Assignment 3 Non Linear Systems

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1 Introduction

Our aim is to study the non-linear system consisting of the heater and the valve. We will find maximum valve coefficient under different conditions. First we will study maximum valve coefficient using the circle criteria with and without saturation and later we study it using the describing function.

2 Method

Our system transfer function is below,

$$G = \frac{1}{(s^2 + s + 1)(s + 3)} \tag{1}$$

with the valve coefficient k_{vs}, the overall dynamic system is given by

$$G = \frac{k_{vs}}{(s^2 + s + 1)(s + 3)} \tag{2}$$

2.1 Study different values of k_{vs}

The valves are sold in fixed sizes and the given valve sizes can be used as valve coefficients as per Schnielder electric Valve coefficient manual

As per page 82 of this manual, let us assume that the water flow across the valve is 4700 kg/hr and pressure drop is 160 kPa,

We get the maximum valve coefficient as 4.31. Source: k_{vs} calculator

Substituting k_{vs} values in equation 2 for the overall system, we get

$$G_1 = 1 * G \tag{3}$$

$$G_{1.6} = 1.6 * G \tag{4}$$

$$G_{2.5} = 2.5 * G \tag{5}$$

$$G_4 = 4 * G \tag{6}$$

$$G_{6.3} = 6.3 * G \tag{7}$$

$$G_{10} = 10 * G \tag{8}$$

$$G_{16} = 16 * G \tag{9}$$

Plotting the nyquist curves for equation 3 to 9, we see that nyquist curve doesnot encircle -1 for maximum $k_{vs} = 4$. See figure 1 in the 'Results' section From the above findings, our maximum valve coeffient is 4 and minimum is 1.

2.2 Circle criteria

1. Without Saturation
   We re-construct the given non-linearity (see figure 2 in Results section) by taking approximate values below

$$x_1 = [0, 0.2, 0.4, 0.6, 0.8, 1] \tag{10}$$

$$y_1 = [0, 0.12, 0.26, 0.42, 0.61, 1] \tag{11}$$

$$y_2 = [0, 0.3, 0.55, 0.75, 0.92, 1] \tag{12}$$

   Our k_1 and k_2 will be the slope of above polynomial curve plotted using below code

$$coef_1 = polyfit(x_1, y_1, 1) \tag{13}$$

$$k_1 = coef_1(1) \tag{14}$$

$$coef_2 = polyfit(x_1, y_2, 1) \tag{15}$$

$$k_2 = coef_2(1) \tag{16}$$

   We find $k_1 = 0.9471$ and $k_2 = 1.0086$, and plot a circle with radius

$$r = \frac{(-1/k_2 + 1/k_1)}{2} \tag{17}$$

   and center

$$m = \frac{(-1/k1 - 1/k2)}{2}; \tag{18}$$

as

$$fi = (-1.2 : 0.1 : 1) * pi \tag{19}$$

$$x = m + r * exp(1i * fi) \tag{20}$$

$$plot(x) \tag{21}$$

in the same plot with nyquist curve for G.

$$nyquist(G) \tag{22}$$

See figure 3 in the 'Results' section. Nyquist curve doesnot encircle this non-linearity circle which means the overall system is stable.

Thus the maximum valve coefficient is $k_{vs} = 1.0086$.

2. With Saturation
   The value with saturation has a function shown below

$$sat(u(t)) = \begin{cases} \text{u(t)} & \text{if } |u(t)| <= 1 \\ 1 & \text{if } u(t) > 1 \\ \text{-1} & \text{if } u(t) < -1 \end{cases} \tag{23}$$

   The slope of this function is 0 and 1. The non-linearity is a line which can be interpreted as circle with infinite radius and the Circle Criterion is satisfied with $k_1 = 0$ and $k_2 = 1$.

   We plot a vertical line with real part -1 and Nyquist curve for G in figure 4. Nyquist curve never goes over -1 to left half plane. This fact guarantees stability.

   The maximum valve coefficient is $k_{vs} = 1$.

2.3 Describing Function

Replacing the continuous valve with a shut-off valve and relay, the relay control equation becomes

$$f(e) = \begin{cases} 1 & \text{if } e > 0 \\ \text{-1} & \text{if } e < 0 \end{cases} \tag{24}$$

and the describing function is given by

$$Y_f(C) = \frac{4}{\pi C} \tag{25}$$

If there exists an ω and C such that

$$Y_f(C)G(i\omega) = -1 \tag{26}$$

then there is a possibility for self-sustained oscillations with amplitude approximately equal to C and frequency approximately equal to ω.

