1. AUTO-COVARIANCE

$$\mathbf{C}_{00}^{G}(k_{1},k_{2}) \simeq \sum_{\ell'_{1},\ell'_{2}} P_{\ell'_{1}}(k_{1}) P_{\ell'_{2}}(k_{2}) \left\{ \frac{(2\ell_{1}+1)(2\ell_{2}+1)}{\mathbf{I}_{22}^{2}} \int_{\hat{\mathbf{k}}_{1},\hat{\mathbf{k}}_{2},\mathbf{x}_{1},\mathbf{x}_{2}} W_{22}(\mathbf{x}_{1}) W_{22}(\mathbf{x}_{2}) e^{-i(\mathbf{x}_{1}-\mathbf{x}_{2})\cdot(\mathbf{k}_{1}-\mathbf{k}_{2})} \right. \\
\left. \times \mathcal{L}_{\ell'_{1}}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{2}}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \right\}, \\
\mathbf{C}_{22}^{G}(k_{1},k_{2}) \simeq \sum_{\ell'_{1},\ell'_{2}} P_{\ell'_{1}}(k_{1}) P_{\ell'_{2}}(k_{2}) \left\{ \frac{(2\ell_{1}+1)(2\ell_{2}+1)}{\mathbf{I}_{22}^{2}} \int_{\hat{\mathbf{k}}_{1},\hat{\mathbf{k}}_{2},\mathbf{x}_{1},\mathbf{x}_{2}} W_{22}(\mathbf{x}_{1}) W_{22}(\mathbf{x}_{2}) e^{-i(\mathbf{x}_{1}-\mathbf{x}_{2})\cdot(\mathbf{k}_{1}-\mathbf{k}_{2})} \right. \\
\left. \times \mathcal{L}_{2}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{1}}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{2}}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \left[\mathcal{L}_{2}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \right] \right\}, \\
\mathbf{C}_{44}^{G}(k_{1}, k_{2}) \simeq \sum_{\ell'_{1},\ell'_{2}} P_{\ell'_{1}}(k_{1}) P_{\ell'_{2}}(k_{2}) \left\{ \frac{(2\ell_{1}+1)(2\ell_{2}+1)}{\mathbf{I}_{22}^{2}} \int_{\hat{\mathbf{k}}_{1},\hat{\mathbf{k}}_{2},\mathbf{x}_{1},\mathbf{x}_{2}} W_{22}(\mathbf{x}_{1}) W_{22}(\mathbf{x}_{2}) e^{-i(\mathbf{x}_{1}-\mathbf{x}_{2})\cdot(\mathbf{k}_{1}-\mathbf{k}_{2})} \right. \\
\left. \times \mathcal{L}_{4}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{1}}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{2}}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \left[\mathcal{L}_{4}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{2}) + \mathcal{L}_{4}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \right] \right\},$$

Shot noise in $\mathbf{C}_{\ell_1\ell_2}(k_1,k_2)$

$$C_{\ell_1\ell_2}^{\text{SN-G}}(k_1, k_2) = \frac{(2\ell_1 + 1)(2\ell_2 + 1)}{\mathbf{I}_{22}^2} \int_{\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2, \mathbf{x}_1, \mathbf{x}_2} \times \left\{ (1 + \bar{\alpha})^2 W_{12}(\mathbf{x}_1) W_{12}(\mathbf{x}_2) e^{-i(\mathbf{k}_1 - \mathbf{k}_2) \cdot (\mathbf{x}_1 - \mathbf{x}_2)} \mathcal{L}_{\ell_1}(\hat{\mathbf{x}}_1 \cdot \hat{\mathbf{k}}_1) \left[\mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_1 \cdot \hat{\mathbf{k}}_2) + \mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_2 \cdot \hat{\mathbf{k}}_2) \right] + (1 + \bar{\alpha}) \sum_{\ell'_1} P_{\ell'_1}(k_1) e^{-i(\mathbf{k}_1 - \mathbf{k}_2) \cdot (\mathbf{x}_1 - \mathbf{x}_2)} W_{12}(\mathbf{x}_1) W_{22}(\mathbf{x}_2) \mathcal{L}_{\ell'_1}(\hat{\mathbf{x}}_2 \cdot \hat{\mathbf{k}}_1)$$

