Electric Circuits I

Thevenin Theorem

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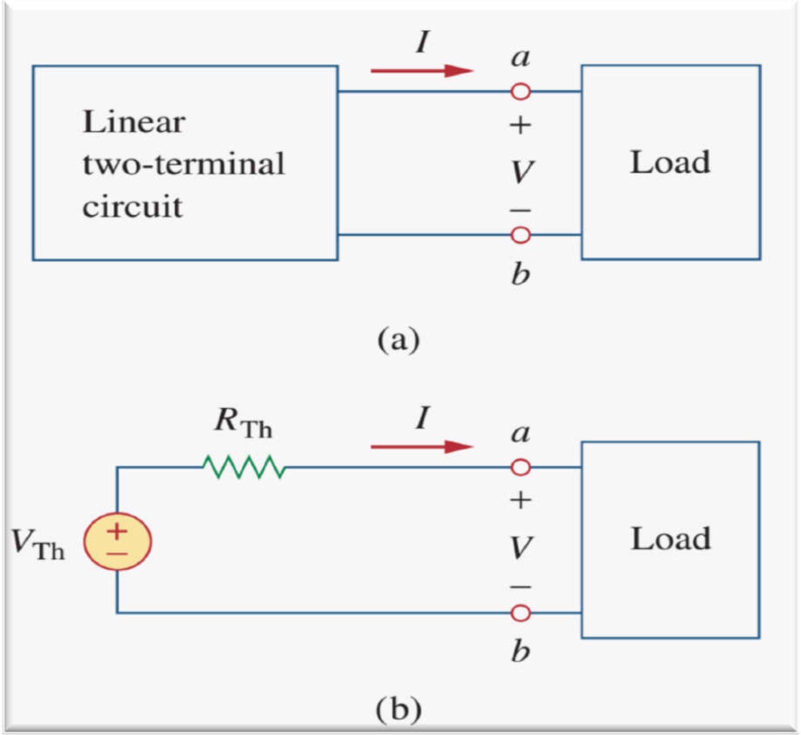
Thevenin Theorem

- Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

- One of the main uses of Thévenin's theorem is the replacement of a large part of a circuit, often a complicated and uninteresting part, with a very simple equivalent. The new, simpler circuit enables us to make rapid calculations of the voltage, current, and power which the original circuit is able to deliver to a load. It also helps us to choose the best value of this load resistance.

Thevenin Theorem

According to Thevenin's theorem, the linear circuit in fig.(a) can be replaced by that in fig.(b) (The load in the figure may be a single resistor or another circuit.). The circuit to the left of the terminals in fig.(b) is known as the Thevenin equivalent circuit.

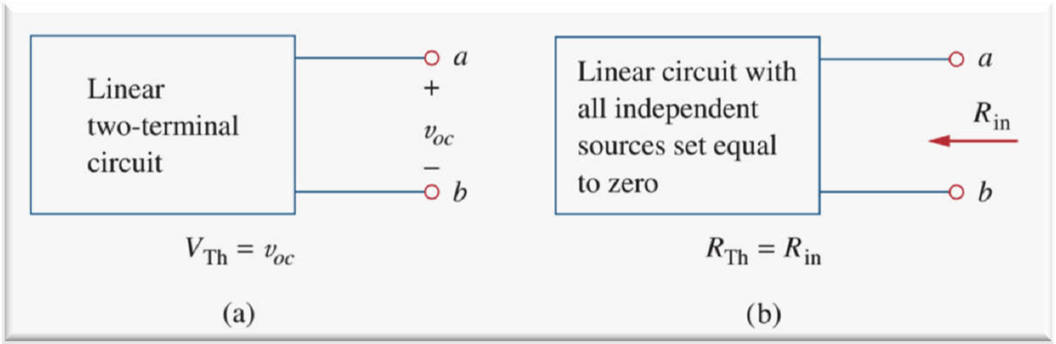


- ***Thevenin's theorem*** **states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.**

Thevenin Theorem

How to find the Thevenin equivalent voltage (V_{Th}) and resistance (R_{Th})

