PUBH 7150/8150
Homework 4 Solutions

Homework should only be submitted to Canvas as a single document. Please make sure to name the document: PUBH7150_Class_First Name_Last Name_HW4.docx. Show your work. If excel is used for calculations, please submit as a second document but with the same document name as the word/pdf document. For probability calculations, please show the probability notation you are solving for (if not provided as apart of the problem, e.g. P(X>x) or P(X=x)) and round to a minimum of 4 decimals (if needed, some answers may be more exact, i.e. 0.25 or 0.5 while others might be 0.212583312, round the latter to 4 decimals), for distributions statistics or "scores", round to 2 decimal places.

1. We are currently working with a researcher who focuses on auditory services. This researcher has data measuring dominant ear scores measuring perceived sounds. This data follows a normal distribution with a mean of 75 and variance of 9. Use this information to answer the following questions.

a) What is the probability a person has an ear score of 81?

P(X=81) = $\frac{1}{\sqrt{9\,x\,2\pi}}\exp\left(\frac{-(81-75)^2}{2\,x\,9}\right)$

$= \frac{1}{\sqrt{18\pi}}\exp\left(\frac{-(6)^2}{18}\right)$

$= \frac{1}{\sqrt{18\pi}}\exp\left(\frac{-36}{18}\right)$

$= \frac{1}{\sqrt{18\pi}}\exp(-2)$

=0.0180

b) What is the probability a person has an ear score of greater than 69?

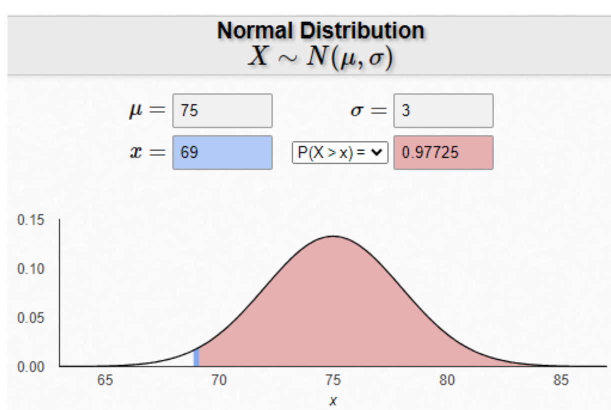
From the question

Mean of the data is (μ) = 75

variance of data is σ2 = 9

Standard deviation σ = 3 , x= 69

Placing the values in the online calculator for normal distribution



$P(Z > 69) = 0.9773$

b) What is the probability a person has an ear score of at most 77?

From the question

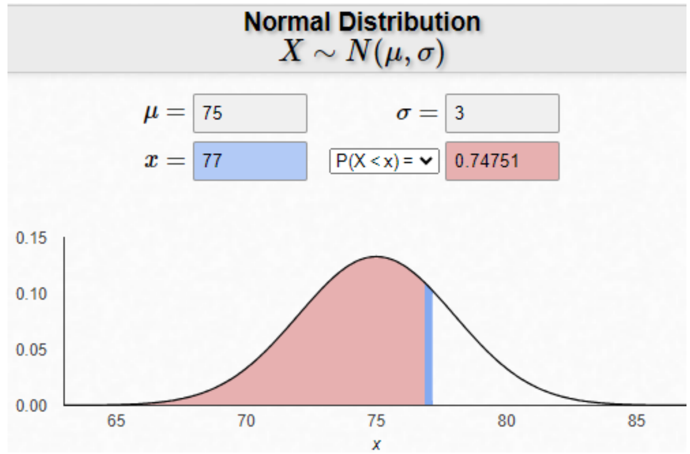
Mean of the data is (μ) = 75

variance of data is σ2 = 9

Standard deviation σ = 3 , x= 77

Placing the values in the online calculator for normal distribution

P(X < 77)=0.7475



c) What is the probability a person has an ear score between 71 and 79?

