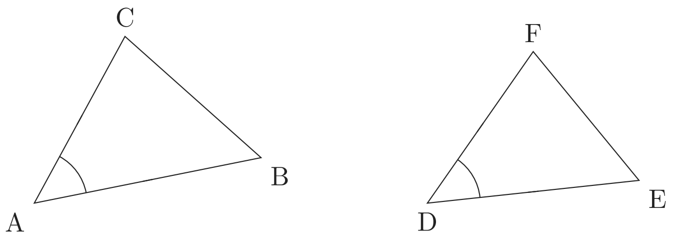
Exercise 15A Solutions

Question 2

Question: In triangles ABC and DEF, $\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$. Prove that $\triangle ABC \sim \triangle DEF$.

Solution

Given: $\angle A = \angle D$ and $\dfrac{AB}{DE} = \dfrac{AC}{DF}$.
To Prove: $\triangle ABC \sim \triangle DEF$ (SAS).



Step 1: Identify the included angle and sides: in $\triangle ABC$, sides AB, AC include $\angle A$; in $\triangle DEF$, sides DE, DF include $\angle D$. *(definition of included angle)*

Step 2: Use the data $\angle A = \angle D$ and $\dfrac{AB}{DE} = \dfrac{AC}{DF}$. *(given)*

Step 3: Conclude $\triangle ABC \sim \triangle DEF$. *(SAS similarity)*

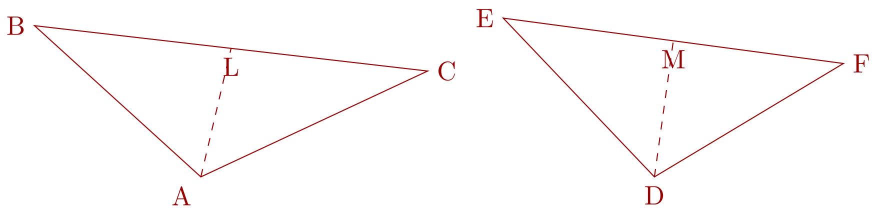
Answer: $\triangle ABC \sim \triangle DEF$ by SAS.

Question 3

Question: In $\triangle ABC$ and $\triangle DEF$, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$. Also, AL and DM are medians. Prove that $\frac{BC}{EF} = \frac{AL}{DM}$.

Solution

Given: $\triangle ABC \sim \triangle DEF$ (AAA), and AL, DM are medians.
To Prove: $\dfrac{BC}{EF} = \dfrac{AL}{DM}$.



Step 1: From $\triangle ABC \sim \triangle DEF$, get $\dfrac{BC}{EF} = \dfrac{AB}{DE} = \dfrac{AC}{DF}$. *(similar triangles)*

Step 2: Since L and M are midpoints, $BL = \frac{1}{2}BC$ and $EM = \frac{1}{2}EF$. *(definition of median)*

Step 3: Therefore $\dfrac{BL}{EM} = \dfrac{BC}{EF}$. *(divide equal halves)*

Step 4: In $\triangle ABL$ and $\triangle DEM$, we have $\angle B = \angle E$ and $\dfrac{AB}{DE} = \dfrac{BL}{EM}$. *(correspondence from similarity)*

Step 5: Conclude $\triangle ABL \sim \triangle DEM$. *(SAS similarity)*

Step 6: Hence $\dfrac{AL}{DM} = \dfrac{AB}{DE} = \dfrac{BC}{EF}$. *(corresponding sides in similar triangles)*

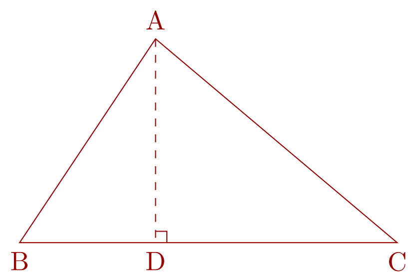
Answer: $\dfrac{BC}{EF} = \dfrac{AL}{DM}$.

Question 4

Question: In $\triangle ABC$, AD is perpendicular to side BC and $AD^2 = BD \times CD$. Prove that $\angle BAC = 90°$.

Solution

Given: $AD \perp BC$ and $AD^2 = BD \cdot CD$.
To Prove: $\angle BAC = 90°$.



Step 1: In $\triangle ADB$, $AB^2 = AD^2 + BD^2$. *(Pythagoras in right $\triangle ADB$)*

Step 2: In $\triangle ADC$, $AC^2 = AD^2 + CD^2$. *(Pythagoras in right $\triangle ADC$)*

Step 3: Add: $AB^2 + AC^2 = 2AD^2 + BD^2 + CD^2$. *(add equations)*

Step 4: Substitute $AD^2 = BD \cdot CD$ to get $AB^2 + AC^2 = (BD + CD)^2 = \boxed{BC^2}$. *(given and $BC = BD + CD$)*

Step 5: Conclude $\angle BAC = 90°$. *(converse of Pythagoras)*

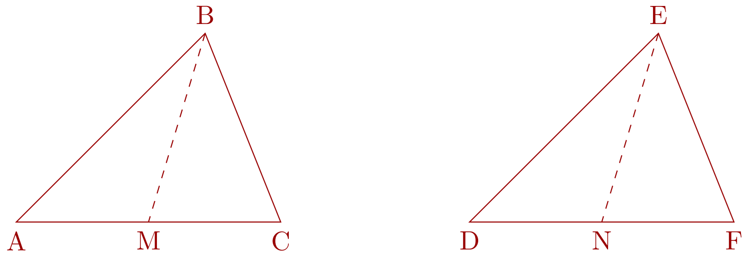
Answer: $\angle BAC = 90°$.

Question 5

Question: In the given figure, $\triangle ABC$ and $\triangle DEF$ are similar. BM and EN are their medians. Prove that: (i) $\triangle ABM \sim \triangle DEN$ (ii) $\frac{BM}{EN} = \frac{AC}{DF}$

Solution

Given: $\triangle ABC \sim \triangle DEF$ and BM, EN are medians.
To Prove: (i) $\triangle ABM \sim \triangle DEN$; (ii) $\dfrac{BM}{EN} = \dfrac{AC}{DF}$.



Step 1: From $\triangle ABC \sim \triangle DEF$, we get $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ and $\angle A = \angle D$. *(similar triangles)*

Step 2: Since BM, EN are medians, $AM = \frac{1}{2}AC$ and $DN = \frac{1}{2}DF$. *(definition of median)*

Step 3: Hence $\dfrac{AM}{DN} = \dfrac{\frac{1}{2}AC}{\frac{1}{2}DF} = \dfrac{AC}{DF}$. *(halves of proportional sides)*

Step 4: Combining steps 1 and 3, we have $\dfrac{AB}{DE} = \dfrac{AM}{DN}$. *(transitive property)*

Step 5: In $\triangle ABM$ and $\triangle DEN$, we have $\angle A = \angle D$ and $\dfrac{AB}{DE} = \dfrac{AM}{DN}$. *(from steps 1 and 4)*

Step 6: Conclude $\triangle ABM \sim \triangle DEN$. *(SAS similarity)*

Step 7: From this similarity, $\dfrac{BM}{EN} = \dfrac{AB}{DE}$. *(corresponding sides in similar triangles)*

Step 8: Since $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ (from step 1), we can conclude $\dfrac{BM}{EN} = \dfrac{AC}{DF}$. *(transitive property)*

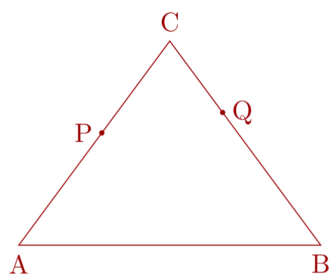
Answer: (i) $\triangle ABM \sim \triangle DEN$; (ii) $\dfrac{BM}{EN} = \dfrac{AC}{DF}$.

