- [Voiceover] Let's
now introduce ourselves
to the idea of a differential equation.
And as we'll see, differential equations
are super useful for
modeling and simulating
phenomena and understanding
how they operate.
But we'll get into that later.
For now let's just think
about or at least look at
what a differential equation actually is.
So if I were to write, so
let's see here is an example
of differential equation,
if I were to write that
the second derivative of y plus two times
the first derivative of y
is equal to three times y,
this right over here is
a differential equation.
Another way we could write it if we said
that y is a function of
x, we could write this
in function notation.
We could write the second
derivative of our function
with respect to x plus two
times the first derivative
of our function is equal to
three times our function.
Or if we wanted to use
the Leibniz notation,
we could also write, the
second derivative of y
with respect to x plus two
times the first derivative
of y with respect to x is
equal to three times y.
All three of these equations
are really representing
the same thing, they're saying
OK, can I find functions
where the second
derivative of the function
plus two times the first
derivative of the function
is equal to three times
the function itself.
So just to be clear,
these are all essentially
saying the same thing.
And you might have just
caught from how I described it
that the solution to a
differential equation
is a function, or a class of functions.
It's not just a value or a set of values.
So the solution here, so the solution
to a differential equation is a function,

or a set of functions,
or a class of functions.
It's important to contrast this relative
to a traditional equation.
So let me write that down.
So a traditional equation,
maybe I shouldn't say
traditional equation,
differential equations
have been around for a while.
So let me write this as
maybe an algebraic equation
that you're familiar with.
An algebraic equation
might look something like,
and I'll just write up a simple quadratic.
Say x squared plus three x
plus two is equal to zero.
The solutions to this algebraic equation
are going to be numbers,
or a set of numbers.
We can solve this, it's
going to be x plus two
times x plus one is equal to zero.
So x could be equal to
negative two or x could
be equal to negative one.
The solutions here are
numbers, or a set of values
that satisfy the equation.
Here it's a relationship
between a function
and its derivatives.
And so the solutions, or the solution,
is going to be a function
or a set of functions.
Now let's make that a
little more tangible.
What would a solution to
something like any of these three,
which really represent the same thing,
what would a solution actually look like?
Actually let me move
this over a little bit.
Move this over a little bit.
So we can take a look at what some
of these solutions could look like.
Let me erase this a little.
This little stuff that
I have right over here.
So I'm just gonna give you
examples of solutions here.
We'll verify that these
indeed are solutions
for I guess this is really
just one differential equation
represented in different ways.
But you'll hopefully
appreciate what a solution
to a differential equation looks like.

And that there is often
more than one solution.
There's a whole class of functions
that could be a solution.
So one solution to this
differential equation,
and I'll just write it as our first one.
So one solution, I'll call it y one.
And I could even write it as y one of x
to make it explicit that
it is a function of x.
One solutions is y one of x is equal to e
to the negative three x.
And I encourage you to
pause this video right now
and find the first derivative of y one,
and the second derivative of
y one, and verify that it does
indeed satisfy this differential equation.
So I'm assuming you've had a go at it.
So let's work through this together.
So that's y one.
So the first derivative of y one,
so we just have to do the chain rule here,
the derivative of negative
three x with respect to x
is just negative three.
And the derivative of e
to the negative three x
with respect to negative three x is just e
to the negative three x.
And if we take the second
derivative of y one,
this is equal to the same exact idea,
the derivative of this is
three times negative three
is going to be nine e
to the negative three x.
And now we could just
substitute these values
into the differential
equation, or these expressions
into the differential
equation to verify that this
is indeed going to be
true for this function.
So let's verify that.
So we first have the
second derivative of y.
So that's that term right over there.
So we have nine e to the negative three x
plus two times the first derivative.
So that's going to be two
times this right over here.
So it's going to be minus
six, I'll just write
plus negative six e to
the negative three x.
Notice I just took this two
times the first derivative.

Two times the first derivative
is going to be equal to,
or needs to be equal to, if
this indeed does satisfy,
if y one does indeed satisfy
the differential equation,
this needs to be equal to three times y.
Well three times y is three
times e to the negative three x.
Three e to the negative three x.
Let's see if that indeed is true.
So these two terms right over here,
nine e to the negative three x,
essentially minus six e
to the negative three x,
that's gonna be three e
to the negative three x.
Which is indeed equal to three
e to the negative three x.
So y one is indeed a solution
to this differential equation.
But as we'll see, it is
not the only solution
to this differential equation.
For example, let's say y
two is equal to e to the x
is also a solution to this
differential equation.
And I encourage you to
pause the video again
and verify that it's a solution.
So assuming you've had a go at it.
So the first derivative of this
is pretty straight-forward,
is e to the x.
Second derivative, one
of the profound things
of the exponential function,
the second derivative here
is also e to the x.
So the second derivative,
let me just do it
in those same colors.
So the second derivative
is going to be e to the x
plus two times e to the x
is indeed going to be equal
to three times e to the x.
This is absolutely going to be true.
E to the x plus two e to the
x is three e to the x.
So y two is also a solution
to this differential equation.
So that's a start.
In the next few videos,
we'll explore this more.
We'll start to see what
the solutions look like,
what classes of solutions are,
techniques for solving them,

visualizing solutions to
differential equations,
and a whole toolkit for
kind of digging in deeper.