Lab 2 Non-Linear System

January 19, 2021

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1 Introduction

Our aim is to study the stability of a non-linear system with different input signals. The input signals we will be using are

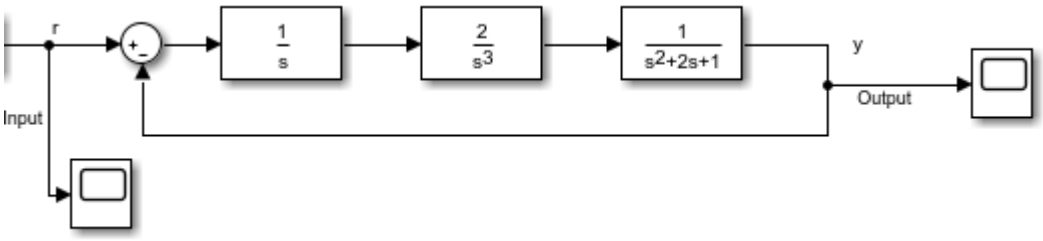
1. Constant
2. Step response
3. Ramp
4. Sine
5. Sine plus ramp
6. Pulse

In the second part, we will switch the quadratic non-linearity of valve to saturation and study the stability under above 6 input signals.

2 Method

2.1 Valve characteristic f(x) = x^2

In this section, we will show the block diagram of quadratic non-linear system with input r. The scope measurements will be shown and discussed in the results sections. Converting the non-linearity in s-domain, x^2 becomes $2/s^3$



1. **Constant input** Constant input of 0.5 gives ouput of $2.3*10^{60}$ at around 190 seconds. Before that the output is zero. See figure 1.1a. Changing input to constant input to 0.1 reduces the shoot in output amplitude. See figure 1.1b at around 190 seconds.

2. **Step input** Step input of 0.5 gives ouput close to zero uptill 190 seconds. Then there is a spike of $1.3*10^{59}$ for around 10 seconds. See figure 1.2a. In the second case, the ouput shoots in the negative direction to $-2*10^{60}$. See figure 1.2a and figure 1.2b.

3. **Ramp input** Ramp input at slope 0.01 gives output spike at around $-2.6*10^{60}$ around 200 seconds. See figure 1.3.

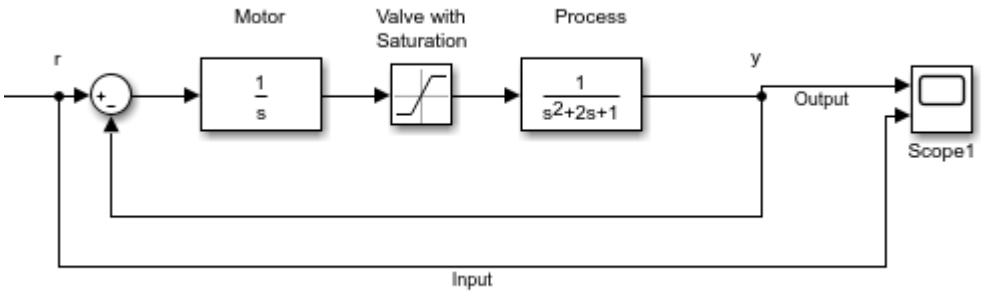
4. **Sine wave input** For sine wave amplitude 0.01, output is zero for 190 seconds and then shoots down to $-3*10^{60}$ around 200 seconds. Figure 1.4a. Increasing the sine wave amplitude to 0.1, shows no change in output.Figure 1.4b.

5. **Sine wave plus ramp input** A ramp with slope 0.01 and sine with amplitude 0.01 is given as input. See figure 1.5a, the ouput shoots down to $-2.6*10^{60}$. We increase the sine amplitude to 0.1, the output is shown in figure 1.5b. The ouput amplitude increases to more than $-3*10^{60}$.

6. **Pulse input** A pulse input with amplitude 0.1, pulse width 50% and period 50 seconds gives an ouput shown in figure 1.6a. When the pulse period is changed to 10 seconds, we get the ouput shown in figure 1.6b.

2.2 Valve characteristic Saturation

In this section, we will show the block diagram of quadratic valve with saturation system having input r. Upper and lower limits of saturation are +0.5 and -0.5 respectively. Any input in a range between -0.5 and 0.5 will give a linear ouput. The scope measurements will be shown and discussed in the results sections.



1. **Constant input** Constant input of 1 gives constant output 0.5. Changing input to constant 0.4 gives ouput at 0.4. See figures 2.1a and 2.1b respectively.

2. **Step input** Step input of 1 gives step output 0.5. i.e. saturation. Step input of 0.4 gives a step output at 0.4. Here the valve is linear and ouput follows input. See figures 2.2a and 2.2b respectively.

3. **Ramp input** A ramp input with slope 1 keeps the output at 0. Figure 2.3a. Changing the input slope to 0.01 makes the output follow input till it reaches the upper saturation limit. Figure 2.3b.

4. **Sine wave input** Sine wave input of amplitude 1 produces sine wave output of amplitude 0.3. Figure 2.4a. But if the amplitude of input is reduced to 0.1, the ouput follows the input. Figure 2.4b.

5. **Sine wave plus ramp input** Keeping the above settings for sine wave amplitude 0.1 and ramp slope 0.01, gives an output that follows both ramp and sine input till the saturation point 0.5. See figure 2.5.

6. **Pulse input** Pulse has amplitude 1 and period width 50%. The output is constant at 0.5. See figure 2.6a.
   Change the pulse amplitude to 0.4, ouput follows the input. See figure 2.6b