To calculate the probabilities in between 71 and 79

$\boldsymbol{we\ calculate\ P(X < 79) - P(X < 71)}$

Calculating probability for $\boldsymbol{P(X < 79)}$

From the question

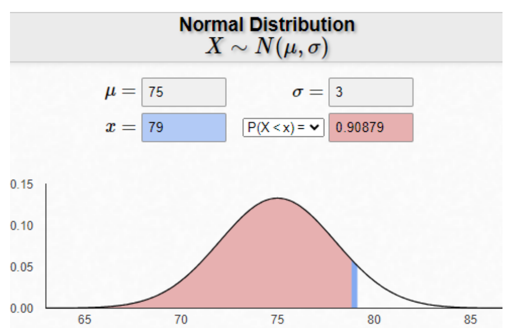
Mean of the data is (μ) = 75

variance of data is σ2 = 9

Standard deviation σ = 3, x= 79

Placing the values in the online calculator for normal distribution

P(X < 79)= $\mathbf{0.9088}$



Calculating probability for $P(X < 79)$

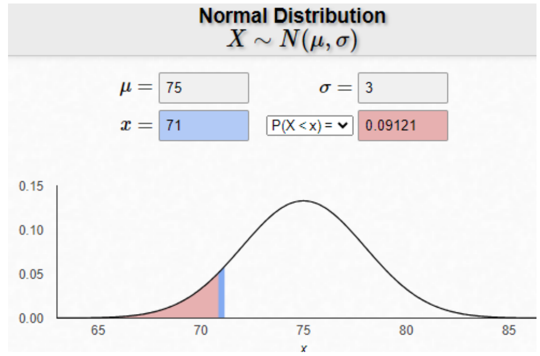
From the question

Mean of the data is (μ) = 75

variance of data is σ2 = 9

Standard deviation σ = 3, x= 71

Placing the values in the online calculator for normal distribution



P(X < 71)= $\mathbf{0.0912}$

P(71<X<79) = $\mathbf{0.9088 - 0.0912 = 0.8176}$

e) What is the probability a person has an ear score of 73?

P(X=73) =$\frac{1}{\sqrt{9\,x\,2\pi}}\,\mathbf{exp}\left(\frac{-(73-75)^2}{2\,x\,9}\right)$

$$= \frac{1}{\sqrt{18\pi}}\,\mathrm{exp}\left(\frac{-(-2)^2}{18}\right)$$

$$= \frac{1}{\sqrt{18\pi}}\,\mathrm{exp}\left(\frac{-4}{18}\right)$$

$$= \frac{1}{\sqrt{18\pi}}\,\mathrm{exp}(-0.22223)$$

=0.1065

2. Let Z follow a standard Normal distribution

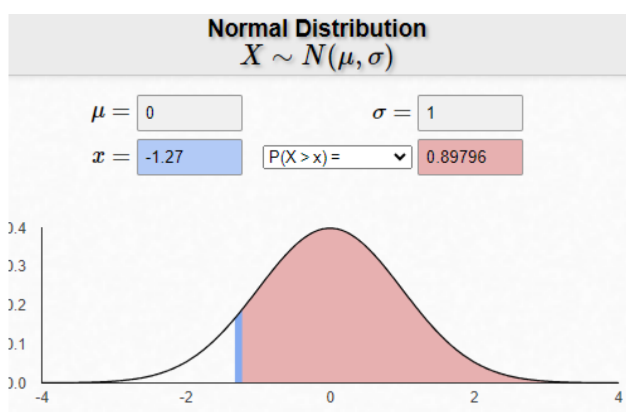
a) Find P(Z > -1.27)=

From the question

Mean of the data is (μ) = 0

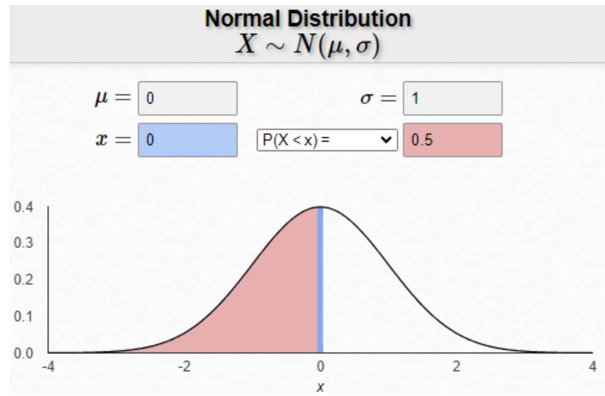
Standard deviation σ = 1, x= -1.27

Placing the values in the online calculator for normal distribution



P(Z > 71)= 0.8980

b) Find P(Z < 0)



