Assignment 1 System description and Analysis

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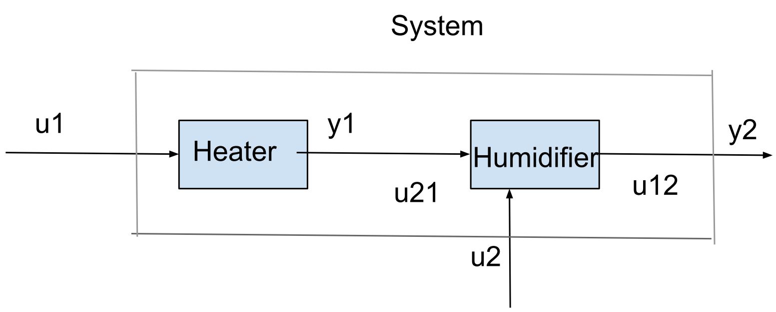
1 Introduction

Our aim is to study an air handling unit with a heater and a humidifier. Two variables air temperature and humidity are controlled by air flow to the heater and humidifier respectively. First we study the open loop system to understand the interdependencies between these two systems using enthalpy entropy chart. We find the transfer function, state space representation, singular values and observability plus controllability for this open loop system.

Later we convert our air handling unit to a closed loop system by introducing a proportional controller for heat and humidity control. We change the controller gain values to see the changes in the closed loop system. We plot the system sensitivity, complementary sensity and input sensitivity functions along with the closed loop system. Finally we study the effects of changes in controller gain values on singular values of these four functions.

2 Method

We represent the open loop system as in figure below

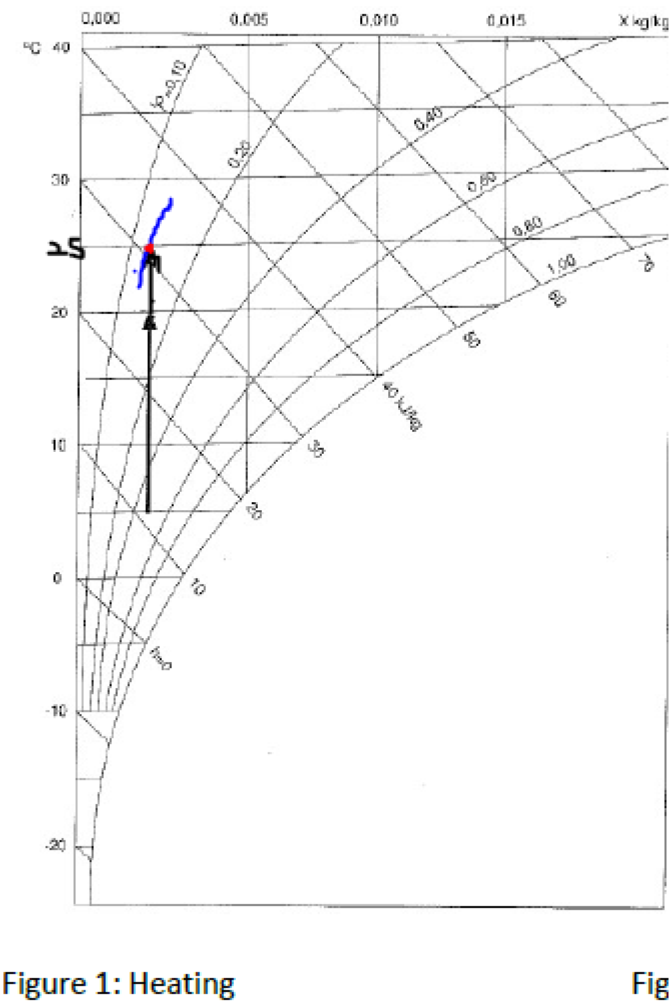


where
u_1 =Heater input temperature 10° C
y_1 =Heater output temperature 25° C
u_2 =Humidity ratio at humidifier input 10% RH
y_2 =Humidity ratio at humidifier output 80% RH

