Linear Algebra 2 Pick 5 questions, 1 pts per question

Singular ValuesIf we perform SVD on an image LaTeX: A\in\mathbb{R}^{512\times 512}A ∈ R 512 × 512 (LaTeX: [U, S, V] = svd(A)[ U , S , V ] = s v d ( A )), we have the singular vectors (LaTeX: UU and LaTeX: VV) of LaTeX: 512\times512512 × 512, and 512 singular values (the diagonal of LaTeX: SS). The image can be reconstructed using only few of the singular values (the rest of singular values are set to 0). Which is the most possible solution to get the image on the right?

lena\_svd.png

A picture containing text, person, clothing, person

Description automatically generated

Correct Answer

Use the 30 largest singular values

Use the 30 smallest singular values

Use the 30 random singular values

Use all the singular values

Projection Vector Properties

Let LaTeX: VV be a finite-dimensional real vector space and let LaTeX: PP be a linear transformation of LaTeX: VV such that LaTeX: P^2=PP 2 = P. Which of the following must be true?

I. LaTeX: PP is invertible

II. LaTeX: PP is diagonalizable

III. LaTeX: PP is either the identity transformation or the zero transformation

Correct Answer

II only

None

I only

III only

II and III

Inverse and Projection

We know that LaTeX: A(A^\top A)^{-1}A^\topA ( A ⊤ A ) − 1 A ⊤ is the matrix of an orthogonal projection.

But LaTeX: (A^\top A)^{-1} = A^{-1}(A^\top)^{-1}( A ⊤ A ) − 1 = A − 1 ( A ⊤ ) − 1 so LaTeX: A(A^\top A)^{-1} A^\top =AA^{-1}(A^\top)^{-1} A^\top = I \cdot I = IA ( A ⊤ A ) − 1 A ⊤ = A A − 1 ( A ⊤ ) − 1 A ⊤ = I ⋅ I = I, so the matrix is always identity. LaTeX: AA is an LaTeX: m \times nm × n matrix.

This argument is:

Correct Answer

Valid when LaTeX: m=nm = n and LaTeX: AA is invertible

Always valid

Always valid when LaTeX: m=nm = n

Invalid because matrix multiplication is not associative

Invalid unless LaTeX: m=nm = n because LaTeX: AA is not a square matrix

Invalid unless LaTeX: m=nm = n because LaTeX: AA is a square matrix but is not invertible

Projection Length

The length of the projection of the vector LaTeX: {\bf x} = \left[ \begin{array}{c}7\\0\\1\end{array} \right]x = [ 7 0 1 ] to the subspace spanned by LaTeX: {\bf v}\_1 = \left[ \begin{array}{c}1\\2\\-1\end{array}\right]v 1 = [ 1 2 − 1 ]

and LaTeX: {\bf v}\_2 = \left[ \begin{array}{c}-1\\1\\1\end{array}\right]v 2 = [ − 1 1 1 ] is:

Correct Answer

LaTeX: \sqrt{18}18

LaTeX: \sqrt{17}17

LaTeX: 44

LaTeX: \sqrt{15}15

Projection L2

In xyz-space, what are the coordinates of the point on the plane LaTeX: 2x+y+3z = 32 x + y + 3 z = 3 that is closest to the origin?

Correct Answer

LaTeX: \left( \frac{3}{7} , \frac{3}{14}, \frac{9}{14} \right)( 3 7 , 3 14 , 9 14 )

LaTeX: (0,0,1)( 0 , 0 , 1 )

LaTeX: \left( \frac{7}{15} , \frac{8}{15}, \frac{1}{15} \right)( 7 15 , 8 15 , 1 15 )

LaTeX: \left( \frac{5}{6} , \frac{1}{3}, \frac{1}{3} \right)( 5 6 , 1 3 , 1 3 )

LaTeX: \left( 1 , 1, \frac{1}{3} \right)( 1 , 1 , 1 3 )

Pseudoinverse and SVD

Which of the following statements is True?