$$(2)$$

 $\left. \times \left[\mathcal{L}_{\ell_1}(\hat{\mathbf{x}}_1 \cdot \hat{\mathbf{k}}_1) \mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_2 \cdot \hat{\mathbf{k}}_2) + \mathcal{L}_{\ell_1}(\hat{\mathbf{x}}_2 \cdot \hat{\mathbf{k}}_1) \mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_1 \cdot \hat{\mathbf{k}}_2) + \mathcal{L}_{\ell_1}(\hat{\mathbf{x}}_1 \cdot \hat{\mathbf{k}}_1) \mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_1 \cdot \hat{\mathbf{k}}_2) + \mathcal{L}_{\ell_1}(\hat{\mathbf{x}}_2 \cdot \hat{\mathbf{k}}_1) \mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_2 \cdot \hat{\mathbf{k}}_2) \right] \right\}$

Shot noise in auto-covariance:

$$\mathbf{C}_{00}^{\text{SN-G}}(k_{1},k_{2}) = \frac{(2\ell_{1}+1)(2\ell_{2}+1)}{\mathbf{I}_{22}^{2}} \int_{\hat{\mathbf{k}}_{1},\hat{\mathbf{k}}_{2},\mathbf{x}_{1},\mathbf{x}_{2}} \left\{ (1+\bar{\alpha})^{2}W_{12}(\mathbf{x}_{1})W_{12}(\mathbf{x}_{2})e^{-i(\mathbf{k}_{1}-\mathbf{k}_{2})\cdot(\mathbf{x}_{1}-\mathbf{x}_{2})} + (1+\bar{\alpha})e^{-i(\mathbf{k}_{1}-\mathbf{k}_{2})\cdot(\mathbf{x}_{1}-\mathbf{x}_{2})}W_{22}(\mathbf{x}_{1})W_{12}(\mathbf{x}_{2}) \left[\frac{1}{2} \sum_{\ell'} P_{\ell'}(k_{1})\mathcal{L}_{\ell'}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{1}) + \frac{1}{2} \sum_{\ell'} P_{\ell'}(k_{2})\mathcal{L}_{\ell'}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) \right] \right\} \\
+ (1+\bar{\alpha})e^{-i(\mathbf{k}_{1}-\mathbf{k}_{2})\cdot(\mathbf{x}_{1}-\mathbf{x}_{2})} \left\{ (1+\bar{\alpha})^{2}W_{12}(\mathbf{x}_{1})W_{12}(\mathbf{x}_{2})e^{-i(\mathbf{k}_{1}-\mathbf{k}_{2})\cdot(\mathbf{x}_{1}-\mathbf{x}_{2})}\mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{2}\cdot\hat{\mathbf{k}}_{2}) \right\} \\
+ (1+\bar{\alpha})e^{-i(\mathbf{k}_{1}-\mathbf{k}_{2})\cdot(\mathbf{x}_{1}-\mathbf{x}_{2})}W_{22}(\mathbf{x}_{1})W_{12}(\mathbf{x}_{2}) \left[\frac{1}{2} \sum_{\ell'} P_{\ell'}(k_{1})\mathcal{L}_{\ell'}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{1}) + \frac{1}{2} \sum_{\ell'} P_{\ell'}(k_{2})\mathcal{L}_{\ell'}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) \right] \\
\times \left[\mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{1})\mathcal{L}_{2}(\hat{\mathbf{x}}_{2}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{1})\mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) \right] \\
\times \left[\mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{1})\mathcal{L}_{2}(\hat{\mathbf{x}}_{2}\cdot\hat{\mathbf{k}}_{1})\mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{2}\cdot\hat{\mathbf{k}}_{2}) \right] \\
\times \left[\mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{1})\mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{$$

Going to higher order in multipoles

$$\mathcal{L}_6 = \frac{11}{5} \mathcal{L}_4 \mathcal{L}_2 - \frac{4}{7} \mathcal{L}_4 - \frac{22}{35} \mathcal{L}_2 \tag{4}$$

