$\begin{aligned} E &= i\hbar \frac{\partial}{\partial t} \\ p &= -i\hbar \frac{\partial}{\partial x}, \quad p^2 = -\hbar^2 \frac{\partial^2}{\partial x^2} \\ V &= V \end{aligned}$$

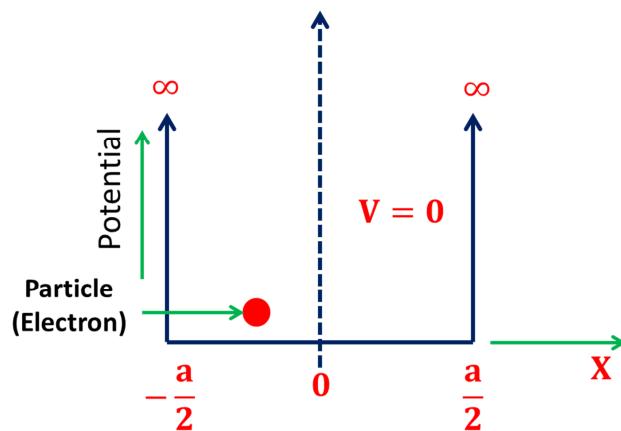
And operator on wave function ψ , then we can write –

$$\begin{aligned} E \psi &= \frac{p^2}{2m} \psi + V \psi \\ \text{i.e. } i\hbar \frac{\partial \psi}{\partial t} &= \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V \psi \end{aligned} \quad \dots \quad (7)$$

This equation (1.6.7) represents the Schrodinger's Time Dependent Equation (STDE).

Physical significance of STIE and STDE is that it is used to find the allowed energy levels of quantum mechanical systems. The associated wave function gives the probability of finding position of the particle. As Newton's laws predict the future behavior of a dynamic system in classical mechanics, Schrodinger's equations are used to predict future behavior in quantum mechanics.

PARTICLE (ELECTRON) IN AN INFINITE POTENTIAL BOX



The particle is free to move within the walls from $-\frac{a}{2}$ to $+\frac{a}{2}$ as there is no potential in that region.

At boundaries, potential is ∞ . Therefore, the particle can neither cross these boundaries nor it can be located at $x = \pm \frac{a}{2}$.

ψ and its derivatives are finite, continuous and single valued within $-\frac{a}{2} < x < +\frac{a}{2}$ but it is discontinuous at boundaries and does not exist outside the boundaries.

Only real part solution is required as the particle must be present somewhere within the well (box).

Schrodinger's Time Independent Equation is –

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \quad \dots \quad (1)$$

for region between $-\frac{a}{2} < x < +\frac{a}{2}$; $V = 0$

$$\begin{aligned} \therefore -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} &= E\psi(x) \\ \therefore \frac{d^2\psi(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi(x) &= 0 \end{aligned} \quad \dots \quad (2)$$

Real solution of the above equation is given by –

$$\begin{aligned} \psi &= A \sin kx + B \cos kx \quad ; \quad -\frac{a}{2} < x < +\frac{a}{2} \\ &= 0 \quad ; \quad |x| \geq \frac{a}{2} \end{aligned} \quad \dots \quad (3)$$

At the boundaries, $\phi(x) = 0$.

$$\therefore \psi(x = -\frac{a}{2}) = -A \sin \frac{ka}{2} + B \cos \frac{ka}{2} = 0 \quad \dots \quad (4)$$

$$\therefore \psi(x = +\frac{a}{2}) = A \sin \frac{ka}{2} + B \cos \frac{ka}{2} = 0 \quad \dots \quad (5)$$

Adding two equations (4) and (5),

$$\begin{aligned} 2B \cos \frac{ka}{2} &= 0 \\ \therefore k_n &= \frac{n\pi}{a} ; \quad n = 1, 3, 5, \dots \dots \end{aligned} \quad \dots \quad (6)$$

Subtracting (1.7.5) from (1.7.4),

$$\begin{aligned} 2B \sin \frac{ka}{2} &= 0 \\ \therefore k_n &= \frac{n\pi}{a} ; \quad n = 2, 4, 6, \dots \dots \end{aligned} \quad \dots \quad (7)$$

Now, momentum of the particle is given by –

$$P_n = \hbar k = \hbar \frac{n\pi}{a} = \frac{n\pi \hbar}{a} ; \quad n = 1, 2, 3, \dots \dots \quad \dots \quad (8)$$

Energy of the particle is given by –

$$E_n = \frac{P_n^2}{2m} = \frac{n^2 \hbar^2 \pi^2}{2m a^2} = \frac{n^2 h^2}{8m a^2} \quad \dots \quad (9)$$

Thus, energy of the particle in infinite potential well (Box) is quantized.

1. An electron is bound in a one dimensional potential well of width $2 A^0$ and of infinite height. Find its energy values in ground state and first two excited states.

Given : $m = 9.1 \times 10^{-31} \text{ kg}$, $a = 2 \times 10^{-10} \text{ m}$

$\hbar = 6.63 \times 10^{-34}$, $E_0, E_1, E_2 = ?$

$$E = \frac{n^2 h^2}{8 m a^2}$$

For ground state, $n = 1, E = E_0$

$$\therefore E_0 = \frac{n^2 h^2}{8 m a^2} = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} = 1.5 \times 10^{-18} \text{ J}$$

For first excited state, $n = 2, E = E_1$

$$\therefore E_1 = \frac{n^2 h^2}{8 m a^2} = \frac{2^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} = 6 \times 10^{-18} \text{ J}$$

For second excited state, $n = 3, E = E_2$

$$\therefore E_2 = \frac{n^2 h^2}{8 m a^2} = \frac{3^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} = 1.35 \times 10^{-17} \text{ J}$$

2. An electron is trapped in a one dimensional box of length 0.1 nm. Calculate the energy required to excite the electron from its ground state to the 4th excited state.

Given : $m = 9.1 \times 10^{-31} \text{ kg}$, $a = 0.1 \times 10^{-9} \text{ m}$

$$h = 6.63 \times 10^{-34}, E_4 - E_0 = ?$$

$$E = \frac{n^2 h^2}{8 m a^2}$$

For ground state, $n = 1$, $E = E_0$

$$\therefore E_0 = \frac{n^2 h^2}{8 m a^2} = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.1 \times 10^{-9})^2} = 6.04 \times 10^{-18} \text{ J}$$

For fourth excited state, $n = 5$, $E = E_4$

$$\therefore E_4 = \frac{n^2 h^2}{8 m a^2} = \frac{5^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.1 \times 10^{-9})^2} = 1.51 \times 10^{-16} \text{ J}$$

$$E_4 - E_0 = 1.51 \times 10^{-16} - 6.04 \times 10^{-18} = 1.45 \times 10^{-16} \text{ J}$$

$$= \frac{1.45 \times 10^{-16}}{1.6 \times 10^{-19}} = 906.25 \text{ eV}$$

3. An electron is bound by a potential which closely approaches an infinite square well of width $2.5 \times 10^{-10} \text{ m}$. Calculate the first lowest permissible energy for the electron.

Given : $m = 9.1 \times 10^{-31} \text{ kg}$, $a = 2.5 \times 10^{-10} \text{ m}$

$h = 6.63 \times 10^{-34}$, For lowest permissible Energy level, $E = E_1$ and $n = 2$

$$E = \frac{n^2 h^2}{8 m a^2}$$

For lowest permissible Energy level, $n = 2$, $E = E_1$

$$\therefore E_1 = \frac{n^2 h^2}{8 m a^2} = \frac{2^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2} = 3.864 \times 10^{-18} \text{ J}$$

Applications of Quantum Physics

- Much of modern technology operates at a scale where quantum effects are significant. Quantum tunnelling is vital in many electronic devices, being the basis for transistor operation. Flash memory chips, found in USB drives, use quantum tunnelling to erase their memory cells.
- The MRI scanner has powerful magnets, which cause the protons of hydrogen atoms in water to align. The signal given off by the protons is processed and used to build up a picture.
- Lasers work using the quantum phenomenon known as stimulated emission.
- Quantum dots are tiny particles of a semiconductor material, with a width of about 50 atoms. By controlling the size of the dot, the light it emits or absorbs can be very precisely controlled.
- Lighting Solar cells Light detectors Security marking
- light-sensing devices used in Modern digital cameras and solar cell
- One of the ultimate applications will be nanotechnology. At nano scale, the quantum effect is dominant.

Quantum Computing

Quantum computing means developing a computer that uses quantum mechanical phenomena to perform operations on data through devices such as superposition and entanglement.

Quantum superposition is a fundamental principle of quantum mechanics. It states that, much like waves in classical physics, any two (or more) quantum states can be added together ("superposed") and the result will be another valid quantum state.

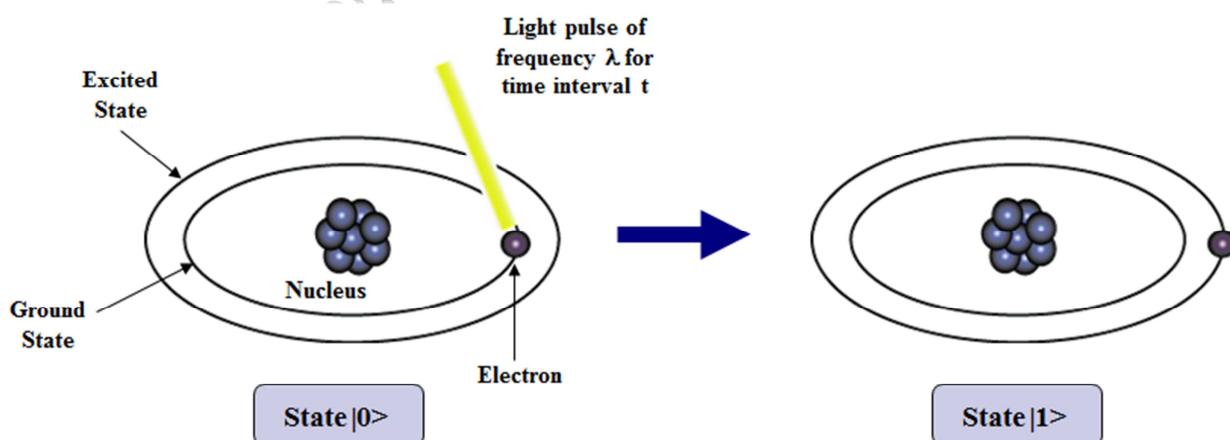
Quantum entanglement is a quantum mechanical phenomenon in which the quantum states of two or more objects have to be described with reference to each other, even though the individual objects may be spatially separated. This leads to correlations between observable physical properties of the systems. For example, it is possible to prepare two particles in a single quantum state such that when one is observed to be spin-up, the other one will always be observed to be spin-down and vice versa, this despite the fact that it is impossible to predict, according to quantum mechanics, which set of measurements will be observed. As a result, measurements performed on one system seem to be instantaneously influencing other systems entangled with it.

Quantum computing uses the power of atoms and molecules to perform memory and processing tasks.

Quantum computing uses quantum bits or qubits which can either take value 0 or 1 or both simultaneously.

A bit of data is represented by a single atom that is in one of two states denoted by $|0\rangle$ and $|1\rangle$. A single bit of this form is known as a **qubit**.

A physical implementation of a qubit could use the two energy levels of an atom. An excited state representing $|1\rangle$ and a ground state representing $|0\rangle$.



Advantages of Quantum Computing

- The main advantage of quantum computing is it can execute any task very fast when compared to the classical computer. So a very high speed can be achieved.
- In quantum computing qubit is the conventional superposition state and so there is an advantage of exponential speedup which is resulted by handling number of calculations.
- The other advantage of quantum computing is even classical algorithm calculations are also performed easily which is similar to the classical computer.
- The size of quantum computers will be very small compared to classical computers and power consumption will also be very less.

Disadvantages of Quantum Computing:

- The main disadvantage of computing is that the technology required to implement a quantum computer is not available at present. The reason for this is the consistent electron is damaged as soon as it is affected by its environment and that electron is very much essential for the functioning of quantum computers.
- The research on this problem is still going on and requires time, efforts and high cost.

	Classical Computer	Quantum Computer
1	It is large scale integrated multi-purpose computer.	It is high speed parallel computer based on quantum mechanics.
2	Information storage is bit based on voltage or charge etc.	Information storage is Quantum bit based on direction of an electron spin.
3	Information processing is carried out by logic gates e.g. NOT, AND, OR etc.	Information processing is carried out by Quantum logic gates.
4	Circuit behaviour is governed by classical physics.	Circuit behaviour is governed explicitly by quantum mechanics.
5	Classical computers use binary codes i.e. bits 0 or 1 to represent information.	Quantum computers use Qubits i.e. 0, 1 and both of them simultaneously to run machines faster.
6	Operations are defined by Boolean Algebra.	Operations are defined by linear algebra over Hilbert Space and can be represented by unitary matrices with complex elements.
7	No restrictions exist on copying or measuring signals	Severe restrictions exist on copying and measuring signals
8	Circuits are easily implemented in fast, scalable and macroscopic technologies such as CMOS.	Circuits must use microscopic technologies that are slow, fragile and not yet scalable e.g. NMR (Nuclear magnetic resonance).

