Computationally Efficient Features for Leaf Image Classification

Abstract

Plant species identification is time consuming, costly, and requires lots of efforts, and expertise knowledge. In the field of medicinal plants identification, most of the research are done recently based on leaf images. The reason is that leaf images are considered as they contain a large number of diverse set of features such as shape, veins, edge features, apices etc. that are useful in idetifying medicinal plants. Image processing, and feature extraction are the most important, challenging, and crucial steps in classifying leaf images. The purpose of image processing is to improve the leaf image by removing undesired distortion. The main image processing steps are i)Convert original image to RGB image, ii)Gray scaling, iii)Gaussian smoothing, iv)Binary thresholding, v)Remove stalk, vi)Closing holes, and vii)Resize image. The next step after image processing is to extract features from plant leaf images. We introduced 52 computationally efficient features to classify plant species. These features are mainly classified in to four groups as shape, color, texture, and scagnostics. Length, width, area, texture correlation, and monotonocity are some of them. The image processing algorithms are developed based on Python. The algorithms regard to shape, color, and texture features are based on Python, and the scagnostic features are extracted using R. In addition, supervised dimensionality reduction techique as Linear Discriminant Analysis (LDA), and unsupervised dimensionality reduction techique as Principal Component Analysis (PCA) are used to visualize the images in feature space. All the applications are based on flavia, and swedish which are the open source repositories. It is observed that LDA perform better than PCA for shapes of leaf image classification using introduced features.

Introduction

Leaf identification is becoming very popular in classifying plant species. Leaf contains significant features that can help people to identify and classify the plant species in developing. In medical perspective, medicinal plants are usually identified by practitioners based on years of experience through sensory or olfactory senses. The other method of recognizing these plants involves laboratory-based testing, which requires trained skills, data interpretation which is costly and time-intensive. Automatic ways to identify medicinal plants are useful especially those that are lacking experience in medicinal plant recognition. Statistical machine learning techniques play a crucial role in the development of automatic system to identify medicinal plants. In this process, image processing, and feature extraction have an important influence, because they are the initial step in identification.

The main aim of image processing is to extract important features by removing undesired noise and distortion (Waldchen and Mader 2018). Image preprocessing plays a vital role, because features have to be clearly found after preprocessing. Image processing steps include image segmentation (Anantrasirichai, Hannuna, and Canagarajah 2017), image orientation, cropping, grey scaling, binary thresholding, noise removal, contrast stretching, threshold inversion, image normalization, and edge recognition are some of image processing techniques applied in recent research. These steps can be applied parallely or individually, on several times until the quality of leaf image reaches a specific threshold.

One of the most challenging part is to extract distinctive leaf features from the images. Therefore most of the time research more focused on neural network models like CNN (Wu et al. 2007; Azlah et al. 2019; Herdiyeni and Wahyuni 2012) which are complicated and hard to understand what happening inside the algorithm. Feature extraction refers to taking measurements, geometric or otherwise, of possibly segmented, meaningful regions in the image (Waldchen and Mader 2018).

A digital image is merely a collection of pixels represented as large matrix of integers corresponding to intensities of colors at different positions of the image (Gonzalez and Woods 2006). The main aim of feature extraction is to reduce dimensionality of this information by obtaining patterns of leaf images. In general shape, colors, and texture contain these patterns. In this paper other than existing features, we are introducing scagnostic features, correlation of cartesian coordinate, number of convex points, and number of minimum and maximum points as new features.

The paper is organized as follows. In the first section, describes about the steps of image processing. Preprocessing is necessary before extracting features from the images. In section 2, discusses about feature extraction in in-detail because features are highly influenced by the plant species to be classified. Under this section, we discuss about mainly four types of features as shape, color, texture, and scagnostics and how to extract them. The next section 3, discuss about applications of leaf features. This section consists of details about the datasets that are used to explain the applications, and visualization of leaf images in the feature space using supervised, and unsupervised dimensionality reduction techniques. In section 4, consist of summary of the software and packages that use to extract the features. Some discussion about the outputs and concluding remarks are given in last section.

Image Processing

Introduction

Image processing is an essential step to reduce noise, background subtraction and content enhancement in the identification process (Goyal, Kapil, and Kumar 2018). The workflow we use to process images in this paper is shown in Figure 1. This includes four main steps. They are converting BGR image to RGB, gray scaling, Gaussian filtering, binary thresholding, remove stalk, close holes, and image resizing. Some of these steps can be applied for the necessary cases like applying remove stalk to the leaf images which only have a stalk.

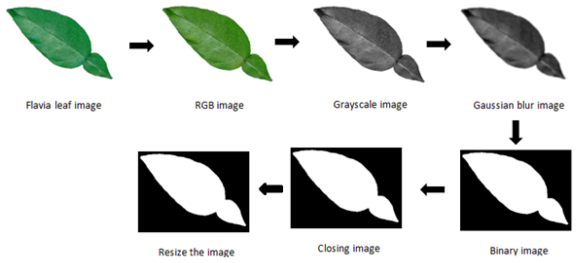


Figure 1: Image processing

In classifying plant species the focus was on the leaf which has a simple arrangement as shown in Figure 2. A single leaf that is never divided into smaller leaflet units is know as a leaf with simple arrangement. That leaf is always attached to a twing by its stem or the petiole. The margins, or edges, of the leaf can be smooth, lobed, or toothed.

Step 1: Converting BGR Image to RGB

BGR (Blue-Green-Red) and RGB(Red-Green-Blue) are conventions for the order of the different colour channels. They are not colour spaces. When converting BGR image to RGB, there is no any computations, just switches around the order. There is a difference in OpenCV and Matplotlib in pixel ordering. OpenCV follows BGR order while Matplotlib follows RGB order. Therefore when we want to display an image which loaded from OpenCV using Matplotlib functions, have to convert it to RGB mode.

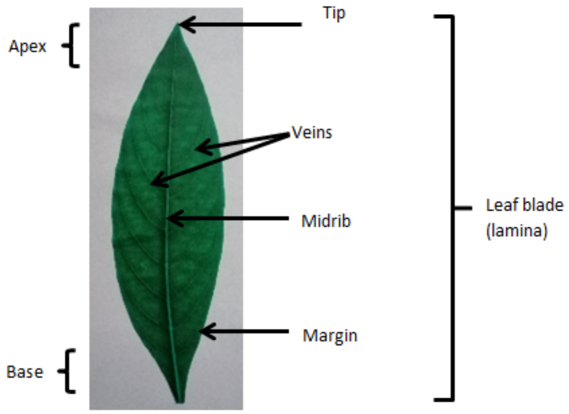


Figure 2: Leaf with simple arrangement

Step 2: Grayscaling

Grayscaling is the process of converting an image to shades of gray from other colour spaces like RGB. Gray conversion of the image is implemented to optimize the contrast and intensity of images (Goyal, Kapil, and Kumar 2018). Grayscaled images have several advantages over RGB images.

(i) Dimension reduction

As an example, there are three colour channels in RGB images and has three dimensions. But in gray scaled images only have one dimension.

(ii) For other algorithms to work

There are many algorithm customized to work only on gray scaled images. Eg: Haralick (Haralick, Shanmugam, and Dinstein 1973) texture features calculation works only on gray scaled images.

(iii) Reduced model complexity

Consider an example of training neural article on RGB images of 10x10x3 pixel. The input layer will have 300 input nodes. Whereas for gray scaled images the same neural network will need only 100 input node.

Step 3: Image Smoothing

Image smoothing is also known as image blurring in OpenCV. Image smoothing refers to making the image less clear or distinct. Image smoothing is done with the help of various low pass filter kernels.

Advantages of Smoothing include

(i) Smoothing techniques help in noise removal. Noise of an image is considered as high pass signal which can be occurred because of the source (camera sensor). Therefore restrict noise by using the low pass kernel.

(ii) Helps in smoothing image.

(iii) To remove low intensity edges.

(iv) Helps in hiding the details when necessary. E:g:- To hide the face of the victim in police cases.

There are many image smoothing techniques that are available in OpenCV. For our research we used Gaussian filtering as the image smoothing technique.

Step 4: Gaussian Filtering (Gaussian Blurring/Gaussian Smoothing)

Gaussian function is used to blur the image. It is a linear filter which is done by using the functions in OpenCV. By specifying the width and height of the Gaussian kernel that must be positive and odd, and specifying the kernel standard deviation along x and y-axis, Gaussian smoothing is established in OpenCV. When the kernel standard deviation along x-axis is specified, kernel standard deviation along y-axis is taken as equal to the the kernel standard deviation along x-axis. But if both kernel standard deviation are given as zeros, they are calculated by using the kernel size. In our research the width and height of the kernel is defined as 55 and the kernel standard deviation along x-axis is assigned as zero.