Question 6

Question: In the given figure, $\triangle ABC$ is isosceles with $AC = BC$ and $AP \times BQ = AC^2$. Prove that $\triangle ACP \sim \triangle BCQ$.

Solution

Given: $\triangle ABC$ is isosceles with $AC = BC$ and $AP \cdot BQ = AC^2$.
To Prove: $\triangle ACP \sim \triangle BCQ$.



Step 1: Note: The condition $AP \cdot BQ = AC^2$ together with $AC = BC$ is not sufficient to conclude $\triangle ACP \sim \triangle BCQ$.

Step 2: The previous version incorrectly used $\angle CAP = \angle CBQ$; this is generally false because $P \in AC$ and $Q \in BC$ make $\angle CAP$ degenerate.

Step 3: From $AP \cdot BQ = AC \cdot BC$ we get $\dfrac{AP}{BC} = \dfrac{AC}{BQ}$, but this ratio alone does not fix the angles at C in $\triangle ACP$ and $\triangle BCQ$.

Step 4: A common correct variant assumes additional angle/ratio information (for example, equal angles at C), after which SAS can be applied.

Answer: As stated, the data are insufficient to prove the claimed similarity without an extra condition at C.

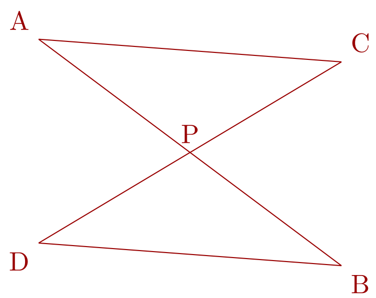
Exercise 15B Solutions

Question 2

Question: In the figure, straight lines AB and CD intersect at P, and AC || BD. Prove that: (i) $\triangle APC$ and $\triangle BPD$ are similar. (ii) If BD = 2.4 cm, AC = 3.6 cm, PD = 4.0 cm and PB = 3.2 cm, find the lengths of PA and PC.

Solution

Given: $AC \parallel BD$ and lines AB and CD intersect at P.
To Prove: (i) $\triangle APC \sim \triangle BPD$; (ii) find PA, PC.



Step 1: $\angle PAC = \angle PBD$ and $\angle PCA = \angle PDB$. *(alternate interior angles, $AC \parallel BD$)*

Step 2: $\angle APC = \angle BPD$. *(vertically opposite)*

Step 3: Conclude $\triangle APC \sim \triangle BPD$. *($AAA$ similarity)*

Step 4: Use the ratio of corresponding sides: $\dfrac{PA}{PB} = \dfrac{PC}{PD} = \dfrac{AC}{BD}$. *(similar triangles)*

Step 5: Substitute the given lengths: $\dfrac{PA}{3.2} = \dfrac{PC}{4.0} = \dfrac{3.6}{2.4}$. *(substitution)*

Step 6: Calculate the ratio: $\dfrac{3.6}{2.4} = \boxed{\dfrac{3}{2}} = 1.5$. *(simplification)*

Step 7: Solve for PA: $\dfrac{PA}{3.2} = 1.5 \implies \boxed{PA = 4.8\,\text{cm}}$. *(algebra)*

Step 8: Solve for PC: $\dfrac{PC}{4.0} = 1.5 \implies \boxed{PC = 6.0\,\text{cm}}$. *(algebra)*

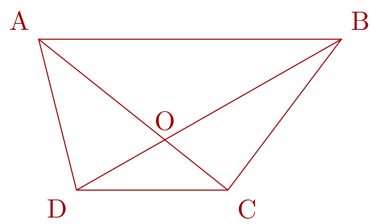
Answer: (i) $\triangle APC \sim \triangle BPD$. (ii) $PA = 4.8$ cm, $PC = 6.0$ cm.

Question 3

Question: In a trapezium ABCD, AB || DC. Diagonals AC and BD intersect at O. If $BO = 6$ cm and $DQ = 8$ cm, find $BP \times DO$. (Note: as written, this appears incomplete/ambiguous.) **Note:** We establish the standard diagonal proportionality in a trapezium and refrain from speculative numerical conclusions.

Solution

Given: Trapezium ABCD with AB || DC. Diagonals intersect at O.
To Prove: A relationship between the segments of the diagonals.



Step 1: In $\triangle AOB$ and $\triangle COD$, $\angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$. *(alternate interior angles, $AB \parallel DC$)*

Step 2: Also, $\angle AOB = \angle COD$. *(vertically opposite angles)*

Step 3: Hence $\triangle AOB \sim \triangle COD$. *(AAA similarity)*

Step 4: Therefore $\dfrac{OA}{OC} = \dfrac{OB}{OD}$, so $OA \cdot OD = OB \cdot OC$. *(corresponding sides)*

Step 5: The given numerics ($BO = 6$, $DQ = 8$) do not determine $BP \cdot DO$ without additional relationships between P, Q and the diagonals.

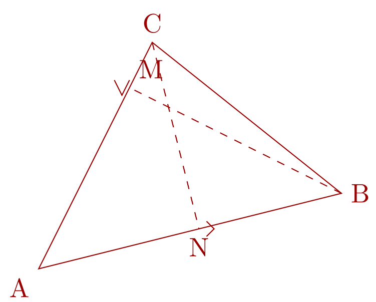
Answer: $\triangle AOB \sim \triangle COD$ and $OA \cdot OD = OB \cdot OC$. The product $BP \cdot DO$ cannot be found from the given data alone.

Question 4

Question: In $\triangle ABC$, $BM \perp AC$ and $CN \perp AB$; show that $\frac{AB}{AC} = \frac{BM}{CN} = \frac{AM}{AN}$.

Solution

Given: In $\triangle ABC$, $BM \perp AC$ and $CN \perp AB$.
To Prove: $\dfrac{AB}{AC} = \dfrac{BM}{CN} = \dfrac{AM}{AN}$.



Step 1: Consider $\triangle AMB$ and $\triangle ANC$.