Quantum Mechanics

Syllabus : de Broglie's hypothesis, group and phase velocity, wave packet, uncertainty principle and applications, Wave function and probabilistic interpretation, one dimensional time dependent Schrodinger equation, reduction to time independent form, application to free particle and particle in a box.

Wave – particle dualism:-

The electromagnetic radiation travels not in the form of a continuous stream of energy but in the form of tiny packets or bundles of energy, following each other in quick succession. These packets of energy are called photons and behaves almost exactly like a material particle.

On the other hand, the phenomenon like interference or diffraction of light or x-rays could not be explained unless electromagnetic radiations were assumed to posses a wave character. This resulted in the acceptance of the dual character of radiation.

When radiation interacts with matter it exhibits its particle character whereas when radiation interacts with radiation, it exhibits a wave nature. Radiation never exhibits both the characters simultaneously.

According to de Broglie, “the wave particle dualism should not be confined to radiation alone but should also be extended to all fundamental entities of physics.”

The electrons, protons, atoms and molecules, when in motion, should have some type of wave motion associated with them. De Broglie was led to this hypothesis from considerations based upon the special theory of relativity and quantum theory.

De Broglie Hypothesis:

The wavelength of these waves depends upon the mass and velocity of particles. De Broglie deduced the connection between the particle and the wave properties using the Planck- Einstein expression for energy of an electromagnetic wave and expression for energy of an electromagnetic wave and expression for the momentum derived in classical wave theory.

$$E = h\nu \dots \quad (1)$$

If a photon posses same mass m which is by virtue of its motion, its energy according to the theory of relativity should be mc^2 .

$$E = mc^2 \dots \quad (2)$$

$$\text{Momentum } P \text{ of the photon} = \frac{mc^2}{c}$$

$$P = \frac{E}{c} \dots \dots \dots \text{(from eq^n (1))}$$

$$P = \frac{h\gamma}{c} \dots \dots \dots (\because \gamma = \frac{c}{\lambda})$$

$$P = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p} \dots \dots \dots \text{(3)}$$

According to de Broglie eq^n. (3) is applicable to photons as well as to any material particle. Thus, **A moving particle has a wave associated with it is called as matter wave or de Broglie wave.**

A particle of mass 'm' moving with velocity v must be associated with a wave of wavelength

$$\lambda = \frac{h}{mv} \dots \dots \dots \text{(4)}$$

This is known as ***de-Broglie equation.***

If a charged particle (electron) is moving with velocity v then the de-Broglie wavelength is given by

$$\text{K. E. of } e \text{ is } \frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}}$$

Substituting this value of v in Eq. (4), we get,

$$\lambda = \frac{h}{\sqrt{2meV}} \quad \text{(where } V \text{ is the applied potential)}$$

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad (\therefore p = \sqrt{2mE})$$

$$\lambda = \frac{h}{\sqrt{2mE}} \dots \dots \dots \text{(5)}$$

Bohr's condition of quantization:

De Broglie also connected his hypothesis with the Bohr's model. He demonstrated that it could provide an explanation for the postulate regarding quantization of electron angular momentum in Bohr's model of atom.

As the electron travels round in one of its circular orbits, the associated matter waves propagate along the circumference again and again. A wave must meet itself after going round one full circumference

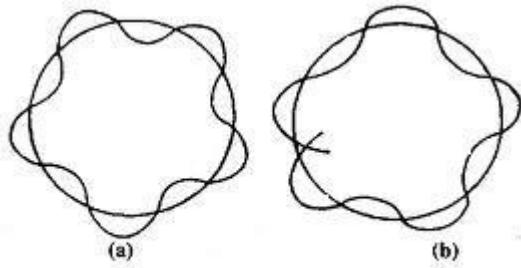
Quantum Mechanics

and form a stationary wave pattern. A stationary wave pattern can form along the circumference if only an integral number of wavelengths fit into the orbit. If a fractional number of wavelengths is placed around an orbit as in Fig. (1B), destructive interference will occur as the waves travel around an orbit, and vibrations will die out rapidly.

This statement combines both the particle and wave characters of the electron since the electron wavelength depends upon the orbital velocity needed to balance the pull of the nucleus.

Since electron in an atom is restricted to certain selected orbits with quantum number n , ($n = 1, 2, 3, \dots$)

An electron can circle a nucleus only if its orbit contains an integral number of de Broglie wavelengths.



de Broglie electron - waves in an orbit

Figure 1: Bohr's Orbit

The circumference of a circular orbit of radius r_n is $2\pi r_n$, so the condition for orbit stability is

$$n\lambda = 2\pi r_n \quad n = 1, 2, 3, 4, \dots \quad (1)$$

r_n designates the radius of the orbit that contains n wavelengths. The integer n is called quantum number of the orbit.

$$n \frac{\hbar}{m_e v} = 2\pi r_n$$

$$L_n = m_e v r_n = \frac{\hbar}{2\pi} n = n\hbar \quad (2)$$

Where L_n is angular momentum of electron in n^{th} orbit and eq. (2) is just Bohr's condition of quantization. De Broglie thus demonstrated that the quantization of angular momentum is a direct consequence of wave nature of electron.

Difference between Electromagnetic Waves and Matter Waves:

	Electromagnetic Waves	Matter Waves
1	Electromagnetic waves are associated with photons, which have zero rest mass.	Matter waves are associated with any material particle having non zero rest mass and which are moving with some velocity.
2	A single electromagnetic wave can be associated with a photon.	A number of waves called wave packet is associated with a material particle.
3	Electric and magnetic fields are real and can be measured experimentally.	Wave function is an abstract mathematical quantity and it has no physical relevance.
4	Square of electric and magnetic field give intensity of electromagnetic radiation.	Square of wave function gives probability of finding particle.

Wave Packet:

Let us assume that a particle like an electron can be described mathematically.

$$\psi = A \sin(kx - \omega t)$$

This wave has no beginning and no end. It is of infinite extent and completely non-localized. But particle (electron) is confined to a very small volume. Therefore, a monofrequency wave cannot represent a particle. **It implies that the de Broglie waves are not harmonic waves but could be a combination of several of waves.** A superposition of individual harmonic waves of slightly different frequencies gives rise to wave packet. Such wave packet possesses both wave and particle properties. The regular separation between successive maxima in a wave packet is the characteristic of a wave and at the same time it has a particle like localization in space.

A wave packet can be described in terms of a superposition of individual harmonic waves of slightly different frequencies centered on a frequency ν_0 . Let a superposition of two waves be

$$\psi_1 = A \sin(k_1 x - \omega_1 t)$$

$$\psi_2 = A \sin(k_2 x - \omega_2 t)$$

$$\therefore \psi = \psi_1 + \psi_2$$

$$= A \sin(k_1 x - \omega_1 t) + A \sin(k_2 x - \omega_2 t)$$

$$= 2A \sin\left[\frac{1}{2}(k_1 + k_2)x - \frac{1}{2}(\omega_1 + \omega_2)t\right] \cdot \cos\left[\frac{1}{2}(k_1 - k_2)x - \frac{1}{2}(\omega_1 - \omega_2)t\right]$$

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$$= 2A \sin(\bar{k}x - \bar{\omega}t) \cdot \cos\left(\frac{\Delta k x}{2} - \frac{\Delta \omega t}{2}\right)$$

Where, $\bar{k} = \frac{k_1 + k_2}{2}$, $\bar{\omega} = \frac{\omega_1 + \omega_2}{2}$, $\Delta k = k_1 - k_2$, $\Delta \omega = \omega_1 - \omega_2$

This is the superposition of two waves. The actual formation of a wave packet requires the superposition of a large number of waves. Thus,

$$\psi(r, t) = \int A(k) \sin(kx - \omega t) dk$$

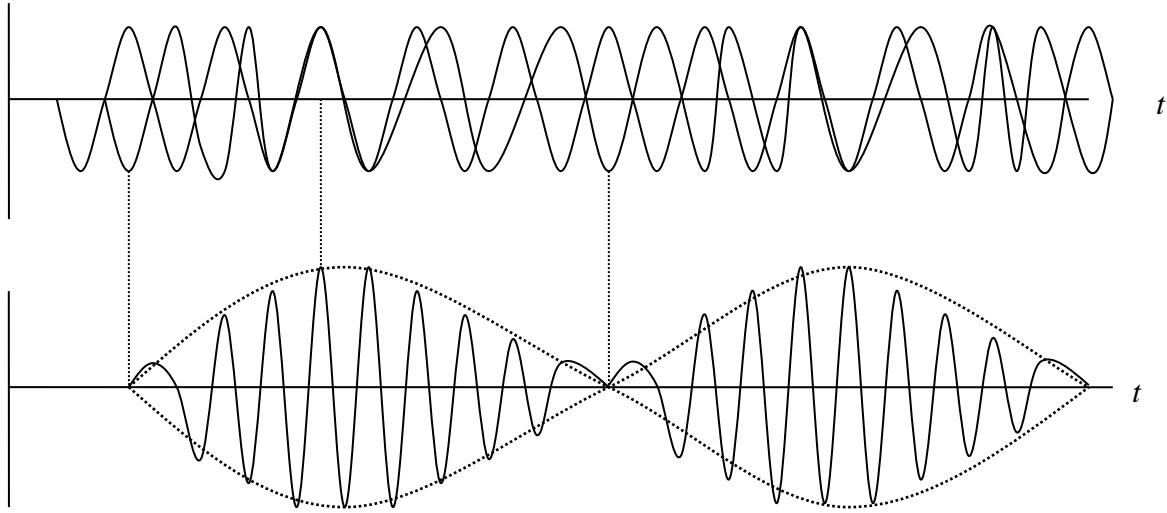


Figure 2

A wave packet produced by such superposition is shown in Fig 2. The packet is centered on γ_0 . As $|\gamma - \gamma_0|$ increases the resultant wave diminishes.

Δx is the spread of wave packet and Δx is inversely proportional to $\Delta \lambda$.

$$\Delta x = \frac{-\lambda^2}{\Delta \lambda}$$

If the velocities of the constituent waves in a wave packet are the same, the wave packet travels with the phase velocity v_p

Phase velocity and Group velocity:-

Phase velocity is also called as Wave velocity. Phase velocity is velocity of individual wave and it is represented by v_p .

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$$v_p = \gamma\lambda = \frac{2\pi\gamma}{\frac{2\pi}{\lambda}} = \frac{\omega}{k}$$

Consider a particle of rest mass m_0 and moving with velocity v . Let ω be the angular frequency and k be the wave no. of de Broglie waves associated with a particle.

$$\omega = 2\pi\gamma = 2\pi\left(\frac{mc^2}{h}\right) = \frac{2\pi m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \times \frac{c^2}{h} \quad (\text{since } E = h\gamma = mc^2)$$

$$k = \frac{2\pi}{\lambda} = 2\pi \frac{mv}{h} = \frac{2\pi}{h} \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{v}{h}$$

$$\text{Wave velocity } v_p = \frac{\omega}{k} = \frac{c^2}{v}$$

As $v < c$, the **phase velocity of the associated wave is always greater than c , the velocity of light**.

However v_p has no physical significance.

Group Velocity:

The velocity with which wave packet travels in a medium is called as **Group velocity** which is given by .

$$v_g = \frac{d\omega}{dk}$$

Relation between particle velocity and group velocity:

$$\text{The group velocity } v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}.$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 c^2}{h} \left(-\frac{1}{2}\right) \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \times \frac{-2v}{c^2} = \frac{2\pi m_0 v}{h \left(1 - \frac{v^2}{c^2}\right)^{3/2}}$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h} \left(1 - \frac{v^2}{c^2}\right)^{-1/2} + \frac{2\pi m_0 v}{h} \left[\left(-\frac{1}{2}\right) \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \cdot \left(\frac{-2v}{c^2}\right) \right]$$

$$\frac{d\omega}{dk} = \frac{2\pi m_0 \left[\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2}{c^2} \right]}{h \left(1 - \frac{v^2}{c^2}\right)^{3/2}}$$

$$= \frac{2\pi n_0}{h \left(1 - \frac{v^2}{c^2}\right)^{3/2}}$$

$$\therefore v_g = \frac{d\omega/dv}{dk/dv} = \frac{2\pi n_0 v}{h [1 - v^2/c^2]^{3/2}} \times \frac{h(1 - v^2/c^2)^{3/2}}{2\pi n_0} = v$$

$$\therefore v_g = v$$

Thus, the de Broglie wave group travels with the same velocity as that of particle.

Wave Function: The quantity with which quantum mechanics is concerned is the **wave function** Ψ of a body. The value of the wave function associated with a moving body at particular point x, y, z in space at the time t is related to the likelihood of finding the body there at the time.

The wave function Ψ has no direct physical significance. There is a simple reason why Ψ cannot be interpreted in terms of an experiment.