Since the systems are dependent on each other,
u_h =relative humidity at heater ouput
u_t =temperature at humidifier output

From the enthalpy-entropy chart,

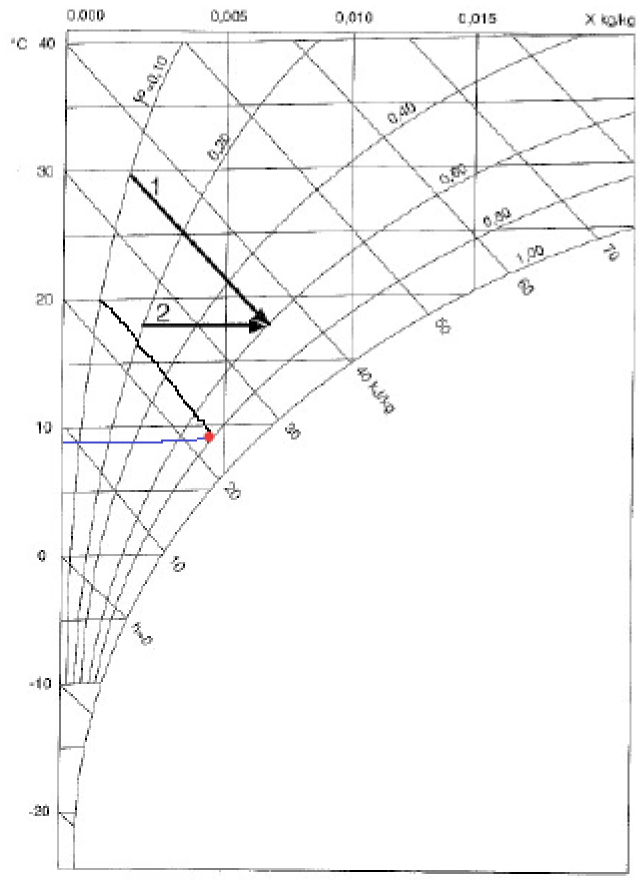
Figure 1: Heating chart



$u_h = 15.5\%$ at the red dot,

Source: Mollier calculator

Figure 2: Humidifying chart



ıre 2: Humidifying with water (1) and steam (2).

$u_t = 9°$ C at the red dot,
$u_1 = 0.01438$ kg/kg
$y_2 = 0.005685$ kg/kg

Source: Mollier calculator

Assuming that the heater and humidifier are described by first order transfer function,

$$G_{11} = \frac{15}{(50s+1)} \tag{1}$$

G_{11} is the heater transfer function with time constant 50s

$$G_{22} = \frac{70}{(10s+1)} \tag{2}$$

G_{22} is the humidifier transfer function with time constant 10s

The system has disturbances

The change in air temperature y_1 is affected by humidity change in the heater u_h.
For example, when the valve is fully open i.e. $u_1 = 1, u_h = 15.5\%$ (refer Figure 1: Heating chart above).

y_1 increases from 10^oC to 25^oC as u_h decreases from 40% to 15.5%. The heater time constant remains the same.

$$G_{12} = \frac{-11}{(10s+1)} \tag{3}$$

where G_{12} is the heater transfer function due to the disturbance from humidity change

The change in humidity ratio y_2 is affected by the temperature change in the humidifier.
For example, when the valve is fully open i.e. $u_2 = 1, u_t = 9^oC$ (refer Figure 2: Humidity chart above).

y_2 increases from $0.01438kg/kg$ to 0.005685kg/kg as u_t changes between 20^oC to 9^oC. The humidity time constant remains the same.

$$G_{21} = \frac{-24.5}{(50s+1)} \tag{4}$$

where G_{21} is the humidifier transfer function due to the disturbance from temperature change

The transfer function of open loop system can be represented as,

$$G = [G_{11}G_{12}; G_{21}G_{22}] \tag{5}$$

State space representation can be derived from transfer function in Matlab with command 'ss',

$$sys = ss(G) \tag{6}$$

Observability can be found using 'obsv' command in Matlab on the state space equation,

$$O = obsv(sys) \tag{7}$$

The system has full rank 2, hence it is observable.