Step 5: Binary Thresholding

Thresholding is a segmentation technique that is used to separate foreground from its background. In thresholding technique, the pixel values are assigned by using the threshold value. By comparing each pixel value with the threshold value, the thresholding technique is worked.If the pixel value is smaller than the threshold value, the pixel value is set as 0, and if not the pixel value is set to a maximum value which is generally 255. Thersholding technique is done on grayscale images in Computer Vision.

We used Otsu's binarization which is an adaptive thresholding after Gaussian filtering to convert colour images to the binary images. In Otsu's method, the threshold value is determined automatically. The algorithm of Otsu's method finds the optimal threshold value which is chosen arbitrary.

Otsu's Thresholding

Nobuyuki Otsu is the investor of Otsu's method which is defined for a gray scale histogram h_I of an input image I. To segment an image I into two subsets of pixels Otsu's method calculates an optimal threshold τ. Image I is defined on a regular carrier Ω containing $|\Omega|$ pixel locations.

The algorithm maximizes the variance σ^2 between the two subsets (Within-class-variance) to find the threshold τ.

$$\sigma^2 = P_1(\mu_1 - \mu)^2 + P_2(\mu_2 - \mu)^2 = P_1 P_2(\mu_1 - \mu_2)^2$$

where μ is the mean of the histogram, μ_1 and μ_2 are the mean values of first and second subset respectively, P_1 and P_2 are the corresponding probabilities of the two clusters, defined by

$$P_1 = \frac{\sum_{i=0}^{u} h_I(i)}{|\Omega|}$$

$$P_2 = \frac{\sum_{i=u+1}^{255} h_I(i)}{|\Omega|}$$

Where u is the candidate threshold and the maximum gray level (G_{max}) is assumed as 255. To find optimal threshold τ for segmenting image I, all candidate thresholds are evaluated this way.

The algorithm of Otsu's method is defined as follows,

```
Compute the histogram of the grayscale image
Set the histogram variance $S_{max} = 0$
while $u < G_{max}$ do
    Compute $\sigma^2 = P_1 P_2 (\mu_1 - \mu_2)^2$
    if $\sigma^2 > S_{max}$ then
        $S_{max} = \sigma^2$
        $\tau = u$
    end if
    Set $u = u + 1$
end while
```

Table 1: Otsu's method

Step 6: Image Resizing

There are two different leaf image datasets (Flavia, Swedish) which have different sizes. To compare the results on different datasets, to improve the memory storage capacity and to reduce computational complexity the leaf images are resized to a fixed resolution. In our study, the leaf images have been resized to [1600 x 1200px] which is the size of Flavia leaf images.

Other than the main image processing techniques, the following two techniques are applied in some or all cases as an image processing techniques after image thresholding.

Remove Stalk

Remove the petiole (stalk) of leaf image is another version of thresholding process. Thresholding is applied after finding the sure foreground area. To find the sure foreground area, distance transform technique is used. Binary image is used as the input of distance transform technique. In distance transform technique, image is created by assigning a number for each object pixel that corresponds to the distance to the nearest background pixel. The distance is calculated using the euclidean distance (1). After finding the sure foreground area, Otsu's binarization is applied again as the thresholding technique.

$$Euclideandistance = \sqrt{\sum_{i=1}^{n} (q_i - p_i)^2} \tag{1}$$

where n = n-space

q_i, p_i = Euclidean vectors, starting from the origin of the space

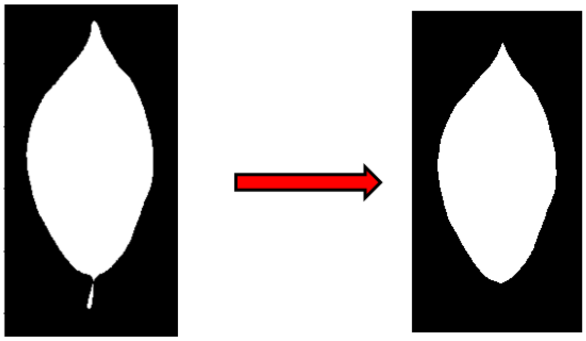


Figure 3: Remove stalk

Closing Holes

Closing holes is followed by noise removal technique which demonstrates the closing effect. Closing effect is used to remove small holes inside the foreground objects. Closing holes is a result of morphological transformation. morphological transformation are the operations based on image shape. Binary image is used as one input in morphological transformation. Kernel or structuring element which decides the nature of the operation is used as the second input. There are two morphological operators as Erosion and Dilation. Closing is a variant form of morphological operators which is used in closing holes process. Closing is also know as Dilation followed by Erosion.

Erosion

The basic idea of Erosion is that erodes away the boundaries of foreground object. Since the input is binary image, a pixels in the original image is either 1 or 0. If all the pixels under the kernel is 1, a pixel of original image is considered as 1, otherwise made to zero (eroded). Which means that depending upon the size of the kernel all pixels near boundary will be discarded. Therefore the thickness or size of the foreground object decreases (White region of the image decreases).

Dilation

Opposite of erosion is defined as Dilation. If at least one pixel under the kernel is 1, the pixel element is 1 in Dilation. It tends to increase the foreground of the image or the white region of the object.

Noise removal is that a technique of erosion is followed by Dilation. In erosion white noise is removed and shrinks the object (dilate) which doesn't come back. But in closing holes approach the white area is increased.

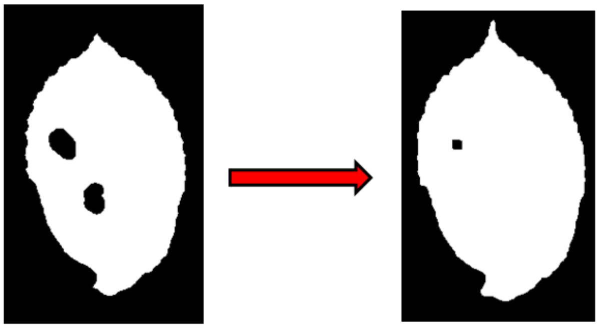


Figure 4: Closing holes

Leaf Features

In identification of plant species by using leaf images, features of the leaves play a main role, because each leaf posses unique feature that it make different from other. In previous research (Azlah et al. 2019; Jeon and Rhee 2017), let the algorithm like CNN to extract features by itself and do the classification. Therefore it is so hard to interpret and generalize the features. We introduced pre-calculate features which can be easy to interpret and generalize. They are also computational efficient. Mainly we focused on four types of features of leaf images as Shape features, Texture features, Color features and Scagnostics features. We identified altogether 52 features.

Shape Features

When identifying real-world objects, the shape is known as an essential sign for humans. A shape measure is a quantity, which relates to a particular shape characteristic of an object in general (Waldchen and Mader 2018).

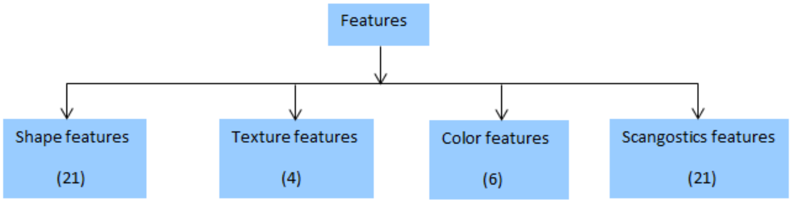


Figure 5: Example of geometric transformation

The main geometric transformation are rotation reflection, scaling and translation (see figure 6). In my research, the defined shape descriptors (shape features) are invariant to the rotation and reflection. But some limitations are applied in translation and scaling.

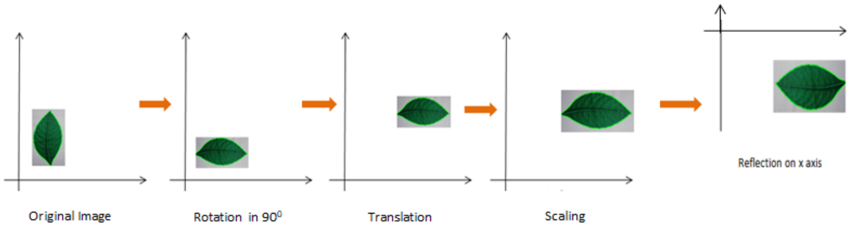
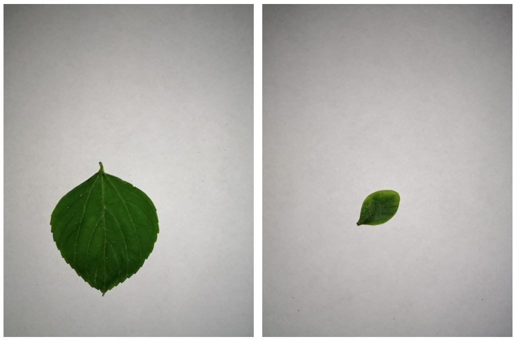


Figure 6: Example of geometric transformation

The finding contour function doesn't work when the center of the leaf image is not in the contour. Therefore if the translation is applied away from the contour **??**, then function can't identify the contour. Inappropriate scaling (see figure **??**) also arises problems in the calculation. If the leaf image is really small, the function also hard to recognize the contour. Therefore taking the closest photo of the leaf images by keeping them in the center of the white paper is more suitable.