The probability that something be in a certain place at a given time must lie between 0 (object is definitely not there) and 1 (the object is definitely there). An intermediate probability, say 0.2, means that there is a 20% chance of finding the object. But the amplitude of any wave is negative as often as it is positive, and a negative probability, say -0.2, is meaningless. Hence Ψ by itself cannot be an observable quantity. This objection does not apply to $|\Psi|^2$, the square of the absolute value of the wave function, which is known as **probability density**: A large value of $|\Psi|^2$ means the strong possibility of the body's presence while a small value of $|\Psi|^2$ means the slight possibility of its presence.

The wave functions are usually complex with both real and imaginary parts. A probability must be a positive real quantity. The probability density $|\Psi|^2$ for a complex function Ψ is therefore taken as the product $\Psi^* \Psi$ of Ψ and its complex conjugate Ψ^* . Therefore $|\Psi|^2$ is a real quantity.

It was shown by scientist Max Born that the square of absolute value of the wave function $|\Psi|^2$, **proportional to the probability of particle being in unit volume of space, centered at the point where Ψ is evaluated at time t** . The probability of finding particle between x and $x + dx$, y and $y + dy$, z and $z + dz$ is

$$P \propto \Psi \Psi^* dx \cdot dy \cdot dz$$

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If $dV = dx dy dz$ is an infinitesimally small volume element surrounding the point (x, y, z) , then the probability of finding the particle in that volume element is given by $|\Psi|^2 dV$.

$$P \propto |\Psi|^2 dV$$

Normalization Condition:

If at all the particle is certainly somewhere in the universe therefore, the probability of finding the particle somewhere in the universe must be unity. Since the probability of its being located in an elemental volume is proportional to $|\Psi|^2 dxdydz$, it is convenient to choose the constant of proportionality such that the sum of the probabilities over all values of x, y, z must be unity. Thus,

$$\int \int \int_{-\infty}^{\infty} \Psi \Psi^* dxdydz = 1$$

This is called as **normalization condition**. A wave function satisfying the above the condition is said to be normalized.

Requirement of an Acceptable Wave Function:

Besides being normalizable an acceptable wave function must fulfill the following requirement.

1. Ψ must be finite everywhere:

If Ψ is infinite, it would mean an infinitely large probability of finding the particle at that point. This would violate the uncertainty principle

$\therefore \Psi$ must have a finite or zero value at any point.

2. Ψ must be single valued:

If Ψ has more than one value at any point it would mean more than one value of probability of finding the particle at that point which is obviously ridiculous.

3. It must be continuous and have a continuous first derivative everywhere:

Only the wave functions with all these properties can yield physically meaningful results when used in calculations, so only such “well - behaved” wave functions are admissible as mathematical representations of real bodies.

Wave Packet and Uncertainty Principle:

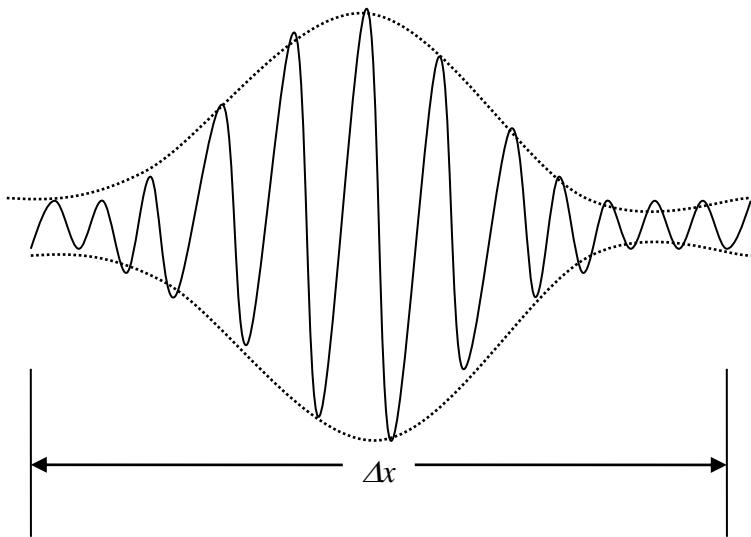
A monochromatic wave is infinite extent so instead of associating a single monochromatic de Broglie wave with a moving particle we associate a wave packet consisting of a group of waves of nearly equal amplitude centered around de Broglie $\lambda=h/p$ of the particle. The wave packet has the dimensions of the localized particle and travels with the same velocity as the particle.

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The association of a group of waves with a moving particle that the position of the particle at any instant of time cannot be specified with any desired degree of accuracy. The particle can be somewhere within the group of waves i.e., within a small region Δx of space. (Δx is linear speed of the wave packet) figure 3.

The probability of finding the particle at the centre of the wave packet is maximum and falls to zero at its ends. Therefore, there is an uncertainty Δx in the position of the particle. A wave packet is formed by waves having a range of wavelength. The spread $\Delta \lambda$ is related to the spread in dimension Δx . As the momentum of the associated particle is related to the wavelength by equation $p = h\lambda = h/\lambda$, there arises an uncertainty in momentum Δp . The two uncertainties are interrelated as spread in momentum depends on the spread in wavelength which in turn depends on the length of the wave packet. The mathematical expression governing this situation is called Heisenberg's uncertainty principle which was derived in 1927. It states that, "**It is not possible to make simultaneous measurement of the position and momentum of a particle to an unlimited accuracy. The product of uncertainty in the measurement of position and uncertainty in the measurement of momentum is equal to or greater than Planck's constant \hbar .**" (\hbar is read as "h-bar" = $h/2\pi$)

i.e., $\Delta x \cdot \Delta p_x \geq h/2\pi$ where Δx is the fundamental error or uncertainty in the measurement of the position and Δp_x if the fundamental error or uncertainty in the measurement of momentum (velocity).



A Wave Packet

Figure 3: wave packet

It means that it is physically impossible to know simultaneously the exact position and exact momentum of the particle. If it is desired to reduce the error in determination of positions i.e., to reduce Δx , then this can only be done at the expense of accuracy in determining momentum and the error involved in Δp will increase.

This principle is applicable to all conjugate or complementary pairs of physical variables the product of which has the dimension of Planck's constants e.g. time and energy and energy and angular momentum.

$$\Delta E \cdot \Delta t \geq \frac{h}{2\pi}$$

and $\Delta L \cdot \Delta \theta \geq \frac{h}{2\pi}$

It is to be noted that the uncertainty principle does not apply to a single measurement on a single particle. It is a statement about a statistical average over lots of measurement of position and momenta.

VERIFICATION OF UNCERTAINTY PRINCIPLE:

Single – slit diffraction method

Quantum Mechanics

Let us consider a stream of electrons moving along x direction passes through a narrow slit of width $d = \Delta y$. If (Δy) is comparable to the wavelength of the electron beam, then the electrons will diffract according to single slit diffraction pattern and form a central maximum and two secondary minimum as shown in Fig. (4)

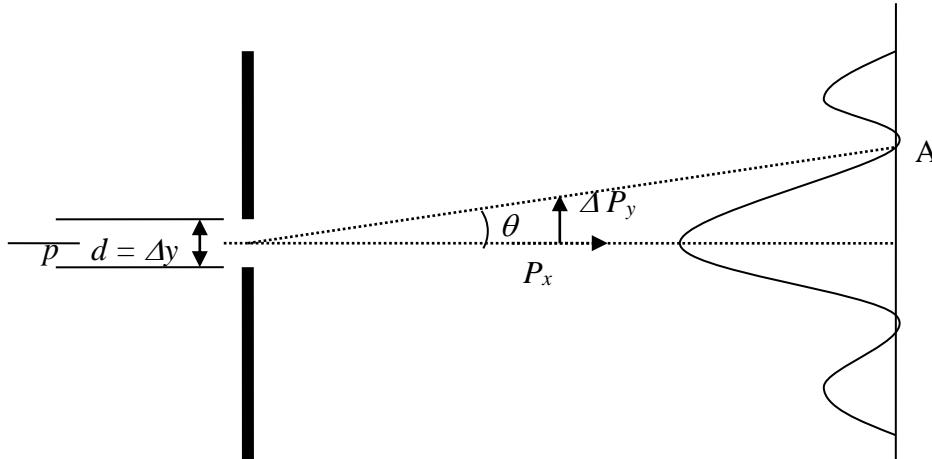


Figure 4: Single-slit Diffraction

According to diffraction theory,

$$d \sin \theta = \lambda$$

or

$$\Delta y = \frac{\lambda}{\sin \theta} \quad \dots \dots \dots (1)$$

(since $d = \Delta y$ = uncertainty in position of electron before being diffracted.)

Before diffraction a slit electron has momentum P_x only along x direction and zero in y - direction. Therefore uncertainty of y component of momentum of $\Delta P_y=0$. Because of the diffraction effect at the slit, the particle acquires a small component of momentum P_y in y direction. The original momentum of the particle in the x-direction P_x decreases so that the resultant momentum P remains constant. The original momentum of the particle in the y – direction was accurately known to be zero. Therefore, ΔP_y is the uncertainty introduced in the y - component of the momentum.

Particles that strike the screen at a point A the first minima, must have left the slit at an angle θ , given

by

$$\tan \theta = \theta = \frac{\Delta P_y}{P_x} \quad \dots \dots \dots (2)$$

Equating (1) and (2)

$$\frac{\Delta P_y}{P_x} = \frac{\lambda}{\Delta y}$$

$$\therefore \Delta y \cdot \Delta P_y = \lambda \cdot P_x$$

$$\therefore \Delta y \cdot \Delta P_y = h \quad \therefore \lambda = \frac{h}{P_x}$$

Which is the uncertainty principle i.e., if we try to improve the accuracy in y, we have to reduce to Δy using a finer slit which results in turn in a wider spectrum. It leads to a larger ΔP_y .

Applications of Uncertainty Principle:

(i) Non-existence of electrons and existence of protons and neutrons in nucleus: The radius of nucleus of any atom is of the order of 10^{-14} m. If an electron is confined inside the nucleus then the uncertainty in the position Δx of the electron is equal to the diameter of the nucleus i.e., $\Delta x = 2 \times 10^{-14}$ m. Using the uncertainty principle,

$$\Delta x \cdot \Delta p_x \geq h / 2\pi$$

$$\therefore \Delta p_x \geq \frac{h}{2\pi \times \Delta x}$$

$$\text{But } \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ J-s} \text{ & } \Delta x = 2 \times 10^{-14} \text{ m}$$

$$\therefore \Delta p_x \geq \frac{1.055 \times 10^{-34}}{2 \times 10^{-14}} = 0.527 \times 10^{-20} \text{ N.sec.}$$

The relativistic formula for energy of electron is

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$\therefore p^2 c^2 > m_0^2 c^4, m_0^2 c^4 \text{ can be neglected}$$

$$E^2 = p^2 c^2 = 2.25 \times 10^{-24} \text{ J}^2$$

$$E = \left[\frac{2.25 \times 10^{-24}}{1.6 \times 10^{-19}} \right]^{1/2} \text{ eV}$$

$$= 1 \times 10^7 \text{ eV}$$

$$= 10 \text{ MeV}$$

Which means if the electrons exist inside the nucleus, their energy must be of the order of 10 MeV. But experiments in Nuclear physics have revealed that even most energetic electrons have energies

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far too less than above value (typically 0.1 MeV) So, no question of electron possessing such high energies and the possibility of electron being a part of nucleus is ruled out.

For protons and neutrons, $m_0 = 1.67 \times 10^{-27}$ kg.

This is non-relativistic problem as v is 3×10^6 m/s.

This K.E. is given by the relation

$$K.E. = \frac{p^2}{2m_0} = \left[\frac{(0.527 \times 10^{-20})^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19}} \right] \text{eV}$$

$$= 52 \text{ KeV}$$

Since this energy is less than the energy carried by these particles emitted by nuclei, both these particles can exist inside the nucleus.

(ii) Binding Energy of an electron in atom: In an atom, the electron is under the influence of electrostatic potential of positively charged nucleus. It is confined to the linear dimensions equal to the diameter of electronic orbit. The uncertainty in the position Δx of an electron is of the order of $2R$ where R is the radius of the orbit and the corresponding uncertainty in momentum component ΔP_x is

given by

$$\Delta P_x \geq \frac{h}{2\pi \times \Delta x}$$

$$\Delta P_x \geq \frac{h}{2\pi \times 2R}$$

Which shows the momentum of electron in an orbit is at least

$$p \approx \Delta P_x \approx \frac{h}{2\pi \times 2R} R \approx 10^{-10} \text{ m}$$

$$\text{K. E. of electron} = \frac{p^2}{2m} = \left(\frac{h}{4\pi R} \right)^2 \times \frac{1}{2m_0} = \frac{h^2}{32\pi^2 m_0 R^2}$$

The potential energy of an electron in the field of nucleus with atomic number z is given by

$$V = \frac{-ze^2}{4\pi\epsilon_0 R}$$

The total energy = K.E. + P.E.

$$= \frac{h^2}{32\pi^2 m_0 R^2} - \frac{ze^2}{4\pi\epsilon_0 R}$$

$$= \frac{(1.055 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} R^2} - \frac{z(1.6 \times 10^{-19})^2}{4 \times 3.14 \times 8.85 \times 10^{-12} \times R}$$

Where, $\frac{h}{2\pi} = 1.055 \times 10^{-34}$

$$E = \frac{10^{-20}}{R^2} - \frac{15 \times 10^{-10}}{R} \text{ ZeV}$$

Taking $R = 10^{-10}$ m we have $E = (1-15z)$ eV

Now, the binding energies of outermost electrons in H and He are -13.6 eV and -24.6 eV respectively. So the value of B.E. derived on the basis of uncertainty principle is acceptable as these are comparable in magnitudes.

