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Synthesis of Linear, Planar, and Concentric Circular Antenna Arrays Using Rao Algorithms

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ABSTRACT

The aim of this paper is to display the efficacy of three newly proposed optimization algorithms named as Rao-1, Rao-2, and Rao-3 in synthesizing antenna arrays. The algorithms are applied to three different antenna array configurations. Thinned arrays with isotropic radiators are considered and the main objective is to find the optimal configuration of ON/OFF elements that produce low side lobe levels. The results of Rao-1, Rao-2, and Rao-3 algorithms are compared with those of improved genetic algorithm (IGA), hybrid Taguchi binary particle swarm optimization (HTBPSO), teaching-learning-based optimization (TLBO), the firefly algorithm (FA), and biogeography-based optimization (BBO). The Rao-1, Rao-2, and Rao-3 algorithms were able to realize antenna arrays having lower side lobe levels (SLL) when compared to the other optimization algorithms.

KEYWORDS

Antenna Arrays, Optimization, Rao Algorithms, Side Lobe Level, Thinned Arrays

1. INTRODUCTION

Antenna arrays are commonly used in communication systems and are preferred over single-antenna systems when there is a need to improve the gain, directivity, have electronic beam steering at different directions, cancel interference and to have diversity (MIMO). The performance of a communication system is dependent also on the antenna system it has and hence a proper and efficacious antenna design is an important task. An antenna array's radiation pattern has a main lobe directed towards the user and side lobes and nulls in the other directions. For efficient operation, the radiation pattern should have high directivity and low side lobe level (SLL). However, these two are conflicting features of an antenna pattern and one cannot be improved without degrading the other. Since side lobes cause wastage of power and can create interference with neighbouring systems it becomes necessary to suppress it and therefore the low SLL criterion is usually fulfilled at the cost of sacrificing gain and beamwidth. The array structure, the inter-element distance, individual amplitude and phase excitations, the number of excited (ON) elements, etc. affect the radiation pattern.

Literature shows that various researchers have applied different types of evolutionary algorithms to antenna problems to achieve certain synthesis goals. Few among them are Moth Flame Optimization (Das, Mandal, Ghoshal & Kar, 2018), comprehensive learning particle swarm optimizer (Ismaiel et al., 2018), Cuckoo search algorithm (Nora, Oudira & Dumond, 2019), probability-based coevolving particle swarm optimization (Pan, Wang, Guo & Wu, 2019), improved fruit-fly optimization algorithm (Darvish & Ebrahimzadeh, 2018), genetic algorithm (GA) (Cen, Yu, Ser & Cen, 2012; Chen, He & Han, 2006), differential evolution (DE) (Lin, Qing & Feng, 2010; Zhang, Jia, & Yao, 2013), particle

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swarm optimization (PSO) (Bhattacharya, Bhattacharyya & Garg, 2012; Khodier & Al-Aqeel, 2009), ant colony optimization (ACO) (Rajo-lglesias & Quevedo-Teruel, 2007), invasive weed optimization (IWO) (Karimkashi & Kishk, 2010), bacteria foraging algorithm (BFA) (Guney & Basbug, 2008), firefly algorithm (FA) (Zaman & Abdul, 2012), biogeography based optimization (BBO) (Singh, 2010), ant lion optimization (ALO) (Saxena & Kothari, 2016), cat swarm optimization (CSO) (Pappula & Ghosh, 2014; Pappula & Ghosh, 2017),etc. All these algorithms try to suppress the SLL or achieve a target SLL by optimizing the inter-element spacing, position of the antenna elements, weights of amplitudes and phases of complex feeding currents, ON-OFF element combinations (thinning), time-modulated pulse width for time-modulated arrays, or placing nulls in the desired interference directions. Few more recent optimization works on antenna arrays are mentioned in section 3.

All the aforementioned algorithms have shown an efficient performance in finding optimal solutions to antenna design problems as well as for other electromagnetic optimization problems. However, their performance is dependent on the tuning of their algorithm-specific parameters. E.g. selection operator, mutation probability, cross-over probability, etc. in GA, mixing ratio in CSO, inertia weight and social and cognitive parameters, etc. in PSO and likewise for other optimization algorithms. The improper tuning of any algorithm's control parameters may lead to convergence at a local optimum or increase the time for convergence. This parameter tuning is in addition to that of the common control parameters such as population size and the number of iterations. The user effort increases since many trial runs have to be run to get a proper combination of the algorithm-specific parameters and the effort is, obviously, proportional to the number of algorithm-specific parameters.

Optimization algorithms that use only common control parameters have been proposed in the literature. Rao, Savsani & Vakharia (2011) had introduced the teaching-learning- based algorithm that is based only on common control parameters such as population size and number of iterations and has gained wide acceptance from the research community for producing significant optimization results. Rao (2016) had developed another algorithm-specific parameter-less optimization algorithm known as Jaya algorithm that has also been greatly used in various optimization scenarios (Huang et al.,2018; Rao & More, 2017; Rao & More, 2017; Ravipudi & Neebha, 2018; Singh, Prakash, Singh & Babu, 2017; Rao & Saroj, 2017; Wang & Huang, 2018; Warid, Hizam, Mariun & Abdul, 2018).

There are many meta-heuristic algorithms that are based on the metaphor of a natural process or animal, birds, insects, societies, etc. The authors of these algorithms prove their algorithms to be better than the other existing ones. However, not every algorithm receives enough success or receives a little because of the lack of researches applying the algorithms. Therefore, there cannot be enough progress in the field of optimization if the basis of new optimization algorithms is on some imitation of a natural process rather than focussing on the main idea of developing simple optimization techniques that can give effective solutions. Keeping this in view, Rao (2020) and Rao & Pawar (2020) recently introduced three simple yet powerful metaphor-less optimization algorithms named Rao-1, Rao-2, and Rao-3.

The Rao algorithms (three in number) have not been applied to any electromagnetic problems yet. They are independent of any algorithm-specific parameters that make them easier to implement. These algorithms have good search capability and have a good convergence rate as well. Therefore, the main contribution of this paper is to show their efficacy by applying them to the problem of antenna array synthesis, *for the first time*, with the intention to motivate researchers to apply the same to other optimization problems as well.

In this paper, the Rao-1, Rao-2, and Rao-3 algorithms are used to find the optimum combination of ON-OFF antenna elements of a linear antenna array, a 50*50 planar antenna array, 2-rings concentric circular antenna array, and 10-rings concentric circular antenna array. The codes are written in MATLAB R2018b and an Intel Core i3 computing platform with a 2.30 GHz processor and 4 GB RAM is used.

This structure of the paper is as follows. Some more recent works in antenna array optimization are mentioned in Section 2. Rao-1, Rao-2 and Rao-3 algorithms are briefly described in Section 3.

Section 4.1 discusses two case studies of the concentric circular antenna array. Section 4.2 discusses the case study of a planar antenna array configuration and Section 4.3 discusses the case study of a linear antenna array configuration. The conclusions are presented in Section 5.