From the question

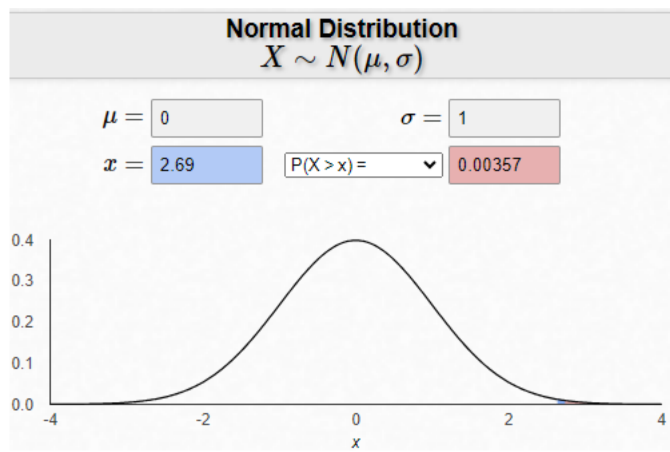
Mean of the data is (μ) = 0

Standard deviation σ = 1, x= -1.27

Placing the values in the online calculator for normal distribution

P(Z < 0)= 0.5

c) Find P(Z > 2.69)



From the question

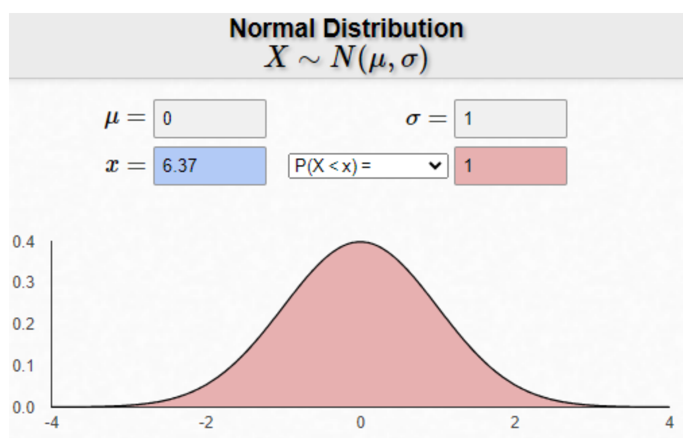
Mean of the data is (μ) = 0

Standard deviation σ = 1, x= 2.69

Placing the values in the online calculator for normal distribution

P(Z > 2.69) =0.0036

d) Find P(Z < 6.37)=**1**



From the question

Mean of the data is (μ) = 0

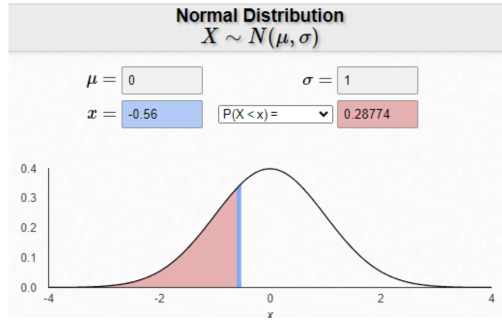
Standard deviation σ = 1, x= 6.37

Placing the values in the online calculator for normal distribution

P(Z < 6.37) =1

e) Find P(-1.78 < Z < -0.56)

Finding probability for P(Z < -0.56)



From the question

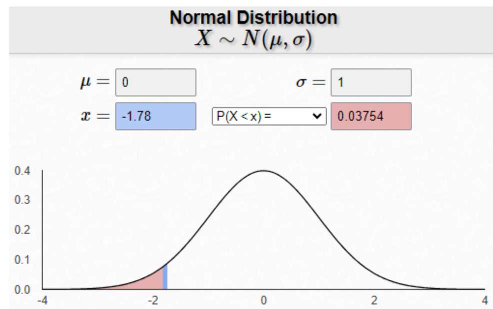
Mean of the data is (μ) = 0

Standard deviation σ = 1, x= -0.56

Placing the values in the online calculator for normal distribution

P(Z < -0.56) =0.2877

Finding probability for P(Z < -0.56)