One – Dimensional Time Dependent Schrodinger Equation:

In 1926, Erwin Schrodinger formulated the wave equation for matter waves which is known as Schrödinger's equation. It plays the same role in Quantum mechanics as Newton's second law does in classical mechanics. The motion of an atomic particle can be determined using Schrödinger's wave equation. In order to find the new equation of mechanics of the quantum systems, we need to propose certain requirements from the wave equation.

They are

- (1) The wave equation must be consistent with the de Broglie's hypothesis and the Planck's formula $E_n = nh\nu$.
- (2) It must be consistent with the energy equation of classical mechanics viz.

$$\text{Total Energy} = \text{Kinetic} + \text{Potential i.e. } \left(E = \frac{p^2}{2m} + V \right)$$

- (3) It must be linear in $\psi(x,t)$ i.e. if $\psi_1(x,t)$ and $\psi_2(x,t)$ are two solutions of the equation then their linear combination must also be the solution of the same equation.
- (4) The wave function must reduce to classical Newtonian equations of motion for higher dimensions (i.e. at the macroscopic level).

Let us consider a micro particle. Let Ψ be the wave function associated with the motion of this microparticle Ψ function represents the wave field of the particle.

For one – dimensional case, the classical wave equation has the following form

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \xi}{\partial t^2} \dots \dots \dots \quad (1)$$

A solution of the above equation is the familiar plane wave

$$\xi(x,t) = A e^{i(kx-\omega t)} \dots \dots \dots \quad (2)$$

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Where $\omega = vk$ and v is phase velocity.

For microparticle, ω and k may be replaced with E & P using Einstein & de Broglie relations.

$$\omega = \frac{E}{\hbar}$$

$$k = \frac{P}{\hbar} \dots\dots\dots\dots\dots(3)$$

Substituting eqns. (3) in (2) & replacing $\xi(x, t)$ with the wave function $\Psi(x, t)$

$$\psi(x, t) = Ae^{-i(Et - px)/\hbar} \dots\dots\dots\dots\dots(4)$$

Differentiating with respect to t

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} E \psi \dots\dots\dots\dots\dots(5)$$

Differentiating (4) with respect to x .

$$\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} P \psi \dots\dots\dots\dots\dots(6)$$

Differentiating again with respect to x .

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{-p^2}{\hbar^2} \psi \dots\dots\dots\dots\dots(7)$$

The kinetic energy and momentum of a free particle are related by the expression.

$$K.E. = \frac{p^2}{2m}$$

The T.E. of a particle is

$$T.E. = K.E. + P.E.$$

$$E = \frac{p^2}{2m} + V \dots\dots\dots\dots\dots(8)$$

Multiply this equation by Ψ .

$$E\Psi = \frac{p^2}{2m}\Psi + V\Psi \dots\dots\dots\dots\dots(9)$$

Substitute $E\Psi$ & $p^2\Psi$ using eqn. (5) & (6) in eqn. (8)

$$\begin{aligned} -\frac{\hbar}{i} \frac{\partial \psi}{\partial t} &= -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi \\ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi &= i\hbar \frac{\partial \psi}{\partial t} \end{aligned} \dots\dots\dots\dots\dots(10)$$

This is one – dimensional time dependent Schrödinger's eqn.

Time Independent Schrödinger's Equation:

In many cases the potential energy V of a particle does not depend on time, it varies only with the position of the particle and the field is said to be stationary. In such cases Schrödinger's time independent wave eqn. is used.

In this case the wave function is written as product of $\psi(x)$ and $\phi(t)$

$$\psi(x,t) = \psi(x)\phi(t) \dots \dots \dots (11)$$

Substitute this in Schrödinger's time dependent wave equation.

$$-\frac{\hbar^2}{2m}\phi(t)\frac{\partial^2\psi(x)}{\partial x^2} + V\psi(x)\phi(t) = i\hbar\psi(x)\frac{\partial\phi(t)}{\partial t}$$

And dividing by $\psi(x,t) = \psi(x)\phi(t)$, we get

$$-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + V(x) = i\hbar\frac{1}{\phi(t)}\frac{\partial\phi(t)}{\partial t} \dots \dots \dots (12)$$

The right hand side of eqn. (12) is a function of t only and the left hand side a function of x only. Therefore, equation (12) must be valid for any x and t , it can be so only if the two sides of eqn. (12) are equal to a constant. By setting this constant equal to energy E . We get,

$$-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + V(x) = E$$

$$\therefore -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$$

.....(13)

This is time independent Schrödinger's equation.

In three dimensions, the time independent Schrodinger eqn. is written as

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(r)\psi = E\psi \dots \dots \dots (14)$$

$$\text{Where, } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Eqn. (13) is frequently written in the form $H\psi = E\psi$

Where H is Hamiltonian operator

Quantum Mechanics

$$H = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V \dots \dots \dots \quad \dots \dots \dots \quad (15)$$

Eigenfunctions and Eigenvalues:

In quantum mechanics it is practice to call equation of the sort

$$A_{op} \psi_n(x, t) = a_n \psi_n(x, t); \quad n = 1, 2, 3, 4, \dots$$

As eigenvalue equation.

The German term Eigen stands for characteristics. Aop in the above equation is the operator of a physical quantity A.

Solution or solutions, $\psi_n(x, t)$ of such an equation are called as eigenfunctions corresponding to a constant a_n called as eigenvalue.

Normally, when the Aop operates on $\psi_n(x, t)$ it gives back these functions multiplied by a constant a_n .

An example of eigenvalue equation is

$$-i\hbar \frac{\partial}{\partial t} [\psi_n(x, t)] = E_n \psi_n(x, t)$$

Applications of the Schrodinger Equation

(i) Motion of a free particle:

Consider an electron moving freely in space in the positive x -direction and not acted upon by any force. Since no force is acting on electron and its potential energy is zero.

Schrödinger's time independent eqn. is,

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V \psi = E \psi \dots \dots \dots \quad (1)$$

$$\frac{d^2 \psi}{dx^2} + \frac{8\pi^2 m}{\hbar^2} E \psi = 0 \dots \dots \dots \quad (2)$$

$$\because \hbar = \frac{h}{2\pi} \quad & \quad V = 0$$

$$\text{If } K^2 = \frac{8\pi^2 m E}{h^2} \dots \dots \dots \quad (3)$$

The above eqⁿ. reduces to

$$\frac{d^2 \psi}{dx^2} + K^2 \psi = 0 \dots \dots \dots \quad (4)$$

Quantum Mechanics

The general solution to the above eqⁿ. is

$$\psi(x) = Ae^{iKx} + Be^{-iKx}$$

Where A and B are constants. As it is assumed that the waves propagating only in the positive x -direction, we can write

$$\psi(x) = Ae^{iKx} e^{-i\omega t}$$

Since the electron is moving freely, there are no boundary conditions and hence no restriction on K . All values of the energy are allowed. The allowed values of energy form a continuum and are given by

$$E = \frac{\hbar^2}{8\pi^2 m} K^2 \quad \text{using eqn. (3)}$$

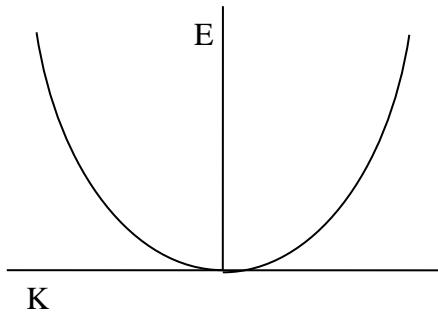


Figure 5 : Parabolic relationship between E & K in case of free electron.

A freely moving electron therefore possesses a continuous energy spectrum.

$$K = \sqrt{\frac{2mE}{\hbar^2}} = \frac{P}{\hbar} = \frac{2\pi}{\lambda}$$

The K vector describes the wave properties of the electrons.

As $E \propto K^2$ the graph between E and K is a parabola as shown in Fig. (5). the momentum is well defined in this case. Therefore, according to uncertainty principle it is difficult to assign a position to the electron i.e., the electron position is indeterminate.

(ii) Particle in a box:

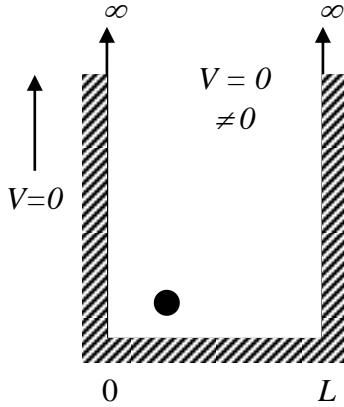


Figure 6: An electron trapped or an electron confined to a one – dimensional potential wall

Consider the motions of electrons in one-dimensional deep potential well bounded by high potential barriers. Electron can propagate along x axis & can get reflected from the walls at \$x = 0\$ and \$x = L\$ as shown in fig. (6) And thus it can propagate in both positive and negative \$x\$ directions within the well the potential energy is zero and at the boundaries i.e., \$x = 0\$ and \$x = L\$, potential is very high almost \$\infty\$. Therefore, the probability of finding the electron outside the well must be zero. i.e., \$\Psi = 0\$ at \$x \leq 0\$ and \$x \geq L\$.

The Schrodinger eqn. is

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

For the electron in well it reduces to

$$\frac{d^2 \psi}{dx^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad \dots \dots \dots \quad (1)$$

$$\frac{d^2 \psi}{dx^2} + K^2 \psi = 0 \quad \text{where } K^2 = \frac{2mE}{\hbar^2} = \frac{8\pi^2 m E}{h^2} \quad \dots \dots \quad (2)$$

The solution of the eqn. is

$$\psi(x) = A \sin Kx + B \cos Kx \quad \dots \dots \dots \quad (3)$$

where A and B are the constants to be determined with the help of boundary conditions which are

at \$x = 0\$, \$\Psi = 0\$

and at \$x = L\$, \$\Psi = 0\$

Substituting (4) in (3) gives, \$B = 0\$

and \$0 = A \sin KL + B \cos KL\$

Quantum Mechanics

$$\therefore E_n = \frac{\hbar^2}{8mL^2} \cdot n^2 \quad (n = 1, 2, 3, \dots) \dots \dots \dots \dots (9)$$

Above Eqn. shows only those values of energy are possible for which n is an integer.

So, $E_1 = \frac{\hbar^2}{8mL^2}$, $E_2 = \frac{\hbar^2}{2mL^2}$, $E_3 = \frac{9\hbar^2}{8mL^2}$, ...etc. are allowed energy states. It is shown in Fig. (7).

The wave function of a particle in a box whose energies are E_n are, from Eq. (3) with $B = 0$,

$$\psi_n = A \sin Kx = A \sin \frac{\sqrt{2mE_n}}{\hbar} x \dots (10)$$

Substituting value of E_n in Eq. (10)

$$\psi_n = A \sin Kx = A \sin \frac{n\pi}{L} x \dots (11)$$

This function Ψ_n satisfies the all requirements for each quantum number n , Ψ_n is a finite, single valued function of x , and $\partial\Psi_n/\partial x$ are continuous. The integral of $|\Psi_n|^2$ over all space is finite, as we can see by integrating $|\Psi_n|^2 dx$ from $x = 0$ to $x = L$ (since the particle confined within these limits).

With the help of trigonometric identity $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$,

$$\begin{aligned} &= \frac{A^2}{2} \left[\int_0^L dx - \int_0^L \cos \left(\frac{2n\pi x}{L} \right) dx \right] \\ &= \frac{A^2}{2} \left[\int_0^L dx - \int_0^L \cos \left(\frac{2n\pi x}{L} \right) dx \right] \\ &= \frac{A^2}{2} \left[x - \left(\frac{L}{2n\pi} \right) \sin \frac{2n\pi x}{L} \right]_0^L = A^2 \left(\frac{L}{2} \right) \dots (12) \end{aligned}$$

To normalize we must assign a value to A such that is equal to the probability Pdx of finding the particle between x and $x + dx$, then it must be true that

$$\int_{-\infty}^{\infty} |\psi_n|^2 dx = 1 \dots (13)$$

Quantum Mechanics

middle of the box, while a particle in the next higher state of $n = 2$ is never there. Classical physics suggests the same probability for the particle being anywhere in the box.