2. CROSS-COVARIANCE

Interchanging ℓ'_1 and ℓ'_2 in Eq. (56)

$$\mathbf{C}_{\ell_{1}\ell_{2}}^{G}(k_{1}, k_{2})
\simeq \sum_{\ell'_{1}, \ell'_{2}} P_{\ell'_{1}}(k_{1}) P_{\ell'_{2}}(k_{2}) \left\{ \frac{(2\ell_{1}+1)(2\ell_{2}+1)}{\mathbf{I}_{22}^{2}} \int_{\hat{\mathbf{k}}_{1}, \hat{\mathbf{k}}_{2}, \mathbf{x}_{1}, \mathbf{x}_{2}} W_{22}(\mathbf{x}_{1}) W_{22}(\mathbf{x}_{2}) e^{-i(\mathbf{x}_{1}-\mathbf{x}_{2})\cdot(\mathbf{k}_{1}-\mathbf{k}_{2})} \right.
\times \mathcal{L}_{\ell_{1}}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{1}}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{2}}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \left[\mathcal{L}_{\ell_{2}}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{2}) + \mathcal{L}_{\ell_{2}}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \right] \right\},$$
(5)

1. C_{02}

$$\mathbf{C}_{20}^{G}(k_{1}, k_{2}) \simeq \sum_{\ell'_{1}, \ell'_{2}} P_{\ell'_{1}}(k_{1}) P_{\ell'_{2}}(k_{2}) \left\{ \frac{(2\ell_{1} + 1)(2\ell_{2} + 1)}{\mathbf{I}_{22}^{2}} \int_{\hat{\mathbf{k}}_{1}, \hat{\mathbf{k}}_{2}, \mathbf{x}_{1}, \mathbf{x}_{2}} W_{22}(\mathbf{x}_{1}) W_{22}(\mathbf{x}_{2}) e^{-i(\mathbf{x}_{1} - \mathbf{x}_{2}) \cdot (\mathbf{k}_{1} - \mathbf{k}_{2})} \times 2\mathcal{L}_{2}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{1}}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{2}}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \right\},$$
(6)

Shot noise in $\mathbf{C}_{20}(k_1, k_2)$

$$C_{20}^{\text{SN-G}}(k_{1}, k_{2}) = \frac{(2\ell_{1} + 1)(2\ell_{2} + 1)}{I_{22}^{2}} \int_{\hat{\mathbf{k}}_{1}, \hat{\mathbf{k}}_{2}, \mathbf{x}_{1}, \mathbf{x}_{2}} \\
\times 2 \left\{ (1 + \bar{\alpha})^{2} W_{12}(\mathbf{x}_{1}) W_{12}(\mathbf{x}_{2}) e^{-i(\mathbf{k}_{1} - \mathbf{k}_{2}) \cdot (\mathbf{x}_{1} - \mathbf{x}_{2})} \mathcal{L}_{2}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) \\
+ (1 + \bar{\alpha}) e^{-i(\mathbf{k}_{1} - \mathbf{k}_{2}) \cdot (\mathbf{x}_{1} - \mathbf{x}_{2})} W_{22}(\mathbf{x}_{1}) W_{12}(\mathbf{x}_{2}) \left[\mathcal{L}_{2}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{1}) \right] \\
\times \left[\frac{1}{2} \sum_{\ell'} P_{\ell'}(k_{1}) \mathcal{L}_{\ell'}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) + \frac{1}{2} \sum_{\ell'} P_{\ell'}(k_{2}) \mathcal{L}_{\ell'}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \right] \right\}$$
(7)

2. C_{40}

$$\mathbf{C}_{40}^{G}(k_{1}, k_{2}) \simeq \sum_{\ell'_{1}, \ell'_{2}} P_{\ell'_{1}}(k_{1}) P_{\ell'_{2}}(k_{2}) \left\{ \frac{(2\ell_{1} + 1)(2\ell_{2} + 1)}{\mathbf{I}_{22}^{2}} \int_{\hat{\mathbf{k}}_{1}, \hat{\mathbf{k}}_{2}, \mathbf{x}_{1}, \mathbf{x}_{2}} W_{22}(\mathbf{x}_{1}) W_{22}(\mathbf{x}_{2}) e^{-i(\mathbf{x}_{1} - \mathbf{x}_{2}) \cdot (\mathbf{k}_{1} - \mathbf{k}_{2})} \times 2\mathcal{L}_{4}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{1}}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{2}}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \right\}, \tag{8}$$