From the question

Mean of the data is (μ) = 0

Standard deviation σ = 1, x= -1.78

Placing the values in the online calculator for normal distribution

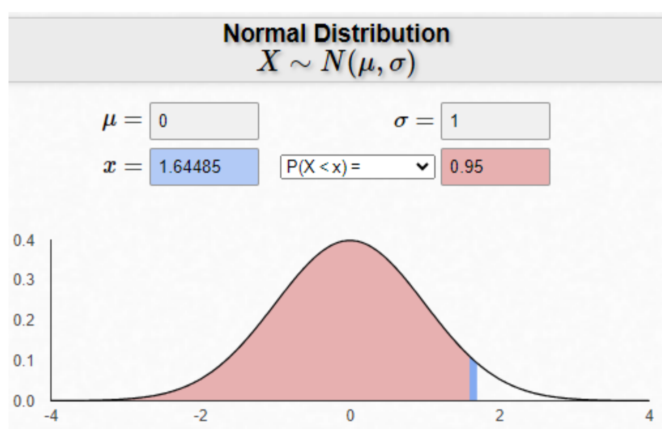
P(Z < -0.56) =0.0375

Calculating P(-1.78 < Z < -0.56)

P(Z < -0.56)-P(Z < -1.78) = 0.2877 - 0.0375 =0.2502

f) Find the constant c such that P(Z < c) = 0.95;

From the question



Mean of the data is (μ) = 0

Standard deviation σ = 1,

Placing the values in the online calculator for normal distribution

P(Z < c) =0.95

So the value of c is 1.6449

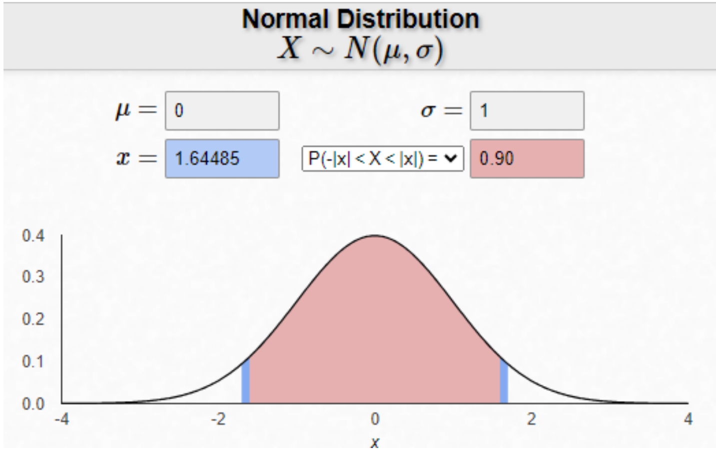
g) Find the constant c such that P(-c< Z < c) = 0.90; **c = 1.645 (+/- 1.645)**

From the question

Mean of the data is (μ) = 0

Standard deviation σ = 1

Placing the values in the online calculator for normal distribution



P(-c< Z < c) =0.90

So the value of c is 1.6449

3. Cochlear implants are set to last "a lifetime", however, manufacture only insure it for up to 10 years. In other words, the failure time for a cochlear implant follows an exponential distribution with a mean of 10 years.

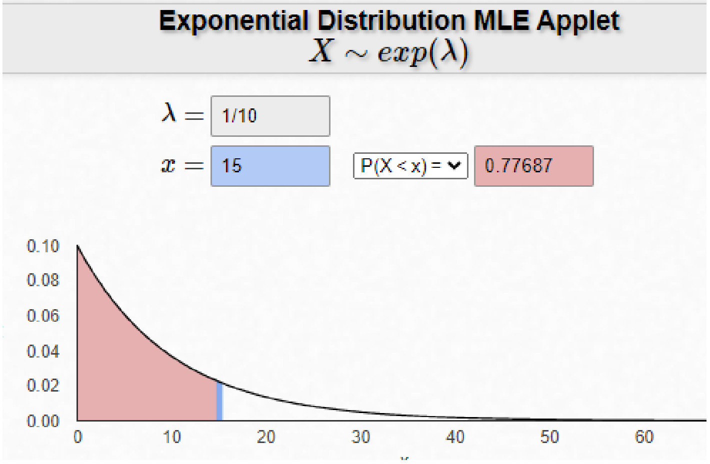
a) What's the probability that the implant will last up to 15 years?