2. RELATED WORKS

In literature, various types of evolutionary algorithms have been applied to synthesize different antenna array structures with different objective functions. To name a few, Das, Mandal & Kar (2020) presented opposition based differential evolution algorithm to find the optimum current excitation weights and the inter-element spacing of a circular antenna array by considering the mutual coupling between the elements; Saxena, Kothari & Saxena (2020) used ant lion optimization (ALO) to optimize current amplitudes for wide null placement and reduced SLL in linear antenna array; Bulgan, Chen, Xue, Fan & Zhang (2020) proposed a variant of brainstorm optimization (BSO) algorithm to reduce SLL and resource usage in thinned linear arrays; Subhashini (2019) used strawberry optimization to minimize SLL in linear and circular arrays; Kailash & Gautam (2020) used quantum particle swarm optimisation (QPSO), teaching-learning based optimisation (TLBO) and symbiotic organism search (SOS) for SLL and peak directivity optimisation in multiple thinned concentric circular arrays; Shen et al. (2019) proposed improved chicken swarm optimization (CSO)) to reduce SLL in planar antenna arrays; Yan, Yang, Xing & Huang (2018) used combination of perturbed compressive sampling and convex optimization to constrain the minimum inter-element spacing in sparse antenna arrays; Almagboul et al. (2019) used atom search optimization to reduce the peak SLL and null steering; Babayigit (2018) used dragonfly algorithm for concentric circular arrays; Liang, Wu & Zhang (2018) used third evolution step of generalized differential evolution (GDE3) algorithm to optimize peak SLL and peak side band level simultaneously in time modulated concentric circular array ring.

However, as mentioned in section 1, the performance of these algorithms (except TLBO algorithm) is dependent on the tuning of their algorithm-specific parameters. The improper tuning of any algorithm's control parameters may lead to convergence at a local optimum or increase the time for convergence. Furthermore, these algorithms are based on some metaphors. However, Rao (2020) mentioned that it would be better if the researchers focus on developing simple optimization techniques that can provide effective solutions to the complex problems instead of looking for developing metaphor based algorithms. Keeping this point in view, the Rao algorithms which are three simple metaphor-less and algorithm-specific parameter-less optimization algorithms are used in this paper for synthesis of linear, planar and concentric circular antenna arrays. The next section describes the proposed algorithms.

3. RAO ALGORITHMS

Let $g(y)$ be the objective function to be minimized (or maximized). Assume that there are '*d*' number of design variables. An initial population of size P is randomly generated within the bounds of the design variables (i.e. p=1,2,…,P number of candidate solutions). *best* denotes the candidate which gives the best solution among all the candidates. The best solution means the minimum value of $g(y)$ for minimization problem and the maximum value of $g(y)$ for a maximization type problem. Similarly, '*worst'* denotes the candidate which gives the worst solution. The notations *j,p,i* are used to denote a variable, a candidate solution and an iteration. Therefore, $Y_{j,p,i}$ represents the value of the j^{th} variable of the k^{th} candidate in the i^{th} iteration. $Y'_{j,p,i}$ denotes the updated value of $Y_{j,p,i}$. For the update process, there are three equations for three Rao algorithms. Equation 1 is used for the Rao-1 algorithm, Equation 2 for the Rao-2 algorithm and Equation 3 for the Rao-3 algorithm. $r_{1,j,i}$ and $r_{2,j,i}$ are the two random numbers in the range [0,1] for the j^{th} variable during the i^{th} iteration.

$$Y'_{j,p,i} = Y_{j,p,i} + r_{1,j,i}\,(Y_{j,best,i} - Y_{j,worst,i}) \tag{1}$$

$$Y'_{j,p,i} = Y_{j,p,i} + r_{1,j,i}\,(Y_{j,best,i} - Y_{j,worst,i}) + r_{2,j,i}\,(\left| Y_{j,p,i} \text{ or } Y_{j,z,i} \right| - \left| Y_{j,z,i} \text{ or } Y_{j,p,i} \right|) \tag{2}$$

$$Y'_{j,p,i} = Y_{j,p,i} + r_{1,j,i}\,(Y_{j,best,i} - \left| Y_{j,worst,i} \right|) + r_{2,j,i}\,(\left| Y_{j,p,i} \text{ or } Y_{j,z,i} \right| - (Y_{j,z,i} \text{ or } Y_{j,p,i})) \tag{3}$$

In Equations (2) and (3), the term "$Y_{j,p,i}$ or $Y_{j,z,i}$" means that the candidate solution *p* is compared with any randomly picked candidate solution *z*. Then, if the objective function value of p^{th} solution is better than that of the z^{th} solution the term "$Y_{j,p,i}$ or $Y_{j,z,i}$" becomes $Y_{j,p,i.}$ and the term "$Y_{j,z,i}$ or $Y_{j,p,i}$" becomes $Y_{j,z,i}$. Else, if the objective function value of z^{th} solution is better than that of the p^{th} solution then the term "$Y_{j,p,i}$ or $Y_{j,z,i}$" becomes $Y_{j,z,i}$ and "$Y_{j,z,i}$ or $Y_{j,p,i}$" becomes $Y_{j,p,i}$.

The Rao algorithms work with the *best* and *worst* solutions in the population and the random interactions between the candidate solutions. The designer does not need to tune the algorithm-specific parameters to get the optimal results. In the introductory paper of Rao algorithms, the algorithms were applied on a variety of unconstrained unimodal, multimodal and fixed dimension multimodal problems and constrained problems (Rao, 2020). A modified version incorporating self-adaptive multi-population idea in Rao algorithms has also been recently applied to various engineering design problems (Rao & Pawar, 2020). An application of the algorithms to multi-objective optimization problems is found in Rao & Keesari (2020). For more details, the readers may refer to: https://sites.google.com/view/raoalgorithms/

The flowchart of the Rao-1 algorithm is shown in Figure 1.

Rao-2 and Rao-3 have similar flowcharts with the difference being the replacement of Equation 1 shown in the flowchart with Equation 2 and Equation 3 respectively.