(a) Inappropriate translation (b) Inappropriate scaling

Contours

Simply contour (see figure 7) is a curve joining all the continuous points (along the boundary), having the same colour or intensity.

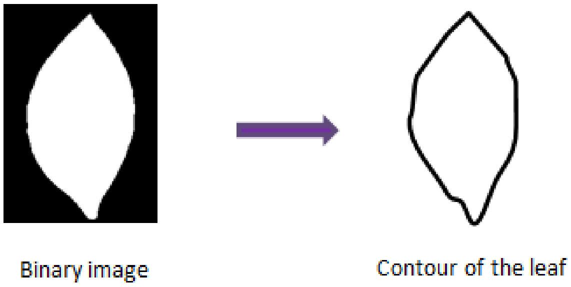


Figure 7: Extract contour of the leaf image

Shape descriptors can be classified into two main categories as contour-based and region-based. Contour-based descriptors extract shape features solely from the contour of a shape (Waldchen and Mader 2018). Whereas, region-based descriptors obtain shape features from the whole region of a shape (Waldchen and Mader 2018). In addition, there also exist some methods, which cannot be classified as either contour-based or region-based (see figure 8).

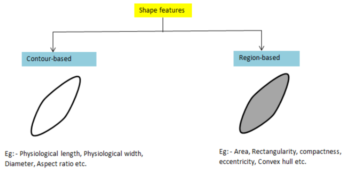

Shape features
Contour-based
Region-based
Eg:- Physiological length, Physiological width, Diameter, Aspect ratio etc.
Eg:- Area, Rectangularity, compactness, eccentricity, Convex hull etc.

Figure 8: Categorization of shape features

Through this research, we restricted our research to the following shape features to identify leaf images (see 2).

Diameter

Diameter is defined as the longest distance between any two points on the margin of the leaf (Waldchen and Mader 2018).

To calculate the diameter of the leaf image, firstly we have to find the contour of the leaf image. Then we have to select all pair of contour points and measure the Euclidean distance (equation 2) between the two points separately. Finally have to find the maximum distance among the calculated distances. (see figure 9)

$$d\left(A,B\right)=\sqrt{\sum_{i=1}^{n}\left(q_i-p_i\right)^2} \tag{2}$$

Physiological length and Physiological width

According to the authors in Azlah et al. (2019), the physiological length is measured based on the main vein of leaf, as it stretches from the main vein to the end tip. The physiological width is the span of leaf viewed from one side to the other, from the leftmost point to the rightmost point of leaf (Azlah et al. 2019).

There are horizontal, vertical and angled leaf images in the datasets (Flavia, Swedish). The straight bounding rectangle is enough to extract physiological length and physiological width of horizontal, and vertical leaf images.

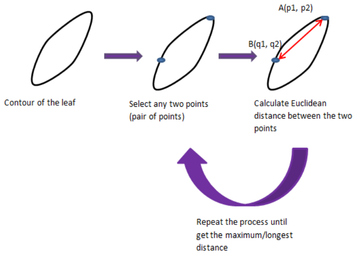

A(p1, p2)
B(q1, q2)
Contour of the leaf
Select any two points (pair of points)
Calculate Euclidean distance between the two points
Repeat the process until get the maximum/longest distance

Figure 9: Logic behind calculation of diameter

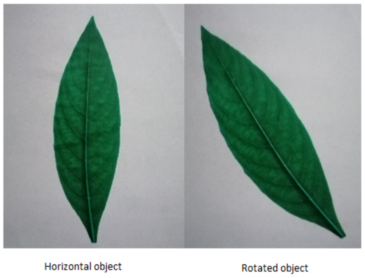
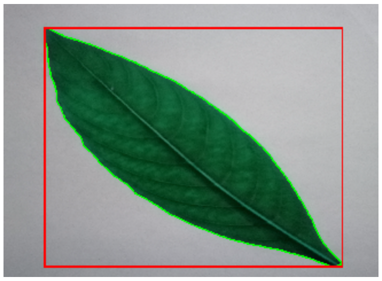
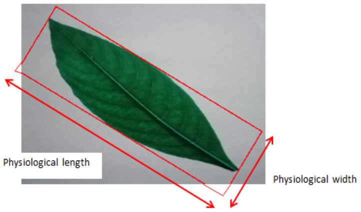


Figure 10: Straight(Horizontal or vertical) and rotated leaf image in Actual leaf image dataset

But straight bounding rectangle of angled image doesn't give the correct values for physiological length and physiological width of leaf images.



(a) Bounded rectangle of rotated leaf image in Actual leaf image dataset



(b) Rotated rectangle of angled leaf image in Actual leaf image dataset

To solve this problem, we considered rotated rectangle rather than bounded rectangle in computing shape features of angled images. The physiological length of the leaf image is the length of the rectangle. The physiological width of the leaf image is the width of the rectangle.

There are two types of bounding rectangles.

- Straight Bounding Rectangle

This is a straight rectangle which doesn't consider the rotation of the object. Therefore area of the bounding rectangle doesn't minimize.

- Rotated Rectangle

This bounding rectangle is drawn with minimum area. Therefore the rotation of the object is also considered.

Area

Area is defined as the product of thresholding process (Azlah et al. 2019). Pixel (Azlah et al. 2019) which is represented by the number 0 defines the leaf area.

OpenCV package in python is used to measure the area of the leaf. Firstly have to extract the best contour and based on the contour area is measured. Number of pixels covered by the contour is the measure of area of leaf image.

Figure 11 shows that the changes of the shape when the area is increasing.

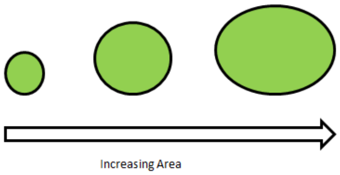


Figure 11: Area

Roundness/ Circularity

According to Waldchen and Mader (2018), roundness is named as aka form factor, circularity, or isoperimetric factor. Roundness of leaf image illustrates the difference between organ and a circle (Waldchen and Mader 2018). Equation 3 is used to calculate roundness.

$$R = \frac{4\pi A}{P^2} \tag{3}$$

Figure 12 shows what happen to the shape when the roundness measure is changed.

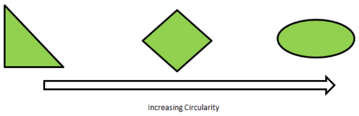


Figure 12: Circularity

Compactness

Compactness is closely related to roundness. Authors in Waldchen and Mader (2018) defined as compactness as the ratio of the perimeter over the leaf image's area (see equation 4).

$$C = \frac{P^2}{A} \tag{4}$$

Compactness measure is increasing (see figure 13), means that the leaf image is fitted to a circle and compact within the circle area.

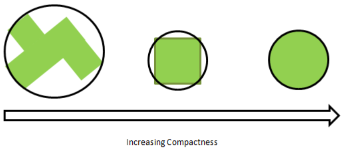


Figure 13: Compactness

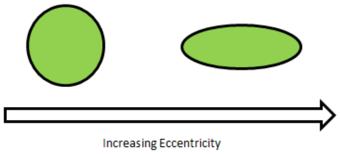


Figure 14: Eccentricity

Eccentricity

Eccentricity is definitive characteristic of any conic section of a leaf (**???**). Eccentricity is calculated using equation 5.

OpenCV package in Python is used to extract the ellipse of the leaf image.

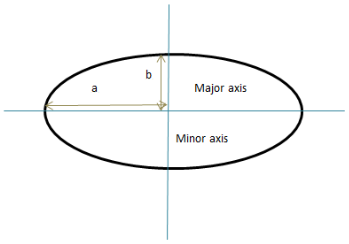


Figure 15: Ellipse

$$Eccentricity = \sqrt{1 - \frac{b^2}{a^2}} \tag{5}$$

Convexity

Convexity measures of leaf images are based on the convex hull that is extracted from the OpenCV package in python.

Convex hull

The more details about convex hull can be found in the convex hull under scagnostics features.

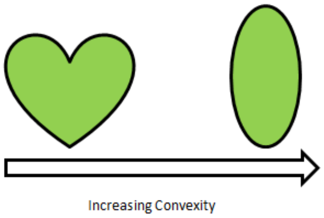


Figure 16: Convexity

Number of Minimum and Maximum Points

Number of minimum, and maximum points is a new measure which is obtained from the polar coordinate of leaf contour. Number of global maximum points are defined as number of maximum points. Number of global minimum points are defined as number of minimum points.