I. We can compute the pseudoinverse of any matrix

II. The singular value decomposition of a real matrix is unique

III. Any LaTeX: n \times nn × n matrix is the pseudoinverse of the matrix LaTeX: {\bf 0}\_{n \times n}0 n × n (LaTeX: n \times nn × n matrix with all components 0)

Correct Answer

Only I

Only II

I and III

II and III

Singular Values

A LaTeX: 5 \times 15 × 1 vector LaTeX: \mathbf{v}v of length 1 (LaTeX: ||\mathbf{v}||=1| | v | | = 1) is transformed by matrix LaTeX: AA such that LaTeX: \mathbf{u}=A\mathbf{v}u = A v. What is the longest length LaTeX: \mathbf{u}u can be?

LaTeX: A = \begin{bmatrix}

9 & -5 & 2 & 10 & 4\\

-7 & -1 & -3 & -10 & 10\\

-4 & 3 & -5 & -5 & 4\\

-6 & 2 & -1 & -5 & -2

\end{bmatrix}A = [ 9 − 5 2 10 4 − 7 − 1 − 3 − 10 10 − 4 3 − 5 − 5 4 − 6 2 − 1 − 5 − 2 ]

Correct Answer

22.1765

1.0000

11.6813

4.0687

SVD vs. Eigendecomposition

Which of the following statements is/are true?

Correct Answer

SVD can be applied over any matrix

If a matrix is symmetric, its eigenvalues are equal to its singular values

The eigenvalues of a matrix are always equal to its singular values

For any matrix, we can compute its eigendecomposition

SVD

Which statement is incorrect about Singular Value Decomposition (SVD)?

Correct Answer

For symmetric matrices, the left and right singular vectors/matrices may not be identical, LaTeX: U \neq VU ≠ V in LaTeX: A = U S V^TA = U S V T

SVD finds only orthogonal vectors

The "magnitudes" of contributions of the left and right vectors are given by the singular values

We can calculate the pseudo-inverse (PINV) via the SVD

Inverse Properties

Let LaTeX: AA and LaTeX: BB be invertible LaTeX: n \times nn × n matrices. Then the inverse of LaTeX: ABA^{-1}A B A − 1 is:

Correct Answer

LaTeX: AB^{-1}A^{-1}A B − 1 A − 1

LaTeX: A^{-1}B^{-1}AA − 1 B − 1 A

LaTeX: B^{-1}B − 1

LaTeX: A^{-1}BAA − 1 B A

Optimization Pick 5 questions, 1 pts per question

Convex Definition

The function LaTeX: f(x) = a^Tx+bf ( x ) = a T x + b, where LaTeX: aa and LaTeX: xx are vectors and LaTeX: bb is a scalar, is

Correct Answer

Both concave and convex

Convex

Concave

None of the above or insufficient information to answer

Derivative Values

Which of the following statements is true?

Correct Answer

The value of the function LaTeX: ff can be positive or negative at any optimal value

If the function LaTeX: ff is maximized at LaTeX: {\bf x}x, then the second derivative of LaTeX: ff at LaTeX: {\bf x}x is positive

If the second derivative of LaTeX: ff at LaTeX: {\bf x}x is negative, then LaTeX: {\bf x}x is a local maximum

If the first derivative of LaTeX: ff at LaTeX: {\bf x}x is zero, then LaTeX: {\bf x}x is either a local minimum or local maximum

General Optimization

Which of the following statements is false?

Correct Answer

Direct optimization methods are useful when the objective function is not differentiable

The first and second derivative of the objective function are used to determine if we have reached an optimal point

The gradient of LaTeX: f(x,y)f ( x , y ) is the a vector pointing in the direction of the steepest slope at that point

The Hessian is the matrix of second-order partial derivatives of a function

Newton's Method

Which of the following is NOT required for using Newton’s method to find the minimum of a given function LaTeX: ff?