From the question

Average rate (λ) = 1/10

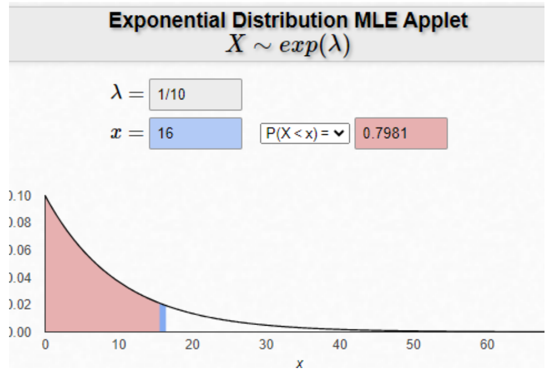
x = 15

Placing the values in the online calculator for exponential distribution



$$\mathbf{P(X \leq 15) = 0.7769}$$

b) What's the probability that the implant will last between 5 and 16 years?



We calculate the probability as $P(X \le 16) - P(X \le 5)$

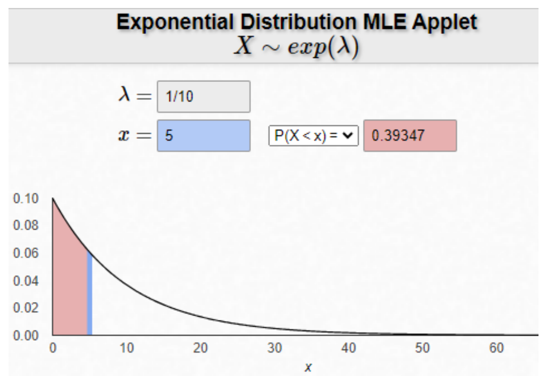
From the question

Average rate (λ) = 1/10

x = 16

Placing the values in the online calculator for exponential distribution

P(X≤ 16) $= 0.7981$



From the question

Average rate (λ) = 1/10

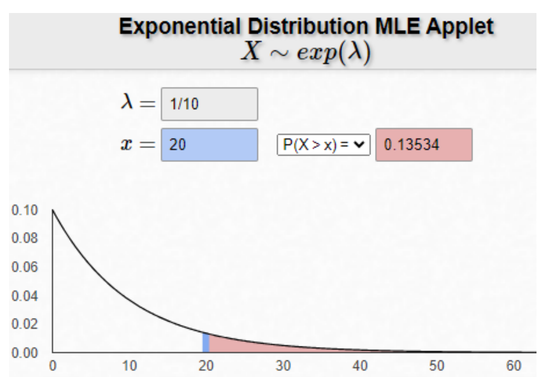
x = 16

Placing the values in the online calculator for exponential distribution

P(X≤ 5) $= 0.3935$

$$P(5 \le X \le 16) = 0.7981 - 0.3935 = 0.4046$$

c) What's the probability that the implant will last beyond 20 years?