SEM-1 Physics Notes



Module 6
Liquid
Crystals

INTRODUCTION TO LIQUID CRYSTALS

Module 6: Engineering Materials and Applications

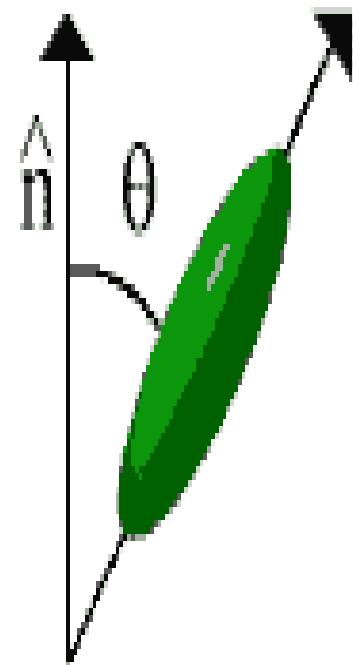
Liquid Crystals : Mesomorphic phases

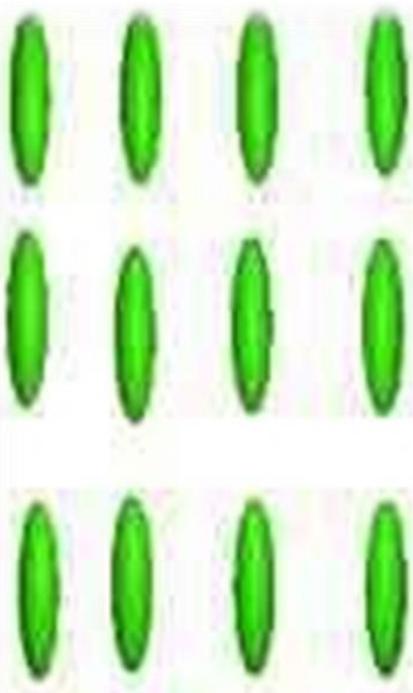
- There are many liquid crystal phases which can be distinguished on the basis of their different optical properties.
- These are also called as mesomorphic phases.
- When viewed under microscope, using polarized light source, different liquid crystal phases appear to have distinct texture.
- Each patch in a texture corresponds to a domain where liquid crystal molecules are oriented in a different direction.
- Within a domain the molecules are well ordered.



ORDER PARAMETER

- Liquid Crystals are soft condensed matters.
- Important in the study of advanced technology.
- Scientist Reinitzer found two melting points for Cholestryk benzoate, one at 145.5°C and another 178.5°C .
- Positional order may be lost, but some of orientational order remains.
- Also referred as *mesophase*.
- Order Parameter:
$$S = \frac{1}{2} [3\cos^2 \theta - 1]$$
where, θ is the angle made by each molecule with the director.

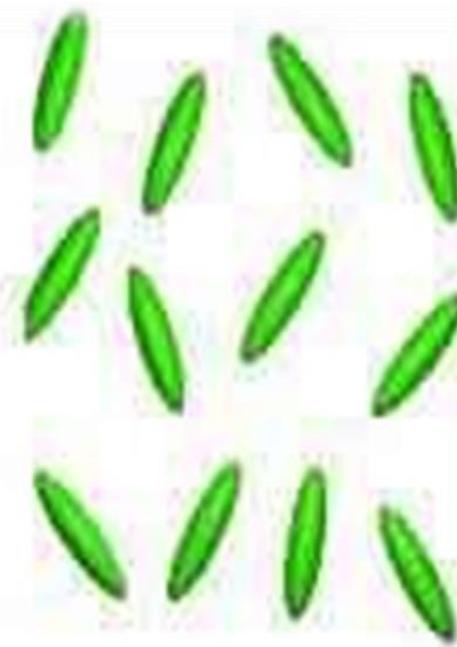




Solid

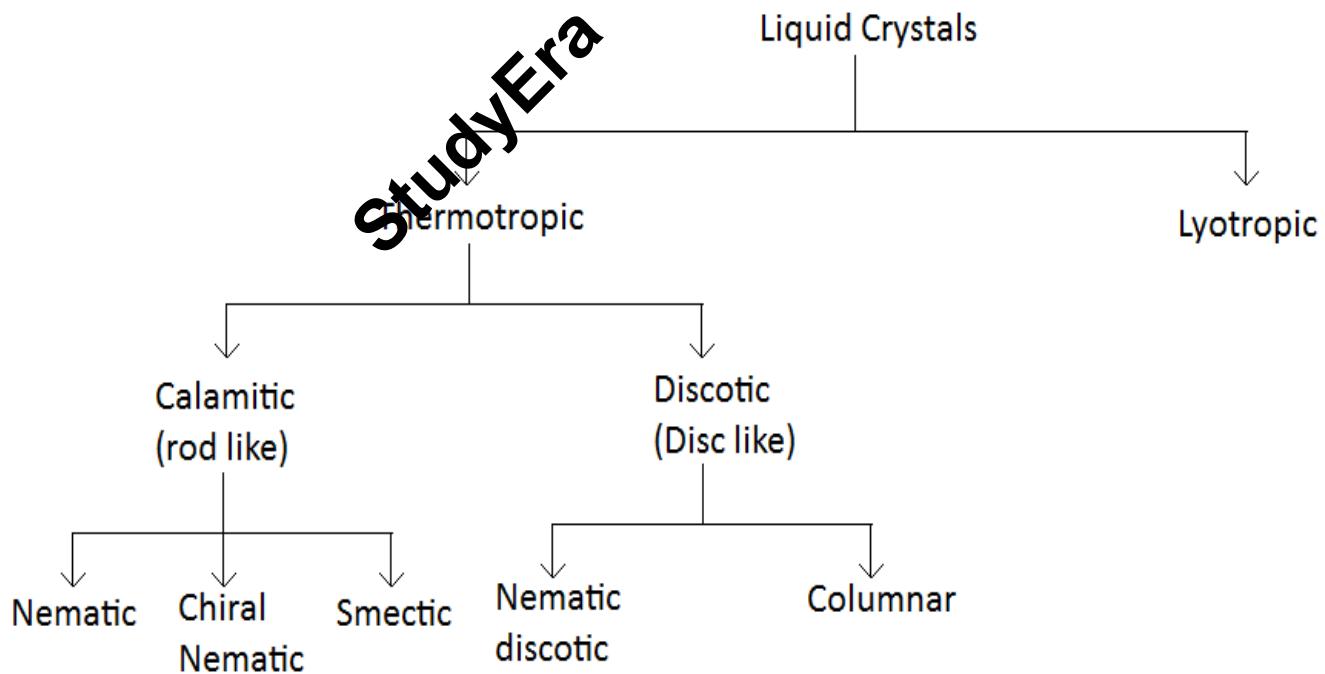


Liquid Crystal



Liquid

CLASSIFICATION OF LIQUID CRYSTALS



Module 6: Engineering Materials and Applications

Liquid Crystals : Mesomorphic phases

Thermo-tropic phases

phase transition occurs
when temperature is
changed

Lyotropic phases

phase transition occurs
when pressure is changed

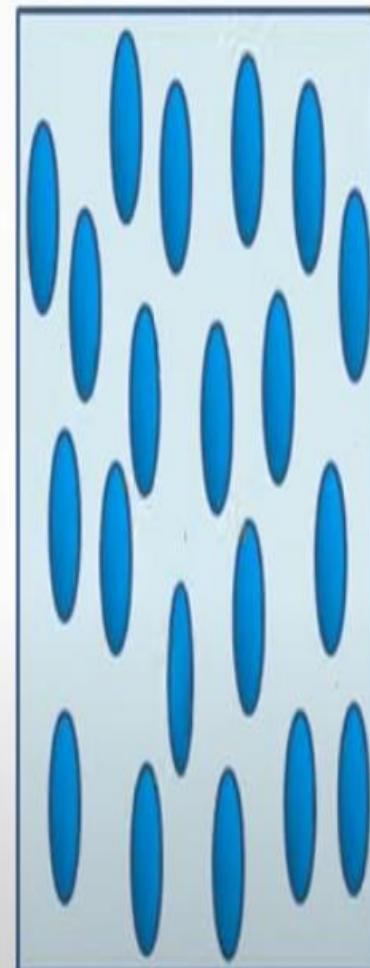
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Module 6: Engineering Materials and Applications

Liquid Crystals : Thermo-tropic phases

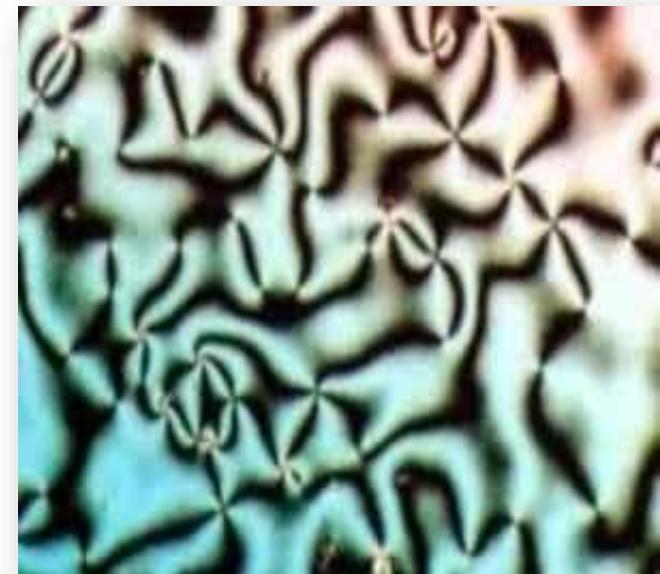
Nematic phase :

- Molecules do not have positional order but they have long range orientational order.
- The molecules flow and their center of mass positions are randomly distributed as in a liquid, but they all point in same direction within each domain.
- Nematics have fluidity similar to that of liquids; but they can be easily aligned by an external magnetic or electric field.
- An aligned nematic is characterized by optical properties due to orientational order which can be altered by using electric field or magnetic field.
- This makes them very useful liquid crystal displays.



NEMATIC LIQUID CRYSTALS

- ▶ means ‘thread like’.
- ▶ tend to orient along the director.



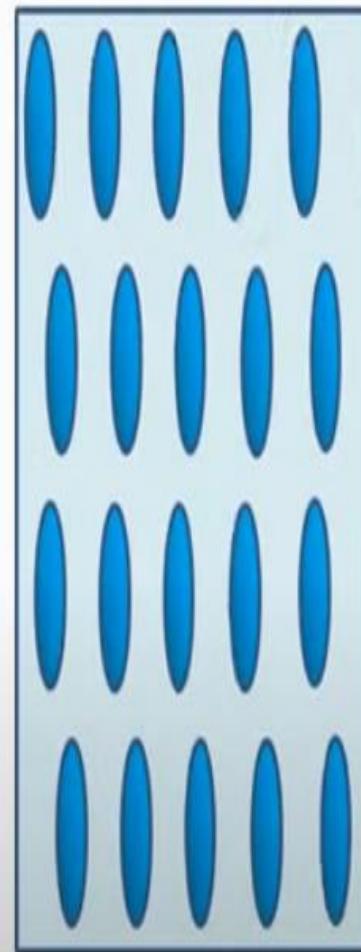
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Module 6: Engineering Materials and Applications

Liquid Crystals : Thermo-tropic phases

Smectic Phases :

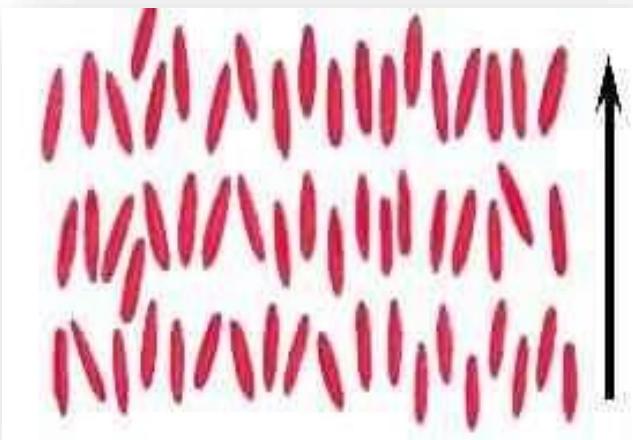
- Smectic phases are found at lower temperatures.
- Aligned molecules are arranged in different layers.
- These form well defined layers that can slide over one another like soap.
- Molecules are ordered along one direction.
- The axes of the molecules are perpendicular to the plane of the layers