Controlability can be found using 'ctrb' command in Matlab on the state space equation,

$$S = ctrb(sys) \tag{8}$$

The system has full rank 2, hence it is controllable.

Singular values of the system can be found using 'sigma' command in Matlab as below,

$$sigma(sys) \tag{9}$$

The plot of singular values is shown in the results section below. The gain of the system lies between 25.4 db and -7.55 db.
We can use command 'db2mag' in Matlab to convert db values to amplitude.

$$Kmax = db2mag(37.6) = 75.8578 \tag{10}$$

$$Kmin = db2mag(20.2) = 10.2329 \tag{11}$$

We introduce a feedback controller F_y = [1 0;0 1] and let $F_r = 1$

In the feedback loop, we add a proportional controller with gain k_{p1} for temperature control and k_{p2} for humidity control.

If G_c is the transfer function of the closed loop system, it will be a 2-input, 2-output equation as below,

$$G_c = [G_{c11} G_{c12}; G_{c21} G_{c22}] \tag{12}$$

where

$$G_{c12} = \frac{(G_{12} + (-G_{12}/G_{11}) * G_{11})}{(1 + k_{p1} G_{11})} \tag{13}$$

and

$$G_{c21} = \frac{(G_{21} + (-G_{21}/G_{22}) * G_{22})}{(1 + k_{p2} G_{22})} \tag{14}$$

We choose a feedforward controller such that the disturbances become zero i.e.

$$F_{12} = \frac{-G_{12}}{G_{11}} \tag{15}$$

and

$$F_{21} = \frac{-G_{21}}{G_{22}} \tag{16}$$

Now we have a decoupled MIMO system whose closed loop transfer function is below,

$$G_c = [G_{c11} 0; 0 G_{c22}] \tag{17}$$

where

$$G_{c11} = \frac{k_{p1}G_{11}}{1 + k_{p1}G_{11}} \tag{18}$$

and

$$G_{c22} = \frac{k_{p2}G_{22}}{1 + k_{p2}G_{22}} \tag{19}$$

Sensitivity function is

$$S = [S_{11}0; 0S_{22}] \tag{20}$$

where

$$S_{11} = \frac{1}{1 + G_{11}k_{p1}}; \tag{21}$$

and

$$S_{22} = \frac{1}{1 + G_{22}k_{p2}}; \tag{22}$$

Complementary sensitivity function is

$$T = [T_{11}0; 0T_{22}] \tag{23}$$

where

$$T_{11} = \frac{G_{11}k_{p1}}{1 + G_{11}k_{p1}}; \tag{24}$$

and

$$T_{22} = \frac{G_{22}k_{p2}}{1 + G_{22}k_{p2}}; \tag{25}$$

Input sensitivity function is

$$S_u = [S_{u11}0; 0S_{u22}] \tag{26}$$

where

$$S_{u11} = \frac{1}{1 + G_{11}k_{p1}}; \tag{27}$$

and

$$S_{u22} = \frac{1}{1 + G_{22}k_{p2}}; \tag{28}$$

Now we choose three values of k_{p1} andk_{p2} to plot the singular values of G_c, S, T, S_u

Since the same four functions are to be used repetatively, we will insert them into a function and call this function with different values of k_{p1} andk_{p2}
This is done in function name $'Singular_Values'$ in Matlab
The function will return values of G_c, S, T, S_u which will be plotted outside the function.