4. CASE STUDIES

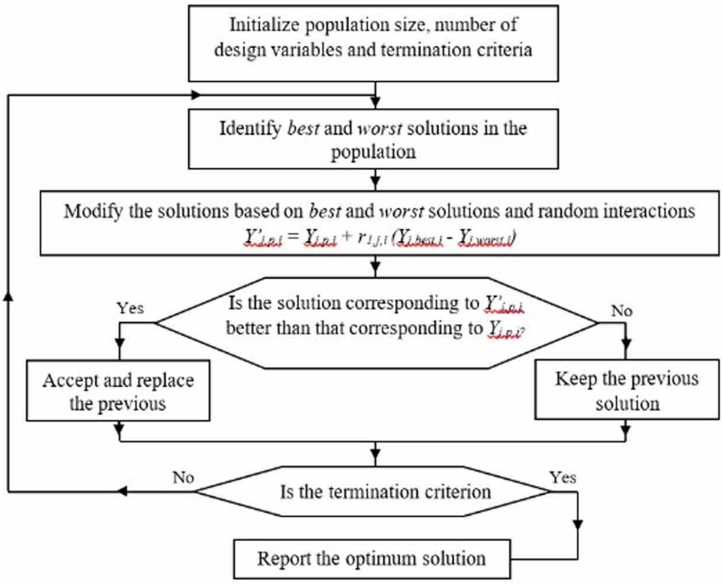
The case studies of synthesis of thinned linear, planar and concentric circular planar antenna arrays presented by Jia and Lu (2019), Suman, Ashwin, Miranda, Gangwar & Gangwar (2018) and Dib and Sharaqa (2014) are considered here for application of the Rao algorithms. The objective functions, fitness functions, ranges of the variables and the conditions used by Jia and Lu (2019), Suman, Ashwin, Miranda, Gangwar & Gangwar (2018) and Dib and Sharaqa (2014) are used as it is in this paper for making fair comparison. It may be mentioned here that although various researchers had done similar works in antenna array synthesis problems by applying various optimization techniques, but the objective functions, fitness functions, ranges of the variables and the conditions used by them were different. Therefore, in this paper, the Rao algorithms are compared only with the algorithms that were mentioned in the case studies of the referred papers.

4.1 Synthesis of a Thinned Concentric Circular Planar Antenna Array (CCAA)

Figure 2 depicts the geometry of a CCAA lying in the x-y plane containing isotropic antenna elements uniformly distributed over M rings. Each m^th ring contains N_m number of elements where m varies from 1 to M. The array factor is given by Equation 4 (Dib & Sharaqa, 2014).

$$AF\left(\theta,\Phi\right) = \sum_{m=1}^{M}\sum_{n=}^{N_m} A_{mn} \exp\left\{ j\left[kr_m sin\theta \cos\left(\varphi - \varphi_{mn}\right) + \alpha_{mn} \right]\right\} \tag{4}$$

Figure 1. Flowchart of Rao algorithms



$$k = 2\pi / \lambda \tag{5}$$

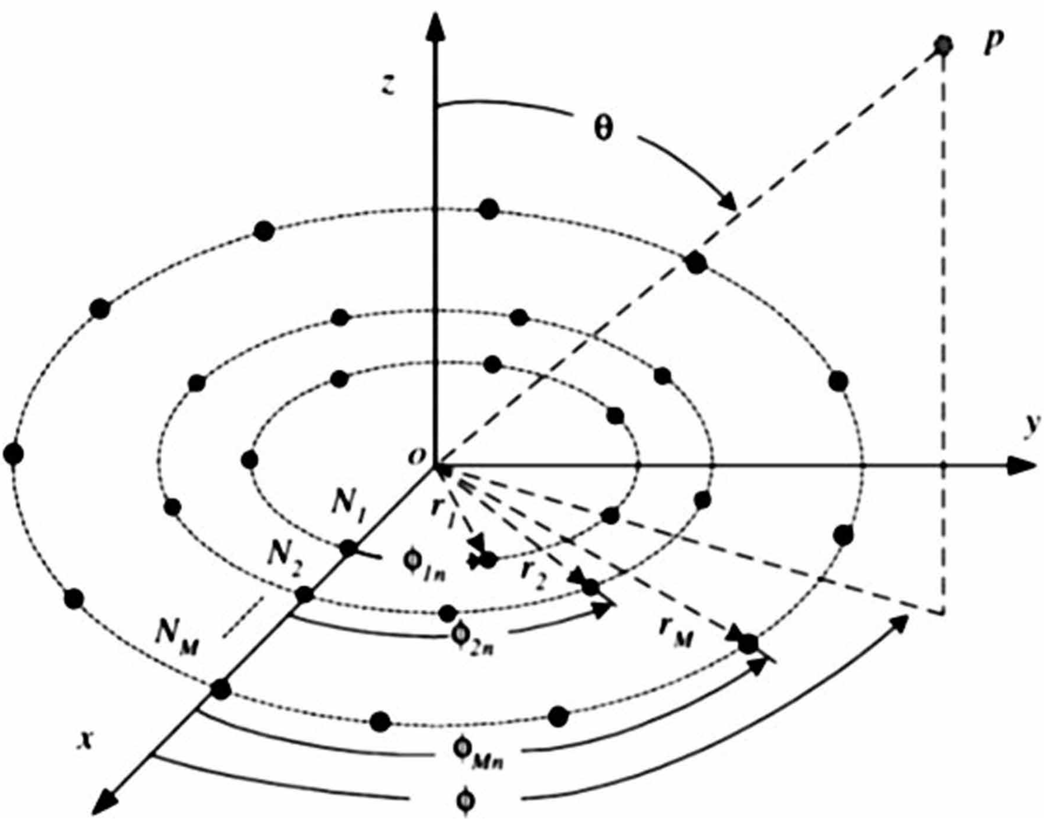
$$\phi_{mn} = 2\pi n / N_m \tag{6}$$

$$\alpha_{mn} = -kr_m \sin(\theta_{o)} \cos\left(\varphi_o - \varphi_{mn}\right) \tag{7}$$

Where A_{mn} is the excitation amplitude, k is the wavenumber, r_m is the radius of the m[th] ring, the angular position of an n[th] element belonging to m[th] ring is φ_{mn} . θ_o and φ_o represents the main lobe direction and is set as 0 to get a broadside pattern. Also, the φ in the array factor equation is chosen to be zero. The fitness function is given in Equation 8 (Dib & Sharaqa, 2014). The design variable is A_{mn} that takes the value 1 to denote that the element is ON and 0 for OFF. The three algorithms find the optimum combination of 1's and 0's which when plugged into the array factor equation and then used in the fitness function, gives the lowest side lobe level.

$$Fitness = C_1 F_1 + C_2 F_2 \tag{8}$$

Figure 2. Geometry of concentric circular antenna array with isotropic elements



$$F_1 = \frac{max\left\{\left|AF\left(\theta_{ms}\right)\right|^2\right\}}{\left|\left(AF_{max}\right)\right|^2} \tag{9}$$

$$F_2 = \left(\left(T_o^{OFF} - T_d^{OFF}\right) / N_{total}\right)^2 \tag{10}$$

Where θ_{ms} is the angle at which maximum side lobe occurs and $AF\left(\theta_{ms}\right)$ is the array factor value at that angle. AF_{max} is the array factor's maximum value. T_o^{OFF} denotes the total number of OFF elements that occur when the optimization code is working, T_d^{OFF} is the total number of desired OFF elements and N_{total} is the total number of elements. C_1 and C_2 are the weights taken as 10 and 1 taken from Dib & Sharaqa (2014).

Two case studies of CCAA containing 2 rings and 10 rings have been considered. Dib and Sharaqa (2014) used the TLBO algorithm to solve this problem (Dib & Sharaqa, 2014). The values of parameters were taken as follows. Population size: 100, number of generations: 300 and number of runs: 20. To have fairness in the comparison of results, the same parameter values have been maintained during the implementation of Rao-1, Rao-2 and Rao-3 algorithms.