3 Results

3.1 Valve characteristic f(x) = x^2

Figure 1.1a

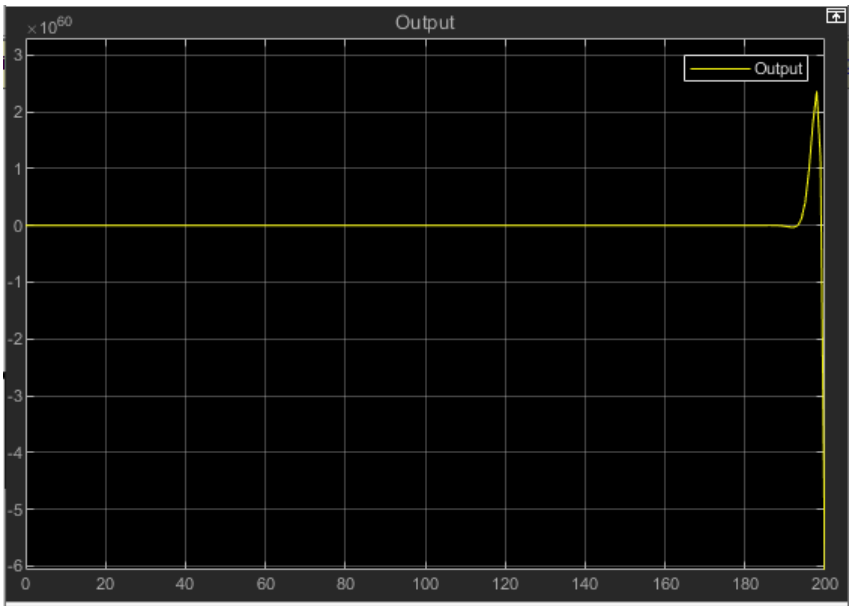


Figure 1.1b

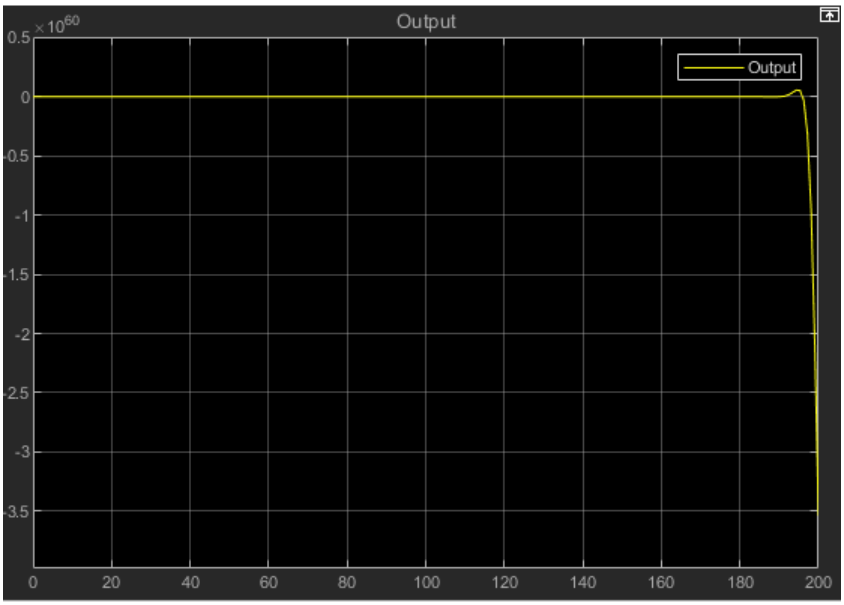


Figure 1.2a

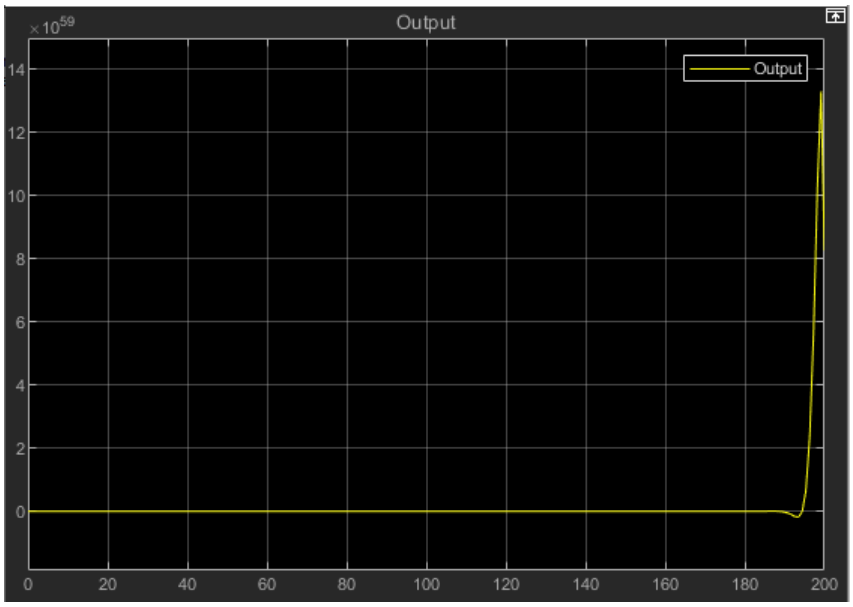


Figure 1.2b

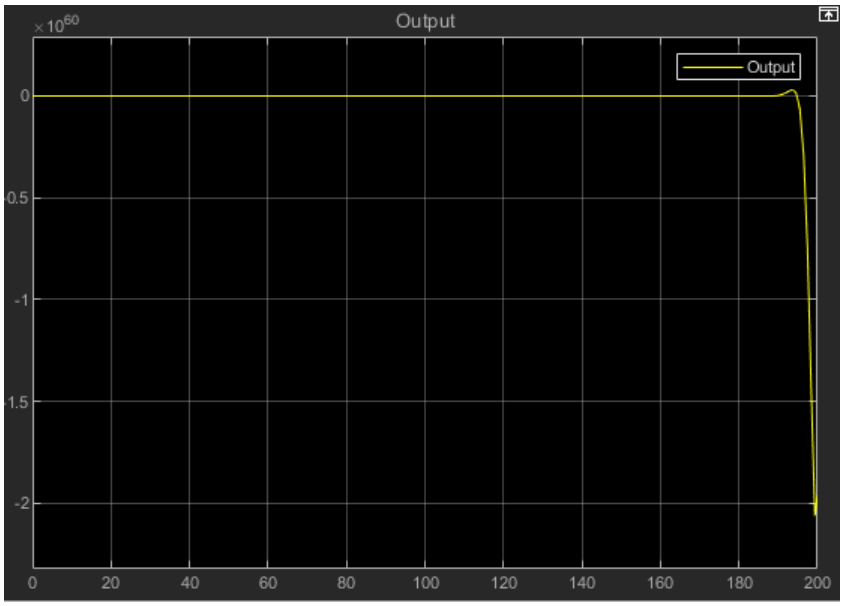