Three controller gain values used are as below,

1. $k_{p1} = 0.4$ and $k_{p2} = 2$
2. $k_{p1} = 2$ and $k_{p2} = 0.4$
3. $k_{p1} = 0.5$ and $k_{p2} = 0.5$

The singular value plots are shown in the results section with figure numbers

1. Figure 2 for $k_{p1} = 0.4$ and $k_{p2} = 2$
2. Figure 3 for $k_{p1} = 2$ and $k_{p2} = 0.4$
3. Figure 4 for $k_{p1} = 0.5$ and $k_{p2} = 0.5$

We also calculate the static gain values for each k_{p1} andk_{p2} to understand the direction of the input which cannot be seen from the diagram.

$$g_{c01} = \begin{bmatrix} \frac{0.4}{1.4} & 0 \\ 0 & \frac{2}{3} \end{bmatrix} \tag{29}$$

$$g_{c02} = \begin{bmatrix} \frac{2}{3} & 0 \\ 0 & \frac{0.4}{1.4} \end{bmatrix} \tag{30}$$

$$g_{c02} = \begin{bmatrix} \frac{2}{3} & 0 \\ 0 & \frac{0.2}{1.2} \end{bmatrix} \tag{31}$$

Input direction dependency can be understood by finding the eigen values of $g_{c0} * g'_{c0}$. We use Matlab command 'eig' as below,

$$[V_{c1}, D_{c1}] = eig(g_{c01} * g'_{c01}) \tag{32}$$

$$[V_{c2}, D_{c2}] = eig(g_{c02} * g'_{c02}) \tag{33}$$

$$[V_{c3}, D_{c3}] = eig(g_{c03} * g'_{c03}) \tag{34}$$

3 Results

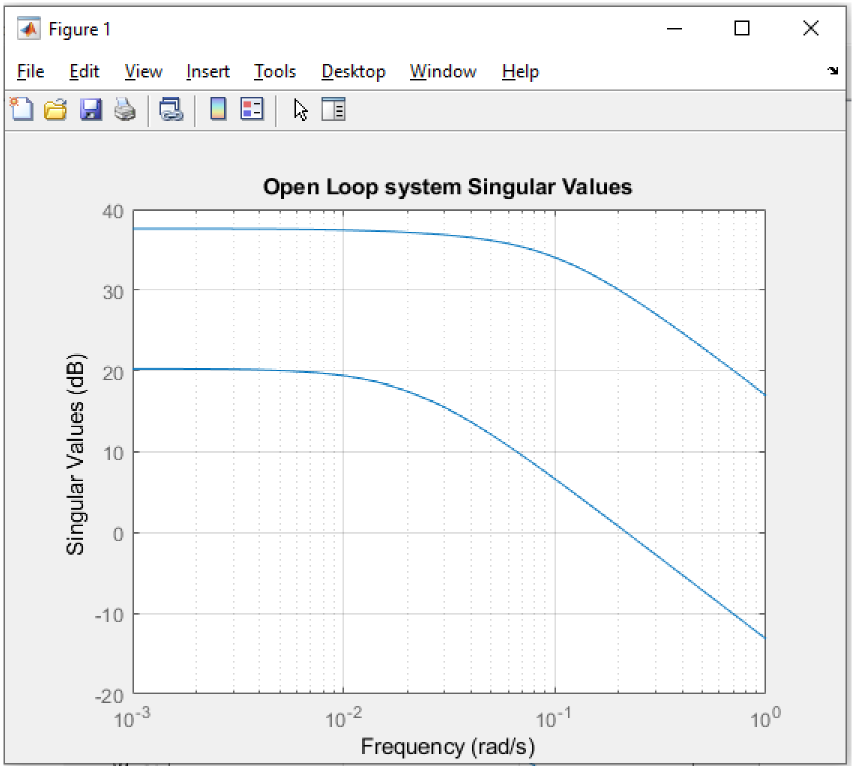


Figure 1
File Edit View Insert Tools Desktop Window Help
Open Loop system Singular Values
Singular Values (dB)
40
30
20
10
0
-10
-20
10^{-3}
10^{-2}
10^{-1}
10^{0}
Frequency (rad/s)

