Summary of Capacity of Hopfield neural network

The main focus of the paper is to determine the capacity of Hopfield neural networks and to recall some measures of error correction. In paper , it is mentioned that neurons have two outcomes , that is ,a bistable state of +1(on) or -1(off). The neuron(i) transmits information to neuron(j) with the interconnection Tij, where Tij is fixed. In paper the state of model is calculated based on Hebbian rule, as follows

$$xi = sgn\{\sum_{j=1}^{n} Tij\ xi\}$$

In paper , they have considered models with two cases asynchronous and synchronous. In both the cases models are insensitive to signal-to-gaussian noise and choosing any model will not impact the capacity of the memory.In Asynchronous case, neurons update the states one at a time randomly, whereas in synchronous case all neurons simultaneously calculate and update the state. The information is globally transmitted through linear synaptic weights Tij and nonlinear operations take place at the neuron itself. There are mainly three properties of associative memory neural networks mentioned in the paper, that is , (i) Parallel computation (ii) distributed information storage and (iii) low computational complexity. Hamming distance is used to check if the model which is assumed is correct. This is ensured by , if the hamming distance obtained is less than n/2, where n is the total number of neurons. With this interpretation, the memory corrects most of the errors in the initial probe vector.

An outer product algorithm is implemented to determine the capacity of the memory.

$$T = \sum_{\alpha = 1}^{m} \mathrm{T}\,\alpha \text{ (Outer Product, T is transpose of row vector)}$$

We assume that once T has been calculated, all other information about the $x(\alpha)$ ($x(\alpha)$ is each memory forming $n * n$ matrix) will be “forgotten.” This is an important point to note when we have to add another memory to the list of things to be remembered, that is, when we have to learn. Sequences were also generated using bernoulli trials, for stability of models (mean/standard deviation) >> 1. To make a model stable, the memory of the model should be very small compared to the number of neurons used to train the model.

In the above algorithm, we only need to know the previous entries themselves, which is no extra burden. In the case of the quantized sum-of-outer products construction which is further discussed in the paper , we have to remember all the mn components of the $x(\alpha)$ (or all the $n(n-1)/2$ sums-of-outer products before quantization) to compute the new T when an $(m+1)st$ memory is to be added. Further, it has been proposed that capacity of memory can be increased by a factor of $1/2log(1/\in)$ times n if ∈ fraction of errors are permitted. But there were few problems observed in various experiments. First one was, (i) If the memories are not fixed points then it could never be recalled by the algorithm and second one, (ii) if there are memories then they may not be the nearest ones. It is assumed in paper that we know at least $(1-\rho)n$ of the components when we probe the memory, so that ρn (or fewer) are wrong. (Here

$0 \leq \rho < 1/2$.) For model stability, the largest ρn is chosen as the radius of attraction. Hamming distance leads to the convergence of the model to the right memory.

At least three possibilities of convergence of the model have been discussed. First, the sphere of radius ρn may be directly or monotonically attracted to its fundamental memory center, meaning that every transition that is actually a change in a component is a change in the right direction. Second, after multiple steps ,with high enough probability, a random step is in the right direction. The third mode of convergence, components can change back and forth during their sojourn, but at least on the average get better,that is, are more likely to be correct after a change than before.

Capacity Heuristics has also been discussed in the paper, which states than when the fundamental memory is presented with m randomly independent -1 and +1 states with probability of occurrence ½ to store with radius of influence ρn, then the capacity of the model is defined by:

$$(1-2\rho)^2 \; n \div 4 \,/\, log\ n$$

If for the second type of convergence, where few wrong moves are allowed then we can have direct convergence, (for any fixed $\rho, 0 < \rho < 1/2$) we get rid of the factor (1 - 2ρ) above. It has been observed that the stable point is towards the boundary of the radius:

$$n/4logn$$

If as above some wrong moves are permitted but no fundamental memory can be exceptional. Atlast, in all of the above, with high probability we are required to arrive at the exact fundamental memory as the stable point with no components in error, in both the models , that is , synchronous and asynchronous.