Correct Answer

An upper bound of LaTeX: ff

The second derivative of LaTeX: ff

The derivative of LaTeX: ff

A good initial estimate that is reasonably close to the optimal

Optimization Constraints

Suppose we have three optimization problems, the first one is minimizing LaTeX: f(x,y)f ( x , y ), and the optimal value is LaTeX: f\_1f 1, the second problem is minimizing LaTeX: f(x,y)f ( x , y ) subject to LaTeX: g(x,y) = 0g ( x , y ) = 0, and the optimal value of this problem is LaTeX: f\_2f 2, the third problem is minimizing LaTeX: f(x,y)f ( x , y ) subject to LaTeX: g(x,y) \geq 0g ( x , y ) ≥ 0, and the optimal value is LaTeX: f\_3f 3. Which of the following is correct about LaTeX: f\_1f 1,LaTeX: f\_2f 2 and LaTeX: f\_3f 3?

Correct Answer

LaTeX: f\_2 \geq f\_3 \geq f\_1f 2 ≥ f 3 ≥ f 1

LaTeX: f\_2 \geq f\_1 \geq f\_3f 2 ≥ f 1 ≥ f 3

LaTeX: f\_1 \geq f\_3 \geq f\_2f 1 ≥ f 3 ≥ f 2

None of these

Regularization

Consider the optimization problem

LaTeX: \text{min } f(x)min f ( x )

If you want to minimize LaTeX: f(x)f ( x ) as well as the number of the non-zero elements in LaTeX: xx, and the distance between LaTeX: xx to the origin, which one should you prefer?

Correct Answer

LaTeX: \text{min } f(x) + \|x\|\_1 + \|x\|\_2^2min f ( x ) + ‖ x ‖ 1 + ‖ x ‖ 2 2

LaTeX: \text{min } f(x) + \|x\|\_0min f ( x ) + ‖ x ‖ 0

LaTeX: \text{min } f(x) + \|x\|\_0 + \|x\|\_2min f ( x ) + ‖ x ‖ 0 + ‖ x ‖ 2

LaTeX: \text{min } f(x) + \|x\|\_0 + \|x\|\_1min f ( x ) + ‖ x ‖ 0 + ‖ x ‖ 1

Newton's Method

Which of the following is true for Newton's method (LaTeX: nn dimensional space).

1) Memory requirement is LaTeX: O(n^2)O ( n 2 )

2) Computational complexity is LaTeX: O(n^3)O ( n 3 )

3) Newton's method is exponentially faster than Gradient descent

Correct Answer

1 and 2

2 and 3

1 and 3

1, 2, and 3

Primal and Dual

Consider the primal problem

LaTeX: \begin{array}{l} \text{min } f(x)\\ \text{s.t. } g(x) \leq 0\end{array} min f ( x ) s.t. g ( x ) ≤ 0

and the dual problem

LaTeX: \begin{array}{l} \text{max } w(\lambda)\\ \text{s.t. } \lambda \geq 0\end{array} max w ( λ ) s.t. λ ≥ 0

Which of the following statements is FALSE?

LaTeX: f(x) \geq w(\lambda)f ( x ) ≥ w ( λ ), LaTeX: \forall g(x) \leq 0 ∀ g ( x ) ≤ 0 and LaTeX: \lambda \geq 0λ ≥ 0

Correct Answer

LaTeX: f(x^\*) = w(\lambda^\*)f ( x ∗ ) = w ( λ ∗ ), even if LaTeX: f(x)f ( x ) is not convex

The primal problem has a global optimal solution or multiple global optimal solutions

If LaTeX: ff is convex, LaTeX: ww is concave

Convex Requirements

Consider functions LaTeX: ff, LaTeX: gg and LaTeX: hh such that LaTeX: h = f(g(x))h = f ( g ( x ) ). Then which of the following is true?

1) LaTeX: hh is convex if LaTeX: ff is convex and non-increasing, and LaTeX: gg is concave

2) LaTeX: hh is concave if LaTeX: ff is concave and non-decreasing, and LaTeX: gg is convex

3) LaTeX: hh is concave if LaTeX: ff is concave and non-increasing, and LaTeX: gg is convex

Only 3

Only 1

Only 2

Correct Answer

1 and 3

General Optimization

Which of the following statements is true?

1) any differentiable function is minimized or maximized if and only if its gradient is zero.

2) a convex function has a unique minimum

3) a convex function is minimized if and only if the gradient is zero

Only 3

Only 1

Correct Answer

Only 2

2 and 3