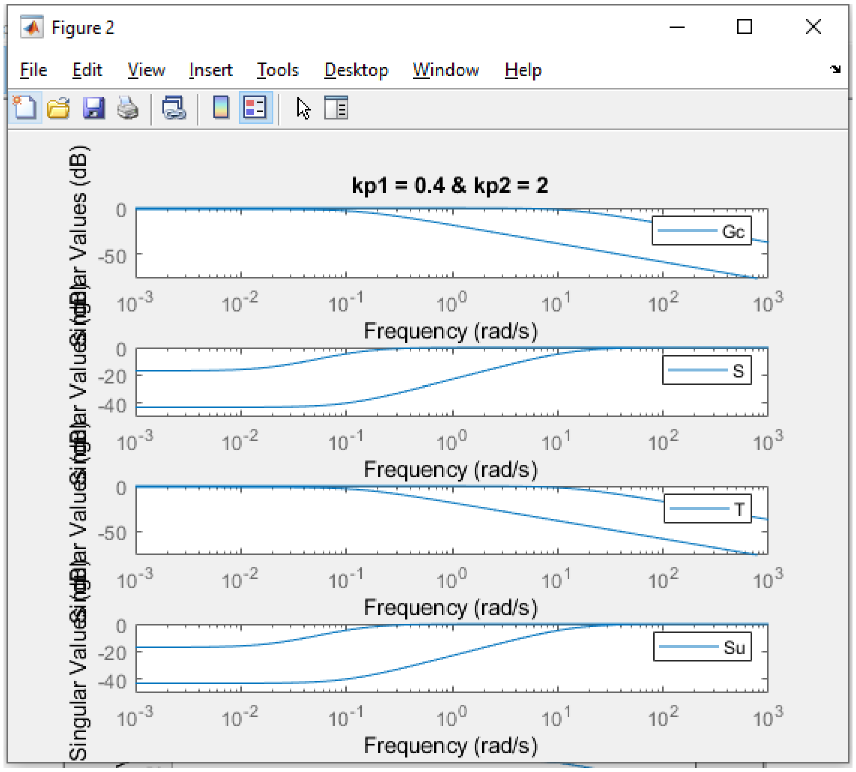


Figure 2
File
Edit
View
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Help
kp1 = 0.4 & kp2 = 2
Singular Values (dB)
Frequency (rad/s)
Gc
S
T
Su
0
-50
-20
-40
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