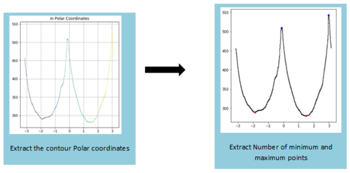


Figure 17: Minimum and Maximum Points

Table 2: Definitions of shape features

Shape feature	Description	Pictogram	Formula
Centroid	Represents the coordinates of the leaf's geometric center		
Physiological length/ Major axis length (L)	Line segment connecting the base and the tip of the leaf		
Physiological width/ Minor axis length (W)	Maximum width that is perpendicular to the major axis		
Diameter (D)	Longest distance between any two points on the margin of the origin		
Area (A)	Number of pixels in the region of the leaf		
Perimeter (P)	Summation of the distance between each adjoining pair of pixels around the border of the leaf		
Aspect ratio (AR)	Ratio of physiological length to physiological width		$AR = \frac{L}{W}$
Roundness/ Circularity (R)	Illustrate the difference between a leaf and a circle		$R = \frac{4\pi A}{P^2}$

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Table 2 – continued from previous page

Shape feature	Description	Pictogram	Formula
Compactness	Ratio of the perimeter over the leaf's area		$C = \dfrac{P^2}{A}$
Rectangularity (N)	Represents how rectangle a shape is, i:e: how much it fits its minimum bounding rectangle		$N = \dfrac{A}{LW}$
Eccentricity (E)	Ratio of the distance between foci of the ellipse (f) and major axis length (a)		$E = \dfrac{f}{a}$
Narrow factor (NF)	Ratio of the diameter over the physiological length		$NF = \dfrac{D}{L}$
Perimeter ratio of diameter (P_D)	Ratio of the perimeter to the dimeter		$P_D = \dfrac{P}{D}$
Perimeter ratio of Major axis length (P_L)	Ratio of the perimeter to the physiological length		$P_L = \dfrac{P}{L}$
Perimeter ratio of Major axis length and Minor axis length (P_{LW})	Ratio of the leaf perimeter over the sum of the physiological length and the physiological width		$P_{LW} = \dfrac{P}{L+W}$
Number of convex points	Number of points to create a convex hull		
Perimeter convexity (P_C)	Ratio of the convex perimeter to the perimeter of the leaf		$P_C = \dfrac{P_{CH}}{P}$
Area convexity (A_{C1})	Normalized difference of the convex hull area and the leaf's area		$A_{C1} = \dfrac{(CH - A)}{A}$
Area ratio of convexity (A_{C2})	Ratio between leaf's area and area of the leaf's convex hull		$A_{C2} = \dfrac{A}{CH}$

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Table 2 – continued from previous page

Shape feature	Description	Pictogram	Formula
Equivalent diameter (D_E)	Diameter of a circle with the the same area as the leaf's area		$D_E = \sqrt{\frac{4 * A}{\pi}}$

Texture Features

Texture is the term used to describe the surface of a given object or appearance and is undoubtedly a main feature used in computer vision and pattern recognition (Waldchen and Mader 2018). Texture can only be assessed for a group of pixels whereas colour is usually a property of a pixel. Generally, texture is associated with the feel of various materials to human touch and texture image analysis is based on visual interpretation (Waldchen and Mader 2018) of this feeling. Leaf surface is a natural texture which has random persistent patterns and do not show detectable quasi-periodic structure (Waldchen and Mader 2018). Therefore, several authors claim fractal theory to be better suited than statistical, spectral, and structural approaches for describing these natural textures (Waldchen and Mader 2018).

The Haralick texture features (Boland 2000; **???**) are functions of the normalized GLCM (Gray Level co-occurrence Matrix - see matrix **??**) which is a common method to represent image texture.

$$GLCM = \begin{bmatrix} p(1,1) & p(1,2) & \cdot & \cdot & \cdot & p(1,N_g) \\ p(2,1) & p(2,2) & \cdot & \cdot & \cdot & p(2,N_g) \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ p(N_g,1) & p(N_g,2) & \cdot & \cdot & \cdot & p(N_g,N_g) \end{bmatrix}$$

The GLCM (**??**) is square with dimension N_g, where N_g is the number of gray levels in the image (Boland 2000). Element [i,j] of the matrix is generated by counting the number of times a pixel with value i is adjacent to a pixel with value j and then dividing the entire matrix by the total number of such comparisons made (Boland 2000). Therefore each entry is considered to be the probability (see figure 19) that a pixel with value i will be found adjacent to a pixel of value j (Boland 2000). Four such matrices can be calculated, because adjacency can be defined to occur in each of four directions in a 2D (see figure 20), square pixel image (horizontal, vertical, left and right diagonals - see equation 18) (Boland 2000).

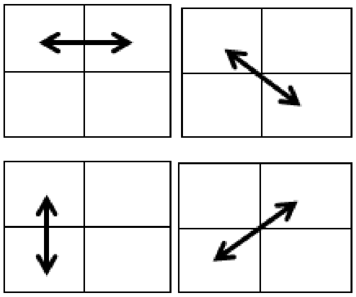


Figure 18: Four directions of adjacency as defined for calculation of the Haralick texture features

The Haralick statistics are calculated for co-occurrence matrices generated using each of these directions (see figure 19) of adjacency (Boland 2000). Haralick then described 14 statistics that can be calculated from the co-occurrence matrix with the intent of describing the texture of the image. Through the research, we only used the following 4 statistics among 14 of them, because most of the researchers used these 4 statistics as texture features (see figure 3) of leaf images.

Texture feature	Description	Formula
Contrast	Measures the scale of difference in the Gray level Co-occurrence matrices.	$\sum_{n=0}^{N_g-1} n^2 \{ \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p(i,j) \} , \|i-j\| = n$
Correlation	Measures of how correlated a pixel is to its neighbour over the whole image in the Gay level Co-occurrences matrices.	$\frac{\sum_i \sum_j (ij)p(i,j) - \mu_x \mu_y}{\sigma_x \sigma_y}$
Entropy	Measures the randomness of the elements of the co-occurrence matrix.	$-\sum_i \sum_j p(i,j) log(p(i,j))$
Inverse Difference Moment	Measure of homogeneity. Measures the closeness of the distribution of elements in the GLCM to the GLCM diagonal.	$\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \frac{1}{1+(i-j)^2} p(i,j)$

Table 3: Definitions of texture features

where p(i, j) = Probability density function of gray - level pairs

$$\mu_x = \sum_i i \sum_j p(i,j)$$

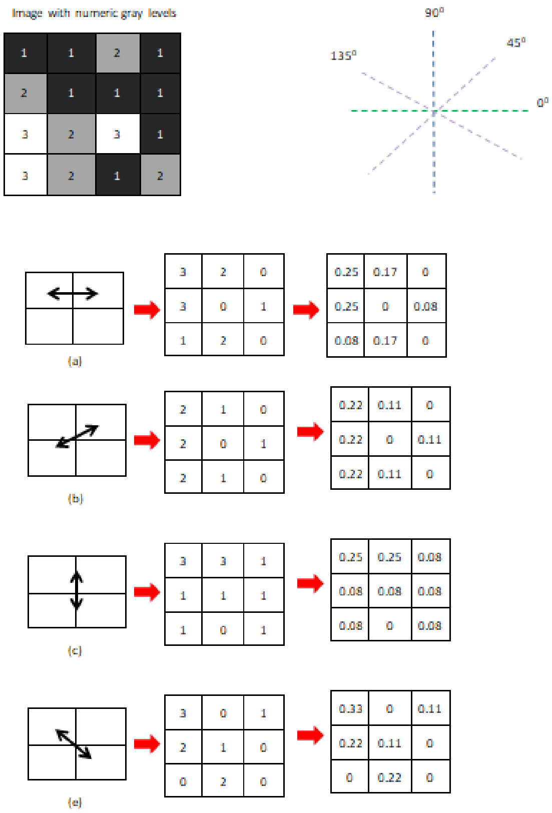
$$\mu_y = \sum_j j \sum_i p(i,j)$$

$$\sigma_x = \sum_i (i - \mu_x)^2 \sum_j p(i,j)$$

$$\sigma_y = \sum_j (j - \mu_y)^2 \sum_i p(i,j)$$

Color Features

Colour is an important feature of images (Waldchen and Mader 2018; Caglayan, Guclu, and Can 2013). Colour properties are defined within a particular colour space like red-green-blue (RGB) (Kodituwakku and S.Selvarajah 2010; Waldchen and Mader 2018). Colour properties can be extracted from images after a colour space is specified. In the field of image recognition, a number of general colour descriptors have been introduced. Colour moments (Kodituwakku and S.Selvarajah 2010; Waldchen and Mader 2018) are the