Figure 1.3

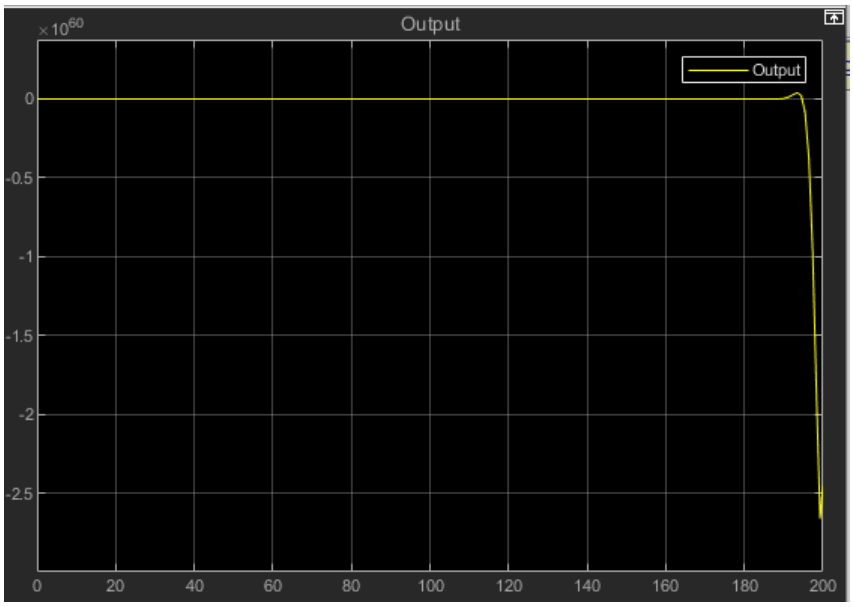


Figure 1.4a

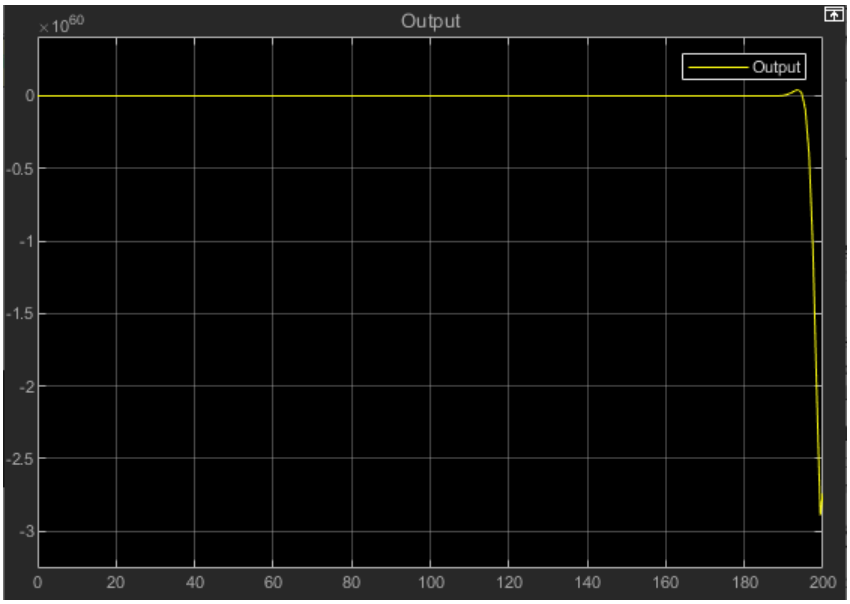


Figure 1.4b

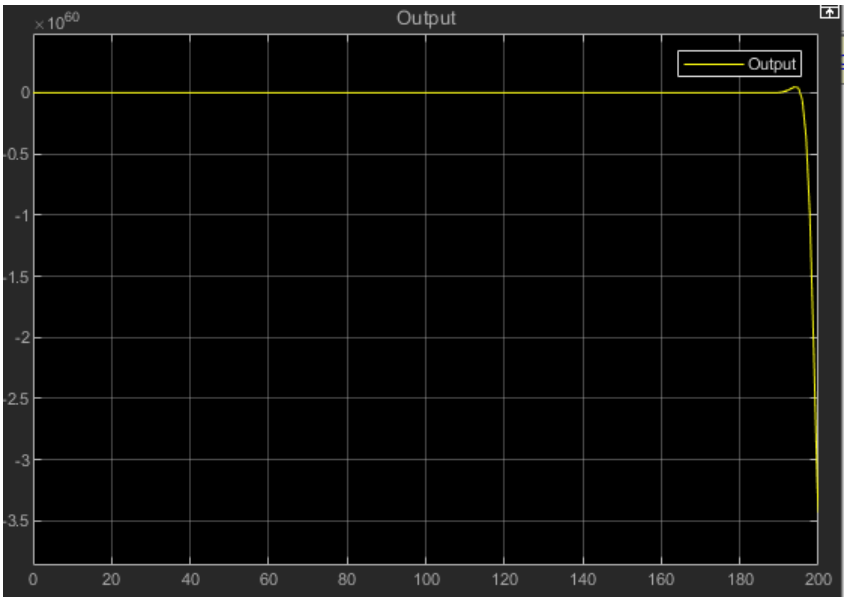


Figure 1.5a

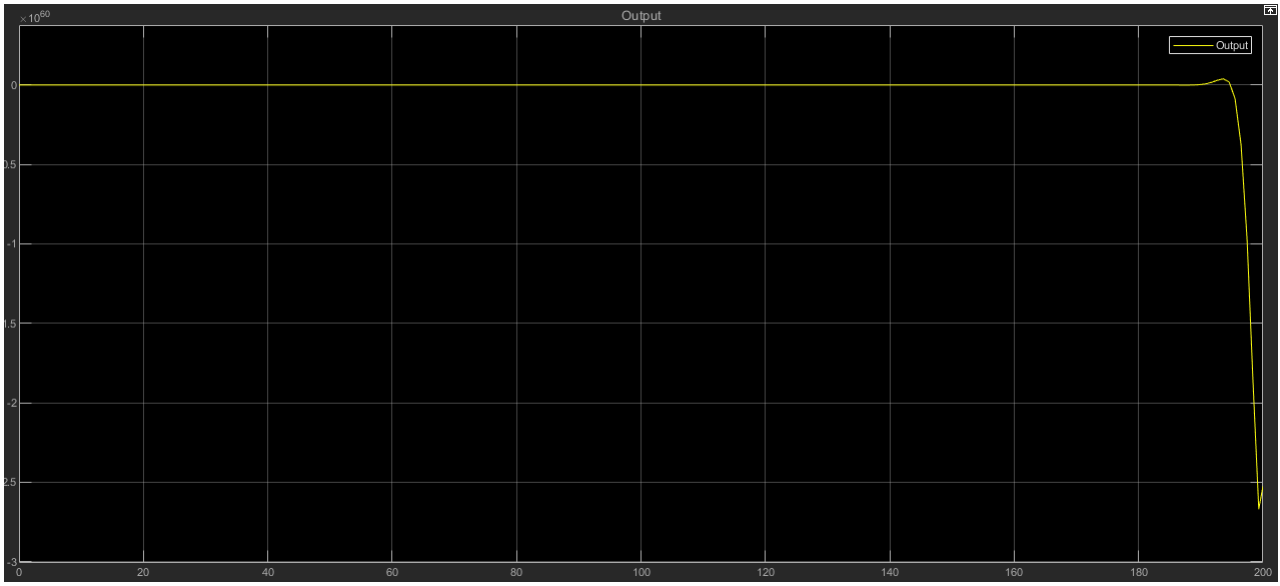


Figure 1.5b