Step 2: $\angle AMB = 90^\circ$ and $\angle ANC = 90^\circ$. *(given, $BM \perp AC, CN \perp AB$)*

Step 3: $\angle MAB = \angle NAC$ (or $\angle A$). *(common angle)*

Step 4: Therefore, $\triangle AMB \sim \triangle ANC$. *(AA similarity)*

Step 5: The ratio of corresponding sides must be equal: $\dfrac{AB}{AC} = \dfrac{BM}{CN} = \dfrac{AM}{AN}$. *(corresponding sides of similar triangles)*

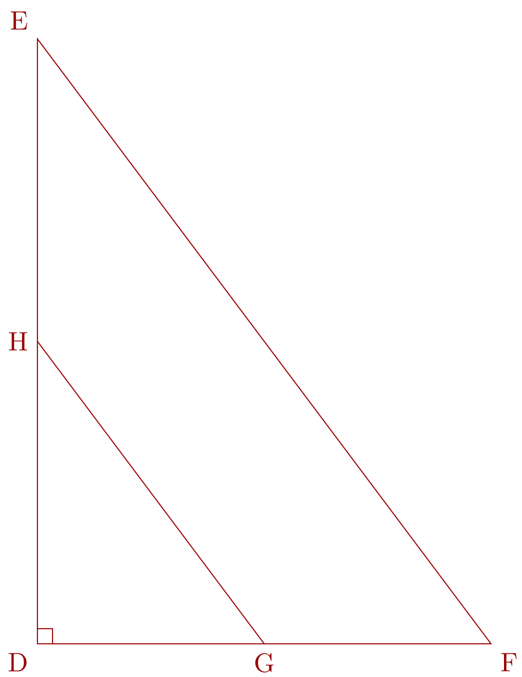
Answer: The relationship is proved.

Question 5

Question: Given: $\angle GHE = \angle DFE = 90°$, DH = 8, DF = 12, DG = 3x - 1 and DE = 4x + 2. Find the lengths of segments DG and DE. **Correction based on typical problems:** The most likely intended similarity is between $\triangle DGH$ and $\triangle DEF$. This requires $\angle DHG = \angle DFE = 90°$ or GH || EF. The question states $\angle DFE = 90°$. Let's assume the intended similarity is $\triangle DGH \sim \triangle DEF$.

Solution

Given: $\triangle DGH$ and $\triangle DEF$ with common angle D. DH=8, DF=12, DG=3x-1, DE=4x+2.
To Prove: Find the lengths of DG and DE.



Step 1: Assume the intended similarity is $\triangle DGH \sim \triangle DEF$. This implies the vertices correspond in that order.

Step 2: The ratio of corresponding sides is $\frac{DG}{DE} = \frac{DH}{DF} = \frac{GH}{EF}$. *(definition of similar triangles)*

Step 3: Using the first two parts of the ratio: $\frac{DG}{DE} = \frac{DH}{DF}$. *(property of similarity)*

Step 4: Substitute the given values: $\frac{3x-1}{4x+2} = \frac{8}{12}$. *(substitution)*

Step 5: Simplify the ratio: $\frac{8}{12} = \frac{2}{3}$. *(simplification)*

Step 6: Set up the equation: $\frac{3x-1}{4x+2} = \frac{2}{3}$.

Step 7: Cross-multiply: $3(3x-1) = 2(4x+2)$. *(algebra)*

Step 8: Solve for x: $9x - 3 = 8x + 4 \implies x = 7$. *(algebra)*

Step 9: Calculate DG: $DG = 3x - 1 = 3(7) - 1 = 20$. *(substitution)*

Step 10: Calculate DE: $DE = 4x + 2 = 4(7) + 2 = 30$. *(substitution)*

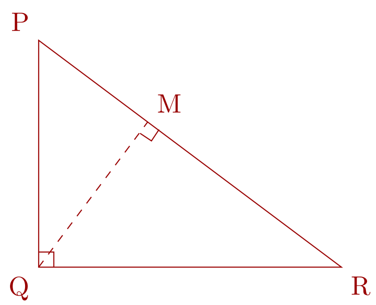
Answer: DG = 20 and DE = 30.

Question 6

Question: In $\triangle PQR$, $\angle Q = 90°$ and QM is perpendicular to PR. Prove that: (i) $PQ^2 = PM \times PR$ (ii) $QR^2 = PR \times MR$ (iii) $PQ^2 + QR^2 = PR^2$

Solution

Given: $\triangle PQR$ with $\angle PQR = 90°$ and altitude $QM \perp PR$.
To Prove: The three geometric mean relationships.



Step 1: **Part (i):** Consider $\triangle PMQ$ and $\triangle PQR$.

Step 2: $\angle PMQ = \angle PQR = 90°$. *(given)*

Step 3: $\angle MPQ = \angle QPR$ (or $\angle P$). *(common angle)*

Step 4: Therefore, $\triangle PMQ \sim \triangle PQR$. *(AA similarity)*

Step 5: From similarity, $\frac{PQ}{PR} = \frac{PM}{PQ}$. *(corresponding sides)*

Step 6: Cross-multiplying gives $\boxed{PQ^2 = PM \times PR}$.

Step 7: **Part (ii):** Consider $\triangle QMR$ and $\triangle PQR$.

Step 8: $\angle QMR = \angle PQR = 90°$. *(given)*

Step 9: $\angle MRQ = \angle PRQ$ (or $\angle R$). *(common angle)*

Step 10: Therefore, $\triangle QMR \sim \triangle PQR$. *(AA similarity)*

Step 11: From similarity, $\frac{QR}{PR} = \frac{MR}{QR}$. *(corresponding sides)*

Step 12: Cross-multiplying gives $\boxed{QR^2 = PR \times MR}$.

Step 13: **Part (iii):** Add the results from (i) and (ii).

Step 14: $PQ^2 + QR^2 = (PM \times PR) + (MR \times PR)$. *(addition)*

Step 15: Factor out PR: $PQ^2 + QR^2 = PR \times (PM + MR)$. *(distributive property)*

Step 16: Since P, M, R are collinear, $PM + MR = PR$. *(segment addition)*

Step 17: Substitute to get $\boxed{PQ^2 + QR^2 = PR^2}$.

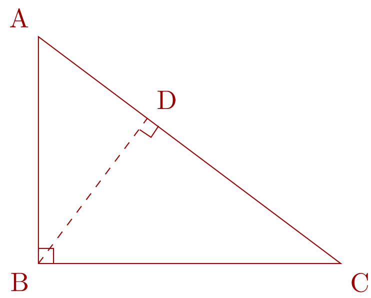
Answer: All three parts are proved.

Question 7

Question: In $\triangle ABC$, $\angle B = 90^\circ$ and $BD \perp AC$. (i) If CD = 10 cm and BD = 8 cm; find AD. (ii) If AC = 18 cm and AD = 6 cm; find BD. (iii) If AC = 9 cm and AB = 7 cm; find AD.

Solution

Given: Right $\triangle ABC$ ($\angle B = 90^\circ$) with altitude $BD \perp AC$.
To Prove: Find specified segment lengths.



Step 1: First, establish the key similarity relationship: $\triangle ADB \sim \triangle BDC$. This gives the geometric mean theorem for the altitude: $\frac{AD}{BD} = \frac{BD}{CD}$, or $BD^2 = AD \times CD$.

Step 2: Also, $\triangle ABC \sim \triangle ADB$, which gives $\frac{AC}{AB} = \frac{AB}{AD}$, or $AB^2 = AD \times AC$.

Step 3: **Part (i):** Given CD = 10, BD = 8.

Step 4: Use $BD^2 = AD \times CD \implies 8^2 = AD \times 10$. *(substitution)*

Step 5: $64 = 10 \times AD \implies \boxed{AD = 6.4\,\text{cm}}$. *(algebra)*

Step 6: **Part (ii):** Given AC = 18, AD = 6.