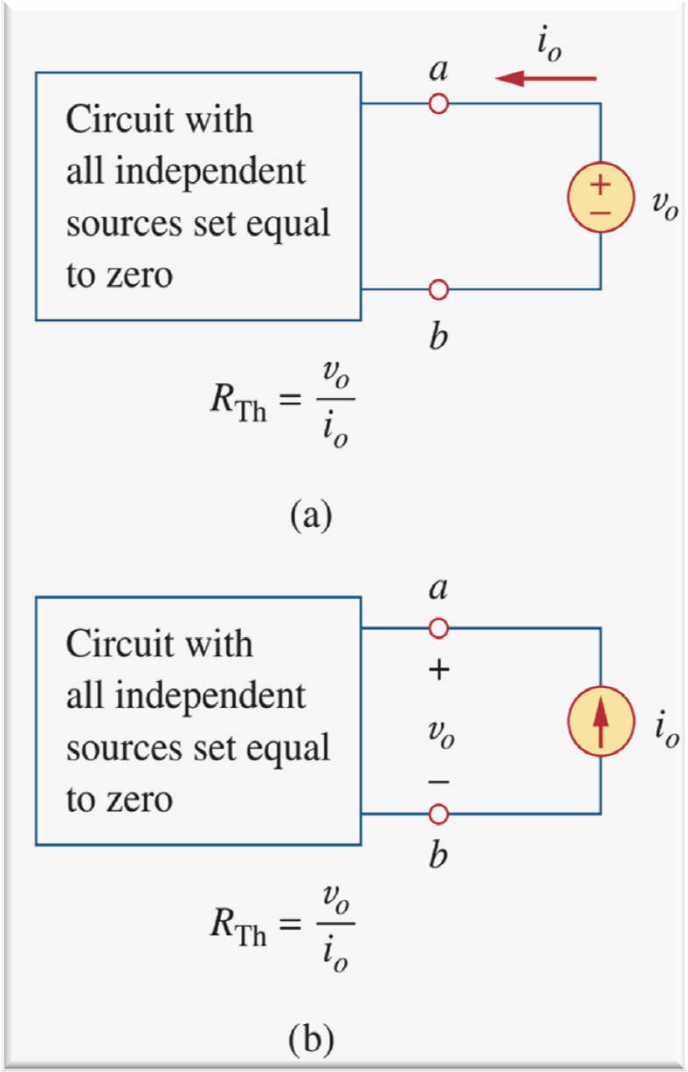
- **Suppose the two circuits in fig.(a)&(b) are equivalent. Two circuits are said to be equivalent if they have the same voltage-current relation at their terminals.**
- **Terminals *a-b* are made open-circuited (by removing the load), no current flows, so that the open-circuit voltage across the terminals in fig(a) must be equal to the voltage source in fig(b), Thus V_{Th} is the open-circuit voltage across the terminals, that is, $V_{Th} = V_{oc}$.**
- **Turn off all independent sources. The input resistance (or equivalent resistance) of the dead circuit at the terminals *a-b* in fig.(a) must be equal to R_{Th} in fig.(b) because the two circuits are equivalent. Thus, R_{Th} is the input resistance at the terminals when the independent sources are turned off. that is, $R_{Th} = R_{in}$.**



Thevenin Theorem

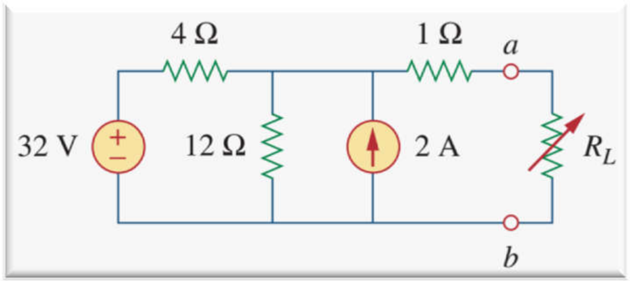
To find the Thevenin resistance R_{Th}, two cases must be considered

- **Case 1: If the network has no dependent sources, turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals *a-b.***
- **Case 2: If the network has dependent sources, we turn off all independent sources. dependent sources are not to be turned off because they are controlled by circuit variables. Apply a voltage source at terminals *a-b* and determine the resulting current i_o. Then, $R_{Th}=v_o/i_o$. *Or* insert a current source at terminals *a-b* as shown in fig.(b) and find the terminal voltage v_o, Then, $R_{Th}=v_o/i_o$. the two approaches will give the same result. We may use $v_o=1V$ or $i_o=1A$.**



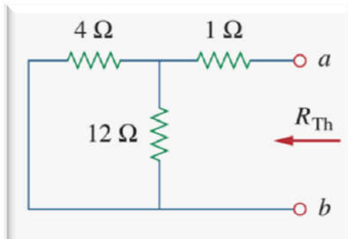
Thevenin Theorem

Example ***(the circuit has only independent sources)*****: Find the Thevenin equivalent circuit of the circuit in shown figure, to the left of the terminals *a-b* Then find the current through R_L=6, 16 and 36Ω.**



Find R_{Th} by turning off the 32V voltage source (replacing it with a short circuit) and the 2A current source (replacing it with an open circuit), The circuit becomes as shown in fig.(a).

$$R_{Th} = 4\|12+1=\frac{4\times 12}{16}+1=4\Omega$$



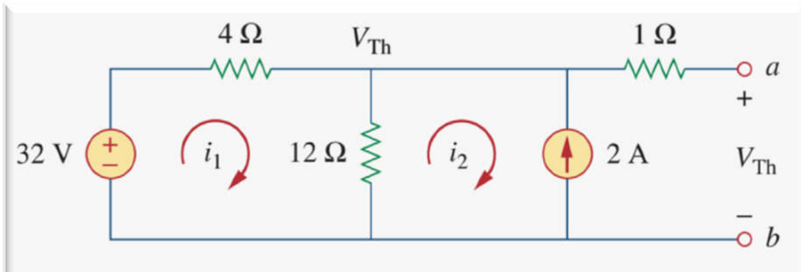
(a)