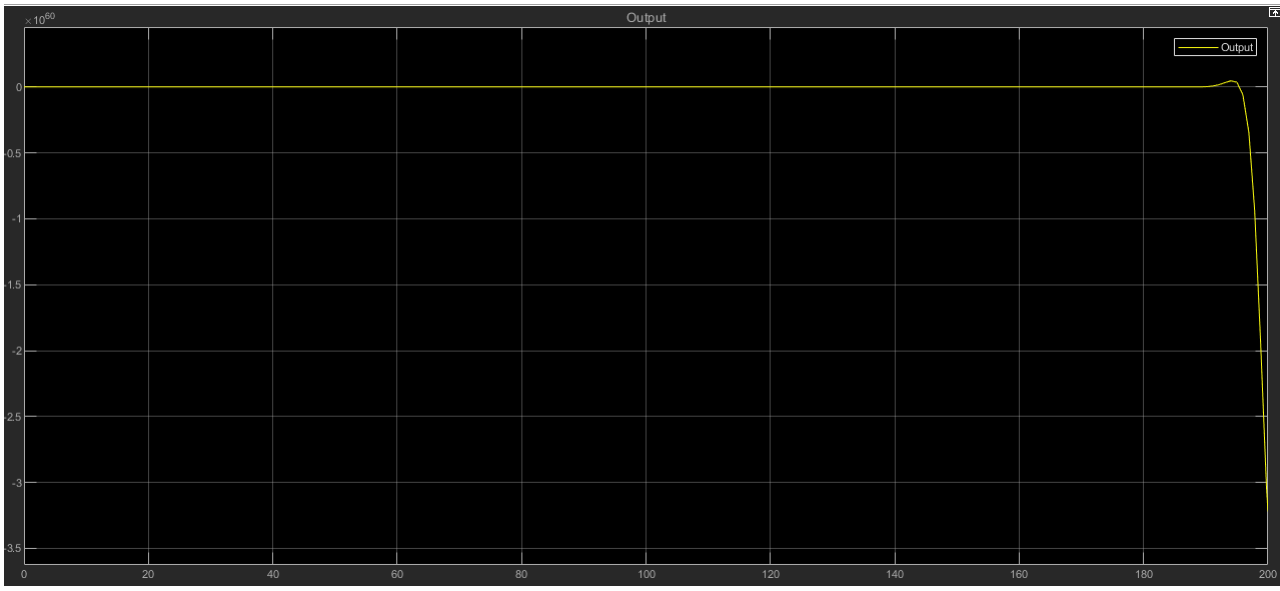

Output
$\times 10^{60}$
Output
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
20
40
60
80
100
120
140
160
180
200

Figure 1.6a

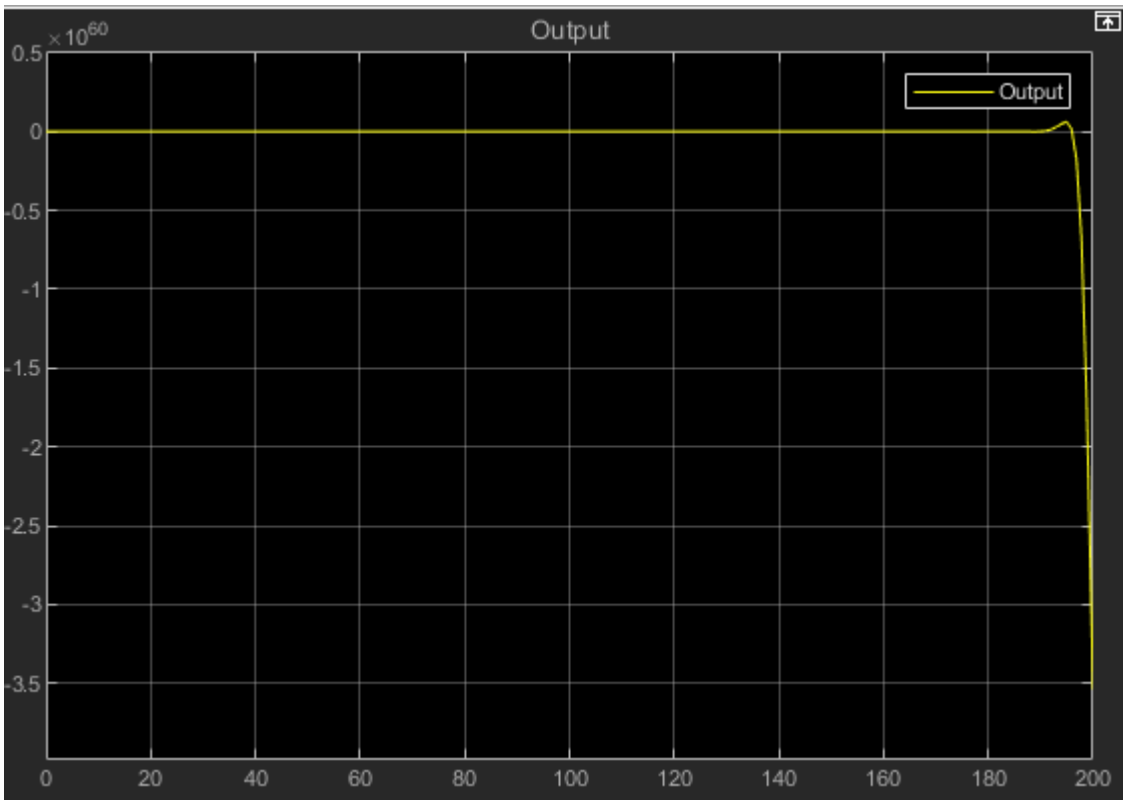
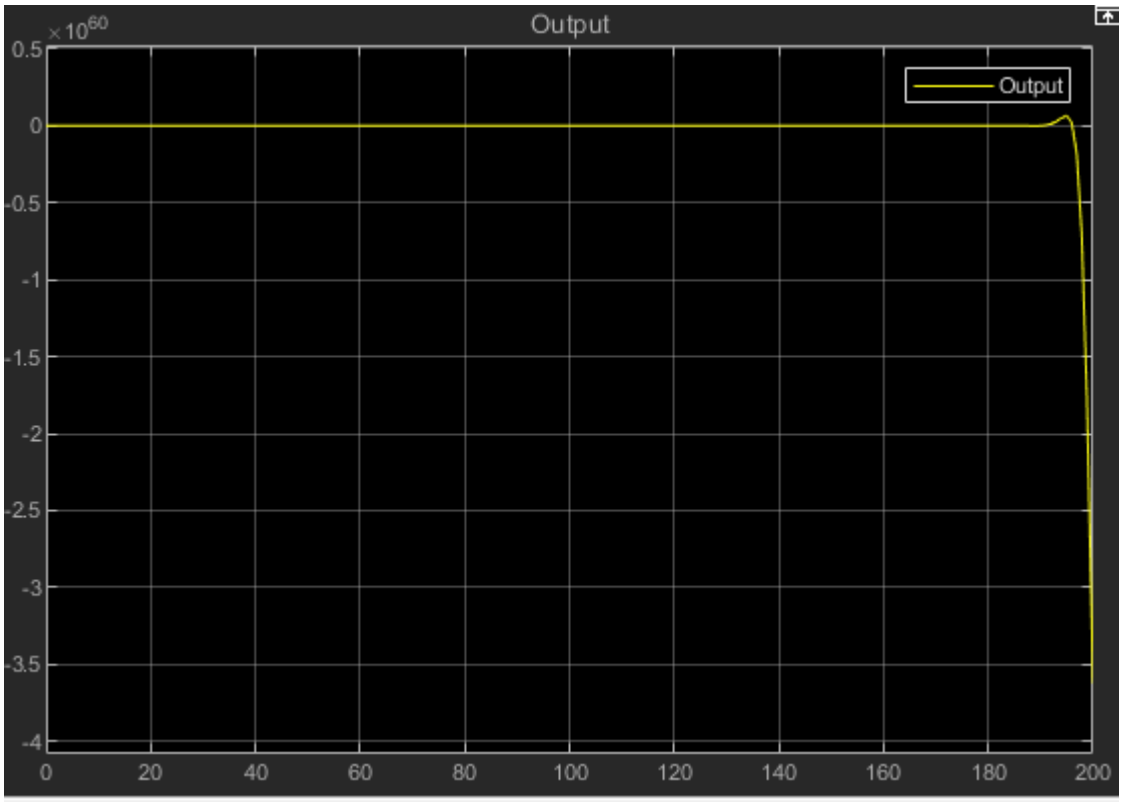