1	1	2	1
2	1	1	1
3	2	3	1
3	2	1	2

3	2	0
3	0	1
1	2	0

0.25	0.17	0
0.25	0	0.08
0.08	0.17	0

2	1	0
2	0	1
2	1	0

0.22	0.11	0
0.22	0	0.11
0.22	0.11	0

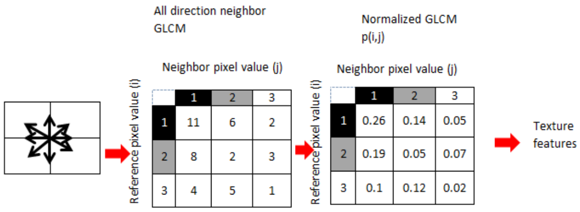
3	3	1
1	1	1
1	0	1

0.25	0.25	0.08
0.08	0.08	0.08
0.08	0	0.08

3	0	1
2	1	0
0	2	0

0.33	0	0.11
0.22	0.11	0
0	0.22	0

Figure 19: Computing the Haralick texture features from a 4 × 4 example image step by step



	1	2	3
1	11	6	2
2	8	2	3
3	4	5	1

	1	2	3
1	0.26	0.14	0.05
2	0.19	0.05	0.07
3	0.1	0.12	0.02

Figure 20: Computing the Haralick texture features from a 4 × 4 example image with all direction

simple descriptor among them. Mean, standard deviation skewness and kurtosis are the comment moments. Colour moments are used for characterizing planar colour patterns, irrespective of viewpoint or illumination conditions and without the need for object contour detection (Waldchen and Mader 2018). Colour moments are convenient for real-time applications because of its low dimension and low computational complexity.

Some of leaf images have very similar shape like Hathawariya (21) and Iramusu (21) in our experiment. Even though shapes are similar in some leaves, there are some differences in colours of leaf images. Therefore in addition to the shape features, we extracted colour based features of leaf images as well.

We used mean (μ) and standard deviation (σ) of intensity values of red, green and blue channels (Caglayan, Guclu, and Can 2013). Mean and standard deviation of each component are calculated as follows:

$$\mu = \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} p_{xy} \tag{6}$$

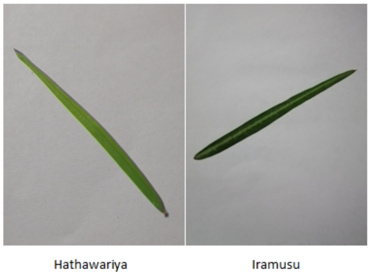


Figure 21

$$\sigma = \frac{1}{MN}\sqrt{\sum_{x=1}^{M}\sum_{y=1}^{N}(p_{xy}-\mu)^2} \tag{7}$$

where M and N are dimensions of a leaf image and p_{xy} is the intensity value of pixel at (x, y) coordinate.

Scagnostic features

Scatterplot diagnostics is a term in Tukey neologism for scagnostics (Wilkinson, Anand, and Grossman 2005; Wilkinson and Wills 2008; Dang and Wilkinson 2014; Dang, Anand, and Wilkinson 2013). Scagnostics are characterizations of the 2D distributions of orthogonal pairwise projections of a set of points in multidimensional Euclidean space (Wilkinson, Anand, and Grossman 2005; Wilkinson and Wills 2008). Measures like density, skewness, shape, outliers, and texture are included in these characterizations.

There are two people who popular in the discussion about scagnostics. John and Paul Tukey, and Wilkinson et al. (Wilkinson, Anand, and Grossman 2005; Wilkinson and Wills 2008) introduce their ideas about scagnostics in two angles. In first place mid of 1980s, an exploratory graphical method called scagnostics was introduced by John and Paul Tukey. A set of measures characterizing a 2D scatterplot (Dang and Wilkinson 2014) was the base of this method. But John and Paul Tukey were never published their ideas.

After some years later, the details collected from the first author's recollection of Institute for Mathematics and its Applications (IMA) and some discussions with Paul Tukey, Wilkinson et al. (2005) developed nine scagnostics measures defined on planar proximity graphs (Wilkinson, Anand, and Grossman 2005; Wilkinson and Wills 2008). These measures were scalable to large datasets and therefore suitable for practical applications (Wilkinson, Anand, and Grossman 2005; Wilkinson and Wills 2008).

Wilkinson et al. was introduced nine scagnostics measures which determined the cells of scatter plot matrix (SPLOM) (Dang and Wilkinson 2014). SPLOM means that the organization of scatterplots in the layout of a covariance matrix. In here the scatterplots are of the scagnostics measures.

By characterizing a large collection of 2D scatterplots through a small number of measures like the area of the peeled convex hull (Tukey 1974) (Wilkinson, Anand, and Grossman 2005; Wilkinson and Wills 2008), the perimeter length of this hull, the area of closed 2D kernel density isolevel contours (Silverman 1986) (Wilkinson, Anand, and Grossman 2005; Wilkinson and Wills 2008), the perimeter length of these contours, and a nonlinearity measure of association based on principal curves (Hastie and Stuetzle 1989) (Wilkinson, Anand, and Grossman 2005; Wilkinson and Wills 2008) of the arrangement of points in these plots was proposed by Tukeys. This was a simple and powerful idea, but when implementing many details wanted to involve.

Wilkinson et al. proposed his method by including the criteria that should met by candidate scagnostics.

1. Distinguish many types of point distributions: multivariate normal, lognormal, multinomial, sparse, dense, convex, clustered, etc.

2. A small number of scagnostics characterizing these distributions.

3. Should have a common scale because want to compare them with each other.
4. Should have a comparable distribution because want to compare them to standard.
5. The intrinsic dimensionality of these scagnostics, when calculated over a large number of heterogeneous scatterplots, to be as large as possible.
6. To be efficiently computable because the scagnostics should be scalable to large numbers of points and dimensions

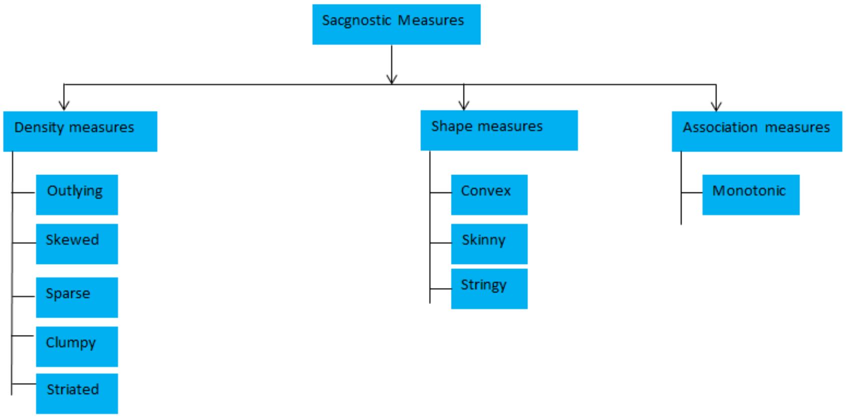


Figure 22: Hierarchy of Scagnostics

Scagnostic measures are based on following definitions.

Geometric Graphs

- Graph

A graph G = (V, E) is defined as a set V (called vertices) together with a relation on V induced by a set E (called edges). A pair of vertices is defined as an edge $e(\nu, \omega)$, with e ∈ E and $\nu, \omega \in$ V.

- Geometric Graph

An embedding of a graph in a metric space S that maps vertices to points and edges to straight line segments connecting pairs of points is defined as a geometric graph G* = [f(V), g(E), S].

From several features of 2D Euclidean geometric graphs, Scagnostic measures are derived.

The Euclidean distance between vertices that connected to edge is defined as the length of an edge, length(e).

The sum of the lengths of edges in graph is known as the length of a graph, length(G).

A list of successively adjacent, distinct edges are known as a path. If first and last vertex are the same of the path, then the path is closed.

A region bounded by a closed path is known as a polygon (P). A polygon bounded by exactly one closed path that has no intersecting edges is known as a simple polygon.

The length of boundary of a simple polygon is known as the perimeter of a simple polygon. The area of interior of a simple polygon is known as the area of a simple polygon.

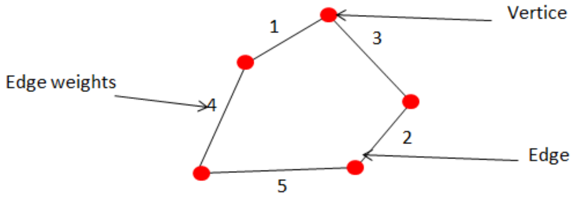


Figure 23: Graph with 5 vertices and 5 edges

Minimum Spanning Tree (MST)

- Tree

A graph in which any two nodes are connected by exactly one path is known as a *tree*.

- Spanning Tree

An undirected graph whose edges are structured as a tree is defines as a *Spanning Tree*.

Spanning Tree of Graph G is: G'(V',E')

$$V' = V$$

$$E' \subset E$$

$$E' = |V| - 1$$

The graph can have more than one spanning tree.