To find v_{Th} consider the circuit in fig.(b). Applying mesh analysis to the two loops, we obtain

$$-32+4i_1+12(i_1-i_2)=0, \qquad i_2=-2A$$

Solving for i_1, we get

$$i_1=0.5A$$

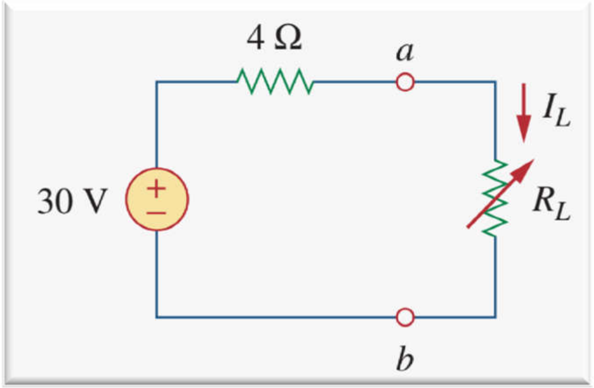


(b)

Thevenin Theorem

$$v_{Th} = 12(i_1 - i_2) = 12(0.5 + 2) = 30V$$

$$i_L = \frac{v_{Th}}{R_{Th} + RL} = \frac{30}{4 + RL}$$



When R_L=6

$$i_L = \frac{30}{4+6} = 3A$$

When R_L=16

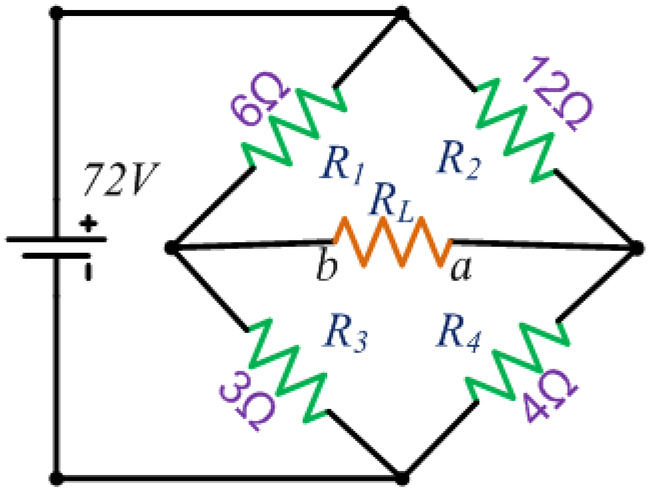
$$i_L = \frac{30}{4+16} = 1.5A$$

When R_L=36

$$i_L = \frac{30}{4+36} = 0.75A$$

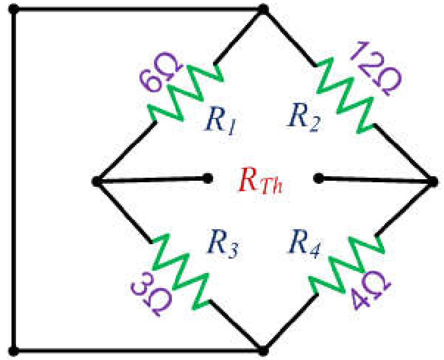
Example ***(the circuit has only independent sources)*****: Find the Thevenin equivalent circuit of the circuit in shown figure at terminals *a-b*.**

Find R_{Th} by turning off the voltage source, the short-circuit replacement of the voltage source E provides a direct connection between c and c′ as shown in fig.(a). Then rearranging the circuit in fig. (a) to get the circuit in fig.(b)

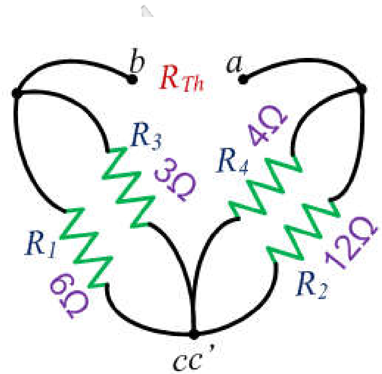


Thevenin Theorem

$R_{Th}=R_1||R_2+R_2||R_4$
$=6\Omega||3\Omega+4\Omega||\Omega12$
$=2\Omega+3\Omega=5\Omega$