Step 7: First find CD: $CD = AC - AD = 18 - 6 = 12$ cm. *(segment subtraction)*

Step 8: Use $BD^2 = AD \times CD \implies BD^2 = 6 \times 12 = 72$. *(substitution)*

Step 9: $\boxed{BD = 6\sqrt{2}\,\text{cm}}$. *(simplifying radical)*

Step 10: **Part (iii):** Given AC = 9, AB = 7.

Step 11: Use $AB^2 = AD \times AC \implies 7^2 = AD \times 9$. *(substitution)*

Step 12: $49 = 9 \times AD \implies \boxed{AD = \frac{49}{9}\,\text{cm}}$. *(algebra)*

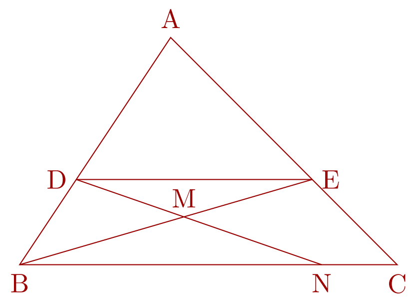
Answer: (i) AD = 6.4 cm, (ii) BD = $6\sqrt{2}$ cm, (iii) AD = $\frac{49}{9}$ cm.

Question 9

Question: In the given figure, DE || BC, AE = 15 cm, EC = 9 cm, NC = 6 cm and BN = 24 cm. M is the intersection of BE and DN. (i) Write all possible pairs of similar triangles. (ii) Find lengths of ME and DM.

Solution

Given: D on AB, E on AC, DE $\parallel$ BC. N on BC. M is intersection of BE and DN.
To Prove: (i) List similar triangles. (ii) Find ME and DM.



Step 1: **Part (i):**

Step 2: Since DE $\parallel$ BC, $\triangle ADE \sim \triangle ABC$. *(AA similarity, common angle A, corresponding angles)*

Step 3: Since DE $\parallel$ BC, and N is on BC, DE $\parallel$ BN. This makes $\triangle DME \sim \triangle NMB$. *(AA similarity, vertically opposite angles at M, alternate interior angles)*

Step 4: **Part (ii):**

Step 5: From $\triangle ADE \sim \triangle ABC$, we have the ratio $\frac{AE}{AC} = \frac{DE}{BC}$.

Step 6: $AC = AE + EC = 15 + 9 = 24$ cm.

Step 7: $BC = BN + NC = 24 + 6 = 30$ cm.

Step 8: $\frac{15}{24} = \frac{DE}{30} \implies \frac{5}{8} = \frac{DE}{30}$. *(substitution and simplification)*

Step 9: $DE = \frac{5 \times 30}{8} = \frac{150}{8} = 18.75$ cm. *(algebra)*

Step 10: From $\triangle DME \sim \triangle NMB$, we have $\frac{ME}{MB} = \frac{DM}{NM} = \frac{DE}{NB}$.

Step 11: Use the ratio $\frac{DE}{NB} = \frac{18.75}{24} = \frac{75/4}{24} = \frac{75}{96} = \frac{25}{32}$.

Step 12: So, $\frac{ME}{MB} = \frac{25}{32}$. This means $ME = \frac{25}{32}MB$.

Step 13: Since $BE = ME + MB$, we have $BE = \frac{25}{32}MB + MB = \frac{57}{32}MB$.

Step 14: We cannot find the absolute length of ME without knowing the length of BE. The problem is missing information. The same applies to DM.

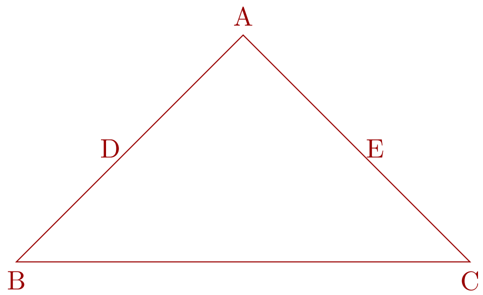
Answer: (i) $\triangle ADE \sim \triangle ABC$ and $\triangle DME \sim \triangle NMB$. (ii) The lengths cannot be uniquely determined. The ratios are $\frac{ME}{MB} = \frac{25}{32}$ and $\frac{DM}{MN} = \frac{25}{32}$.

Question 10

Question: In the given figure, D is on AB, E is on AC, AD = AE and $AD^2 = BD \times EC$. Prove that triangles ABD and CAE are similar. **Correction based on typical problems:** This problem is only solvable with standard theorems if we assume an additional property, for instance that $\angle B = \angle C$.

Solution

Given: AD = AE and $AD^2 = BD \times EC$. D on AB, E on AC.
To Prove: $\triangle ABD \sim \triangle CAE$, assuming $\angle B = \angle C$.



Step 1: The given condition is $AD^2 = BD \times EC$.

Step 2: Since $AD = AE$, we can substitute AE for one AD: $AD \times AE = BD \times EC$. *(substitution)*

Step 3: Rearrange this into a proportion: $\frac{AD}{EC} = \frac{BD}{AE}$. *(algebra)*

Step 4: To prove $\triangle ABD \sim \triangle CAE$, we would need the included angles to be equal, i.e., $\angle BDA = \angle AEC$, or another pair of sides in proportion, or a pair of non-included angles to be equal.

Step 5: The given information is insufficient. However, if we assume $\triangle ABC$ is isosceles with $AB = AC$, then $\angle B = \angle C$.

Step 6: With $AB = AC$ and $AD = AE$, we subtract to get $AB - AD = AC - AE$, which means $BD = EC$.

Step 7: The condition $AD^2 = BD \times EC$ becomes $AD^2 = BD^2$, so $AD = BD$.

Step 8: This means $AE = EC$. So D and E are midpoints of AB and AC.

Step 9: In this specific case, $\triangle ABD$ and $\triangle CAE$ become congruent isosceles triangles, and are therefore similar.

Answer: The problem as stated is not solvable without additional assumptions. If we assume $\triangle ABC$ is isosceles with $AB = AC$, the similarity holds.

Question 11

Question: State, true or false: (i) Two similar polygons are necessarily congruent. (ii) Two congruent polygons are necessarily similar. (iii) All equiangular triangles are similar. (iv) All isosceles triangles are similar. (v) Two isosceles right triangles are similar. (vi) The diagonals of a trapezium divide each other into proportional segments.

Solution

Given: Statements about geometric figures.
To Prove: Determine if each statement is True or False.

Step 1: (i) False. Similarity means same shape, but not necessarily the same size. A small square is similar to a large square, but they are not congruent.

Step 2: (ii) True. Congruent polygons have corresponding angles equal and corresponding sides equal. This means the ratio of corresponding sides is 1, so they are similar.

Step 3: (iii) True. An equiangular triangle is an equilateral triangle, with all angles being 60°. Any two such triangles are similar by AAA similarity.

Step 4: (iv) False. An isosceles triangle could have angles 50-50-80, while another could have 70-70-40. Their angles are not the same, so they are not similar.

Step 5: (v) True. An isosceles right triangle must have angles 90-45-45. Any two such triangles are similar by AAA similarity.