4.1.1 Case Study 1

The synthesis of 2-rings thinned CCAA is analysed in this case 1. The inner ring consists of 35 isotropic elements and the outer one has 70. The inter-element spacing is kept as $\lambda / 2$. T_d^{OFF} is taken as 55. The peak SLL obtained by Rao algorithms and the TLBO algorithm is tabulated in Table 1.

Dib and Sharaqa (2014) reported some best SLL values of all the runs. The minimum of these is taken for comparison purposes Rao-1 algorithm minimized the SLL from -18.3611 dB to -25.7275 dB (a reduction of 7.3664 dB). Rao-2 algorithm minimized the SLL to -28.0976 dB (a reduction of 9.7364 dB). Rao-3 algorithm minimized the SLL to -28.1490 dB (a reduction of 9.7879 dB). Also, the three percentage thinning values yielded by TLBO are 43%, 50% and 55%. Rao algorithms obtained a thinning percentage of » 50% which is less when compared to the 55% case obtained by TLBO. However, the SLL values obtained by all three Rao algorithms are better. The array pattern generated by Rao algorithms and the convergence behaviour is shown in Figure 3 and Figure 4.

From Figure 3, the FNBW obtained by Rao algorithms looks larger than the TLBO pattern but the FNBW value was not reported by Dib and Sharaqa (2014) so as to use in the process of array

Table 1. Optimized ON-OFF combination, number of ON elements and SLL obtained by Rao-1, Rao-2, Rao-3, FA and TLBO algorithms for minimizing SLL of a 2-ring CCAA array

Algorithm	Ring 1	Ring 2	No. of ON elements	Max. SLL (dB)
TLBO†	101111101111100 101100011110110 00001	1100111111000101110001001111111001 110100000011101111011011011100000 00101	60	-18.3611
	011000111110110 001000010010111 00000	1110111101010111111110011111100101 110000010100111001000011011100000 00000	52	-18.2082
	01010011110010100 00100010000101010	1001001011010110111110011000110001 100000100111001110011000000101101 00001	47	-17.2953
FA†	010111111110101 100000011011011 11010	11010101110100111110111111011001 0 110100101111101111111011011111101 01001	69	-18.2600
Rao-1	0 1 1 1 1 1 1 0 1 1 0 0 1 0 0 1 0 0 1 1 0 0 0 1 1 1 1 1 0 0 0 0 0 0 1	0 0 0 0 0 0 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 0 0 0 1 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 0 0 1 0 0 0	52	**-25.7275**
Rao-2	1 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 0 1 1 1 1 0 1 1 1 1 1 0 1 0 0 0 1	1 0 0 0 0 1 0 0 1 1 1 1 1 0 1 0 1 0 1 1 1 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 0 0 0 0 0 0	53	**-28.0976**
Rao-3	0 1 0 1 1 1 1 1 1 1 0 1 1 0 1 0 0 1 0 0 0 1 0 1 1 1 1 1 1 1 0 0 1 0 0	0 0 0 0 0 0 1 1 1 1 1 0 1 0 1 0 1 0 1 1 0 0 1 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 0 1 1 0 0 0 0 1 0 0 0 0 1	53	**-28.1490**
† The results of TLBO and FA are taken from Dib and Sharaqa (2014). The better performance of the algorithm is depicted using bold font values.				

Figure 3. Radiation pattern of 2-rings CCAA obtained using (A) Rao-1 algorithm (B) Rao-2 algorithm (C) Rao-3 algorithm

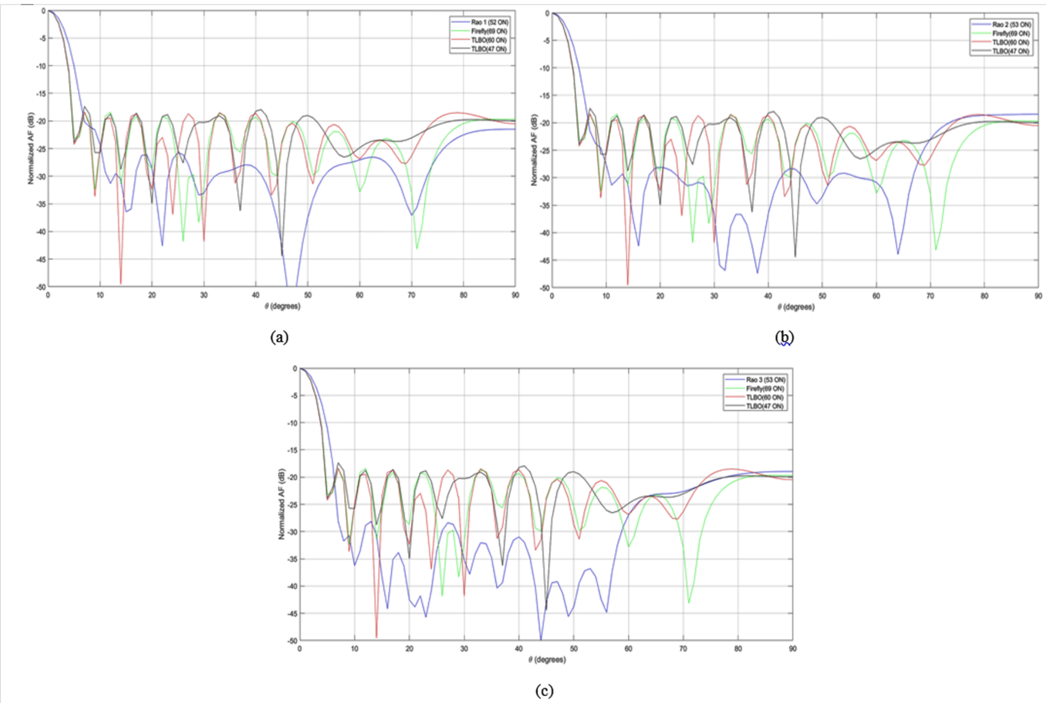
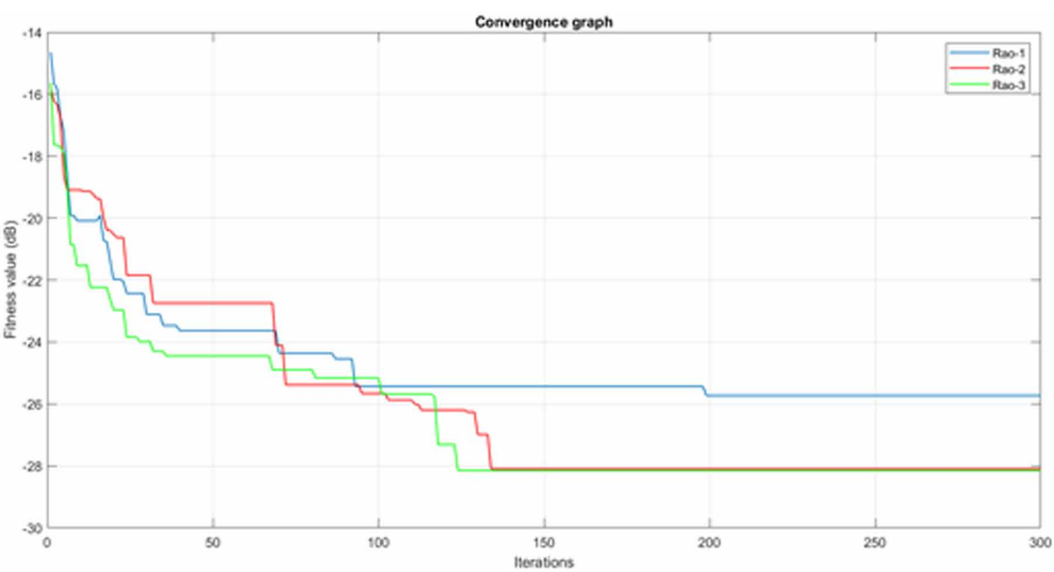