SMECTIC LIQUID CRYSTALS

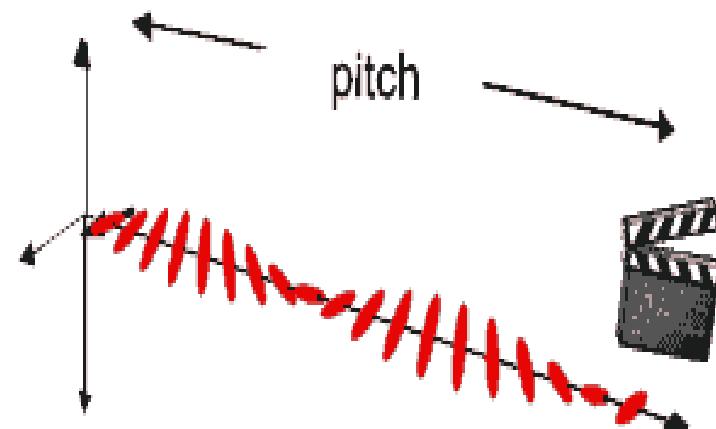
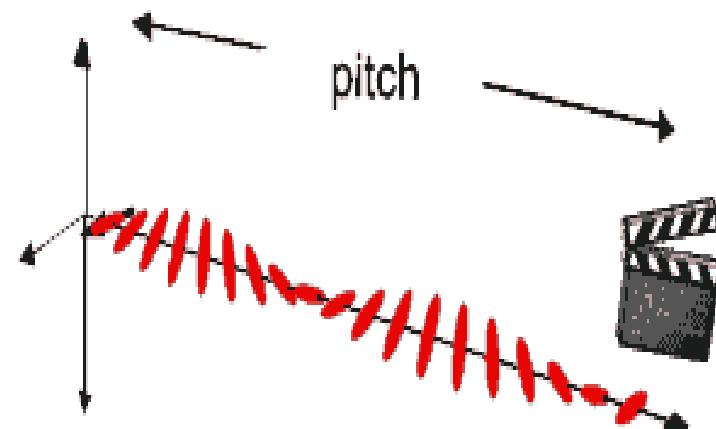
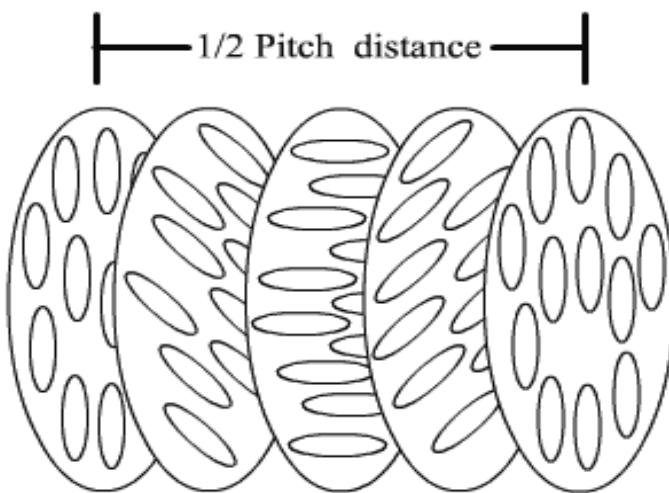
- ▶ means ‘soapy’.
- ▶ general orientation[↑] of order of nematic but also tend to align themselves in layers or planes.

Study[↑]



CHOLESTERIC LIQUID CRYSTALS

- ▶ formed by compounds having Chiral centers.
- ▶ directors actually form in a continuous helical pattern .



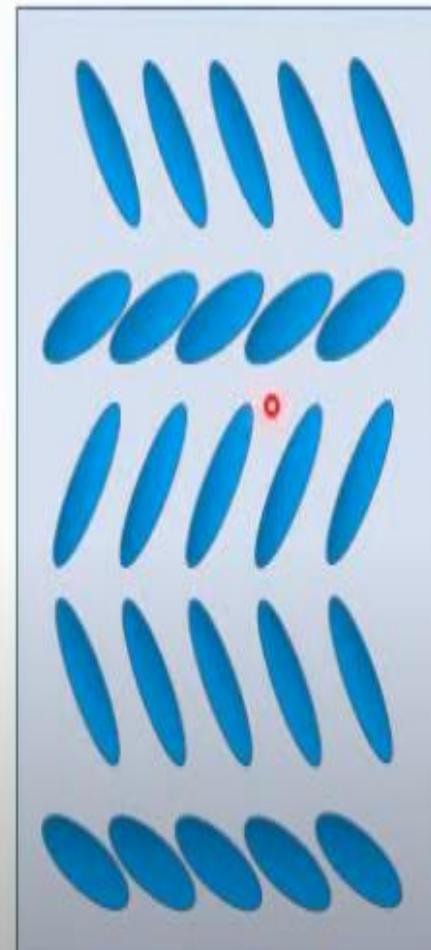
Module 6: Engineering Materials and Applications

Liquid Crystals : Thermo-tropic phases

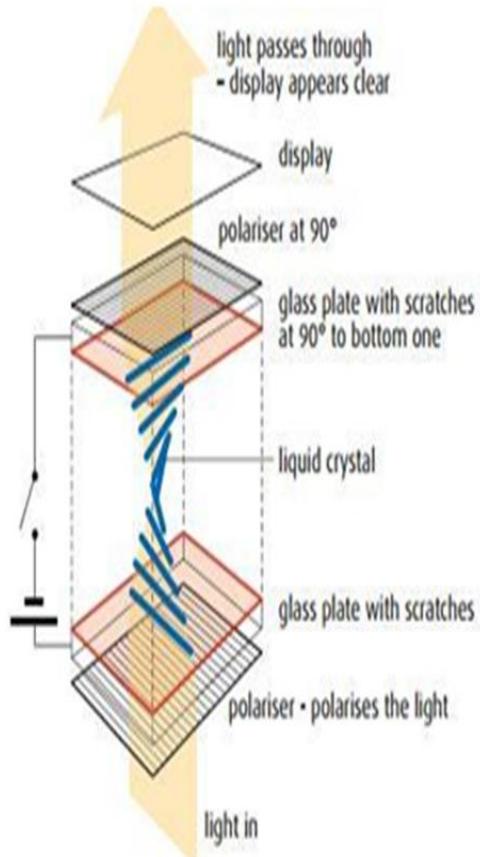
Cholesteric Phase :

- Aligned molecules are arranged in different layers.
- This phase exhibits twisting of molecules perpendicular to the director, with the molecular axis parallel to the director.
- The axes of the molecules are parallel to the plane of the layers.

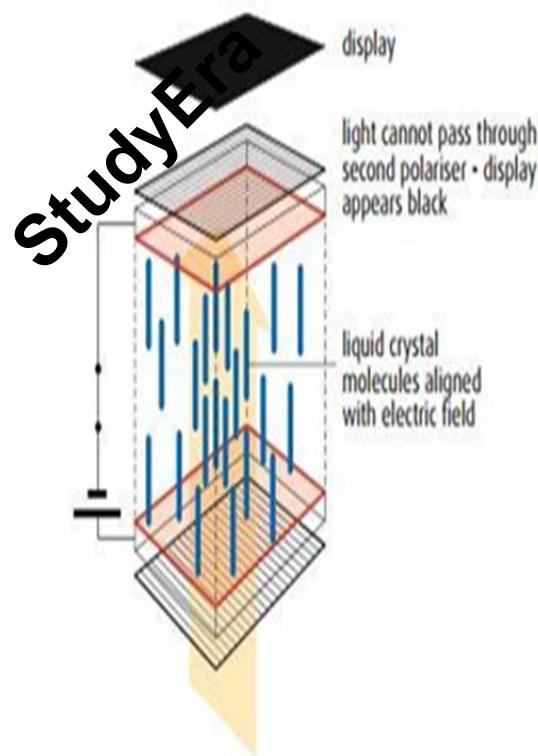
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Liquid Crystal Display



switch open • no voltage



switch closed • voltage applied
across the liquid crystal



StudyEra

Sem-1 Physics Notes



**Module-5
SUPERCONDUCTORS**

SUPERCONDUCTIVITY

Q1. Define superconductivity.

(M.U. Dec 2007, 2008, 2009, 2011, 2012, 2016, May 2008, 2010, 2012, 2013, 2014, 2015, 2016) [2 Marks]

When normal metals are cooled their resistivity decreases with temperature. In some materials at a lower temperature resistivity suddenly drops to zero, they are called superconductors.

Superconductivity is thus a phenomenon of sudden disappearance of electrical resistance (zero resistance) and expulsion of magnetic flux occurring in certain materials when they are cooled below a characteristic low temperature.

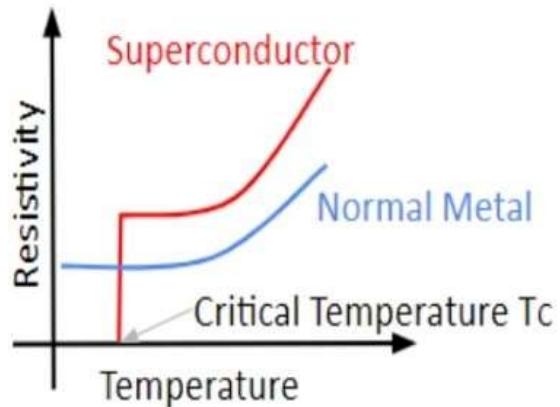


Figure 1: Resistivity Vs temperature

Q2. Define Critical temperature.

(M.U. May 2008, 2010, 2012, 2013, 2016, 2017; Dec. 2007, 2010, 2016) [2 Marks]

When a superconducting material is cooled below a certain temperature, its resistance suddenly drops to zero and it goes into the superconducting state from the normal state. The temperature at which a material transforms into a superconducting state is called **critical temperature 'Tc'** for that material.

Different materials have different critical temperatures. The transition is reversible. When the temperature of the material is increased above the critical temperature, it passes into the normal state. For elementary solids (in extremely pure form), critical temperature (T_c) is found to be very low (e.g. Tungsten = 0.015°K , Zinc = 0.85°K , Niobium = 9.46°K), whereas for compounds or alloys, T_c is found relatively high. {NbTi $\rightarrow 10^\circ\text{K}$, Nb₃Ge $\rightarrow 23.2^\circ\text{K}$, etc.}

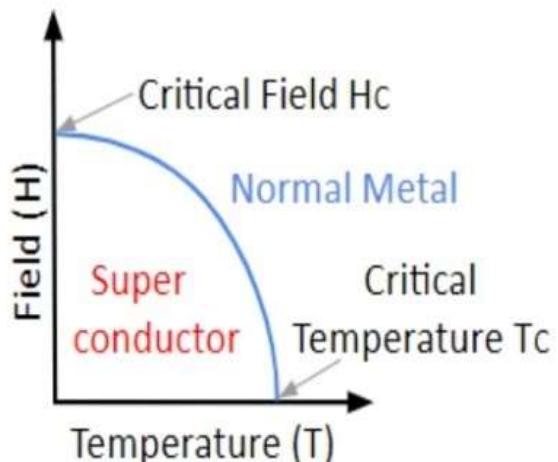


Figure 2: T_c and H_c for Superconductor

Q3. Define Critical Field.

(M.U. Dec. 2010, 2016) [1 Mark]

A material in its superconducting state, behaves like a diamagnetic material when placed in a weak magnetic field. But if the field strength is increased, the material may lose superconductivity even below critical temperature (T_c).

The critical field (H_c) for a superconducting material is the minimum field value at which normal resistivity is regained by the material and it loses its superconducting state.

Naturally, H_c value depends on the temperature of the superconductor which is placed in an external magnetic field as shown in *Figure 2* and hence it is a function of 'T'. At any temperature 'T' less than T_c , the superconducting material when placed in an external magnetic field, remains superconductor below H_c

(T) and becomes normal above the H_c (T). The maximum value of critical field is the field at absolute zero H_0 , critical field at a temperature 'T' in terms of H_0 is given by $H_c(T)$ which is mathematically,

$$H_c(T) = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

For example, pure mercury, $H_c(0) = 0.04$ Tesla --- at $T \approx 0^{\circ}\text{K}$

$H_c(3) = 0.02$ Tesla --- at $T \approx 3^{\circ}\text{K}$.

Q4. Explain Meissner Effect with the help of diagrams.

(M.U. May 2008, 2012; Dec. 2012, 2018, 2019; June 2019) [5 Marks]

When a superconductor is in normal state the external magnetic field lines are able to penetrate through its body but when it is in a superconducting state the field lines (i.e. magnetic flux) are strongly expelled from the body.