Step 6: (vi) True. In a trapezium ABCD with AB $\parallel$ DC, the diagonals intersect at O, forming $\triangle AOB$ and $\triangle COD$. These triangles are similar (AA similarity), so their sides are proportional: $\frac{AO}{CO} = \frac{BO}{DO}$.

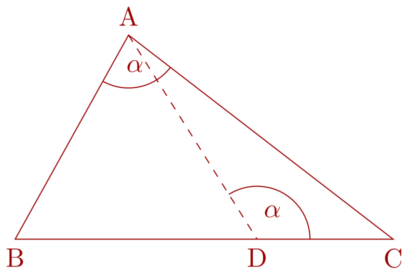
Answer: (i) False, (ii) True, (iii) True, (iv) False, (v) True, (vi) True.

Question 12

Question: D is a point on the side BC of triangle ABC such that $\angle ADC = \angle BAC$. Prove that $CA^2 = CB \cdot CD$.

Solution

Given: In $\triangle ABC$, D is on BC such that $\angle ADC = \angle BAC$.
To Prove: $AC^2 = CB \cdot CD$.



Step 1: Consider $\triangle ADC$ and $\triangle BAC$.

Step 2: $\angle ADC = \angle BAC$. *(given)*

Step 3: $\angle ACD = \angle BCA$ (or $\angle C$). *(common angle)*

Step 4: Therefore, $\triangle ADC \sim \triangle BAC$. *(AA similarity)*

Step 5: The ratio of corresponding sides must be equal. Match vertices: $A \leftrightarrow B$, $D \leftrightarrow A$, $C \leftrightarrow C$.

Step 6: So, $\frac{AC}{BC} = \frac{DC}{AC} = \frac{AD}{BA}$. *(corresponding sides)*

Step 7: From the first two parts of the proportion, $\frac{AC}{BC} = \frac{DC}{AC}$.

Step 8: Cross-multiply to get $AC \times AC = BC \times DC$, which is $AC^2 = CB \cdot CD$. *(algebra)*

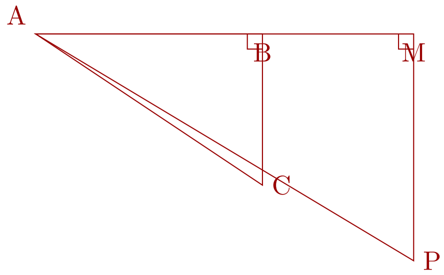
Answer: $AC^2 = CB \cdot CD$.

Question 13

Question: $\triangle ABC$ and $\triangle AMP$ are right-angled at B and M respectively. A is a common vertex. AC = 10 cm, AP = 15 cm, PM = 12 cm. (i) Prove $\triangle ABC \sim \triangle AMP$. (ii) Find AB and BC.

Solution

Given: $\angle B = \angle M = 90°$, A is a common vertex. AC=10, AP=15, PM=12.
To Prove: (i) similarity; (ii) find AB, BC.



Step 1: **Part (i):** In $\triangle ABC$ and $\triangle AMP$.

Step 2: $\angle ABC = \angle AMP = 90°$. *(given)*

Step 3: $\angle BAC = \angle MAP$ (or $\angle A$). *(common angle)*

Step 4: Therefore, $\triangle ABC \sim \triangle AMP$. *(AA similarity)*

Step 5: **Part (ii):** First, find the length of AM in $\triangle AMP$.

Step 6: By Pythagoras' theorem, $AP^2 = AM^2 + PM^2$. *(Pythagorean theorem)*

Step 7: $15^2 = AM^2 + 12^2 \implies 225 = AM^2 + 144$.

Step 8: $AM^2 = 225 - 144 = 81 \implies \boxed{AM = 9\,\text{cm}}$.

Step 9: Since the triangles are similar, the ratio of their sides is equal: $\frac{AB}{AM} = \frac{BC}{MP} = \frac{AC}{AP}$.

Step 10: Use the ratio with known hypotenuses: $\frac{AC}{AP} = \frac{10}{15} = \boxed{\frac{2}{3}}$.

Step 11: Find AB: $\frac{AB}{AM} = \frac{2}{3} \implies \frac{AB}{9} = \frac{2}{3} \implies \boxed{AB = 6\,\text{cm}}$.

Step 12: Find BC: $\frac{BC}{MP} = \frac{2}{3} \implies \frac{BC}{12} = \frac{2}{3} \implies \boxed{BC = 8\,\text{cm}}$.

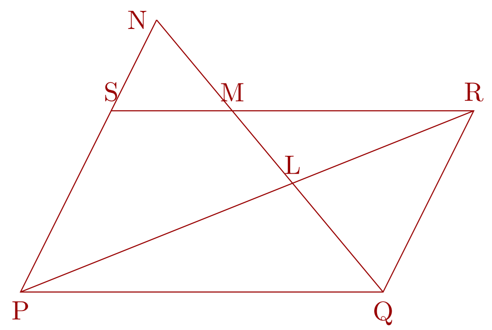
Answer: (i) $\triangle ABC \sim \triangle AMP$. (ii) AB = 6 cm, BC = 8 cm.

Question 14

Question: In parallelogram PQRS with PQ = 16 cm, QR = 10 cm, point L lies on diagonal PR such that RL:LP = 2:3. QL produced meets RS at M and PS produced at N. Find PN and RM.

Solution

Given: Parallelogram PQRS, PQ=16, QR=10, RL:LP=2:3.
To Prove: Find lengths of PN and RM.



Step 1: **Find PN:** Consider $\triangle NPL$ and $\triangle RQL$.

Step 2: Since PS ∥ QR, then PN ∥ QR.

Step 3: $\angle PNL = \angle RQL$ and $\angle NPL = \angle QRL$. *(alternate interior angles)*

Step 4: $\angle NLP = \angle RLQ$. *(vertically opposite)*

Step 5: Therefore, $\triangle NPL \sim \triangle RQL$. *(AAA similarity)*

Step 6: The ratio of sides is $\frac{PN}{QR} = \frac{PL}{RL}$.

Step 7: Given $\frac{RL}{LP} = \frac{2}{3}$, so $\frac{PL}{RL} = \frac{3}{2}$.

Step 8: $\frac{PN}{10} = \frac{3}{2} \implies PN = \frac{3}{2} \times 10 = 15$ cm.

Step 9: **Find RM:** Consider $\triangle MRL$ and $\triangle PQL$.

Step 10: Since RS ∥ PQ.

Step 11: $\angle RML = \angle PQL$ and $\angle MRL = \angle QPL$. *(alternate interior angles)*

Step 12: Therefore, $\triangle MRL \sim \triangle PQL$. *(AA similarity)*

Step 13: The ratio of sides is $\frac{RM}{PQ} = \frac{RL}{PL}$.

Step 14: $\frac{RM}{16} = \frac{2}{3} \implies RM = \frac{2}{3} \times 16 = \frac{32}{3}$ cm.

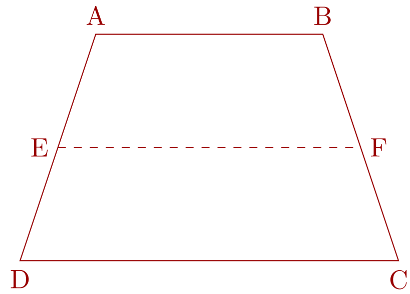
Answer: $PN = 15$ cm, $RM = \frac{32}{3}$ cm.