Figure 4. Convergence graph for 2-rings case study



pattern generation by Rao algorithms. The Rao-1 algorithm has a faster convergence than Rao-2 and Rao-3 algorithms in this case but the latter provides much-reduced SLL values.

4.1.2 Case Study 2

The synthesis of 10-rings thinned CCAA is analysed in this case 2. Each ring has 8*m elements where m varies from 1 to 10. The fixed inter-element spacing ($\lambda / 2$) case is considered. T_d^{OFF} is taken as 230. The peak SLL obtained by Rao algorithms and the TLBO algorithm for minimizing SLL is tabulated in Table 2.

Table 2. The number of ON elements, SLL and percentage thinning obtained by Rao-1, Rao-2, Rao-3, BBO and TLBO algorithms for minimizing SLL of a 10-ring CCAA array

Algorithm	No. of ON elements	Max. SLL (dB)	% thinning
TLBO†	204	-26.0685	53.63%
	216	-26.2457	51%
BBO†	216	-26.3679	51%
Rao-1	208	**-31.6115**	52.72%
Rao-2	203	**-31.8172**	53.86%
Rao-3	198	**-31.4770**	55%
†The results of TLBO and BBO are taken from Dib and Sharaqa (2014). The better performance of the algorithm is depicted using bold font values.			

Dib and Sharaqa (2014) reported some best SLL values of all the runs. The minimum of these is taken for comparison purposes Rao-1 algorithm minimized the SLL from -26.2457 dB to -31.6115 dB (a reduction of 5.3658 dB). Rao-2 algorithm minimized the SLL to -31.8172 dB (a reduction of 5.5715 dB). Rao-3 algorithm minimized the SLL to -31.4770 dB (a reduction of 5.2313 dB). The two percentage thinning values yielded by TLBO are 53.63% and 51%. Rao algorithms obtained thinning percentages as 52.72%, 53.86% and 55%. Rao-1 yielded a slightly smaller thinning percentage of 52.72% when compared to the 53.63% case obtained by TLBO. However, the SLL values obtained by Rao-1 is better. The array pattern generated by Rao algorithms and the convergence behaviour is shown in Figure 5 and Figure 6.

4.2 Synthesis of a Thinned Linear Antenna Array

Figure 7 depicts a symmetric linear antenna array configuration (Jia & Lu, 2019). Equation 11 gives the array factor for this configuration in the azimuth plane.

$$AF\left(\theta\right) = 2\sum_{n=1}^{N} a_n \cos\left\{\left(\frac{2n-1}{2}\right)\beta d cos\theta\right\} \tag{11}$$

Where N is taken as 100, a_n is the amplitude of the nth antenna element, β is the progressive phase shift and d is the inter-element spacing taken as λ/2. The fitness function is given by Equation 12. The design variable is a_n (excitation amplitude) that takes the value 1 to denote that the element is

Figure 5. Radiation pattern of 10-rings CCAA obtained using Rao-1, Rao-2, and Rao-3 algorithms

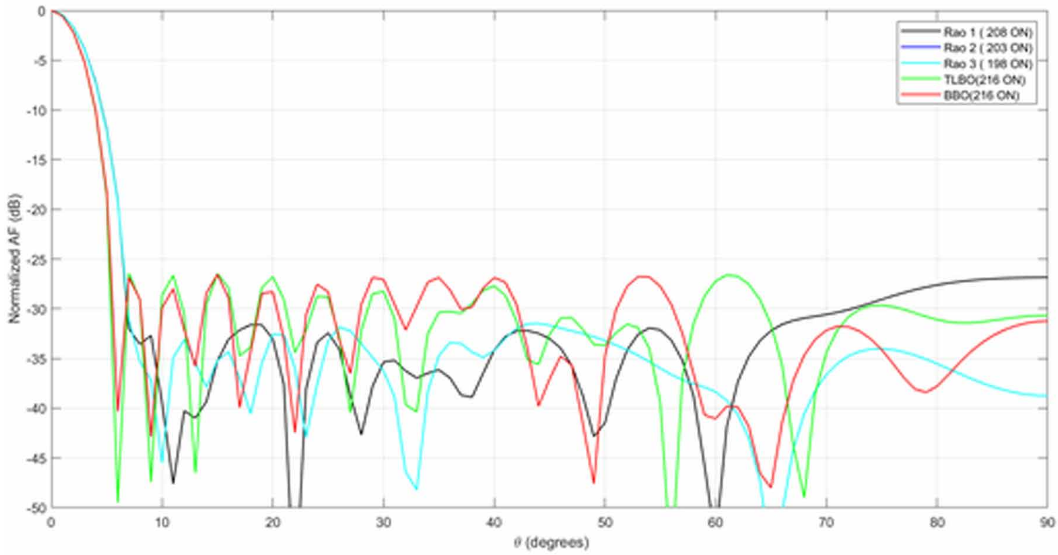
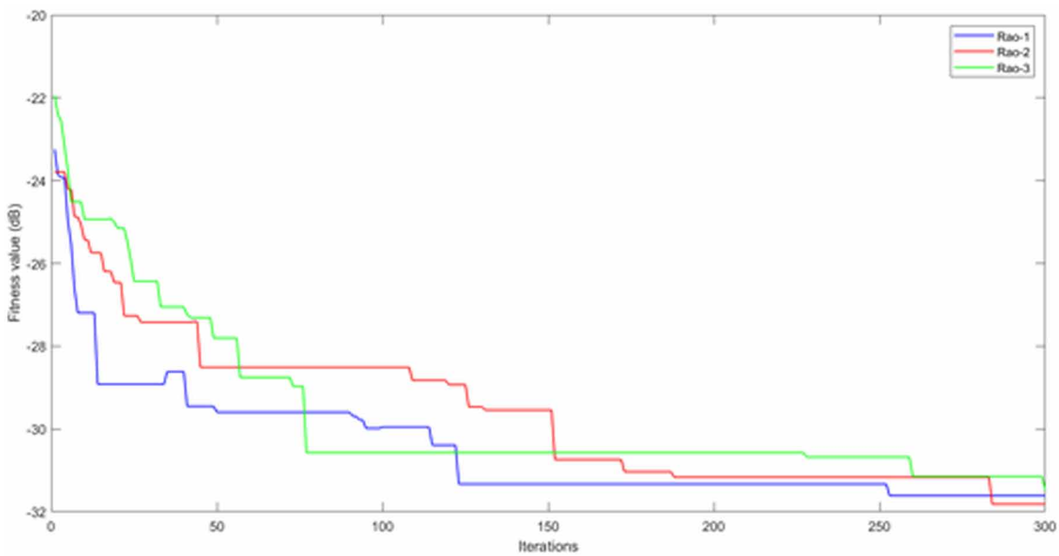