This observation explains a superconductor in a superconducting state is a perfect diamagnetic material. Thus, magnetic flux is excluded by superconducting material, because an equal and opposite

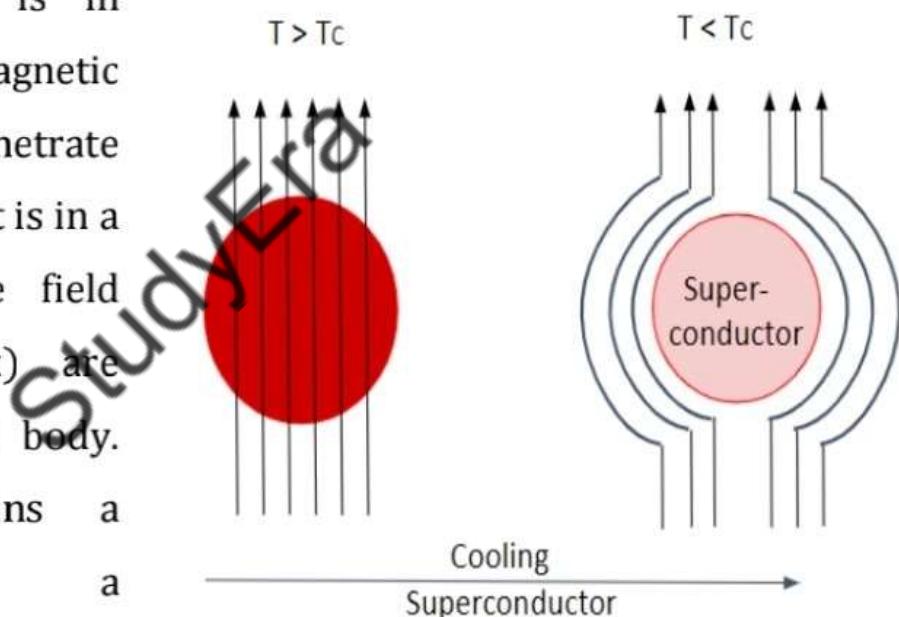


Figure 4: Meissner Effect

- Thus at the critical field (H_c), magnetization abruptly drops down to zero and material returns to normal state as shown in *Figure 7b*.
- Type I superconductors generally have a very low value of critical field ($H_c \sim 0.05 \text{ wb/m}^2$).
- Due to this they are very susceptible to stray magnetic fields and hence their applications are limited.

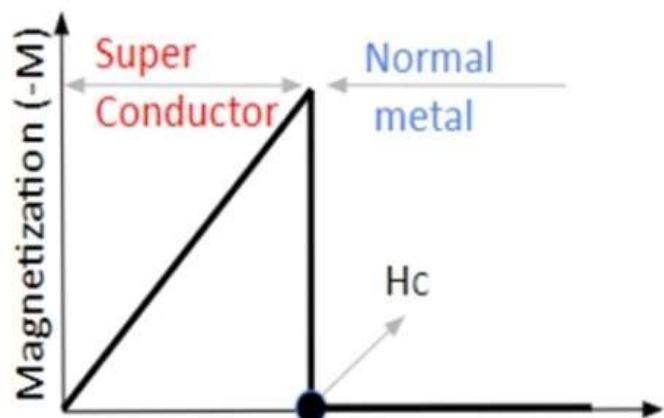


Figure 7b: Magnetization Vs Field

Type II superconductors:

- Type-II superconductors are characterized by two critical field values H_{c1} and H_{c2} . *Figure 7c* is a plot of magnetic field versus temperature indicating that the material is in its **superconducting state below H_{c1}** , the superconductor enters its **normal state after H_{c2}** , whereas the superconducting material exists in a **mixed state between H_{c1} and H_{c2}** .

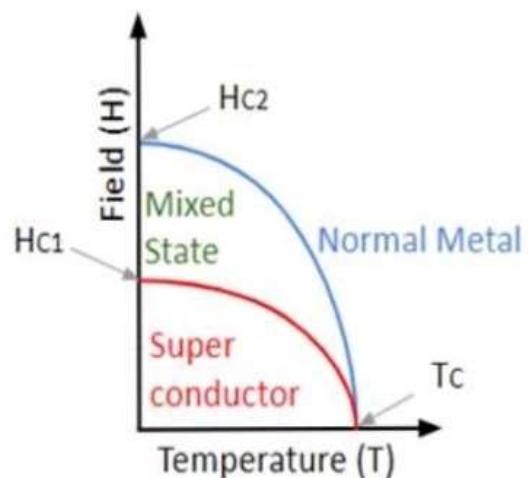


Figure 7c: Field Vs Temperature

Thus, the magnetic susceptibility is negative, and hence the superconductor is called a perfect diamagnet.

Q7. What are Type-I and Type-II Superconductors?

(M.U. May 2014, 2015; Dec. 2008 2009, 2011, 2012, 2019) [5 Marks]

Superconductors are classified as Type-I and Type-II superconductor based on their magnetic behavior.

Type I superconductors:

- *Figure 4a* shows the plot of magnetic field versus temperature for Type I superconductor indicating the region where it exists in a superconducting state and the region above critical field (H_c) and critical temperature (T_c) where it enters a normal state.
- Type I superconductors demonstrate a perfect diamagnetic behavior when in their superconducting state
- They get magnetized in the direction opposite to that of the applied field and their magnetization increases with the increase in magnetic field.
- When the applied field is increases beyond the critical field (H_c), they lose superconductivity and also its diamagnetic property thereby entering a normal state.

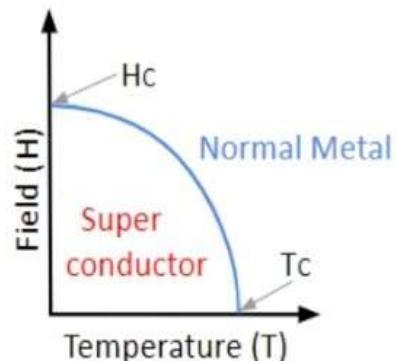


Figure 7a: Field Vs Temperature

allow any friction among the surfaces and a high rotation speed can be achieved without any power loss against friction.

Q6. Show that Superconductor is a perfect diamagnet

Susceptibility ($\chi = \frac{M}{H}$) is defined as the magnetization per unit magnetic field.

Diamagnetic materials get magnetized in the direction opposite to the applied field and hence the sign of internal magnetization 'M' is opposite to applied field 'H'. This implies that the susceptibility for diamagnetic materials is negative.

The magnetic induction inside a material is given by :

$$B = \mu_0 (H + M) \text{----(1)}$$

Where, 'H' denotes the external field applied and 'M' denotes the magnetization produced within the specimen.

For superconductor to be in superconducting state at $T < T_c$, flux should be expelled out of the body of the superconductor and hence the net flux inside the body should be zero.

$$B \text{ (inside)} = 0 \text{ ----(2)}$$

Combining (1) and (2) we get,

$$\mu_0 (H + M) = 0$$

' μ_0 ' is a non-zero constant and hence,

$$H + M = 0$$

$$M = -H \text{-----(3)}$$

$$\chi = \frac{M}{H} = -1$$

magnetization is induced inside its body that opposes the applied field and throws it out.

When a superconducting material is cooled in a weak magnetic field ($H < H_c$), at a certain low temperature which is the critical temperature (T_c) for that material, the field flux which was penetrating the material before is now suddenly expelled from the body of the superconducting material. The transition from penetration to expulsion due to cooling is shown in [Figure4](#). This phenomenon of sudden flux expulsion is called Meissner's effect. It was discovered by Meissner and Ochsenfeld in 1933. The existence of Meissner's effect is the demonstration of diamagnetic behavior of the superconducting state of the material.

Q5. Enlist applications of Meissner Effect

Thus, Meissner's effect is useful in :

1. **Confirming superconductivity:** When a material being cooled in a weak magnetic field ($H < H_c$) demonstrates flux expulsion, we can confirm that the material has a superconducting behavior. This is because suddenly the magnetic field lines are pushed out of the body which were initially penetrating through the body.
2. **Levitated frictionless bearings:** The machines with rotating shafts ball bearings are covered with lubricants to reduce friction and thereby increase rotations per minute (rpm). With a magnetic shaft and superconducting bearings (or vice versa) the levitation effect will not

- When H_{c1} is crossed, few magnetic field lines penetrate some part of the material, this part gets back to normal state whereas the remaining part of the material is still in the superconducting state. When the applied field reaches value H_{c2} the number of field lines penetrating the body increase in number and occupy the whole body of the superconducting material transforming it into a normal state.
- As shown in *Figure 7d* the magnetization of Type-II superconductor **increases** with the increase in applied magnetic field till the lower critical field H_{c1} .
- After this the magnetization **decreases gradually** with increasing applied field, till it reaches zero magnetization at field value H_{c2} .
- In such superconductors the transition from superconducting state to normal state occurs gradually **between H_{c1} and H_{c2}** meanwhile the superconductor remains in the mixed state in which the material is neither in a perfect superconductor nor completely normal.
- Type-II Superconductor provides a liberty of enjoying a partially superconducting state up to H_{c2} , which is relatively larger ($H_{c2} \sim 20$ wb/m²). Hence, Type-II superconductors are widely used industries.

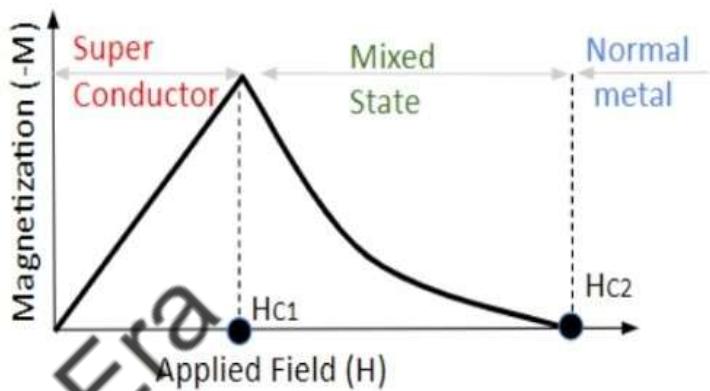


Figure 7d: Magnetization Vs Field (Type-II)

Q8. Explain properties like energy density and power density. Compare Supercapacitors with batteries and regular capacitors based on these properties.

Energy density is the amount of energy that can be stored per unit mass or per unit volume in any given device which is equivalent to the amount of work the device can perform.

$$W = \frac{1}{2} CV^2$$

Where, W – Energy ; C – Capacitance; V – Voltage

This basically means if a device has a high energy density value, then it has the capacity to store a large amount of energy in very small mass or volume or in simple words a small device can do a lot of work. Usually this quantity is measured in **Watt-hours/kilogram (Wh/kg)** or **Watt-hours/litre (Wh/L)**.

Power density is the amount of flow of energy per unit mass or volume, per unit time, in any given device. It gives the measure of how quickly a device can give its energy, relative to its size.

$$P = \left(\frac{1}{4}\right) \frac{V^2}{R}$$

Where, P – Power; V - Voltage applied; R – Internal Resistance

If a device has a high-power density, it can give off large amounts of energy in a very small period of time. It basically tells us numerically, how quickly the stored energy can be utilized. In simple words how quick the work can be done by a device. This quantity is mostly measured in units of **kilowatts/kilogram (kW/kg)**.

Table 8.1: Comparison of various performance parameters for a battery capacitor with supercapacitors

Performance Parameters	Li-ion Battery	Ordinary Capacitor	Supercapacitor
Energy density (Whr/kg)	100-180	0.01-0.02	4-10
Power density (kW/kg)	300-3000	>200	6000-60000
Time to charge	15-60 mins	5-15s	2-10s
Lifespan	5-10 years	15-20 years	10 - 15 years

As shown in the *Table 8.1*

Batteries have energy density higher than ordinary capacitors. This means batteries can store more amount of energy, than an ordinary capacitor.

Even though Batteries may have a high value of energy density they have a low power density. This means they cannot output their energy quickly and take a much longer time to re-charge as compared ordinary capacitors that can give off energy and re-charge much more quickly because of their small size.