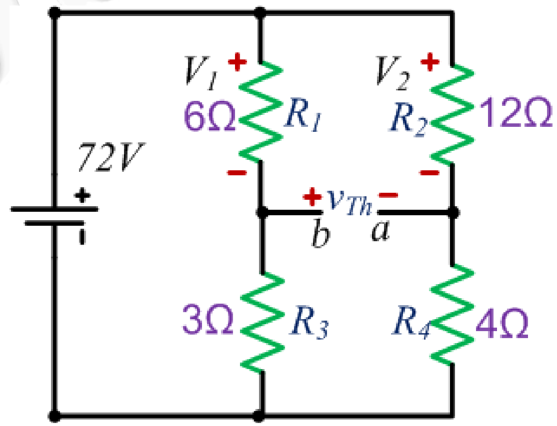


(a)



(b)

To get v_{Th}, the circuit is redrawn as in fig.(c). The absence of a direct connection between *a* and *b* results in a circuit with three parallel branches. The voltages V_1 and V_2 can therefore be determined using the voltage divider rule:



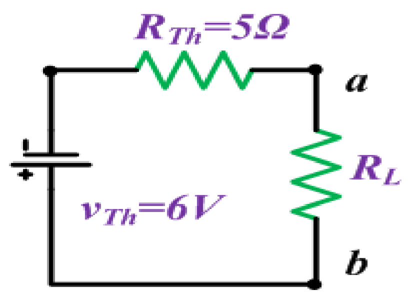
(c)

$$V_1=\frac{R_1E}{R_1+R_3}=\frac{6\times72}{6+3}=\frac{432}{9}=48V$$

$$V_1=\frac{R_2E}{R_2+R_4}=\frac{12\times72}{12+4}=\frac{864}{16}=54V$$

applying KVL to the top loop

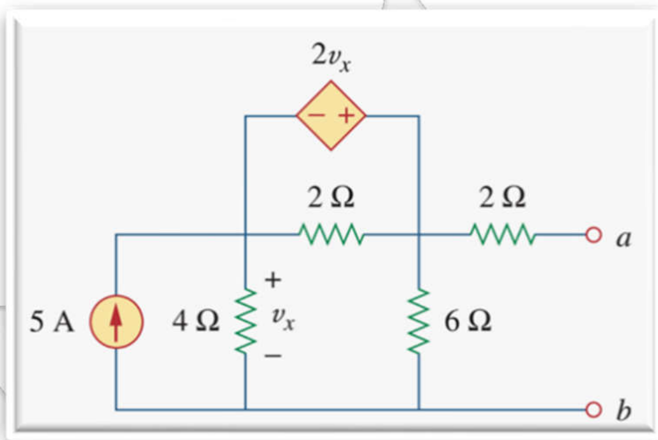
$v_{Th}=V_2-V_1=54-48=6V$



(d)

Thevenin Theorem

Example ***(the circuit has independent and dependent sources)*** **: Find the Thevenin equivalent of the circuit in shown figure at terminals *a-b*.**



To find R_{Th}, set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, excite the network with a voltage source connected to the terminals as indicated in fig.(a). You may set v_o=1V.

Applying mesh analysis to loop 1 in the circuit of fig(a) results in.

$$-2v_x+2(i_1-i_2)=0, \quad or \quad vx=i_1-i_2$$

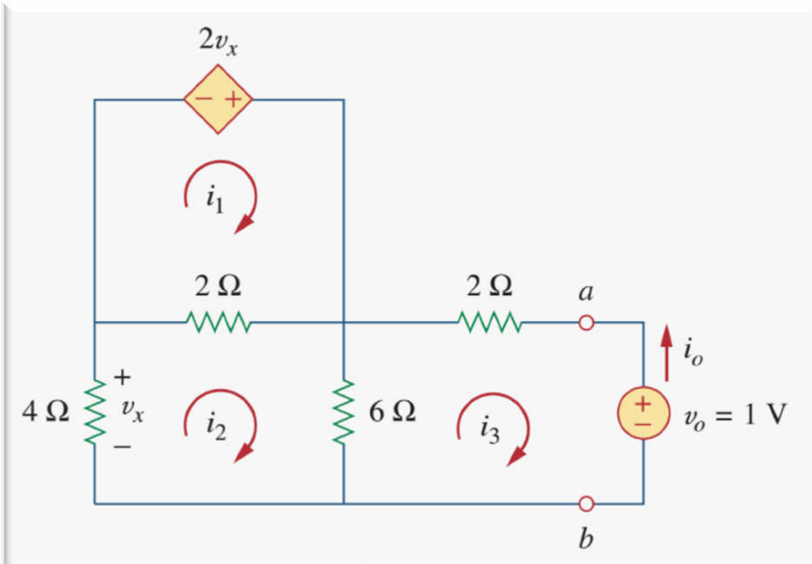
But $v_x=-4i_2=i_1-i_2$

$$i_1=-3i_2$$

For loop 2 and 3, applying KVL produces

$$4i_2+2(i_2-i_1)+6(i_2-i_3)=0$$

$$6(i_3-i_2)+2i_3+1=0$$



(a)