From the question

Average rate (λ) = 1/10

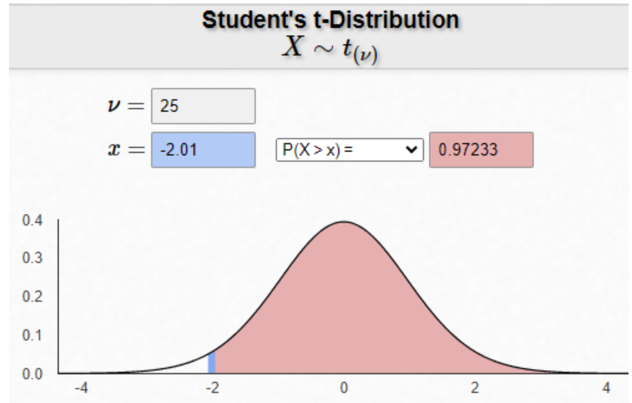
x = 20

Placing the values in the online calculator for exponential distribution

P(X≥ 20) $= 0.1353$

4. Find the following statistic or probability:

a) $P(t_{(25)} \geq -2.01)$



From the question

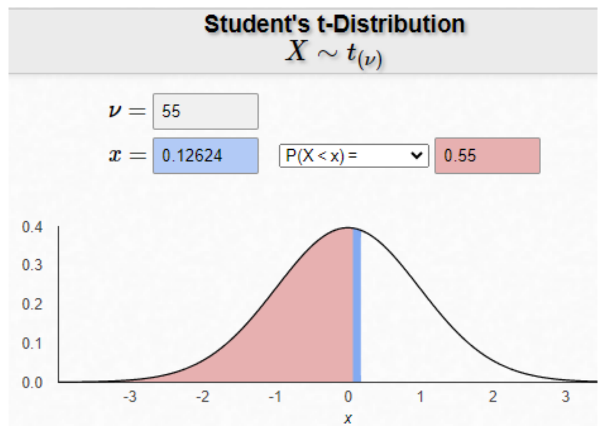
Shape (v) = 25

x = -2.01

Placing the values in the online calculator for student's T distribution

$\mathbf{P(t_{(25)} \geq -2.01) = 0.9723}$

b) $t_{(55),0.55}$



From the question

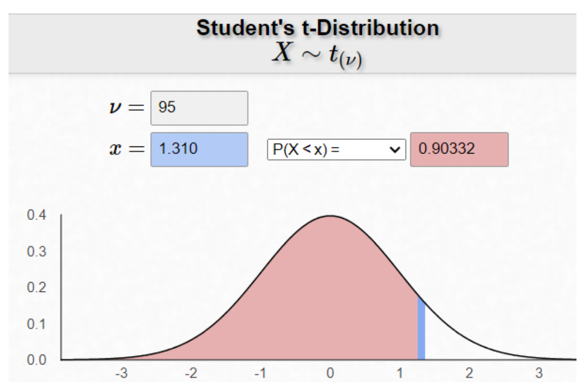
Shape (v) = 55

P = 0.55

Placing the values in the online calculator for student's T distribution

The value of x is 0.1262

c) $P(0.528 \leq t_{(95)} \leq 1.301)$



From the question

Shape (v) = 95

x = 1.301

Placing the values in the online calculator for student's T distribution

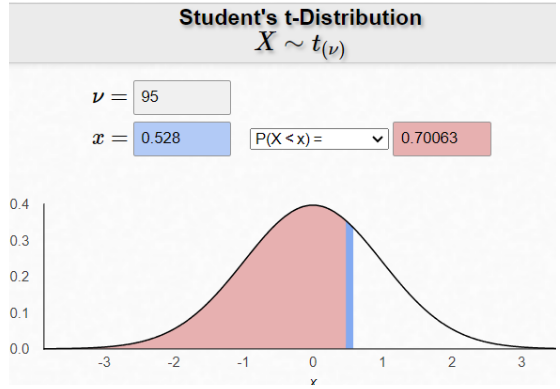
$\mathbf{P(t_{(95)} \geq 1.301) = 0.9018}$