With the given G we get

$$G(i\omega) = \frac{k_{vs}}{((i\omega)^2 + i\omega + 1)(i\omega + 3)} \tag{27}$$

Since Y_f isreal and positive, Equation 19 can be satisfied if $G(i\omega)$ is real and negative. This implies $\omega = 2$.

$$G(i2) = \frac{-k_{vs}}{13} \tag{28}$$

Substituting equation 21 and equation 18 in equation 19, we get

$$\frac{4}{\pi C}\frac{-k_{vs}}{13} = -1 \tag{29}$$

showing

$$C = \frac{4k_{vs}}{13\pi} \tag{30}$$

For $k_{vs} = 1$

$$C_1 = \frac{4k_{vs}}{13\pi} \tag{31}$$

we predict an oscillation with amplitude $C_1 = 0.089$.

For $k_{vs} = 4$

$$C_2 = \frac{4k_{vs}}{13\pi} \tag{32}$$

we predict an oscillation with amplitude $C_2 = 0.38$.

For $k_{vs} = 6.3$

$$C_3 = \frac{4k_{vs}}{13\pi} \tag{33}$$

we predict an oscillation with amplitude $C_3 = 0.56$. See figure 5 in 'Results' section.

Hence the maximum valve coefficient that keeps the magnitude of the oscillation below 0.5 is $k_{vs} = 4$.