Thevenin Theorem

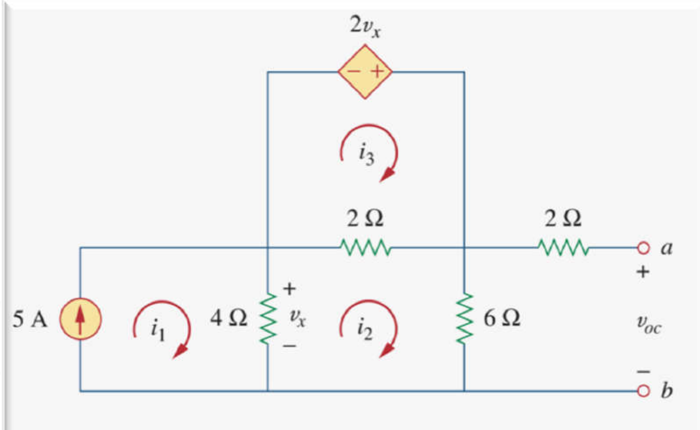
Solving these equations gives

$i_3 = -\frac{1}{6}A$

But $io = -i_3 = \frac{1}{6}A$

$R_{Th} = \frac{1}{i_o}$=6Ω

To get v_{Th}, **find** v_o **in the circuit of fig.(b). Applying mesh analysis, we get.**



(b)

$i_1 = 5A$ **(1)**

$-2v_x + 2(i_3 - i_2) = 0,\quad or \quad vx = i_3 - i_2$ **(2)**

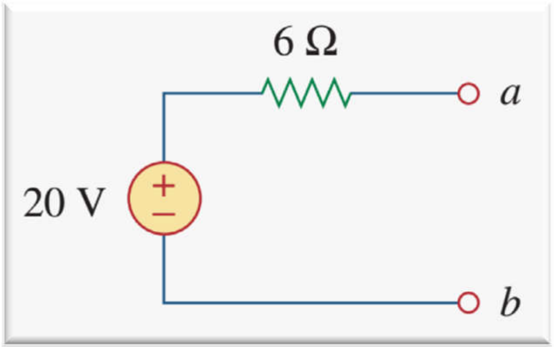
$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$

$12i_2 - 4i_1 - 2i_3 = 0$

But $v_x = 4(i_1 - i_2)$, **solving these equations leads to**

$i_2 = 10/3A$

$v_{Th} = v_{oc} = 6i2 = 20V$



The Thevenin equivalent circuit

Thevenin Theorem

Example ***(the circuit has independent and dependent sources)*****: Determine the Thevenin equivalent of the circuit in the shown circuit at terminals *a-b*.**

To find R_{Th} excite the circuit with either <u>voltage source</u> or current source.

Applying mesh analysis for fig.(b)
Assuming v_o=1kV.

$$2000i_1+3000i_2+1000=0$$

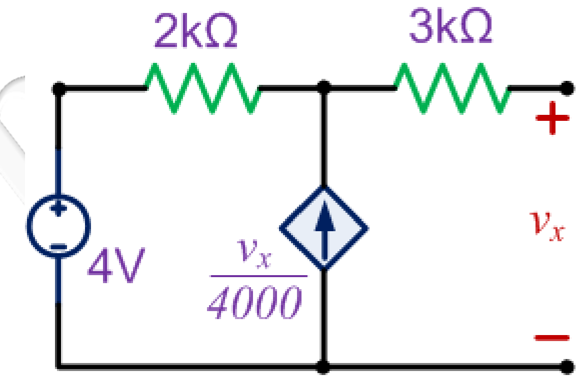
$$2i_1+3i_2=-1 \quad (1)$$

$$-i_1+i_2=\frac{vx}{4000}$$

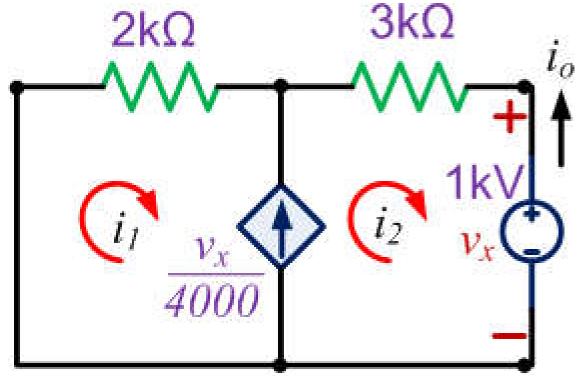
But $v_x=v_o=1000$

$$-i_1+i_2=\frac{vx}{4000}=\frac{1000}{4000}=\frac{1}{4} \qquad (2)$$

Use elimination technique to solve eq.(1) and eq.(2) to get i_2.