From the question

Shape (v) = 95

x = 0.528

Placing the values in the online calculator for student's T distribution



$P(t_{(95)} \geq 0.528) = 0.70063$

$P(t_{(95)} < 1.310) - P(t_{(95)} < 0.528) = 0.9018 - 0.7006 = 0.2016$

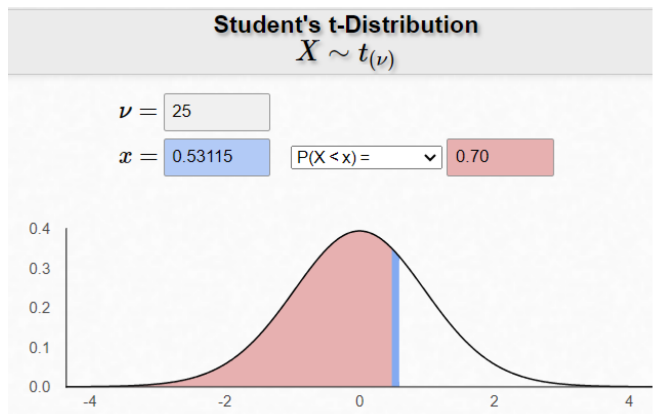
d) $t_{(25),0.70} = \mathbf{0.5312}$

From the question

Shape (v) = 25

P = 0.70

Placing the values in the online calculator for student's T distribution



$t_{(25),0.70} = 0.9723$

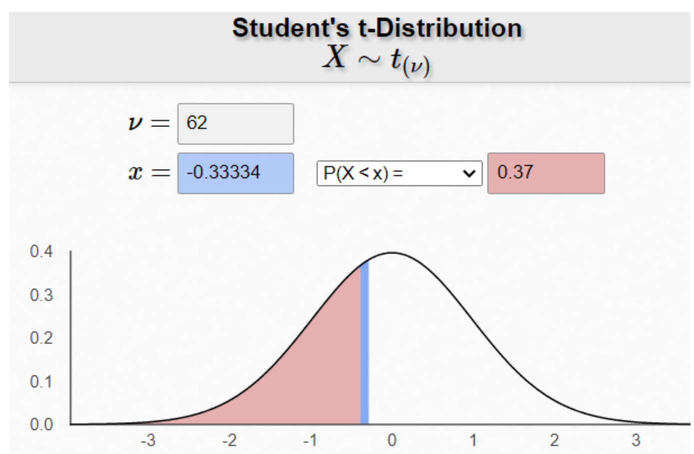
e) $P(t_{(62)} \leq c) = 0.37$

From the question

Shape (v) = 62

P = 0.37

Placing the values in the online calculator for student's T distribution



$P(t_{(62)} \leq c) = 0.37$

The value of c is -0.3334

f) $P(\chi^2_{15} > 11.59)$

From the question

ν = 15

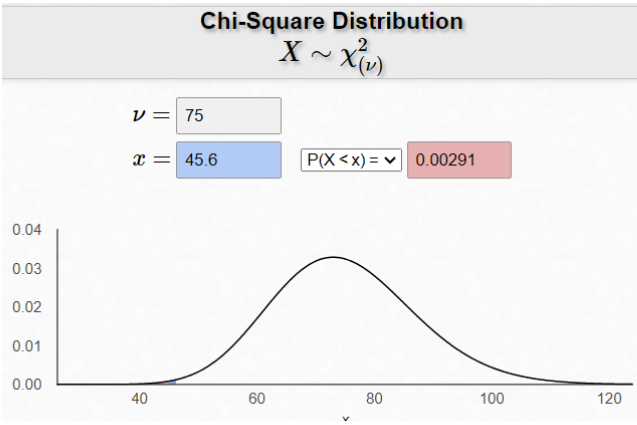
x = 11.59

Placing the values in the online calculator for chi-Square distribution

$\boldsymbol{P(\chi^2_{15} > 11.59)}$**= 0.71**

The value of P is 0.71

g) $P(\chi^2_{75} < 45.6)$



From the question

ν = 75

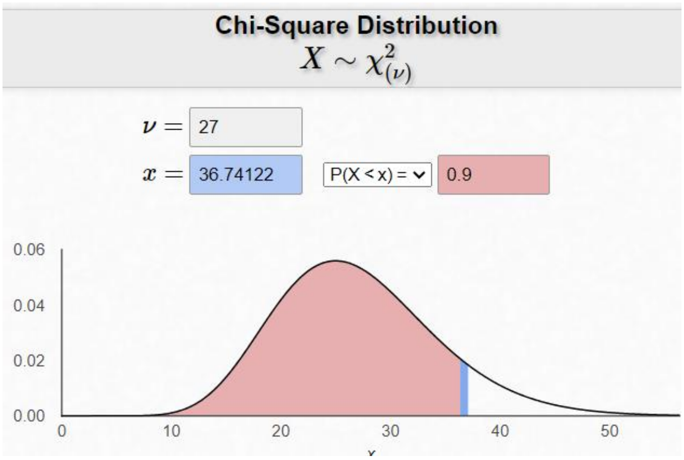
x = 45.6

Placing the values in the online calculator for chi-Square distribution

$\boldsymbol{P(\chi^2_{75} < 45.6)}$ **= 0.0029**

The value of P is 0.0029

h) $\chi^2_{27,0.90}$



From the question

ν = 27

P = 0.9

Placing the values in the online calculator for chi-Square distribution

$\chi^2_{27,0.90}$ = **36.74**

The value of x is 36.74

i) $P(F_{(55,35)} < 3.07)$ = **0.9997**

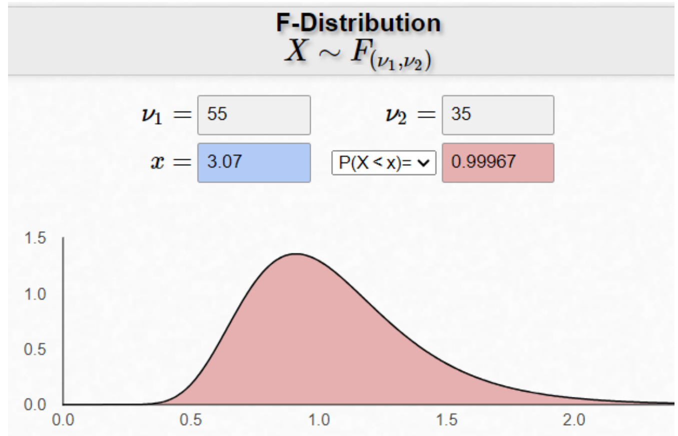
From the question

Numerator degrees of freedom v1 = 55

Denominator degrees of freedom v2 = 35

x = 3.07