Output
×10^60
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
20
40
60
80
100
120
140
160
180
200
Output

Figure 1.6b



3.2 Valve characteristic saturation

Figure 2.1a

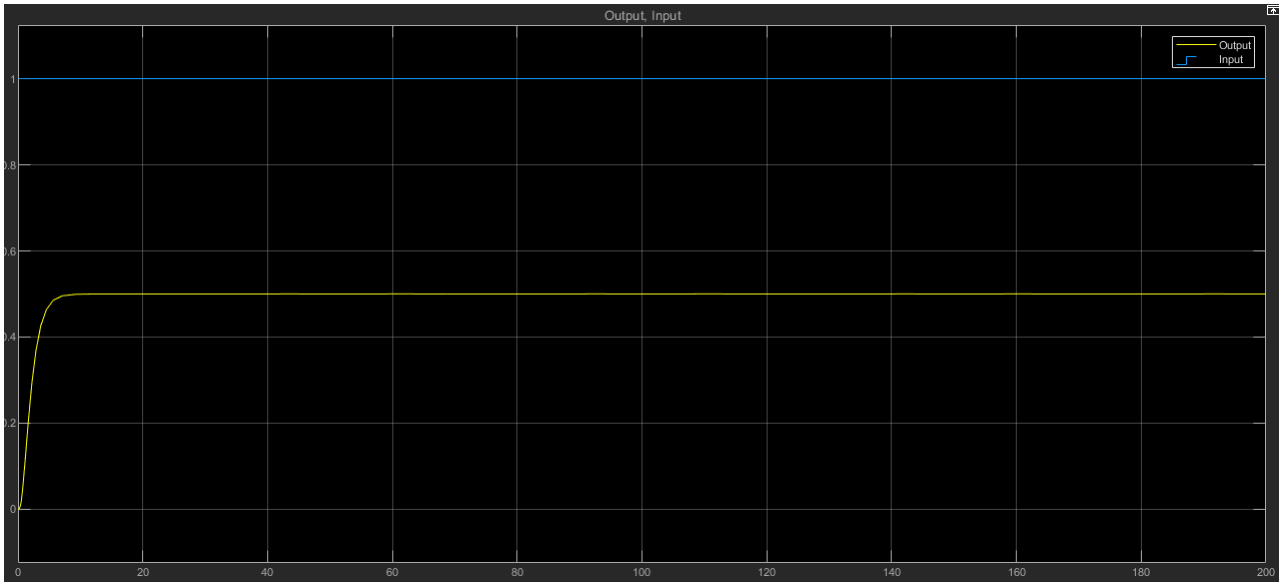


Figure 2.1b

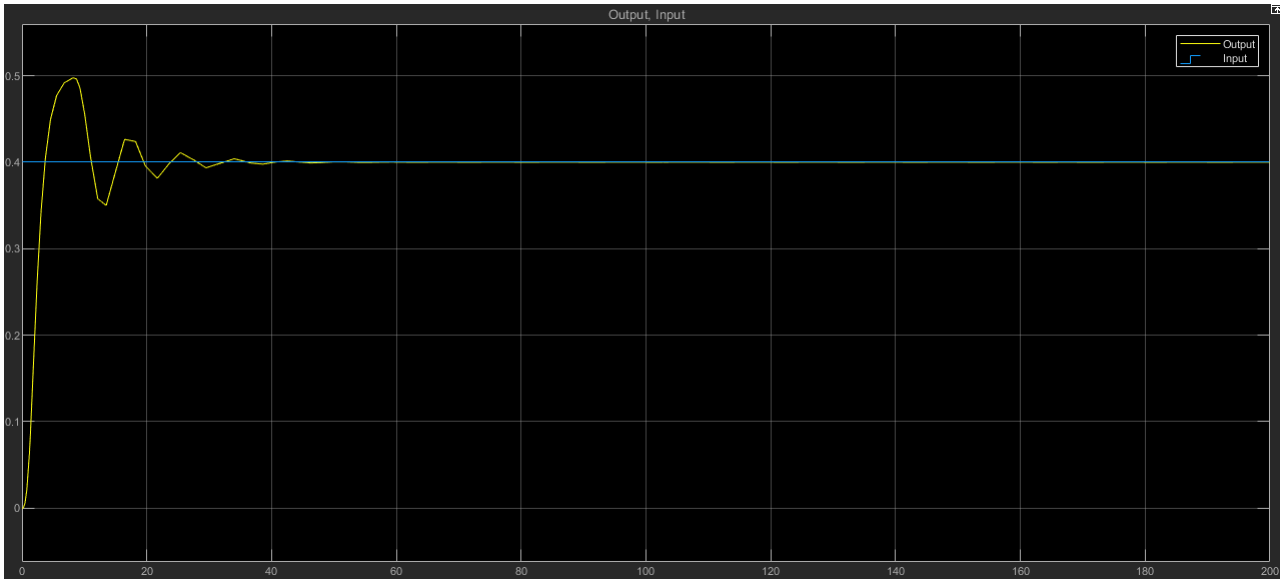


Figure 2.2a

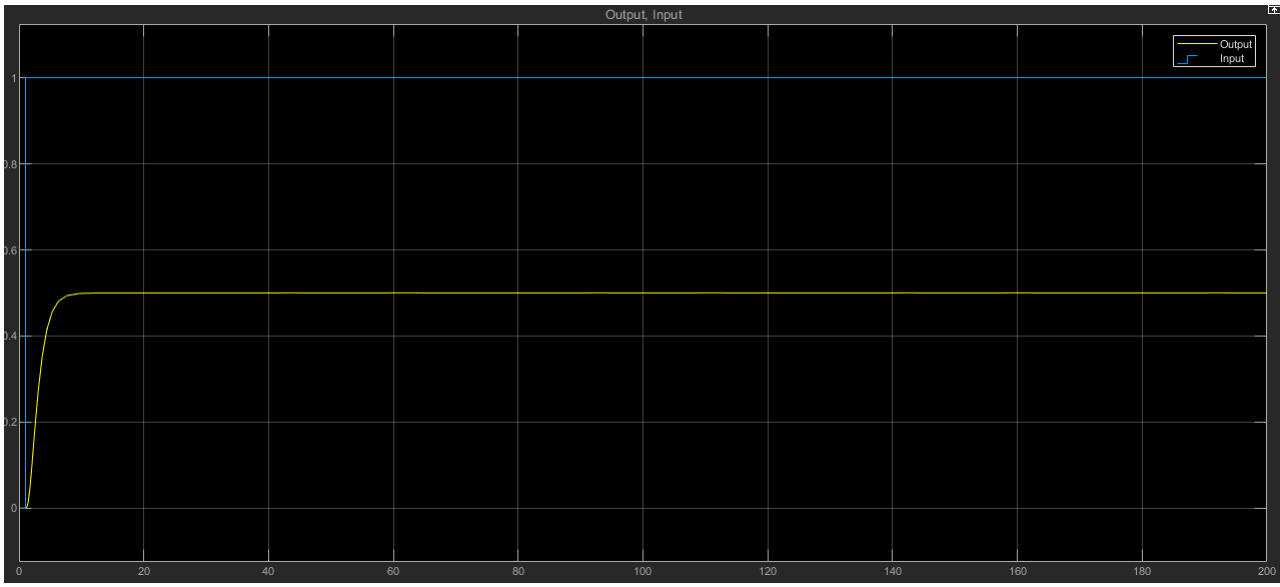


Figure 2.2b