Shot noise in $C_{40}(k_1, k_2)$

$$C_{40}^{\text{SN-G}}(k_{1}, k_{2}) = \frac{(2\ell_{1} + 1)(2\ell_{2} + 1)}{I_{22}^{2}} \int_{\hat{\mathbf{k}}_{1}, \hat{\mathbf{k}}_{2}, \mathbf{x}_{1}, \mathbf{x}_{2}} \times 2 \left\{ (1 + \bar{\alpha})^{2} W_{12}(\mathbf{x}_{1}) W_{12}(\mathbf{x}_{2}) e^{-i(\mathbf{k}_{1} - \mathbf{k}_{2}) \cdot (\mathbf{x}_{1} - \mathbf{x}_{2})} \mathcal{L}_{4}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) + (1 + \bar{\alpha}) e^{-i(\mathbf{k}_{1} - \mathbf{k}_{2}) \cdot (\mathbf{x}_{1} - \mathbf{x}_{2})} W_{22}(\mathbf{x}_{1}) W_{12}(\mathbf{x}_{2}) \left[\mathcal{L}_{4}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) + \mathcal{L}_{4}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{1}) \right] \times \left[\frac{1}{2} \sum_{\ell'} P_{\ell'}(k_{1}) \mathcal{L}_{\ell'}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) + \frac{1}{2} \sum_{\ell'} P_{\ell'}(k_{2}) \mathcal{L}_{\ell'}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \right] \right\}$$
(9)

3. C_{42}

$$\mathbf{C}_{42}^{G}(k_{1}, k_{2}) \simeq \sum_{\ell'_{1}, \ell'_{2}} P_{\ell'_{1}}(k_{1}) P_{\ell'_{2}}(k_{2}) \left\{ \frac{(2\ell_{1} + 1)(2\ell_{2} + 1)}{\mathbf{I}_{22}^{2}} \int_{\hat{\mathbf{k}}_{1}, \hat{\mathbf{k}}_{2}, \mathbf{x}_{1}, \mathbf{x}_{2}} W_{22}(\mathbf{x}_{1}) W_{22}(\mathbf{x}_{2}) e^{-i(\mathbf{x}_{1} - \mathbf{x}_{2}) \cdot (\mathbf{k}_{1} - \mathbf{k}_{2})} \right. \\
\times \mathcal{L}_{4}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{1}}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{1}) \mathcal{L}_{\ell'_{2}}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \left[\mathcal{L}_{2}(\hat{\mathbf{x}}_{2} \cdot \hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{1} \cdot \hat{\mathbf{k}}_{2}) \right] \right\}, \tag{10}$$

Shot noise in $\mathbf{C}_{42}(k_1, k_2)$

$$\mathbf{C}_{\ell_{1}\ell_{2}}^{\text{SN-G}}(k_{1}, k_{2}) = \frac{(2\ell_{1}+1)(2\ell_{2}+1)}{\mathbf{I}_{22}^{2}} \int_{\hat{\mathbf{k}}_{1}, \hat{\mathbf{k}}_{2}, \mathbf{x}_{1}, \mathbf{x}_{2}} \\
\times \left\{ (1+\bar{\alpha})^{2} W_{12}(\mathbf{x}_{1}) W_{12}(\mathbf{x}_{2}) e^{-i(\mathbf{k}_{1}-\mathbf{k}_{2})\cdot(\mathbf{x}_{1}-\mathbf{x}_{2})} \mathcal{L}_{4}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{1}) \left[\mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{2}(\hat{\mathbf{x}}_{2}\cdot\hat{\mathbf{k}}_{2}) \right] \right. \\
\left. + (1+\bar{\alpha}) e^{-i(\mathbf{k}_{1}-\mathbf{k}_{2})\cdot(\mathbf{x}_{1}-\mathbf{x}_{2})} W_{22}(\mathbf{x}_{1}) W_{12}(\mathbf{x}_{2}) \left[\frac{1}{2} \sum_{\ell'} P_{\ell'}(k_{1}) \mathcal{L}_{\ell'}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{1}) + \frac{1}{2} \sum_{\ell'} P_{\ell'}(k_{2}) \mathcal{L}_{\ell'}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) \right] \right. \\
\times \left[\mathcal{L}_{4}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{1}) \mathcal{L}_{2}(\hat{\mathbf{x}}_{2}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{4}(\hat{\mathbf{x}}_{2}\cdot\hat{\mathbf{k}}_{1}) \mathcal{L}_{2}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{4}(\hat{\mathbf{x}}_{1}\cdot\hat{\mathbf{k}}_{2}) + \mathcal{L}_{4}(\hat{\mathbf{x}}_{2}\cdot\hat{\mathbf{k}}_{1}) \mathcal{L}_{2}(\hat{\mathbf{x}}_{2}\cdot\hat{\mathbf{k}}_{2}) \right] \right\} \tag{11}$$