3 Results

Figure 1

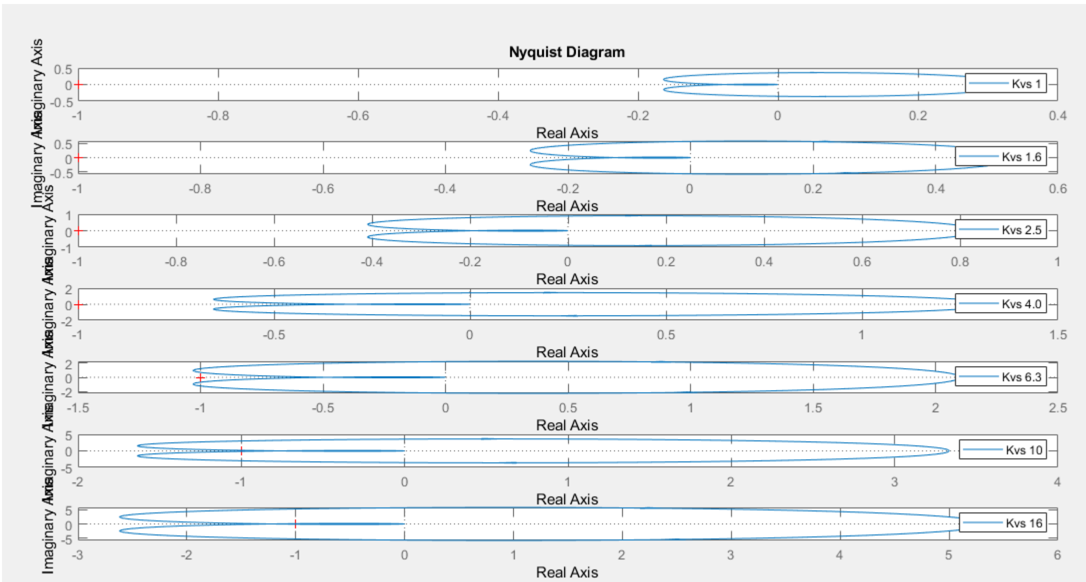


Figure 2 Non-linearity

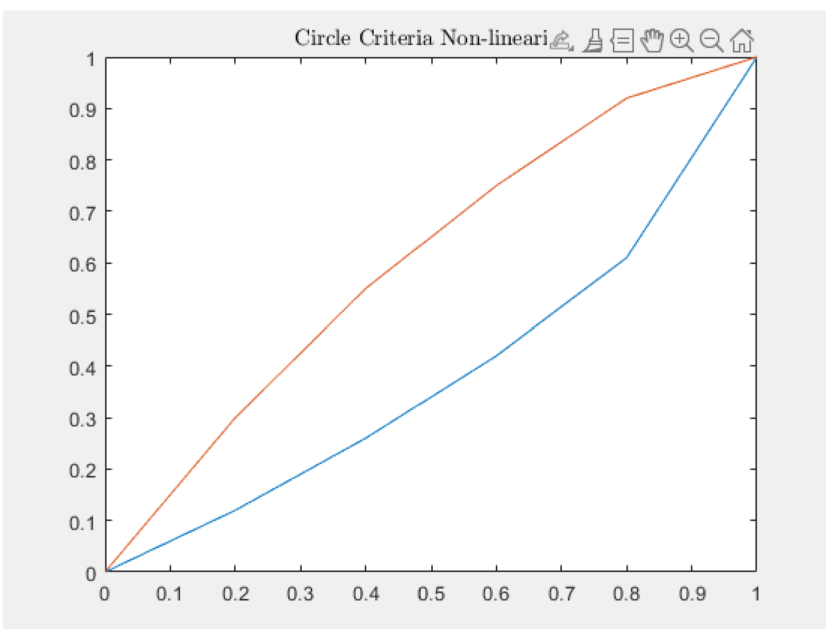


Figure 3 Circle Criteria without saturation

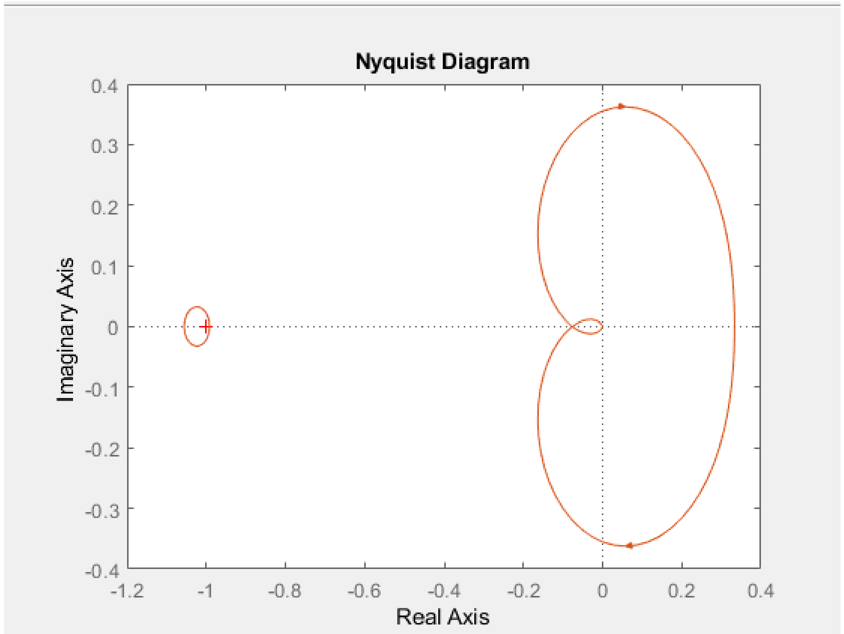


Figure 4 Circle criteria with saturation

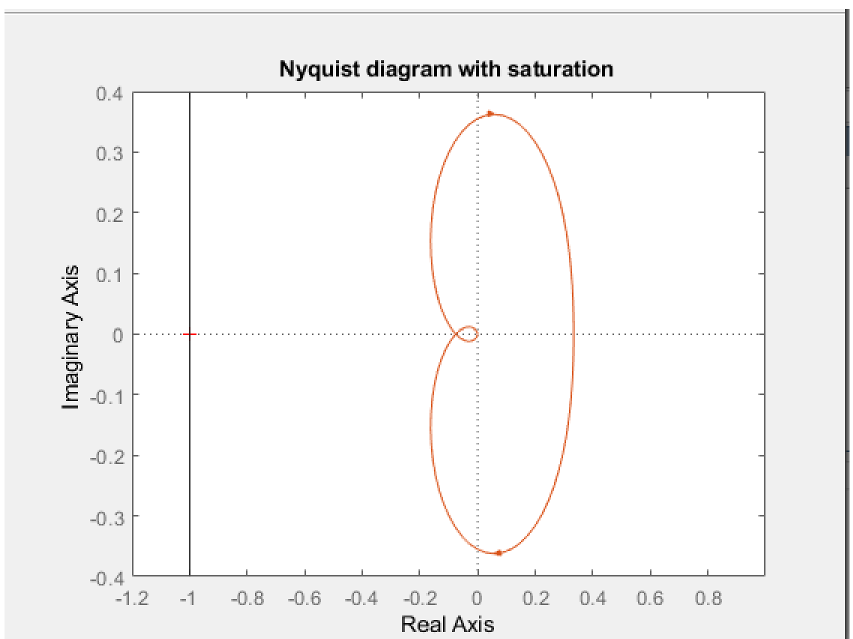
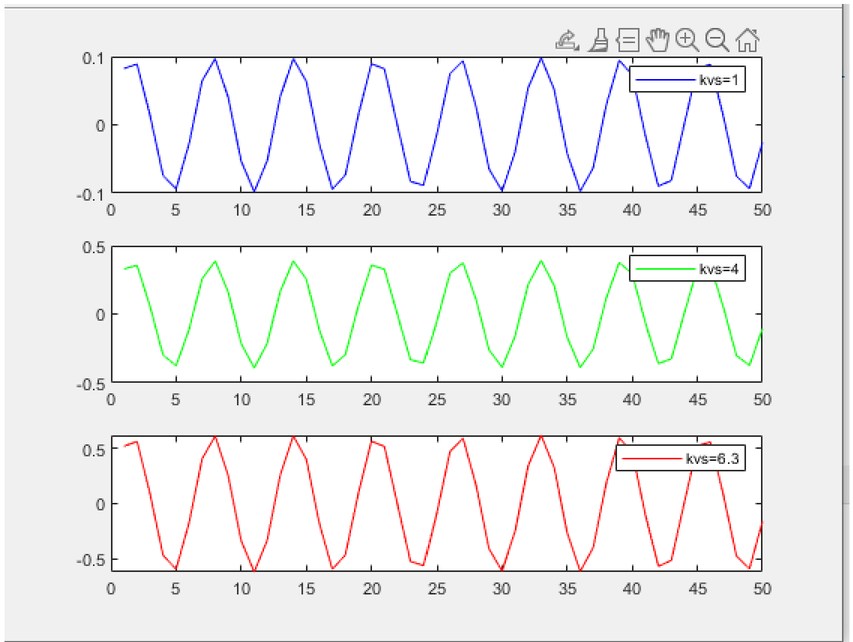