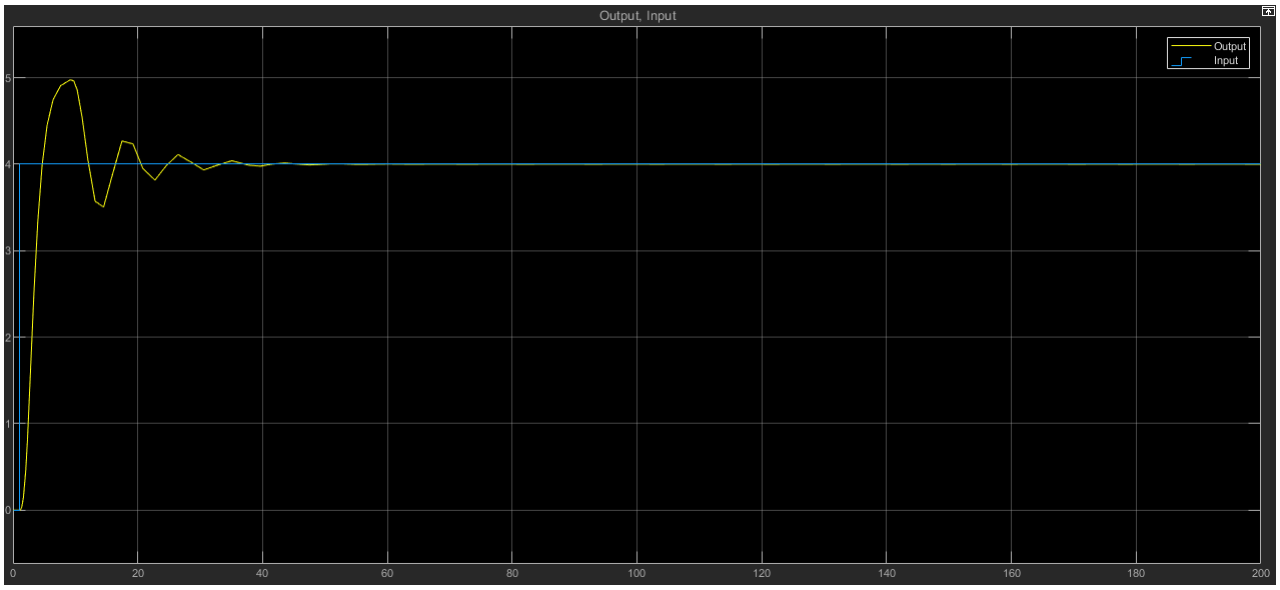


Figure 2.3a

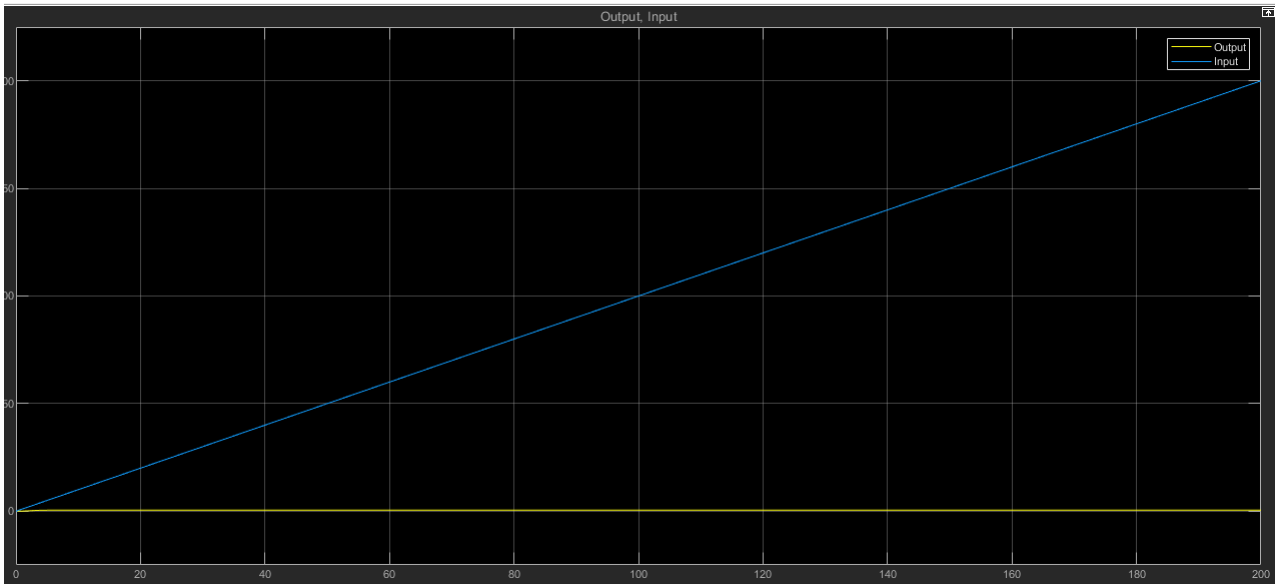


Figure 2.3b

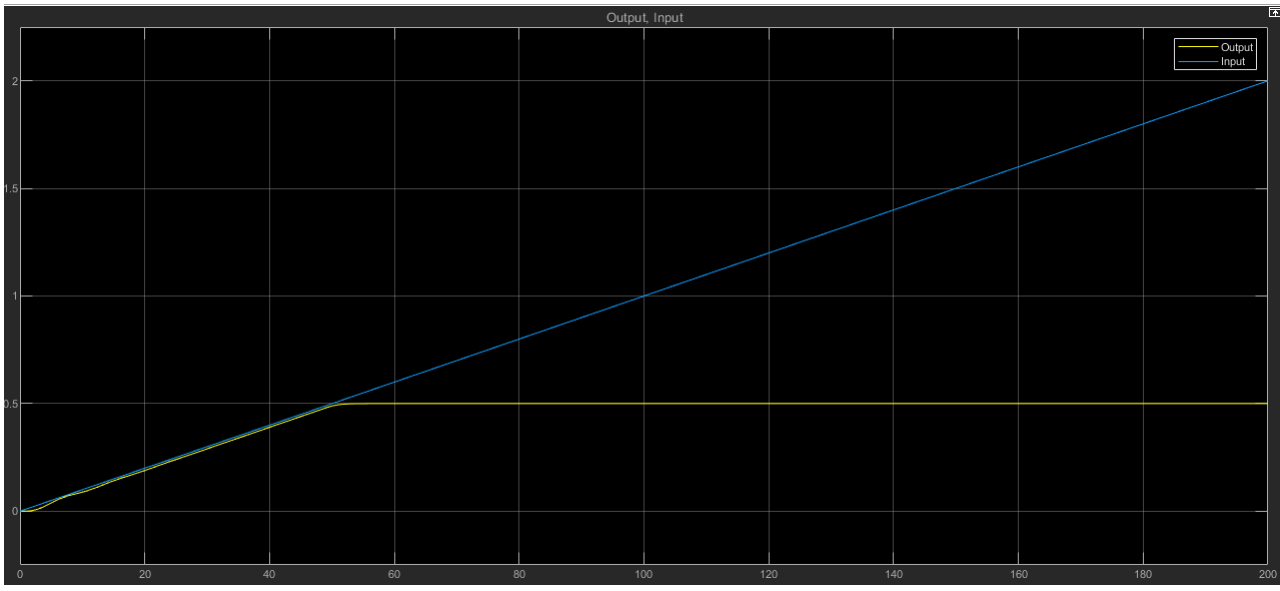


Figure 2.4a

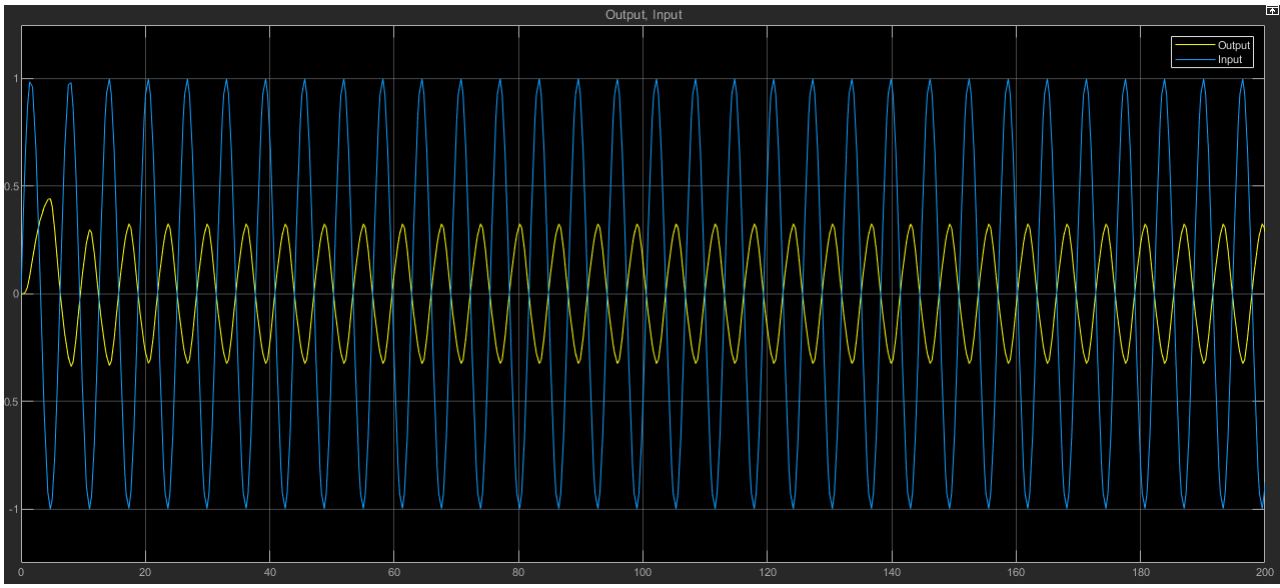


Figure 2.4b

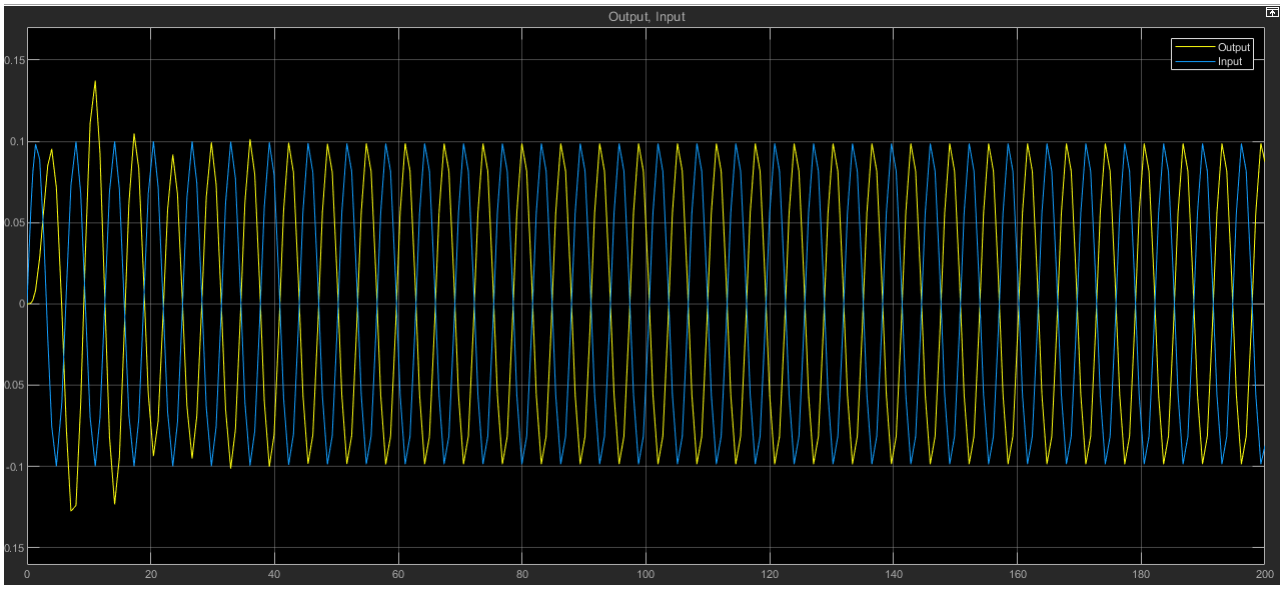


Figure 2.5

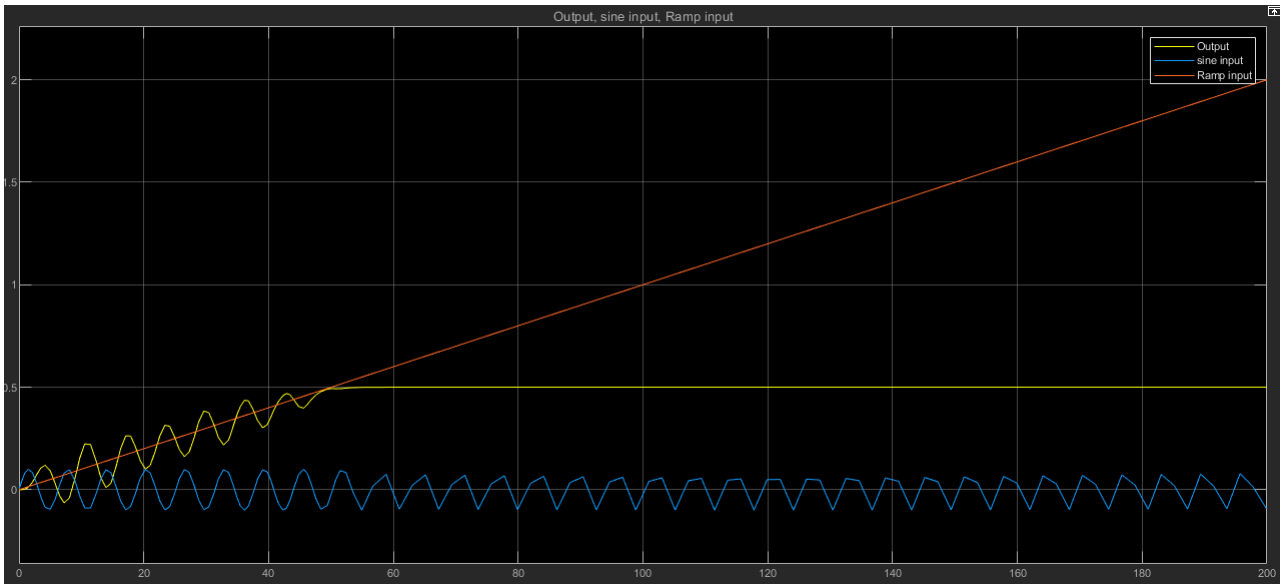


Figure 2.6a

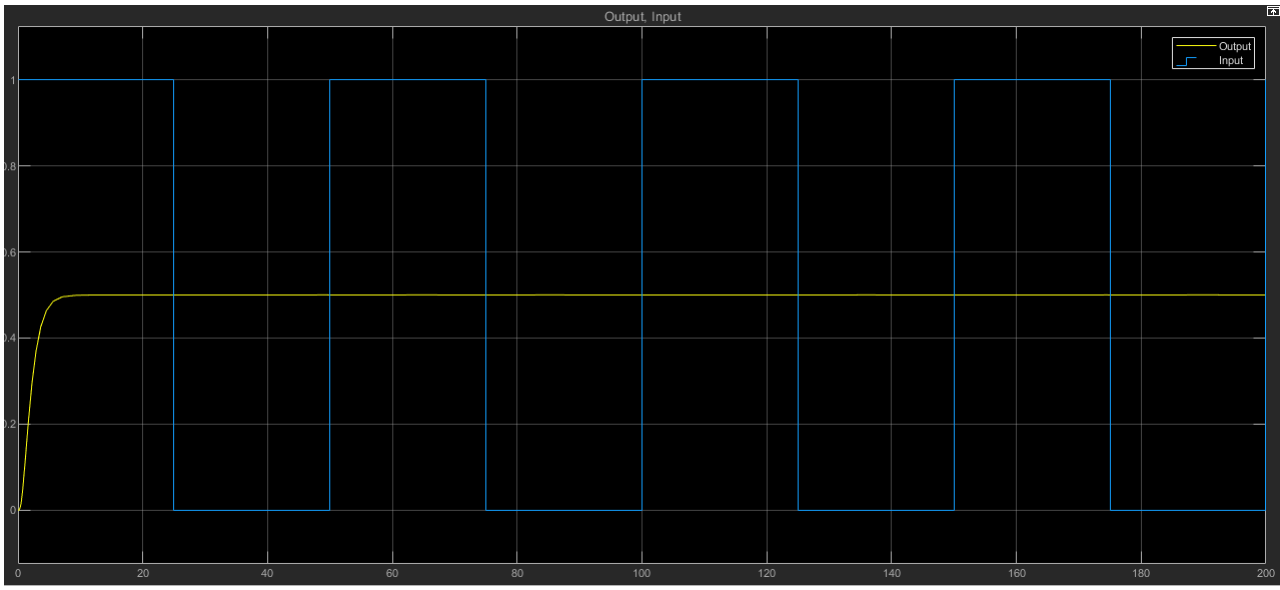