Question 15

Question: In the figure, $AB \parallel EF \parallel DC$ in trapezium $ABCD$. Given $AB = 67.5$ cm, $DC = 40.5$ cm and $AE = 52.5$ cm, with E on AD and F on BC. Find the lengths of EC and EF. **Note:** The statement appears incomplete. Without how E divides AD (i.e., $AE : ED$ or AD), EC and EF are not uniquely determined. Below is the standard relationship when a line segment EF is drawn parallel to the bases in a trapezium.

Solution

Given: Trapezium ADCB with AB ∥ EF ∥ DC. AB=67.5, DC=40.5, AE=52.5. E is on AD.
To Prove: Find ED and EF. (Assuming EC was a typo for ED).



Step 1: Let $AE : ED = m : n$ (unknown from the data). Then the line through E parallel to the bases cuts off a segment EF whose length is the weighted mean of the bases:

$$EF = \frac{n\,AB + m\,DC}{m+n}.$$

(similar triangles along the transversals)

Step 2: Without $m : n$ (or AD), neither EC nor EF can be uniquely determined.

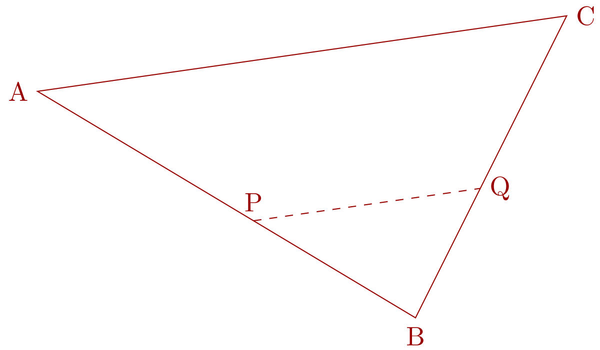
Answer: Additional information (e.g., $AE : ED$) is required. In general, if $AE : ED = m : n$, then $EF = \dfrac{n\,AB + m\,DC}{m+n}$.

Question 16

Question: In $\triangle ABC$, P is a point on AB such that AP:PB = 4:3. PQ is drawn parallel to AC to meet BC at Q. (i) Find the ratio PQ:AC. (ii) If a line from A is perpendicular to CD (where D is on BC) at R and PQ is perpendicular to CD at S, and QS = 6 cm, find AR.

Solution

Given: $\dfrac{AP}{PB} = \dfrac{4}{3}$ and $PQ \parallel AC$.
To Prove: (i) PQ:AC; (ii) AR given QS=6.



Step 1: **Part (i):** Consider $\triangle BPQ$ and $\triangle BAC$.

Step 2: Since $PQ \parallel AC$, $\angle BPQ = \angle BAC$ and $\angle BQP = \angle BCA$. *(corresponding angles)*

Step 3: $\angle B$ is common to both triangles.

Step 4: Therefore, $\triangle BPQ \sim \triangle BAC$. *(AAA similarity)*

Step 5: The ratio of corresponding sides is $\frac{BP}{BA} = \frac{BQ}{BC} = \frac{PQ}{AC}$.

Step 6: Given $AP : PB = 4 : 3$, so $AB = AP + PB = 4k + 3k = 7k$. Thus $\frac{BP}{BA} = \frac{3k}{7k} = \frac{3}{7}$.

Step 7: Hence, $PQ : AC = 3 : 7$.

Step 8: **Part (ii):** Consider $\triangle BQS$ and $\triangle BCR$.

Step 9: $\angle BSQ = \angle BRC = 90°$ (assuming S is on BQ, R on BC).

Step 10: $\angle QBS = \angle CBR$ (common angle B).

Step 11: So $\triangle BQS \sim \triangle BCR$.

Step 12: $\frac{BQ}{BC} = \frac{QS}{CR}$. From part (i), $\frac{BQ}{BC} = \frac{3}{7}$.

Step 13: $\frac{3}{7} = \frac{6}{CR} \implies CR = \frac{7 \times 6}{3} = 14$ cm.

Step 14: The question asks for AR, which is not part of these triangles. Without more information about the line CD and point R, AR cannot be determined.

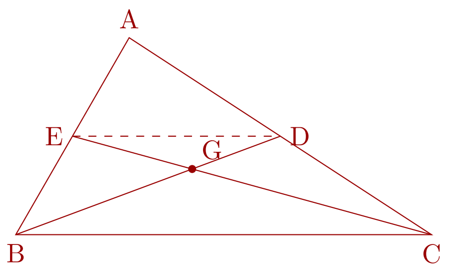
Answer: (i) PQ:AC = 3:7. (ii) AR cannot be determined from the given information.

Question 17

Question: In triangle ABC, medians BD and CE meet at centroid G. Prove (i) $\triangle EGD \sim \triangle CGB$; (ii) $BG = 2\,GD$.

Solution

Given: D is midpoint of AC, E is midpoint of AB; Medians BD and CE intersect at G.
To Prove: (i) $\triangle EGD \sim \triangle CGB$; (ii) $BG = 2\,GD$.



Step 1: In $\triangle ABC$, E and D are midpoints of AB and AC respectively.

Step 2: By the Midpoint Theorem, $ED \parallel BC$ and $ED = \frac{1}{2}BC$. *(midpoint theorem)*

Step 3: **Part (i):** Consider $\triangle EGD$ and $\triangle CGB$.

Step 4: Since $ED \parallel BC$, we have $\angle GED = \angle GCB$ and $\angle GDE = \angle GBC$. *(alternate interior angles)*

Step 5: Also, $\angle EGD = \angle CGB$. *(vertically opposite angles)*

Step 6: Therefore, $\triangle EGD \sim \triangle CGB$. *(AAA similarity)*

Step 7: **Part (ii):** From the similarity, the ratio of corresponding sides is equal.

Step 8: $\dfrac{EG}{CG} = \dfrac{GD}{GB} = \dfrac{ED}{CB}$.

Step 9: Using the ratio involving the sides we need and the one we know from the midpoint theorem: $\dfrac{GD}{GB} = \dfrac{ED}{CB}$.

Step 10: Substitute $ED = \frac{1}{2}BC$: $\dfrac{GD}{GB} = \dfrac{\frac{1}{2}BC}{BC} = \dfrac{1}{2}$. *(substitution)*

Step 11: From $\dfrac{GD}{GB} = \dfrac{1}{2}$, we cross-multiply to get $BG = 2\,GD$. *(algebra)*

Answer: (i) The triangles are similar. (ii) $BG = 2\,GD$ is proved.