Supercapacitors bridge the gap between the two. They have both, a high-power density, as well as a high energy density. This comparison is also shown in

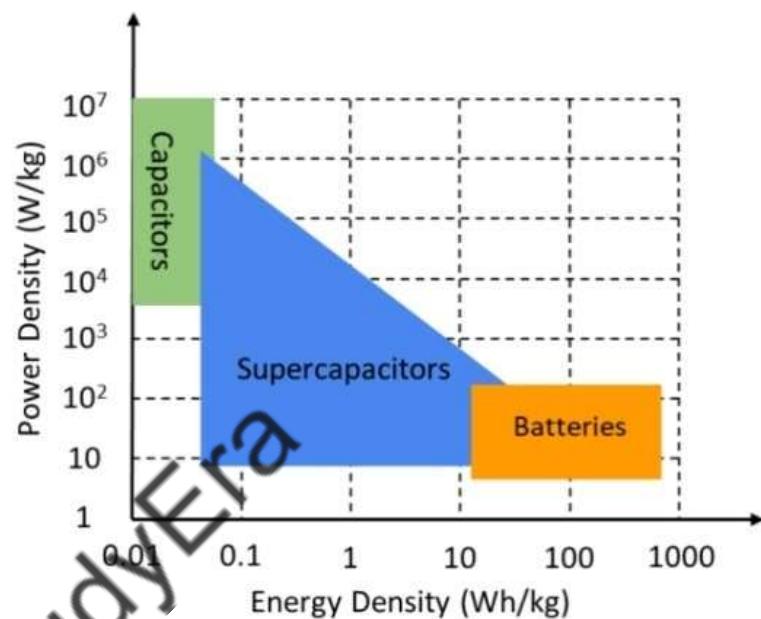


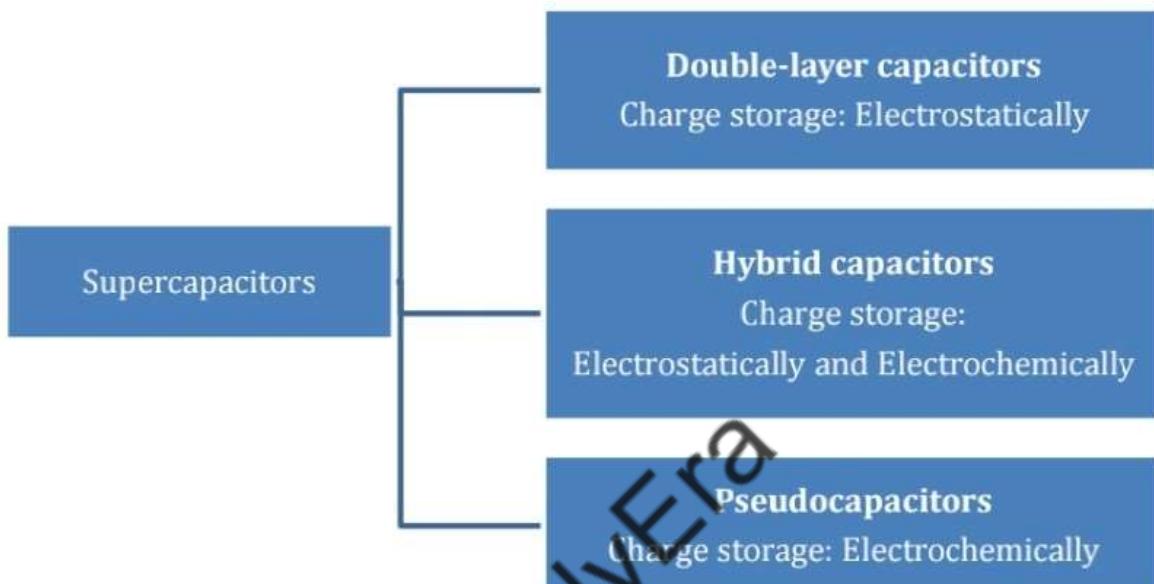
Figure 8: power density vs energy density

Figure 8. They can store and release a large amount of energy as output and are also able to re-charge quickly.

Q9. Explain different types of Supercapacitors.

Supercapacitors are classified as follows:

Table 9.1: Classification of Supercapacitors

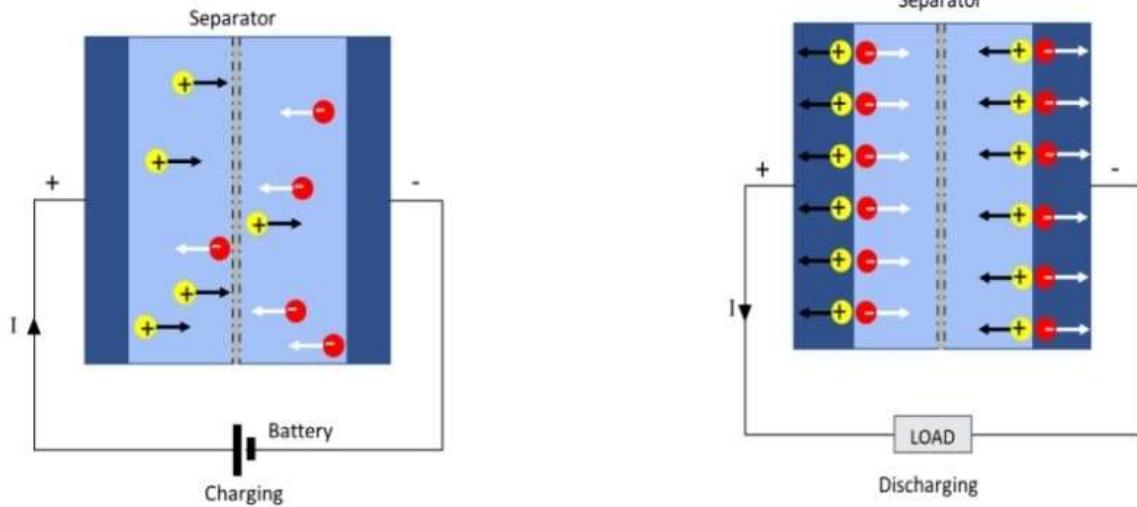


- 1. Double Layer Capacitors:** These capacitors achieve energy storage with activated carbon electrodes. Thus, we can say that charge storage is done electrostatically.
- 2. Pseudo-capacitors:** These capacitors store electrical energy by electron charge transfer between electrodes and electrolyte. Thus, we can say that charge storage is done electrochemically. Metal oxide or conductive polymer electrodes are used.

Applications of Supercapacitors are:

1. Cellular Devices
2. Uninterruptible Power Supplies (UPS)
3. Industrial Lasers
4. Medical Equipment
5. Wireless communication Systems
6. VCRs, CD Players
7. Security systems, Computer Scanners
8. Microwaves, Coffee Makers

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Q8. State the advantages, disadvantages and applications of Supercapacitors.

Advantages of Supercapacitors are:

1. Lasts for a number of years with minimal damage
2. Takes minimum amount of time to charge
3. It is not costly
4. Fast release of battery
5. Stores large amount of energy

Disadvantages of Supercapacitors are:

1. Low voltage for a single cell
2. Energy density is low
3. During discharge complete energy is not used
4. Self - discharge is high
5. Series connection of multiple capacitors is required for high voltage.

electrolyte eventually increases the storage capacity for charge. To top this up the space between them 'd' gets **effectively reduced to accommodate the unique insulating separator**. Opposite charges get deposited on either side of the separator, thus creating a double layer of charge as shown in *Figure 10a*. Hence, such capacitors are also called as double layer supercapacitor which is the most commonly used supercapacitor. In this way a Supercapacitor achieves a much higher value of capacitance than any regular capacitor.

Working:

A supercapacitor can be charged and discharged unlimited number of times. When the supercapacitor is not charged, charges in the electrolyte are distributed randomly. In order to charge a supercapacitor, it is connected to a voltage source. While charging positive charges are attracted to the negative terminal and negative charges attracted to the positive terminal as shown in *Figure 10b*. When all the charges are deposited on the electrodes the supercapacitor is said to be fully charged as shown in *Figure 10a*. Once charged the supercapacitor can be connected to a load for discharging as shown in *Figure 10c*.

3. **Hybrid Supercapacitor:** These have two electrodes; one stores charge electrostatically while the other exhibits electrochemical capacitance. Thus, it's a combination of Double layer capacitor and Electrochemical pseudo capacitor and hence named is Hybrid supercapacitors.

Q10. Explain construction and working of a Supercapacitor.

(M.U. Dec. 2019) [5 Marks]

A supercapacitor is a capacitor which has very high capacitance (of the order of Farads) as compared to a normal capacitor (of the order of 10^{-6} Farads). It is capable of **charging and storing energy at a higher density** than normal capacitor. It is also capable of **discharging to use stored energy to do work faster than the normal battery**.

Construction:

A normal capacitor has two plates which are called as its electrodes, Capacitance is directly proportional to the area of plates 'A' and inversely proportional to the distance between the plates 'd'. A **porous substance** used to coat the metallic plates of a supercapacitor due to this the plates of a **supercapacitor have a much larger effective surface area 'A'**. The larger surface area of electrodes which soaked in

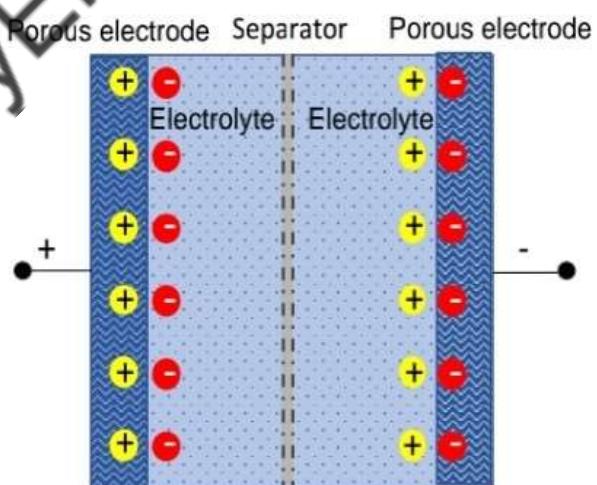


Figure 10a: Supercapacitor construction

SUPERCONDUCTIVITY

Q1. The critical field of niobium is 10^5 A/m at 8K and $2 \times 10^5 \text{ A/m}$ at 0K . calculate the critical temperature of the element.

Given: - $T = 8\text{K}$, $H_C(0) = 2 \times 10^5 \text{ A/m}$, $H_c(T) = 2 \times 10^5 \text{ A/m}$

Formula: - $H_c(T) = H_c(0) \left(1 - \frac{T^2}{T_c^2}\right)$

Solution: -

$$T_c = T_c \left[1 - \frac{H_c(T)}{H_c(0)}\right]^{1/2}$$

$$T_c = 8 \times \left[1 - \frac{10^5}{2 \times 10^5}\right]^{-1/2}$$

$$T_c = 11.3137^\circ \text{ K}$$

Ans: - critical temperature is 11.3137° K

Q2. The transition temperature for Pb is 7.2K . however at 5K it loses the superconducting property if subjected to magnetic field of $3.3 \times 10^4 \text{ A/m}$. Find the maximum value of H which will allow the metal to retain its superconductivity at 0K .

Given: - $T_c = 7.2\text{K}$, $H_c(T) = 3.3 \times 10^4 \text{ A/m}$, $T = 5\text{K}$

Formula: - $H_c(T) = H_c(0) \left(1 - \frac{T^2}{T_c^2}\right)$

Solution: - $H_c(0) = H_c(T) \left(1 - \frac{T^2}{T_c^2}\right)^{-1}$

$$= 3.3 \times 10^4 \left(1 - \frac{5^2}{7.2^2}\right)^{-1}$$

$$= 6.3737 \times 10^4 \text{ A/m}$$

Ans: - the critical field at 0K is $6.3737 \times 10^4 \text{ A/m}$

Q3. The transition temperature for lead is 7.26K. The maximum critical field for the material is 8×10^5 A/m. Lead has to be used as a superconductor subjected to a magnetic field of 4×10^4 A/m. What precaution will have to be taken?

Given: -

$$T_C = 7.26\text{K}, H_C(0) = 8 \times 10^5 \text{ A/m}, H_c(T) = 4 \times 10^4 \text{ A/m}$$

Formula: - $H_c(T) = H_c(0) \left(1 - \frac{T^2}{T_c^2}\right)$

Solution: -
$$\begin{aligned} T &= T_c \left[1 - \frac{H_c(T)}{H_c(0)}\right]^{1/2} \\ &= 7.26 \left[1 - \frac{4 \times 10^4}{8 \times 10^5}\right]^{1/2} \\ &= 7.0761 \end{aligned}$$

Ans: - The temperature of lead should be maintained below 7.0761K above will lose its superconductivity.

Q4. A superconductor has a critical temperature 3.7K at zero magnetic field. At 0K, the critical magnetic field is 0.0306 Tesla. What is the critical magnetic field at temperature 2K?

Given: - $T_C = 3.7\text{K}, H_C(0) = 0.0306\text{T}, T = 2\text{K}$

Formula: - $H_c(T) = H_c(0) \left(1 - \frac{T^2}{T_c^2}\right)$

Solution: -
$$H_c(T) = 0.0306 \left(1 - \frac{2^2}{3.7^2}\right) = 0.0217 \text{ T}$$

Ans: - The critical magnetic field is 0.0217 T

Superconductivity

1. For a certain metal the critical magnetic field is $5 \times 10^3 \text{ A/m}$ at 6K and $2 \times 10^4 \text{ A/m}$ at 0K. Determine its transition temperature.

(Ans: $T_c = 6.93\text{K}$)

2. Calculate the critical current for a wire of lead having a diameter of 1mm at 4.26K. The critical temperature for lead is 7.18K and $H_c(0) = 6.5 \times 10^4 \text{ A/m}$.

(Ans:

$H_c(T) = 4.2118 \times 10^4 \text{ A/m}$)

3. A Lead superconductor with $T_c = 7.2\text{K}$ has critical magnetic field of $6.5 \times 10^3 \text{ A/m}$ at absolute zero. What

Would be the magnitude of critical magnetic field at 5K temperature?

(Ans: $H_c(T) = 3.365 \times 10^3 \text{ A/m}$)

4. The critical magnetic field for vanadium is 10^5 A/m at 8.58K and $2 \times 10^5 \text{ A/m}$ at 0K. Determine its critical temperature.

(Ans: $T_c = 12.133\text{K}$)

5. A Lead wire has a critical magnetic field of $6.5 \times 10^3 \text{ A/m}$ at 0K. The critical temperature is 7.18K. at what temperature the critical field will drop to $4.5 \times 10^3 \text{ A/m}$?

(Ans: 3.9827K)