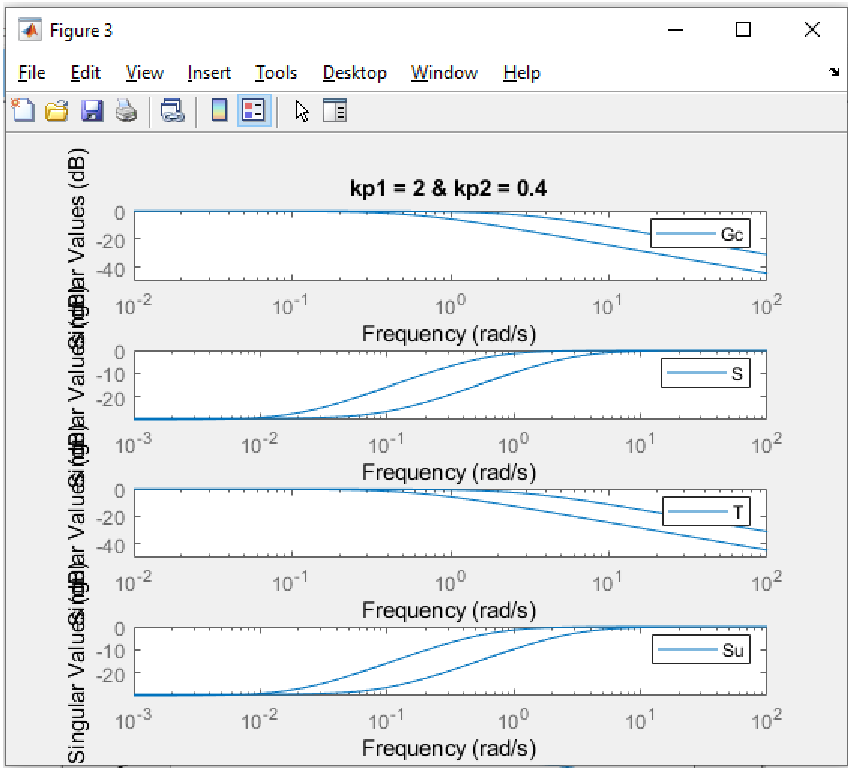


Figure 3
File
Edit
View
Insert
Tools
Desktop
Window
Help
kp1 = 2 & kp2 = 0.4
Singular Values (dB)
Frequency (rad/s)
Gc
S
T
Su

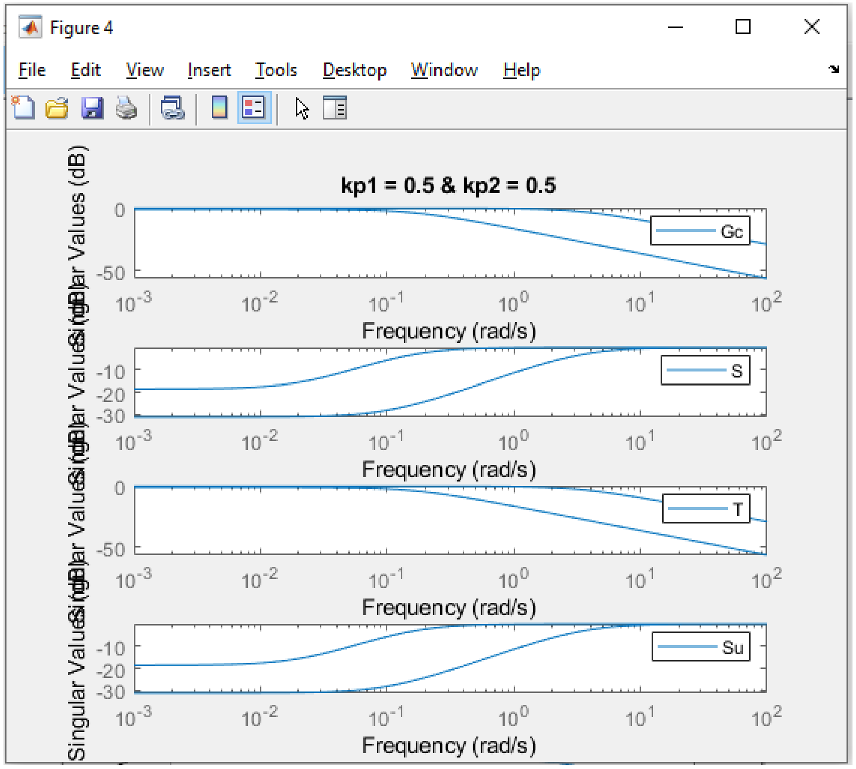
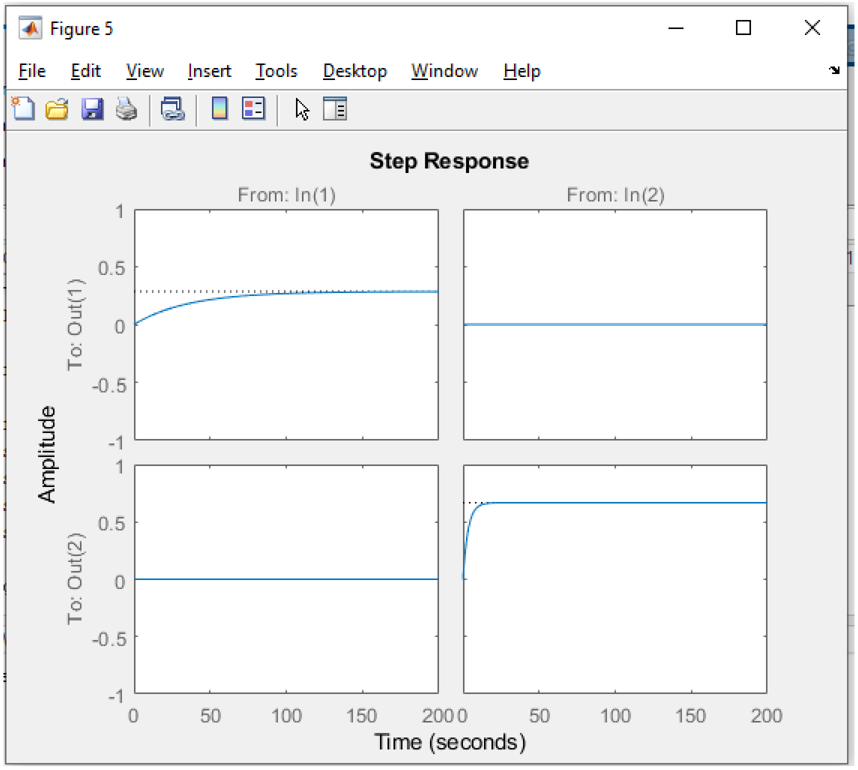