(a)



(b)

Thevenin Theorem

$$2i_1+3i_2=-1 \qquad (1)$$

$$-i_1+i_2=\frac{1}{4} \qquad (2)$$

Multiply eq.(2) by 2 then add the resulting equation to eq.(1)

$$2i_1+3i_2=-1 \qquad (1)$$

$$-2i_1+2i_2=\frac{2}{4}$$

$$5i_2=-\frac{1}{2} \qquad \Rightarrow \qquad i_2=-0.1A$$

$$i_o=-i_2=0.1A$$

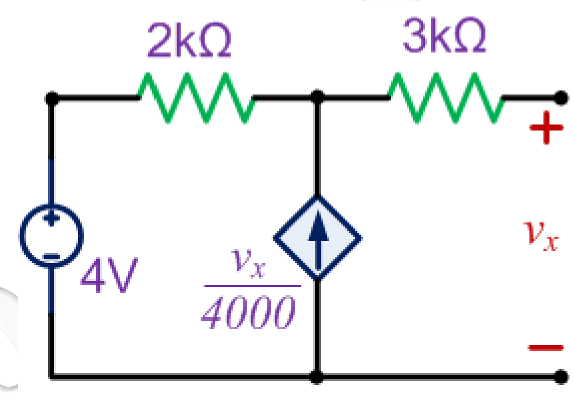
$$R_{Th}=\frac{v_o}{i_o}=\frac{1000}{0.1}=10k\Omega$$

To get v_{Th}, apply KVL around the outer loop in fig.(a)

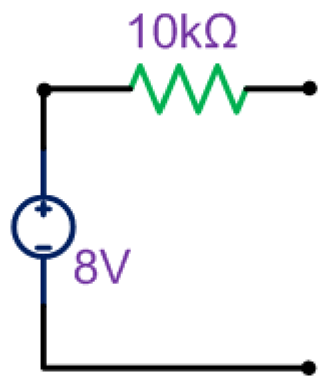
$$-4+2\times 10^3\left(-\frac{v_x}{4000}\right)+3\times 10^3(0)+v_x=0$$

$$v_x=8V=v_{Th}$$

Thévenin equivalent of fig.(c) is obtained.



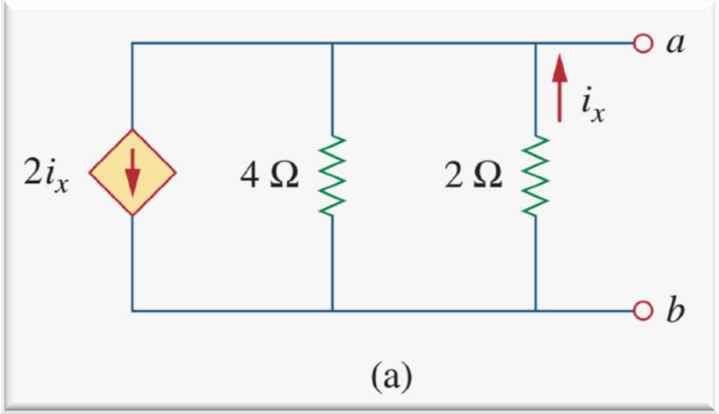
(a)



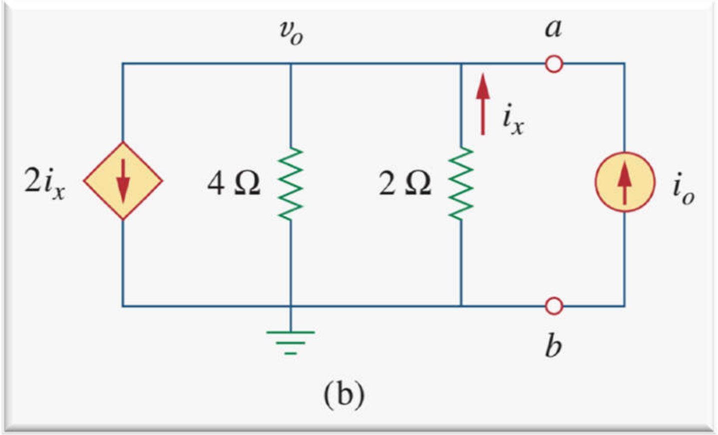
(c)

Thevenin Theorem

Example ***(the circuit has only dependent sources)*****: Determine the Thevenin equivalent of the circuit in the shown circuit at terminals *a-b*.**



(a)



(b)

Excite the circuit with either a *1V* voltage source or a *1A* current source.

writing the nodal equation at a in fig.(b) Assuming $i_o=1A$.