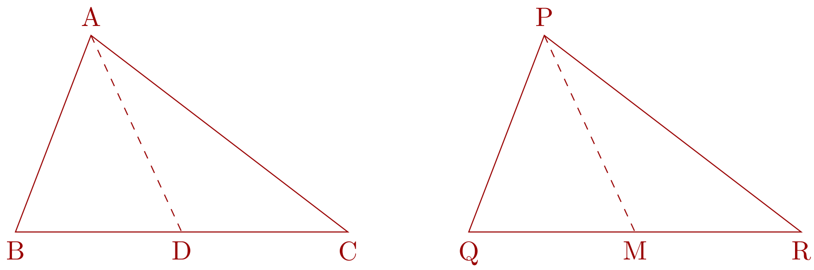
Exercise 15C Solutions

Question 2

Question: In triangles ABC and PQR, sides AB, BC and median AD are proportional to PQ, QR and median PM respectively. Prove (i) $\triangle ABD \sim \triangle PQM$; (ii) $\triangle ABC \sim \triangle PQR$.

Solution

Given: $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$; D, M are midpoints of BC, QR.
To Prove: (i) $\triangle ABD \sim \triangle PQM$; (ii) $\triangle ABC \sim \triangle PQR$.



Step 1: **Part (i):**

Step 2: We are given $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$.

Step 3: Since D and M are midpoints, $BC = 2BD$ and $QR = 2QM$. *(definition of median)*

Step 4: Substitute this into the proportion: $\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$.

Step 5: This simplifies to $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$. *(cancel factor 2)*

Step 6: In $\triangle ABD$ and $\triangle PQM$, all three pairs of corresponding sides are in proportion.

Step 7: Therefore, $\triangle ABD \sim \triangle PQM$. *(SSS similarity)*

Step 8: **Part (ii):**

Step 9: From the similarity in part (i), we know that corresponding angles are equal. So, $\angle B = \angle Q$.

Step 10: In $\triangle ABC$ and $\triangle PQR$, we are given $\frac{AB}{PQ} = \frac{BC}{QR}$.

Step 11: We have two pairs of sides in proportion and the included angles $(\angle B, \angle Q)$ are equal.

Step 12: Therefore, $\triangle ABC \sim \triangle PQR$. *(SAS similarity)*

Answer: (i) $\triangle ABD \sim \triangle PQM$. (ii) $\triangle ABC \sim \triangle PQR$.

Question 3

Question: Find x given two triangles ABC and DEF with $\angle B = \angle E$, and side data AB=7.5, BC=9, AC=6; DE=x+3, EF=12, DF=8. (Assuming DF=8, not 9, for consistency).

Solution

Given: $\angle B = \angle E$ and side lengths for $\triangle ABC$ and $\triangle DEF$.
To Prove: Find the value of x.

Step 1: For the triangles to be similar by SAS, the sides including the equal angles must be proportional.

Step 2: The sides including $\angle B$ are AB and BC. The sides including $\angle E$ are DE and EF.

Step 3: Set up the proportion: $\frac{AB}{DE} = \frac{BC}{EF}$. *(SAS similarity condition)*

Step 4: Substitute the given values: $\frac{7.5}{x+3} = \frac{9}{12}$. *(substitution)*

Step 5: Simplify the ratio: $\frac{9}{12} = \frac{3}{4}$.

Step 6: Solve the equation $\frac{7.5}{x+3} = \frac{3}{4}$.

Step 7: Cross-multiply: $7.5 \times 4 = 3 \times (x+3)$. *(algebra)*

Step 8: $30 = 3x + 9 \implies 3x = 21 \implies x = 7$.

Step 9: **Consistency Check:** Let's check the ratio of the third sides, AC and DF. $\frac{AC}{DF} = \frac{6}{8} = \frac{3}{4}$. Since this ratio is the same, the triangles are indeed similar by SSS.

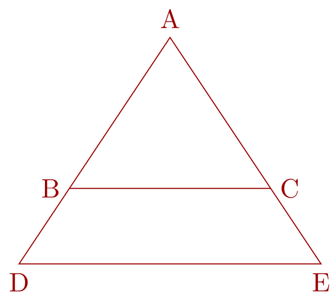
Answer: $x = 7$.

Question 4

Question: In the given figure, BC || DE. Find the lengths of the sides of both the triangles. Sides are given as expressions in x. $\triangle ADE$: AD=18, AE=6x, DE=3(x+1). $\triangle ABC$: AB=2x+1, AC=4x, BC=?.

Solution

Given: A is the top vertex, DE is a horizontal line below BC, and BC || DE. Side lengths in terms of x.
To Prove: Find the value of x and the lengths of all sides.



Step 1: Since BC || DE, by the property of similar triangles, $\triangle ABC \sim \triangle ADE$. *(AA similarity)*

Step 2: The ratio of corresponding sides must be equal: $\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$.

Step 3: Substitute the given expressions: $\frac{2x+1}{18} = \frac{4x}{6x}$.

Step 4: From the second part of the ratio, we can find the similarity constant: $\frac{4x}{6x} = \frac{4}{6} = \frac{2}{3}$.

Step 5: Now use this ratio to solve for x: $\frac{2x+1}{18} = \frac{2}{3}$.

Step 6: Cross-multiply: $3(2x+1) = 18 \times 2$. *(algebra)*

Step 7: $6x + 3 = 36 \implies 6x = 33 \implies x = 5.5$.

Step 8: Now calculate the lengths of the sides for both triangles.

Step 9: **For $\triangle ABC$:**

Step 10: AB $= 2x + 1 = 2(5.5) + 1 = 11 + 1 = 12$ cm.

Step 11: AC $= 4x = 4(5.5) = 22$ cm.

Step 12: To find BC, use the ratio $\frac{BC}{DE} = \frac{2}{3}$. First find DE.

Step 13: DE $= 3(x+1) = 3(5.5+1) = 3(6.5) = 19.5$ cm.

Step 14: $BC = \frac{2}{3} \times DE = \frac{2}{3} \times 19.5 = 13$ cm.

Step 15: **For $\triangle ADE$:**

Step 16: AD $= 18$ cm.

Step 17: AE $= 6x = 6(5.5) = 33$ cm.

Step 18: DE $= 19.5$ cm.

Answer: $x = 5.5$.

Sides of $\triangle ABC$: AB=12 cm, AC=22 cm, BC=13 cm. Sides of $\triangle ADE$: AD=18 cm, AE=33 cm, DE=19.5 cm.

Question 5

Question: In $\triangle ABC$ and $\triangle DEF$, AB = 3x · DF, BC = 3x · DE and AC = 3x · EF. Show the triangles are similar and name them properly.

Solution

Given: Relationships between the sides of $\triangle ABC$ and $\triangle DEF$.
To Prove: Show similarity and provide the correct correspondence.

Step 1: From the given equations, form ratios of corresponding sides.

Step 2: From $AB = 3x \cdot DF$, we get $\frac{AB}{DF} = 3x$.

Step 3: From $BC = 3x \cdot DE$, we get $\frac{BC}{DE} = 3x$.

Step 4: From $AC = 3x \cdot EF$, we get $\frac{AC}{EF} = 3x$.

Step 5: We have $\frac{AB}{DF} = \frac{BC}{DE} = \frac{AC}{EF} = 3x$.

Step 6: Since the ratios of all three pairs of sides are equal, the triangles are similar. *(SSS similarity)*

Step 7: To name them properly, match the corresponding vertices from the ratios:

Step 8: Side AB corresponds to DF.

Step 9: Side BC corresponds to DE.

Step 10: Side AC corresponds to FE.