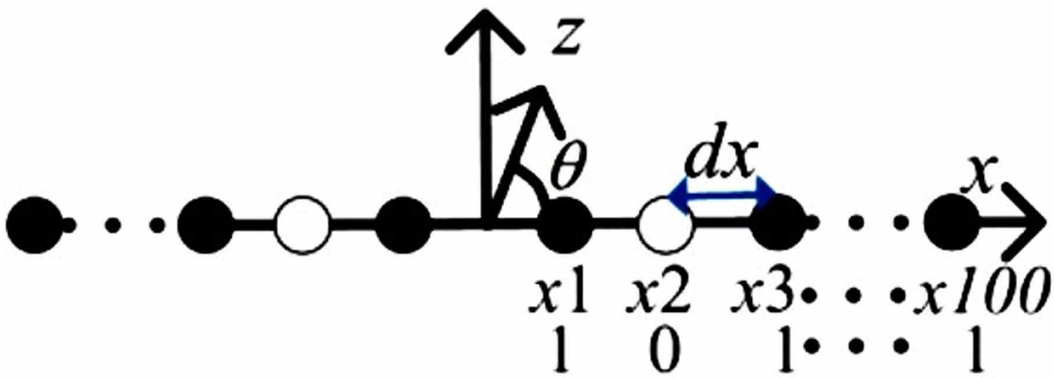
Figure 6. Convergence graph for 10-rings CCAA case



ON and 0 for OFF. The optimum combination of 1's and 0's giving the lowest SLL is found using the three algorithms.

$$\text{Fitness}= min\left(\max\left[20\log\left|\frac{AF\left(\theta,\Phi\right)}{AF_{max}}\right|\right]\right)_{\forall\theta\hat{I}R} \tag{12}$$

Figure 7. Geometry of linear antenna array with isotropic elements



Where R is the region excluding the main lobe and AF_{max} is the maximum value of the array factor over the entire region.

4.2.1 Case Study 3

The synthesis of a thinned linear antenna array to achieve reduced peak side lobe levels is analysed in this case 3. This problem was solved by Jia and Lu (2019) using a hybrid Taguchi binary particle swarm optimization and the values of parameters were taken as follows. Population size: 30, number of generations: 15 and number of runs: 20. The same parameter values have been used for the implementation of Rao-1, Rao-2 and Rao-3 algorithms. The inter-element spacing is $\lambda / 2$.

The peak SLL obtained by Rao algorithms, HTBPSO algorithm, IBPSO, IPBSO without OA and BPSO is tabulated in Table 3.

As HTBPSO obtained the minimum SLL value among the reported BPSO variants it is used for comparison purposes with the Rao algorithms. Rao-1 algorithm decreased the SLL from -22.68 dB to -23.435315 dB (a reduction of 0.755315 dB). Rao-2 algorithm decreased the SLL to -24.047709 dB (a reduction of 1.367709 dB). Rao-3 algorithm decreased the SLL to -24.391125 dB (a reduction of 1.711125 dB). Rao algorithms obtained thinning percentages of 26%, 21% and 21% respectively.

Table 3. Best, Worst and Mean side lobe level (SLL) values obtained by Rao-1, Rao-2, Rao-3, IBPSO, IBPSO w/o OA and HTBPSO algorithms for minimizing SLL of a thinned linear antenna array

Algorithm	SLL (dB) [Best]	SLL (dB) [Worst]	SLL (dB) [Mean]
†IBPSO	-19.44	-17.78	-18.6
†IBSO w/o OA	-19.35	-17.71	-18.5
†BPSO	-22.33	-19.78	-21.29
†Hybrid Taguchi BPSO	-22.68	-22	-22.4
Rao-1	**-23.435315**	-19.601750	-21.327807
Rao-2	**-24.047709**	-19.948224	-21.928780
Rao-3	**-24.391125**	-20.342173	-22.060306
†The results of IBPSO, IBPSO w/o OA and HTBPSO algorithms are taken from Jia and Lu (2019). The better performance of the algorithm is depicted using bold font values.			

The percentage thinning by HTBPSO was not reported in Jia and Lu (2019) and hence could not be used for comparison. The array pattern obtained by Rao algorithms is shown in Figure 8.

The ON-OFF combination obtained using HTBPSO was not reported and hence, the corresponding array pattern could not be reproduced superimposing the array patterns of Rao algorithms.

4.3 Synthesis of a Thinned Planar Antenna Array

A 50*50 linear thinned planar antenna (LTPA) array located symmetrically in the X-Y plane is considered as depicted in Figure 9 (Jia & Lu, 2019).

Equation 13 gives the field radiated by this configuration along the z-axis. The fitness function is given by Equation 15 (Jia & Lu, 2019). The design variable is A_{mn} (excitation amplitude) that takes the value 1 to denote that the element is ON and 0 for OFF. The optimum combination of 1's and 0's giving the lowest SLL is found using the three algorithms.

$$AF\left(\theta,\Phi\right)=\sum_{m=1}^{M}\sum_{n=1}^{N}A_{mn}\cos\left[\frac{\left(2m-1\right)}{2}\beta d_{y}sin\theta cos\Phi\right]*cos\left[\frac{\left(2n-1\right)}{2}\beta d_{x}sin\theta sin\Phi\right] \quad (13)$$

Figure 8. Radiation pattern of linear antenna array obtained using Rao-1, Rao-2 algorithm and Rao-3 algorithms