$$2i_x+(v_o-0)/4+(v_o-0)/2+(-1)=0 \quad (1)$$

$$i_x=-v_o/2 \quad (2)$$

Substitute eq.(1) in eq.(2) yields

$$2(-v_o/2)+(v_o-0)/4+(v_o-0)/2+(-1)=0 \quad (2)$$

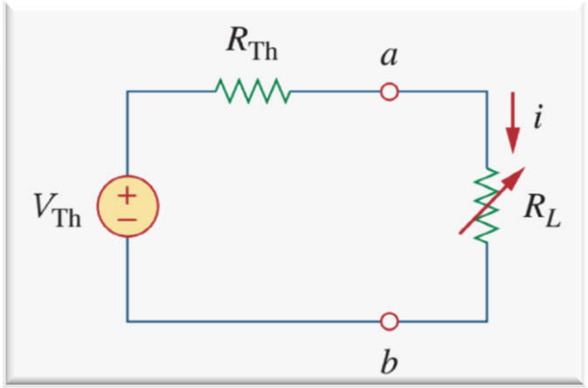
$$-1+1/4+1/2)v_o-1=0 \qquad \boxed{v_o=-4V}$$

$$\boxed{R_{Th}=v_o/i_o=-4/1=-4\Omega}$$

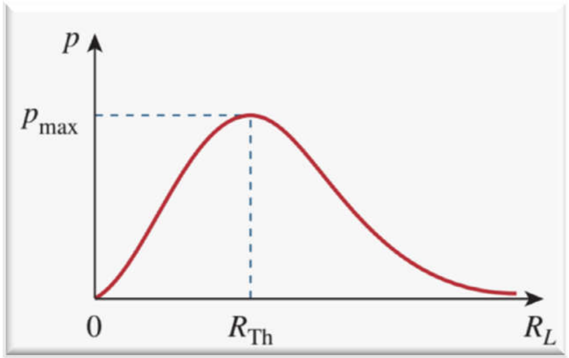
Maximum Power Transfer

- **The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load.**
- **If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in the figure, the power delivered to the load is**

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$



- **For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance the power delivered to the load varies as sketched in the show figure.**



- **If R_L varies from 0 to ∞, the maximum power delivered to the load occurs when R_L is equal to R_{Th}. This is known as the maximum power theorem.**

Maximum Power Transfer

Maximum power **is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load (R_L=R_{Th}).**

The Proof

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

differentiate p in the above equation with respect to R_L and set the result equal to zero.

$$\frac{dp}{dR_L} = {V_{Th}}^2\left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right]$$

$$= {V_{Th}}^2\left[\frac{R_{Th}+R_L-2R_L}{(R_{Th}+R_L)^3}\right] = {V_{Th}}^2\left[\frac{R_{Th}-R_L}{(R_{Th}+R_L)^3}\right] = 0$$

This implies that R_{Th}=R_L

Substitute the result in power equation to get the maximum power.

$$p_{max} = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

$$p_{max} = \left(\frac{V_{Th}}{R_{Th}+R_{Th}}\right)^2 R_{Th}$$

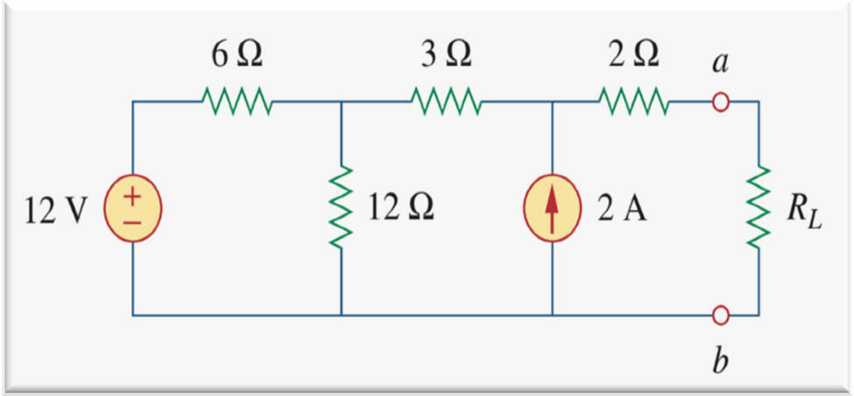
$$p_{max} = \frac{{V_{Th}}^2}{(2R_{Th})^2} R_{Th}$$

$$p_{max} = \frac{{V_{Th}}^2}{4{R_{Th}}^2} R_{Th}$$

$$p_{max} = \frac{{V_{Th}}^2}{4R_{Th}}$$

Maximum Power Transfer

Example: Find the value of for maximum power transfer in the circuit of the shown figure. Find the maximum power.



At maximum power $R_{Th}=R_L$, *so*, we need to find the Thevenin resistance and the Thevenin voltage across the terminals *a-b* .