Figure 4
File
Edit
View
Insert
Tools
Desktop
Window
Help
kp1 = 0.5 & kp2 = 0.5
Singular Values (dB)
Gc
S
T
Su
Frequency (rad/s)



Eigen values for

<u>$k_{p1} = 0.4 \;\; and \; k_{p2} = 2$</u>

$$V_{c1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{35}$$

$$D_{c1} = \begin{bmatrix} 0.0816 & 0 \\ 0 & 0.4444 \end{bmatrix} \tag{36}$$

<u>$k_{p1} = 2 \;\; and \; k_{p2} = 0.4$</u>

$$V_{c2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{37}$$

$$D_{c2} = \begin{bmatrix} 0.0816 & 0 \\ 0 & 0.4444 \end{bmatrix} \tag{38}$$

$\underline{k_{p1} = 0.5 \ and \ k_{p2} = 0.5}$

$$V_{c3} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{39}$$

$$D_{c3} = \begin{bmatrix} 0.1111 & 0 \\ 0 & 0.1111 \end{bmatrix} \tag{40}$$

4 Discussion

1. Figure 5 shows the step response of G_c. The humidifier is faster to step response than the heater.

2. From equation 12, we saw that the closed loop system is absolutely stable since it has two poles at $\frac{-1}{50}$, $\frac{-1}{10}$ each

3. System is internally stable as S, S_u, F_r, $G_{wuy} = \frac{G}{1+GF_y}$, $G_{wu} = \frac{-F_y}{1+F_yG}$ are stable since poles of these equations lie in the left half plane

4. From all three k_{p1} and k_{p2} scenarios, we see as time $\rightarrow \infty$, closed loop system G_c and complementary sensitivity function T goes to 0. Sensitivity S and input sensitivity S_u goes to 1

5. T determines the robustness to noise and S determines the suppression of load disturbances. Ideally we would like them zero. Both cannot be zero at the same time since S+T=1. In our case, the model is robust since T is small. This is from our theoretical choice of feedforward controller which makes G_{12} and G_{21} zero. The real world model will not have T=0 instead S and T will be values between 0 and 1.

6. Lastly, we will show the directional dependency of input from first scenario at $k_{p1} = 0.4$ and $k_{p2} = 2$ but the other two scenarios can also be interpreted in the same way
   G_c function plot shows the gains begin from 1 (0db) and approach 0. Directional dependency can be seen from eigen vector matrix V_{c1} and diagnol matrix D_{c1}. Singular value $\sqrt{0.0816} = 0.2857$ is associated with eigenvector $[1\ 0]^T$ and singular value $\sqrt{0.4444} = 0.6666$ is associated with eigenvector $[0\ 1]^T$. Both eigen vectors have same direction. Input step parallel to both $[1\ 0]^T$ and $[0\ 1]^T$ means that temperature of air flow to heater and humidity of air flow to humidifier will be increased. The increase in humidity will be more than the increase in temperature which confirms with the step response.