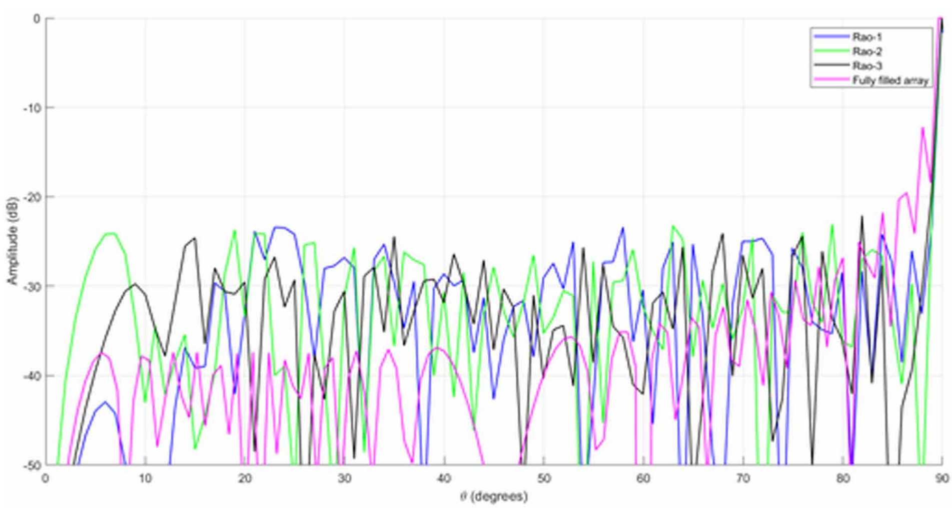
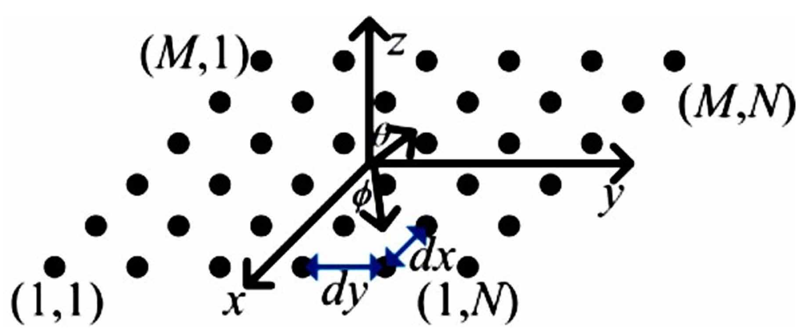


Figure 9. Geometry of planar antenna array with isotropic elements



$$f\left(\theta,\Phi\right) = \max\left[20\log\left|\frac{AF\left(\theta,\Phi\right)}{AF_{max}}\right|\right]_{\forall\theta\hat{I}R} \tag{14}$$

$$\text{Fitness}= \min\left\{f\left(\theta,0^{o}\right)+f\left(\theta,90^{o}\right)\right\} \tag{15}$$

4.3.1 Case Study 4

The synthesis of a linear thinned 50*50 planar antenna array for decreased side lobe levels is analysed in this case 4. This problem was solved by Suman, Ashwin, Miranda, Gangwar & Gangwar (2018) using an improved genetic algorithm (IGA) with the number of iterations as 200 (as per their convergence graph) (Suman et al., 2018). Now, Rao-1, Rao-2 and Rao-3 algorithms are used to solve this problem.

The peak SLL obtained by Rao algorithms and IGA algorithm for minimizing SLL in $\Phi=0^o$ and $\Phi=90^o$ are tabulated in Table 4 and Table 5.

In $\Phi=0^o$ plane, the Rao-1 algorithm decreased the SLL from -30.56 dB to -32.47 dB (a reduction of 1.91dB). Rao-2 algorithm decreased the SLL to -33.92 dB (a reduction of 3.36 dB). Rao-3 algorithm decreased the SLL to -34.03 dB (a reduction of 3.47 dB). The array pattern obtained by

Table 4. Maximum side lobe level (SLL) values obtained by Rao-1, Rao-2, Rao-3 and IGA Algorithms for minimizing SLL of a thinned planar antenna array in *Φ=0º* plane

Algorithm	Max. SLL (dB)
†IGA	-30.56
Rao-1	**-32.474058**
Rao-2	**-33.921020**
Rao-3	**-34.032212**
†The result of the IGA algorithm is reproduced from Suman et al. (2018). The better performance of the algorithm is depicted using bold font values.	

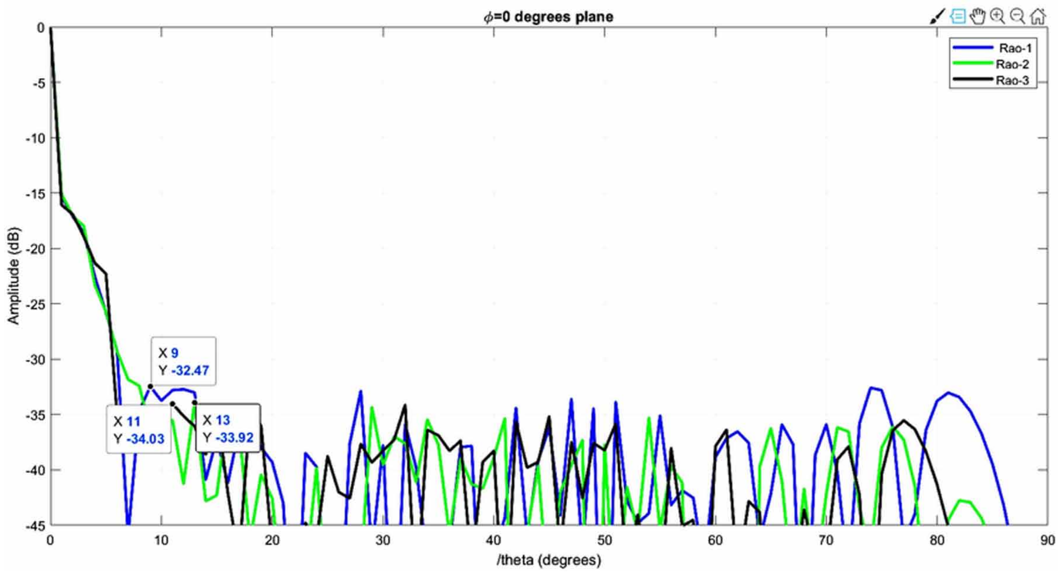
Table 5. Maximum side lobe level (SLL) values obtained by Rao-1, Rao-2, Rao-3 and IGA Algorithms for minimizing SLL of a thinned planar antenna array in *Φ=90º plane*

Algorithm	Max. SLL (dB)
†IGA	-31.20
Rao1	**-32.39**
Rao2	**-34.75**
Rao3	**-34.03**
†The result of the IGA algorithm is reproduced from Suman et al. (2018). The better performance of the algorithm is depicted using bold font values.	

Rao algorithms is shown in Figure 10.