$R_{Th}=2+3+6||12=5+(6\times12)/18=9\Omega$

To get V_{Th} use mesh analysis

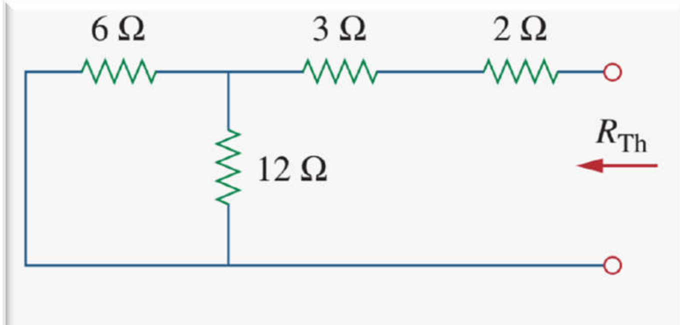
$-12+18i_1-12i_2=0, \qquad i_2=-2A$

Solve the above equations to get $i_1=-2/3A$

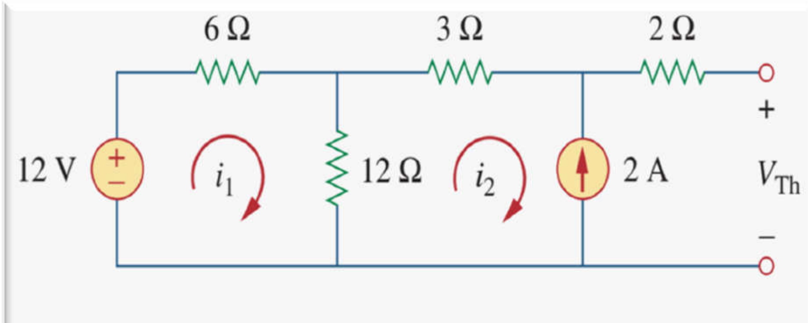
Applying KVL around the outer loop to get V_{Th} across terminals *a-b.*

$-12+6i_1+3i_2+2\times 0+V_{Th}=0 \qquad \Rightarrow \qquad V_{Th}=22V$

$\boxed{R_L=R_{Th}=9\Omega}$ $\qquad \boxed{p_{max}=\frac{22^2}{4\times9}=13.44W}$



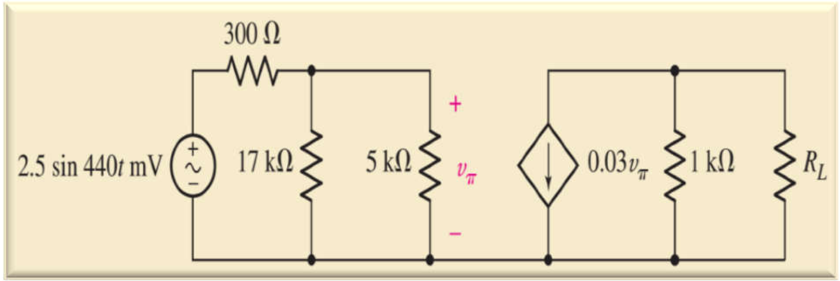
(a)



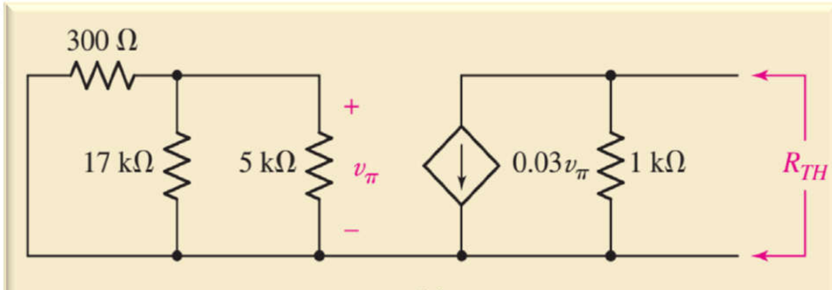
(b)

Maximum Power Transfer

Example: Find the value of for maximum power transfer in the circuit of the shown figure. Find the maximum power.



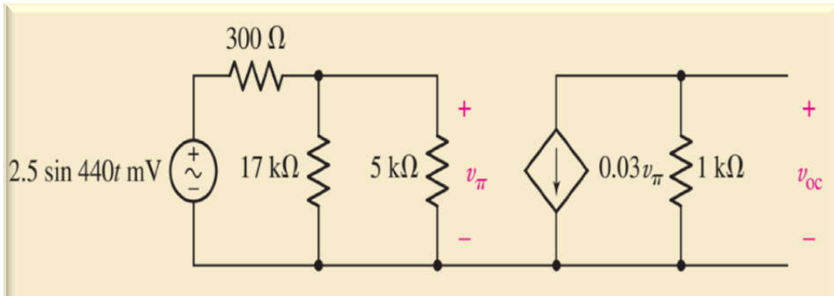
Since $v_\pi = 0$, the dependent current source is an open circuit, and R_{TH}=1 kΩ.



(a)

$$v_{oc}=-0.03v_\pi(1000)=-130v_\pi$$

$$v_\pi = (2.5 \times 10^{-3} sin440t)\left(\frac{3864}{300+3864}\right)$$



(b)

$$v_{oc}=-69.6\sin 440t \text{ mV}$$

$$\boxed{R_L=R_{Th}=1k\Omega}$$

$$\boxed{p_{max} = \frac{V_{Th}^{\;2}}{4RTh} = 1.211\sin^2 440t\ \mu W}$$

19