Step 11: From $AB \leftrightarrow DF$ and $BC \leftrightarrow DE$, vertex B must correspond to vertex D.

Step 12: From $BC \leftrightarrow DE$ and $AC \leftrightarrow FE$, vertex C must correspond to vertex E.

Step 13: This leaves vertex A corresponding to vertex F.

Step 14: Therefore, the correct similarity statement is $\triangle ABC \sim \triangle FDE$.

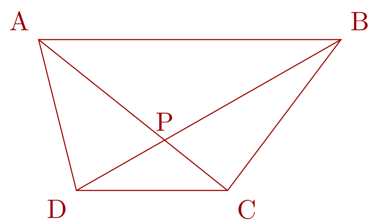
Answer: The triangles are similar by SSS. The correct correspondence is $\triangle ABC \sim \triangle FDE$.

Question 6

Question: In trapezium ABCD with AB ∥ DC, diagonals intersect at P. Prove (i) $\triangle APB \sim \triangle CPD$; (ii) $PA \cdot PD = PB \cdot PC$.

Solution

Given: Trapezium ABCD with AB ∥ DC, diagonals AC and BD intersect at P.
To Prove: (i) $\triangle APB \sim \triangle CPD$; (ii) $PA \cdot PD = PB \cdot PC$.



Step 1: **Part (i):** Consider $\triangle APB$ and $\triangle CPD$.

Step 2: Since AB ∥ DC, $\angle PAB = \angle PCD$. *(alternate interior angles)*

Step 3: Similarly, $\angle PBA = \angle PDC$. *(alternate interior angles)*

Step 4: Also, $\angle APB = \angle CPD$. *(vertically opposite angles)*

Step 5: Therefore, $\triangle APB \sim \triangle CPD$. *(AAA similarity)*

Step 6: **Part (ii):** From the similarity proved in part (i), the ratio of corresponding sides is equal.

Step 7: $\frac{PA}{PC} = \frac{PB}{PD} = \frac{AB}{CD}$.

Step 8: Taking the first two parts of the proportion: $\frac{PA}{PC} = \frac{PB}{PD}$.

Step 9: Cross-multiply to get $PA \cdot PD = PB \cdot PC$. *(algebra)*

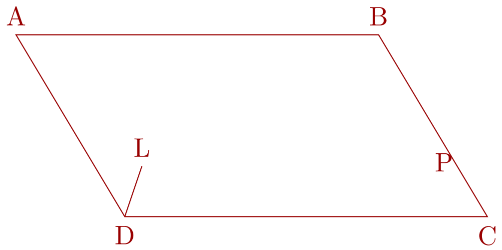
Answer: (i) The triangles are similar. (ii) The product equality is proved.

Question 7

Question: In parallelogram ABCD, P is a point on BC. DP is extended to meet AB extended at L. Prove: (i) DP:PL = DC:BL (ii) DL:DP = AL:DC.

Solution

Given: Parallelogram ABCD, P on BC, L is the intersection of extended DP and extended AB.
To Prove: (i) $\frac{DP}{PL} = \frac{DC}{BL}$; (ii) $\frac{DL}{DP} = \frac{AL}{DC}$.



Step 1: **Part (i):** Consider $\triangle DPC$ and $\triangle LPB$.

Step 2: Since AL is an extension of AB, and AB ∥ DC, we have AL ∥ DC.

Step 3: $\angle PDC = \angle PLB$ (or $\angle PLA$). *(alternate interior angles)*

Step 4: $\angle PCD = \angle PBL$. *(alternate interior angles, since $AD \parallel BC$)*

Step 5: Therefore, $\triangle DPC \sim \triangle LPB$. *(AA similarity)*

Step 6: The ratio of corresponding sides is $\frac{DP}{LP} = \frac{PC}{PB} = \frac{DC}{LB}$.

Step 7: From this, we get the required relation: $\frac{DP}{PL} = \frac{DC}{BL}$.

Step 8: **Part (ii):** Start with the result from part (i): $\frac{PL}{DP} = \frac{BL}{DC}$. *(inverting the ratio)*

Step 9: Add 1 to both sides: $\frac{PL}{DP} + 1 = \frac{BL}{DC} + 1$. *(algebra)*

Step 10: Combine terms: $\frac{PL + DP}{DP} = \frac{BL + DC}{DC}$.

Step 11: Since D, P, L are collinear, $PL + DP = DL$.

Step 12: Since A, B, L are collinear, $BL + AB = AL$. Also, $AB = DC$ in a parallelogram.

Step 13: Substitute these into the equation: $\frac{DL}{DP} = \frac{BL + AB}{DC} = \frac{AL}{DC}$.

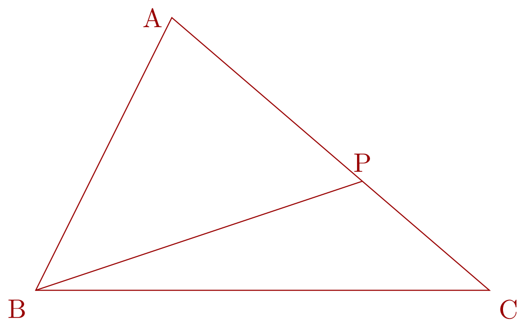
Answer: Both proportions are proved.

Question 8

Question: In $\triangle ABC$, $\angle ABC = 2\angle ACB$. The bisector of $\angle ABC$, BP, meets AC at P. Show: (i) CB:BA = CP:PA (ii) $AB \cdot BC = BP \cdot CA$.

Solution

Given: BP bisects $\angle ABC$; $\angle ABC = 2\angle ACB$.
To Prove: (i) CB:BA = CP:PA; (ii) $AB \cdot BC = BP \cdot CA$.



Step 1: **Part (i):**

Step 2: In $\triangle ABC$, BP is the bisector of angle $\angle B$.

Step 3: By the Angle Bisector Theorem, the bisector divides the opposite side in the ratio of the other two sides.

Step 4: Therefore, $\frac{CB}{BA} = \frac{CP}{PA}$. This is equivalent to CB:BA = CP:PA.

Step 5: **Part (ii):**

Step 6: Let $\angle ACB = y$. Then the given condition is $\angle ABC = 2y$.

Step 7: Since BP bisects $\angle ABC$, we have $\angle ABP = \angle PBC = y$.

Step 8: Now consider $\triangle BPC$. We have $\angle PBC = y$ and $\angle PCB = y$.

Step 9: Since two angles are equal, $\triangle BPC$ is an isosceles triangle with $BP = PC$.

Step 10: Now consider $\triangle ABC$ and $\triangle APB$.

Step 11: $\angle BAC = \angle PAB$ (Common Angle A).

Step 12: $\angle ACB = y$ and $\angle ABP = y$. So, $\angle ACB = \angle ABP$.

Step 13: Therefore, $\triangle ABC \sim \triangle APB$. *(AA similarity)*

Step 14: The ratio of corresponding sides is $\frac{AB}{AP} = \frac{BC}{PB} = \frac{AC}{AB}$.

Step 15: From $\frac{BC}{PB} = \frac{AC}{AB}$, we cross-multiply to get $AB \cdot BC = PB \cdot AC$.

Step 16: Note: The question has $BP \cdot CA$, which is the same.

Answer: Both parts are proved.