Spanning tree should not be disconnected and not contain any cycle. By removing one edge from the Spanning tree will make it disconnected. By adding one edge to the Spanning tree will create a loop. A complete (Each vertices connected with each other) undirected graph can have n^{n-2} number of spanning trees where n is the number of vertices. Every connected and undirected graph has at least one Spanning Tree. Disconnected graph doesn't have any spanning tree. From a complete graph by removing $max(edges - n + 1)$ edges we can construct a spanning tree.

- Minimum Spanning Tree

A spanning tree whose total length is least of all spanning trees on a given set of points is known as a Minimum Spanning Tree (MST).

If each edge has distinct weights then there will be only one and unique MST.

- Remark

The geometric MST computed from Euclidean distances between points in a 2D Euclidean geometric graph is the restriction.

Convex Hull

A collection of the boundaries of one or more simple polygons that have a subset of the points for their vertices and that collectively contain all the points, is defined as a hull of a set of points embedded in 2D Euclidean space.

If a hull contains all the straight line segments connecting any pair of points in its interior, is known as a convex hull. The convex hull bounds a single polygon. After deleting the points on the convex hull, a convex hull called peeled convex hull is computed.

Alpha Hull

Most of proximity graphs (neighborhood graph) represent the nonconvex shape of a set of points on the plane. A geometric graph whose edges are determined by an indicator function based on distances between a given set of points in a metric space, is known as a proximity graph. An open disk D is used to define the indicator function.

If a point is on the boundary of D then D *touches* a point and if a point is in D then D *contains* a point. An open disk of radius r is defined as $D(r)$.

An alpha shape (Dang and Wilkinson 2014) is a collection of one or more simple polygons (Wilkinson and Wills 2008; Wilkinson, Anand, and Grossman 2005). An edge exists between any pair of points that can be touched by an open disk $D(\alpha)$ containing no points, is defined as an alpha shape graph.

A value of α to be the average value of the edge lengths in the MST (Wilkinson and Wills 2008; Wilkinson, Anand, and Grossman 2005). The large values like 90th percentile of the MST edge lengths are used, because to reduce noise. If the percentile exceeds a tenth, clamp the value at one-tenth the width of a frame, because it prevents in including sparse or striated point sets in a single alpha graph.

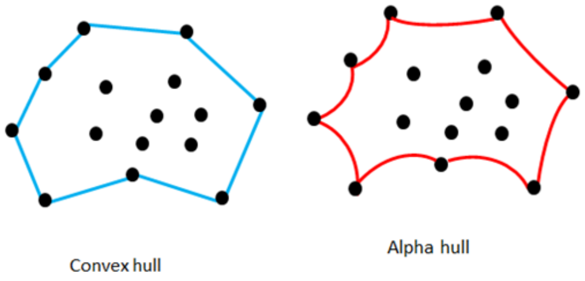


Figure 24: Convex hull and alpha hull

Preprocessing

To improve the performance of the algorithm and robustness of the measures, preprocessing techniques as binning and deleting outliers are used before computing geometric graphs.

- Binning

As the first step of binning, the data are normalized to the unit interval. Then use a 40 by 40 hexagonal grid to aggregate the points in each scatterplot. Reduce the bin size by half and rebin until no more than 250 non emty cells, if there are more than 250 non empty cells. By efficiency (too many bins slow down calculations of the geometric graphs) and sensitivity (too few bins obscure features in the scatterplots), the choice of bin size is constrained.

To improve the performance, hexagon binning is used. To manage the problem of having to many points that start to overlap, hexagon binning is used. The plots of hexagonal binning are density rather than points. To use hexagons instead of squares for binning a 2D surface as a plane, there are many reasons. Hexagons are more similar to circle than square.

To keep scagnostics orientation-independent this bias reduction is important. To attenuate the influence of binning, stabilizing transformation is used when computing scagnostics from binned data.

The weight function is defined as;

$$\omega = 0.7 + \frac{0.3}{1+t^2} \tag{8}$$

where $t = \frac{n}{500}$. (n is the number of vertex)

If $n > 2000$ then this function is fairly constant. By using hex binning the shape and the parameters of the function is determined. In computing Sparse, Skewed and Convex scagnostics this weight function is used to adjust for bias.

- Deleting Outliers

To improve robustness of the scagnostics, deleting outliers can be used. A vertex whose adjacent edges in the MST all have a weight (length) greater than ω is defined as an outlier in this context. By considering nonparametric criterion for the simplicity and Tukey's idea choose the following weight calculation.

$$\omega = q_{75} + 1.5(q_{75} - q_{25}) \tag{9}$$

where q_{75} is the 75th percentile of the MST edge lengths and $(q_{75} - q_{25})$ is the Interquartile range of the edge lengths.

Degree of a Vertex

The degree of a vertex in an undirected graph is know as the number of edges associated with the vertex.

Eg:- Vertices of degree 2 There are 2 edges associated with each vertex.

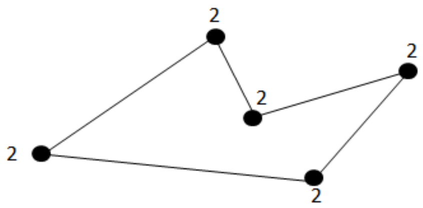


Figure 25: Vertices of degree 2

Geometric Graphs	Notation
Convex Hull	H
Alpha Hull	A
Minimum Spanning Tree	T

Table 4: Notations of Geometric Graphs

Density Measures

Detect different distributions of scattered points in density measures.

- Outlying

$$C_{outlying} = \frac{length(T_{outliers})}{length(T)} \tag{10}$$

The outlying measure calculate before deleting the outliers for the other measures. The proportion of the total edge length of the minimum spanning tree accounted for by the total length of edges adjacent to outlying points is used to calculate the outlying measure.

- Skewed

$$q_{skew} = \frac{q_{90} - q_{50}}{q_{90} - q_{10}} \tag{11}$$

$$C_{skew} = 1 - \omega(1 - q_{skew}) \tag{12}$$

where ω is the weight function (8).

The skewed measure is the first measure of relative density which is a relatively robust measure of skewness in the distribution of edge lengths. After adaptive binning skewed tends to decrease with n.

- Sparse

$$C_{sparse} = \omega q_{90} \tag{13}$$

where ω is the weight function (8) and q_{90} is the 90th percentile of the distribution of edge lengths in the MST.

The second relative density measure is Sparse measure that measures whether points in a 2D scatterplot are confined to a lattice or a small number of locations on the plane.

If the number of points is extremely small or tuples are produced by the product of categorical variables, then sparse can be happen.

$$q_{90} = \alpha statistic \tag{14}$$

The α statistic exceeds unity (e.g., when all points fall on either of the two diagonally opposing vertices of a square), clamp the value to 1 in the extremely rare event (Wilkinson and Wills 2008, @inproceedings44).

- Clumpy

$$C_{clumpy} = max_j[1 - \frac{max_k[length(e_k)]}{length(e_j)}] \tag{15}$$

Clustering points are not indicated by an extreme distribution of MST edge lengths. Therefore RUNT statistic (Wilkinson and Wills 2008, @inproceedings44) which is another measure based on the MST, is introduced. The smaller of the number of leaves of each of the two subtrees joined at that node isdefined as the runt size of a dendogram node. There is an association between runt size (r_j) each edge (e_j) in the MST because there is an isomorphism between a single-linkage dendrogram and the MST.

The smaller of the two subsets of edges that are still connected to each of the two vertices in e_j after deleting edges in the MST with lengths less than length(e_j), is known as the RUNT graph (R_j) (Wilkinson and Wills 2008, @inproceedings44).

The RUNT-based measure responds to clusters with small maximum intracluster distance relative to the length of their nearest-neighbor inter-cluster distance (Wilkinson and Wills 2008, @inproceedings44). In the formula j runs over all edges in MST and k runs over all edges in RUNT graph.

- Striated

$$C_{striated} = \frac{1}{|V|} \sum_{\nu \in V^{(2)}} I(\cos\theta_{e(\nu,a)e(\nu,b)} < -0.75) \tag{16}$$

where $V^{(2)} \subseteq V$ and $I()$ be an indicator function.

Striated define the coherence in a set of points as the presence of relatively smooth paths in the minimum spanning tree.

The measure is based on the number of adjacent edges whose cosine is less than minus 0.75.

Shape Measures

Both topological and geometric aspects of shape of a set of scattered points is considered. As an example, a set of scattered points on the plane appeared to be connected, convex and so forth, want to know under the shape measures. By definition scattered points are not like this. Therefore to make inferences additional machinery (based on geometric graphs) is needed. By measuring the aspects of the convex hull, the alpha hull, and the minimum spanning tree is determined.

- Convex

$$C_{convex} = \omega[area(A)/area(H)] \tag{17}$$

where ω is the weight function (8).