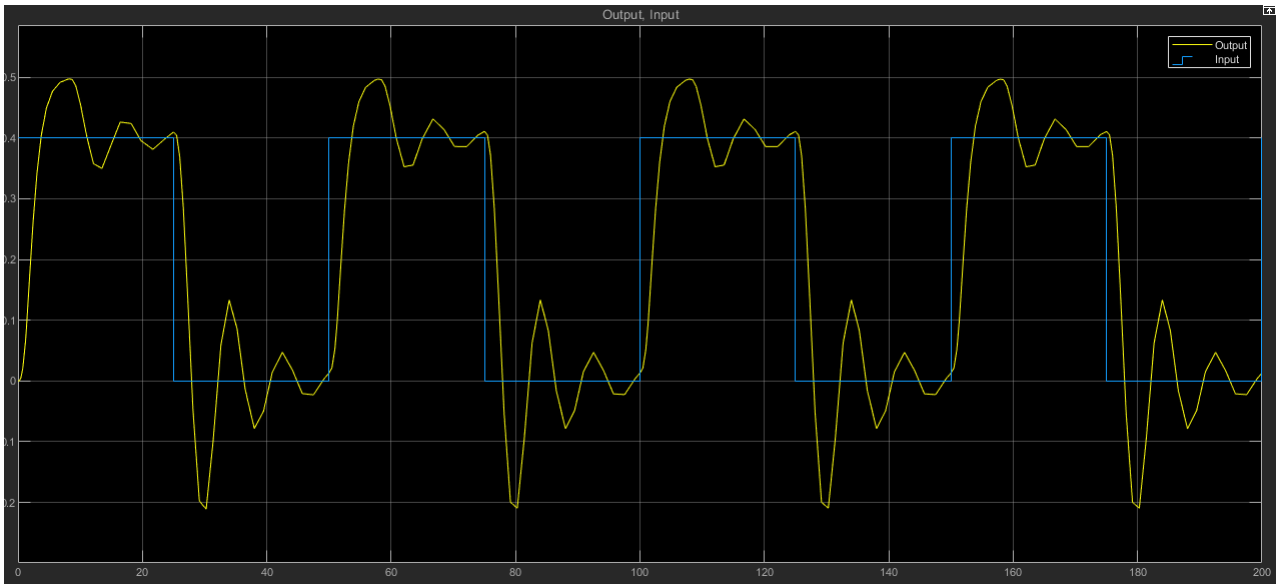


Figure 2.6b



4 Discussion

1. **Valve characteristic f(x) = x^2**
   From the figures 1.1 to 1.6, we see that the small changes in input leads to very big changes in the ouput. We can say that the system is unstable for all 6 input signals.

2. **Valve characteristic Saturation**
   The saturation system is bounded by limits -0.5 and +0.5 i.e.

$$sat(f(x)) = \begin{cases} \text{x} & \text{if } -0.5 < (x) < 0.5 \\ 0.5 & \text{if } x >= 0.5 \\ \text{-0.5} & \text{if } x <= -0.5 \end{cases} \quad (1)$$

   The system is stable in the region -0.5 < (x) < 0.5 for all 6 input signals .i.e. small changes in input leads to small changes in the output.