Placing the values in the online calculator for F distribution



$\boldsymbol{P(F_{(55,35)} < 3.07)}$

The value of probability is 0.9997

$j)$ $P(F_{(50,75)} > c)$ =0.32, **c=1.12**

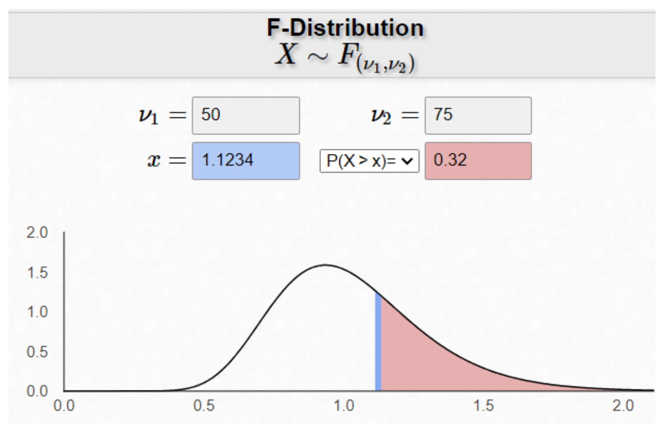
From the question

Numerator degrees of freedom v1 = 50

Denominator degrees of freedom v2 = 75

Probability is 0.32

Placing the values in the online calculator for F distribution



$\boldsymbol{P(F_{(50,75)} > c)}$ **= 0.32**

The value of x is 0.9997

5. Given the population distribution μ=75, σ^2 =5, if the sample size n= 75, find the standard deviation of the sampling distribution of $\overline{x}$.

From the question, the population distribution parameters are

Mean μ = 75,

Variance $\boldsymbol{\sigma^2}$ =5,

Standard deviation σ =$\sqrt{\mathbf{5}}$,

Sample size = 75

Finding sample variance

$$\boldsymbol{\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} = \frac{5}{75} = 0.0667}$$

$$\boldsymbol{\sigma_{\bar{x}} = \sqrt{\sigma_{\bar{x}}^2} = \sqrt{0.0667} = 0.2583}$$

So, the standard deviation of sample is $\mathbf{0.2583}$

6. Given the population distribution, μ=50, σ =5, if the sample size n= 100, find the variance of the sampling distribution of $\bar{x}$.

μ = 50, σ =50, n = 100

Mean μ = 50,

Standard deviation σ =5`,

Sample size = 100

Finding sample variance

$$\boldsymbol{\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} = \frac{5^2}{200} = \frac{25}{100} = 0.25}$$

7. Given the population distribution, μ=5, σ^2 =15, if the sample size n= 28, find the mean of the sampling distribution of $\bar{x}$.

The mean of the sampling distribution is $\boldsymbol{\mu_{\bar{x}}}$ = 5