The ratio of the area of the alpha hall(A) and the area of the convex hull(H) is the base of measuring convexity.

$$Ratio = \frac{area(A)}{area(H)} \begin{cases} (=1); \text{ if the nonconvex hull and convex hull have identical areas} \\ \\ (\neq 1); \text{ otherwise} \end{cases}$$

‘

- Skinny

$$C_{skinny} = 1 - \frac{\sqrt{4\pi area(A)}}{perimeter(A)} \tag{18}$$

Roughly, the skinny is measured by using the corrected and normalized ratio of perimeter to area of a polygon measures.

$$C_{Skinny} = \begin{cases} \text{0; if circle} \\ \\ \text{Near 1; if skinny} \end{cases}$$

- Stringy

$$C_{stringy} = \frac{|V^{(2)}|}{|V| - |V^{(1)}|} \tag{19}$$

where V is the number of vertices.

A skinny shape with no branches is known as a stringy shape. By counting the vertices of degree 2 in the minimum spanning tree and comparing them to the overall number of vertices minus the

number of single-degree vertices, skinny measure is calculated.

To adjust for negative skew in its conditional distribution of n, cube the stringy measure.

Association Measure

Symmetric and relatively robust measure of association are interested.

- Monotonic

$$C_{monotonic} = r^2_{Spearman} \tag{20}$$

To assess the monotonicity in a scatter plot, the squared spearman correlation coefficient is used. This is the only coefficient not based on a subset of the Delaunay graph (Wilkinson and Wills 2008).

In calculating monotonicity, squared the coefficient because to consider the large values and to remove the distinction between positive and negative coefficients (Because assume that the investigators are more interested in strong relationships rather than negative or positive).

We measured the scagnostic features for Cartesian and Polar coordinates separately.

Example

There are two color image leaf datasets (Flavia ,Swedish) are used. After passing through the required image processing steps, binary image is extracted which has a white foreground and black background.

The leaf images are taken as the closest ones. Therefore to find the best contour among several contours, can use the contour which contains the center of leaf image. Identify the best contour is really important when extracting shape features.

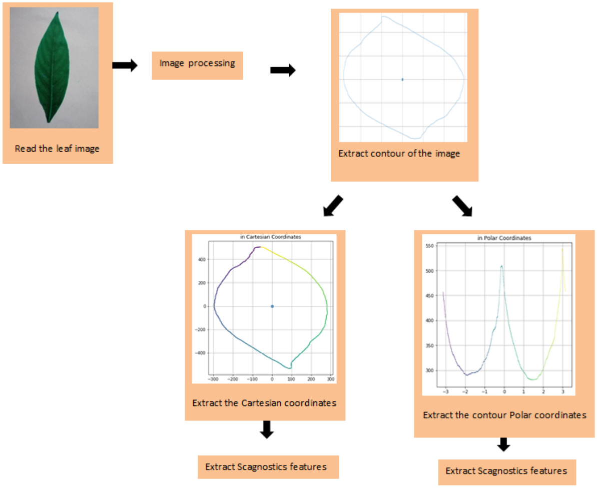

Read the leaf image
Image processing
Extract contour of the image
in Cartesian Coordinates
400
200
0
−200
−400
−300
−200
−100
0
100
200
300
Extract the Cartesian coordinates
Extract Scagnostics features
in Polar Coordinates
550
500
450
400
350
300
−3
−2
−1
0
1
2
3
Extract the contour Polar coordinates
Extract Scagnostics features

Figure 26: Preprocessing for Scagnostics

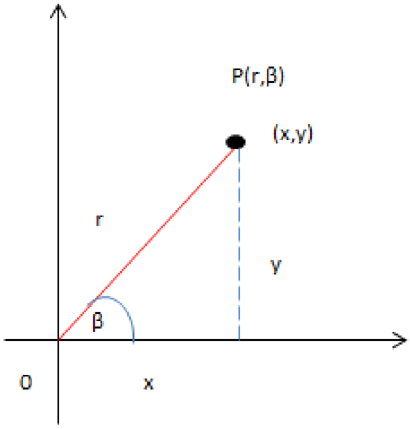


Figure 27: Polar coordinate

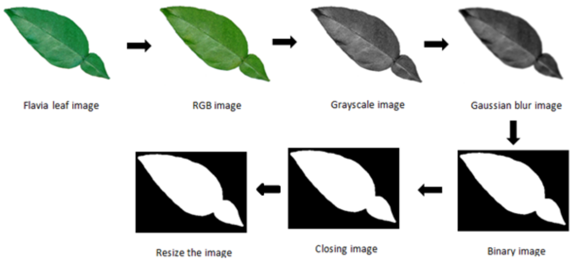


Figure 28: Image processing steps of Flavia dataset

Application of features to classify images

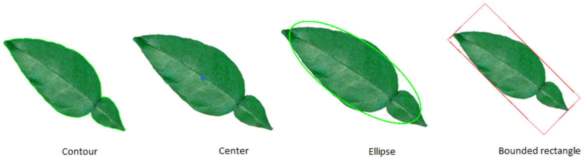


Figure 29: Example feature extraction of Flavia dataset

Data sets

We use two publicly available datasets for demonstrate the applications of features.

Flavia Leaf Image Dataset

The Flavia dataset contains 1907 leaf images. There are 32 different species and each have 50-77 images. Scanners and digital cameras are used to acquire the leaf images on plain background. The isolated leaf images contain blades only, without petiole. These leaf images are collected from the most common plants in Yangtze, Delta, China (Waldchen and Mader 2018). Those leaves were sampled on the campus of the Nanjing University and the Sun Yat-Sen arboretum, Nanking, China (Waldchen and Mader 2018). (https://sourceforge.net/projects/flavia/files/Leaf%2520Image%2520Dataset/)

Figure 30: Sample of Flavia dataset

Swedish Leaf Image Dataset

The Swedish dataset contains 1125 images. The images of isolated leaf scans on a plain background of 15 Swedish tree species, with 75 leaves per species. This dataset has been captured as part of a joined leaf classification project between the Linkoping University and the Swedish Museum of Natural History (Waldchen and Mader 2018). (https://www.cvl.isy.liu.se/en/research/datasets/swedish-leaf/)

Visualization of Leaf Images in the Feature Space

We used LDA, and PCA to visualize leaf images in feature space. LDA is a supervised dimensionality reduction technique, and PCA is unsupervised dimensionality reduction technique. In this section, we visualize, and compare the results obtained using LDA, and PCA on flavia and swedish datasets. To visualize LDA projection shape label is taken as the response variable. There are mainly 5 shape categories as diamond, simple round, round, needle, and heart shape. More details about the shape categories are discussed in our next paper.

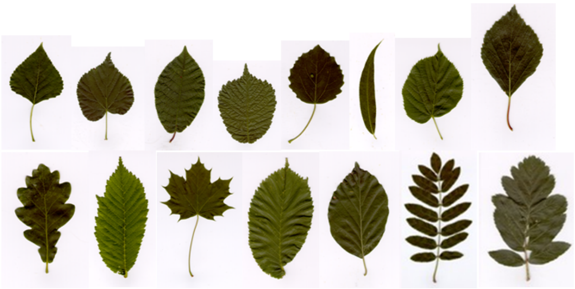
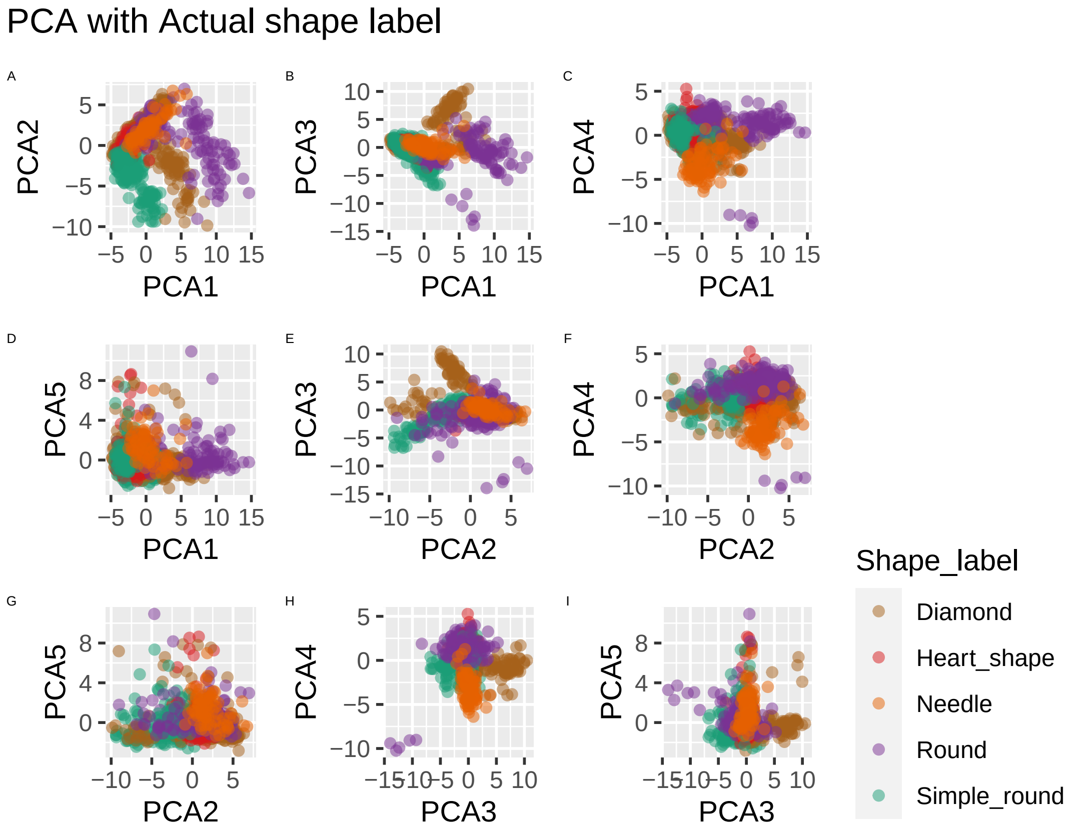