5 Appendix

```
clear all
set(0, 'defaulttextinterpreter','Latex');

s = tf('s');
G11 = 15/(50*s+1);
G22 = 70/(10*s+1);
G12 = -11/(10*s+1);    %temperature changes due to
   humidity disturbance
G21 = -24.5/(50*s+1);  %humidity changes due to heater
    disturbance

G = [G11 G12;G21 G22];
sys = ss(G);             %State space representation

O = obsv(sys);
rank(O);               %Observability
S = ctrb(sys);
rank(S);               %Controllability

%Singular values
figure(1);sigma(sys);grid;title('Open Loop system
   Singular Values')

Kmax = db2mag(37.6);
Kmin = db2mag(20.2);
%Closed loop system
Gd = [G11 G12;G21 G22];
%Adding a proportional controller with temperature
   control gain kp1 and humidity control gain kp2.
kp1 = 0.4;
kp2 = 2;
[Gc,S,T,Su] = Singular_Values(kp1,kp2,G11,G22);

figure(2);
subplot(411);sigma(Gc);legend('Gc');title('kp1 = 0.4 &
    kp2 = 2')
subplot(412);sigma(S);legend('S');title('')
subplot(413);sigma(T),legend('T');title('')
subplot(414);sigma(Su);legend('Su');title('')

gc01 = [0.4/1.4 0;0 2/3];  %Static gain at kp1 = 0.4 &
    kp2 = 2
[Vc1,Dc1]=eig(gc01*gc01');
```

```
kp1 = 2;
kp2 = 0.4;
[Gc,S,T,Su] = Singular_Values(kp1,kp2,G11,G22);

figure(3)
subplot(411);sigma(Gc);legend('Gc');title('kp1 = 2 &
   kp2 = 0.4')
subplot(412);sigma(S);legend('S');title('')
subplot(413);sigma(T),legend('T');title('')
subplot(414);sigma(Su);legend('Su');title('')

gc02 = [2/3 0;0 0.4/1.4];   %Static gain at kp1 = 2 &
   kp2 = 0.4
[Vc2,Dc2]=eig(gc02*gc02');

kp1 = 0.5;
kp2 = 0.5;
[Gc,S,T,Su] = Singular_Values(kp1,kp2,G11,G22);

figure(4)
subplot(411);sigma(Gc);legend('Gc');title('kp1 = 0.5 &
    kp2 = 0.5')
subplot(412);sigma(S);legend('S');title('')
subplot(413);sigma(T),legend('T');title('')
subplot(414);sigma(Su);legend('Su');title('')

gc03 = [0.5/1.5 0;0 0.5/1.5];      %Static gain at kp1
    = 0.5 & kp2 = 0.5
[Vc3,Dc3]=eig(gc03*gc03');

function [Gc,S,T,Su] = Singular_Values(kp1,kp2,G11,G22
   )
Gc11 = kp1*G11/(eye(1)+kp1*G11);
S11 = inv(eye(1)+G11*kp1);
T11 = G11*kp1/(eye(1)+G11*kp1);
Su11 = inv(eye(1)+kp1*G11);

Gc22 = kp2*G22/(eye(1)+kp2*G22);
S22 = inv(eye(1)+G22*kp2);
T22 = G22*kp2/(eye(1)+G22*kp2);
Su22 = inv(eye(1)+kp2*G22);

Gc = [Gc11 0;0 Gc22];
S = [S11 0; 0 S22];
T = [T11 0;0 T22];
```

```
Su = [Su11 0;0 Su22];
figure(5),step(Gc);
end
```