In $\Phi=90^o$ plane, the Rao-1 algorithm decreased the SLL from -31.20 dB to -32.39 dB (a reduction of 1.19 dB). Rao-2 algorithm decreased the SLL to -34.75 dB (a reduction of 3.55 dB). Rao-3

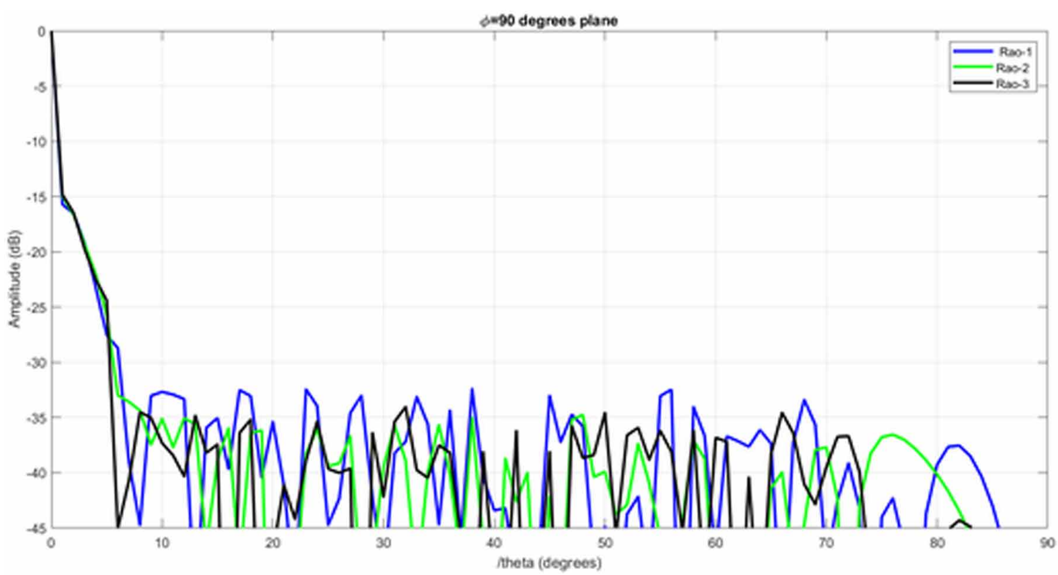
Figure 10. Radiation pattern of linear antenna array obtained using Rao-1, Rao-2 algorithm and Rao-3 algorithms (Φ=0°)



algorithm decreased the SLL to -34.03 dB (a reduction of 2.83 dB). The array pattern obtained by Rao algorithms is shown in Figure 11.

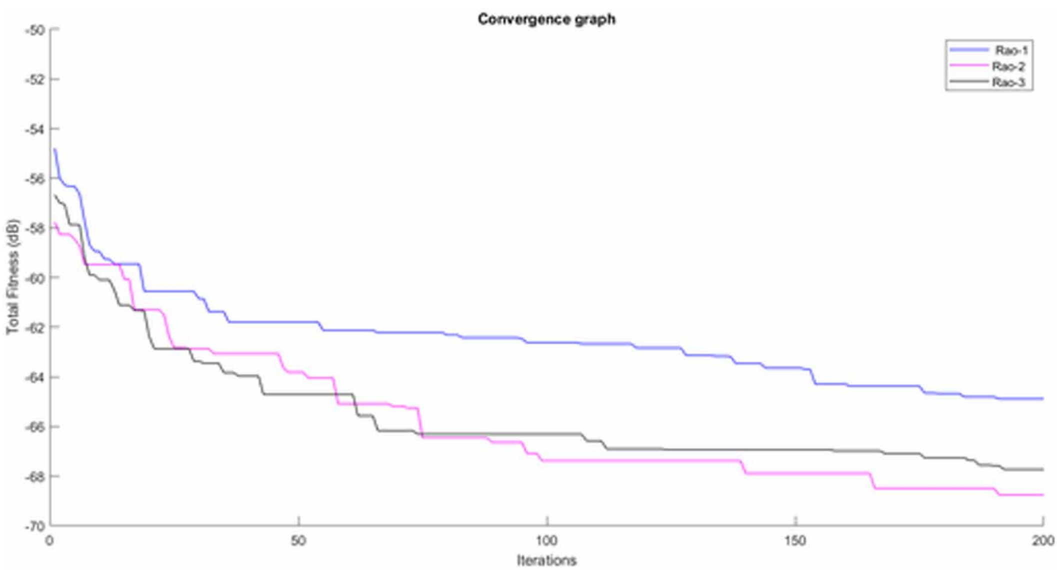
Although Rao-3 algorithms obtained a bit smaller reduction than Rao-2, it may be mentioned that the purpose of this paper is not to compare the three Rao algorithms among themselves but rather applying them to case studies that were studied with different algorithms and show the efficacy of Rao algorithms.

Figure 11. Radiation pattern of linear antenna array obtained using Rao-1, Rao-2 algorithm and Rao-3 algorithms (Φ=90°)



The ON-OFF combination obtained using IGA was not reported probably due to a large number of values (50*50=2500) involved and hence, the corresponding array pattern could not be reproduced superimposing the patterns of Rao algorithms. The convergence graph considering $\Phi=0^o$ and $\Phi=90^o$ planes together is shown in Figure 12.

Figure 12. Convergence graph of Rao algorithms for linear antenna array case



It may be observed from these case studies that the Rao algorithms produce good results and have good exploration capabililites thus making them a good choice for solving the optimization problems. These are believed to have the ability to perform well in diverse applications where other algorithms have already been used such. To name a few, applications in IoT (Armentano, Bhadoria, Chatterjee & Deka, 2017), Big data analytics (Arya, Bhadoria, & Chaudhari, 2018), wireless communications such as 5G networks, adhoc networks, cognitive radio networks, sensor networks etc. (Mazumder, Bhadoria & Deka, 2017), power cable systems (Ocłoń et al., 2018), job-shop scheduling (Buddala & Mahapatra, 2018), structural damage identification (Du, Vinh, Trung, Quyen & Trung, 2017), etc. are some of the diverse applications to which the Rao algorithms may be attempted in near future.

5. CONCLUSION

The purpose of this paper is to portray the effectiveness of Rao-1, Rao-2 and Rao-3 optimization algorithms in synthesizing antenna arrays with low side lobe levels. For this purpose, different configurations of antenna arrays like linear, planar, 2-rings concentric circular and 10-rings concentric circular are considered. Rao algorithms are applied to these configurations to obtain the optimal ON-OFF combinations that yield lower side lobe levels. The obtained results are compared with the results of HTBPSO, IGA, FA, TLBO, and BBO optimization algorithms. It is noticed that the three Rao algorithms performed better in achieving the goal.

In the first case study, Rao algorithms are used to synthesize a thinned linear antenna array with fixed inter-element spacing. The obtained combination of ON-OFF elements by Rao algorithms gave array patterns that have lower SLL as compared to the pattern obtained using HTBPSO. In the second case study, Rao algorithms are used to synthesize a 50*50 thinned planar antenna array with fixed inter-element spacing. The array patterns yielded by Rao algorithms have lower SLL as compared to that of the conventional array pattern and IGA. In the third case study, Rao algorithms are used to synthesize a 2-rings concentric circular antenna array with fixed inter-element spacing. Rao algorithms performed better when compared to the TLBO and FA results. In the fourth case study, Rao algorithms are used to synthesize a large 10-rings concentric circular antenna array with fixed inter-element spacing. The percentage thinning obtained using Rao algorithms is comparable to the TLBO result but the side lobe levels achieved using Rao algorithms are lower than those of TLBO and BBO results.

The results display that the Rao-1, Rao-2, and Rao-3 algorithms can effectively synthesize linear antenna arrays, planar antenna arrays, and concentric circular antenna. The Rao algorithms have an easy execution and are independent of any algorithm-specific parameters. The algorithms have good search abilities and can be applied to find solutions to other electromagnetic optimization problems.

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