Figure 31: Sample of Swedish leaf dataset

Swedish Dataset

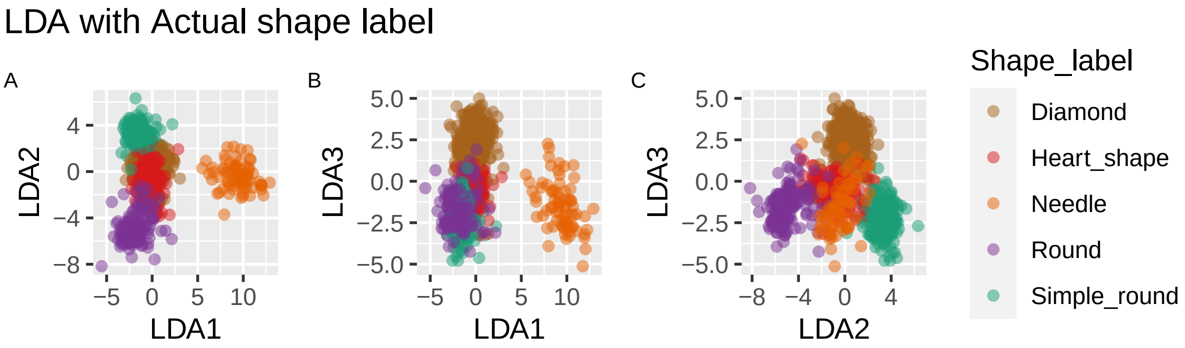
PCA Projection

The results of the PCA1 and PCA2, and PCA1 and PCA3 projection on swedish dataset shows clear separation among the shapes.



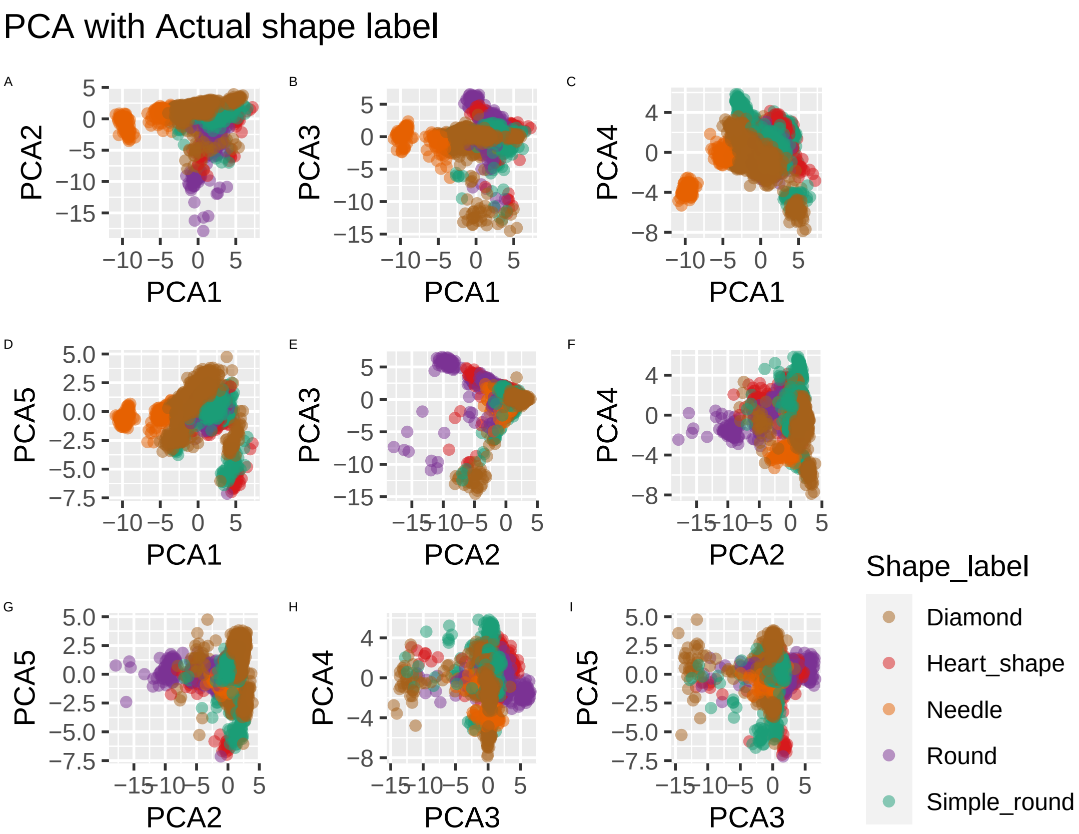
LDA Projection

When consider the LDA results on swedish dataset, LDA1, LDA2, and LDA3 shows the separation of shapes of leaf images.

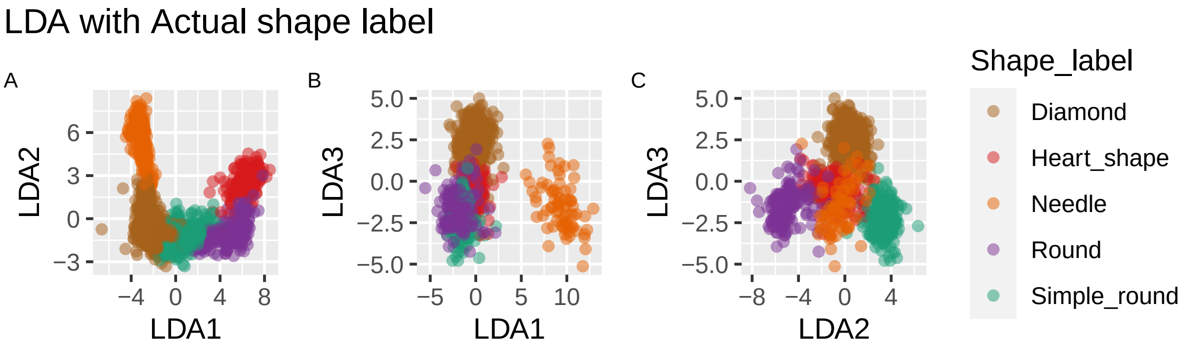


Flavia Dataset

PCA projection


PCA with Actual shape label

LDA Projection



The projected results of LDA1 and LDA2 on flavia dataset shows clear classification of leaf shapes than PCAs.

Software

Feature extraction algorithms are developed using R and python software. The following table 5 shows the package in software that used to extract each feature.

Discussion and Conclusions

There are mainly four categories of features that are used to classify leaf images. Many research were based on shape, color, and texture features. In this research paper, we introduced new feature category called scagnostics. Other than that correlation of cartesian coordinate, number of convex points, number of minimum and maximum points are introduced as new shape features. To visualize the leaf images on feature space LDA, and PCA are used. The results are obtained based on flavia and swedish datasets. LDA is performed better than PCA when classifying leaf shapes. More details about the classification of leaf images are described in our next paper.

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Feature name	Software	Package
Scagnostic features for polar and cartesian coordinates	R	binostics
Number of minimum and maximum points, Correlation of cartesian coordinate	R	-
Diameter	Python	combinations, numpy
Physiological length and width, Eccentricity, Area, Perimeter, Number of convex points, Perimeter of convex hull, Area of convex hull	Python	OpenCV (cv2)
Aspect ratio, Rectangularity, Compactness, Narrow factor, Perimeter ratio diameter, Perimeter ration length, Perimeter ratio length and width	Python	-
x and y coordinates of center	Python	scipy.ndimage
Circularity, Equivalent diameter	Python	numpy
Texture features	Python	mahotas
Color features		numpy

Table 5: Software in feature extraction

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