Figure 5 Describing function



4 Discussion

1. **Circle Criteria**
   It is possible to prove stability for different nonlinearities by choosing different circles in the circle criterion. If the Nyquist curve of G doesnot encircle the circle whose diameter is defined by points $\frac{-1}{k_1}$ and $\frac{-1}{k_2}$, then the system is input output stable.

2. **Describing function**
   Let us see our amplitude C in relation to C_0 i.e. the point where $\frac{-1}{Y_C}$ intersects the Nyquist curve G. For $k_{vs} = 4$,,

   $$C_0 = \frac{1}{Y_C} = (pi * C_2)/4 = 0.3077 \tag{34}$$

   Our $C_2 = 0.38 > C_0$. This means that C_2 is to the left of the intersection. Replacing the nonlinearity with a constant gain would give a stable system according to the Nyquist criterion. The amplitude should decrease and return to C_0 when perturbed.

   For $k_{vs} = 6.3$, even if the amplitude $C_3 = 0.56$, C_0 at this amplitude is 0.4846 i.e. less than 0.5. We can say that the amplitude should eventually return to 0.4846 but we select our maximum valve coefficient to be $k_{vs} =$ 4.

5 Appendix

```
set(0, 'defaulttextinterpreter','Latex');

s = tf('s');
G = 1/((s^2+s+1)*(s+3));

%Study different values of kvs.
figure(1)
subplot(711)
G_1 = 1*G;
nyquist(G_1); legend('Kvs 1');
subplot(712)
G_16 = 1.6*G;
nyquist(G_16);legend('Kvs 1.6');title('');
subplot(713)
G_25=2.5*G;
nyquist(G_25);legend('Kvs 2.5');title('');
subplot(714)
G_4=4*G;
nyquist(G_4);legend('Kvs 4.0');title('');
subplot(715)
G_63=6.3*G;
nyquist(G_63);legend('Kvs 6.3');title('');
subplot(716)
G_10 = 10*G;
nyquist(G_10);legend('Kvs 10');title('');
subplot(717)
G_160 = 16*G;
nyquist(G_160);legend('Kvs 16');title('');

%Circle criterion - without saturation
x_1 = [0,0.2,0.4,0.6,0.8,1];
y_1=[0,0.12,0.26,0.42,0.61,1];  %Lower curve
y_2=[0,0.3,0.55,0.75,0.92,1];   %Upper curve
figure(2)
plot(x_1,y_1);title('Circle Criteria Non-linearity');
   hold on;
plot(x_1,y_2);hold off;

coef_1 = polyfit(x_1,y_1,1);
k_1 = coef_1(1);          %Slope lower curve
coef_2 = polyfit(x_1,y_2,1);
k_2 = coef_2(1);          %Slope upper curve
```

```
r=(-1/k2+1/k1)/2;
m = (-1/k1-1/k2)/2;

figure(3)
hold on;
nyquist(G);title('Nyquist diagram without saturation')
    ;
fi=(-1.2:0.1:1)*pi;
x=m+r*exp(1i*fi);
plot(x);
hold off;

%Circle criterion - with saturation
figure(4)
hold on;
nyquist(G); title('Nyquist diagram with saturation');
xlim([-1.2 1]);
xline(-1);
hold off;

%Describing function
kvs_1=1;
C_1=4*kvs_1/(13*pi);
kvs_2=4;
C_2=4*kvs_2/(13*pi);
kvs_3=6.3;
C_3=4*kvs_3/(13*pi);

figure(5)
t = 1:50;
y_1 = C_1 *sin(t);
y_2 = C_2 *sin(t);
y_3 = C_3 *sin(t);
subplot(311);plot(t, y_1,'b-'); legend('kvs=1');
subplot(312);plot(t,y_2,'g-');legend('kvs=4');
subplot(313);plot(t,y_3,'r-');legend('kvs=6.3')

C_0 = (pi*C_2)/4;   % C_2 > C_0
```