Hexagonal binning algorithm

Computing hexagon grid configurations

Given range of embedding component, r_2, number of bins along the x-axis, b_1, and buffer proportion, q, hexagonal starting point coordinates, $s_1 = -q$, and $s_2 = -q \times r_2$. The purpose is to find width of the hexagon. a_1, and number of bins along the y-axis, b_2.

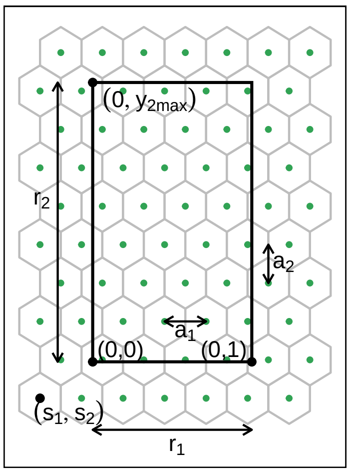

Figure 1: The components of the hexagon grid illustrating notation.

Geometric arguments give rise to the following constraints.

min a_1 s.t.

$$s_1 - \frac{a_1}{2} < 0, \tag{1}$$

$$s_1 + (b_1 - 1) \times a_1 > 1, \tag{2}$$

$$s_2 - \frac{a_2}{2} < 0, \tag{3}$$

$$s_2 + (b_2 - 1) \times a_2 > r_2. \tag{4}$$

Since a_1 and a_2 are distances,

$$a_1, a_2 > 0.$$

Also, $(s_1, s_2) \in (-0.1, -0.05)$ as these are multiplicative offsets in the negative direction.

Equation 1 can be rearranged as,

$$a_1 > 2s_1$$

which given $s_1 < 0$ and $a_1 > 0$ will *always* be true. The same logic follows for Equation 3 and substituting $a_2 = \frac{\sqrt{3}}{2}a_1$, and $s_2 = -q \times r_2$ to Equation 3 can be written as,

$$a_1 > -\frac{4}{\sqrt{3}}qr_2$$

Also, substituting $a_2 = \frac{\sqrt{3}}{2}a_1$, $s_2 = -q \times r_2$ and rearranging Equation 4 gives:

$$a_1 > \frac{2(r_2 + qr_2)}{\sqrt{3}(b_2 - 1)}. \tag{5}$$

Similarly, substituting $s_1 = -q$ Equation 2 becomes,

$$a_1 > \frac{(1 + q)}{(b_1 - 1)}. \tag{6}$$

This is a linear optimization problem. Therefore, the optimal solution must occur on a vertex. So, by setting Equation 5 equals to Equation 6 gives,

$$\frac{2(r_2 + qr_2)}{\sqrt{3}(b_2 - 1)} = \frac{(1 + q)}{(b_1 - 1)}.$$

After rearranging this,

$$b_2 = 1 + \frac{2r_2(b_1 - 1)}{\sqrt{3}}$$

and since b_2 should be an integer,

$$b_2 = \left\lceil 1 + \frac{2r_2(b_1 - 1)}{\sqrt{3}} \right\rceil. \tag{7}$$

Furthermore, with known b_1 and b_2, by considering Equation 2 or Equation 4 as the *binding* or *active constraint*, can compute a_1.

If Equation 2 is active, then,

$$\frac{(1+q)}{(b_1 - 1)} < \frac{2(r_2 + qr_2)}{\sqrt{3}(b_2 - 1)}.$$

Rearranging this gives,

$$r_2 > \frac{\sqrt{3}(b_2 - 1)}{2(b_1 - 1)}.$$

Therefore, if this equality is true, then $a_1 = \frac{(1+q)}{(b_1-1)}$, otherwise, $a_1 = \frac{2r_2(1+q)}{\sqrt{3}(b_2-1)}$.

Binning the data

Points are assigned to the bin they fall into based on the nearest centroid. If a point is equidistant from multiple centroids, it is assigned to the centroid with the lowest hexagonal bin ID.

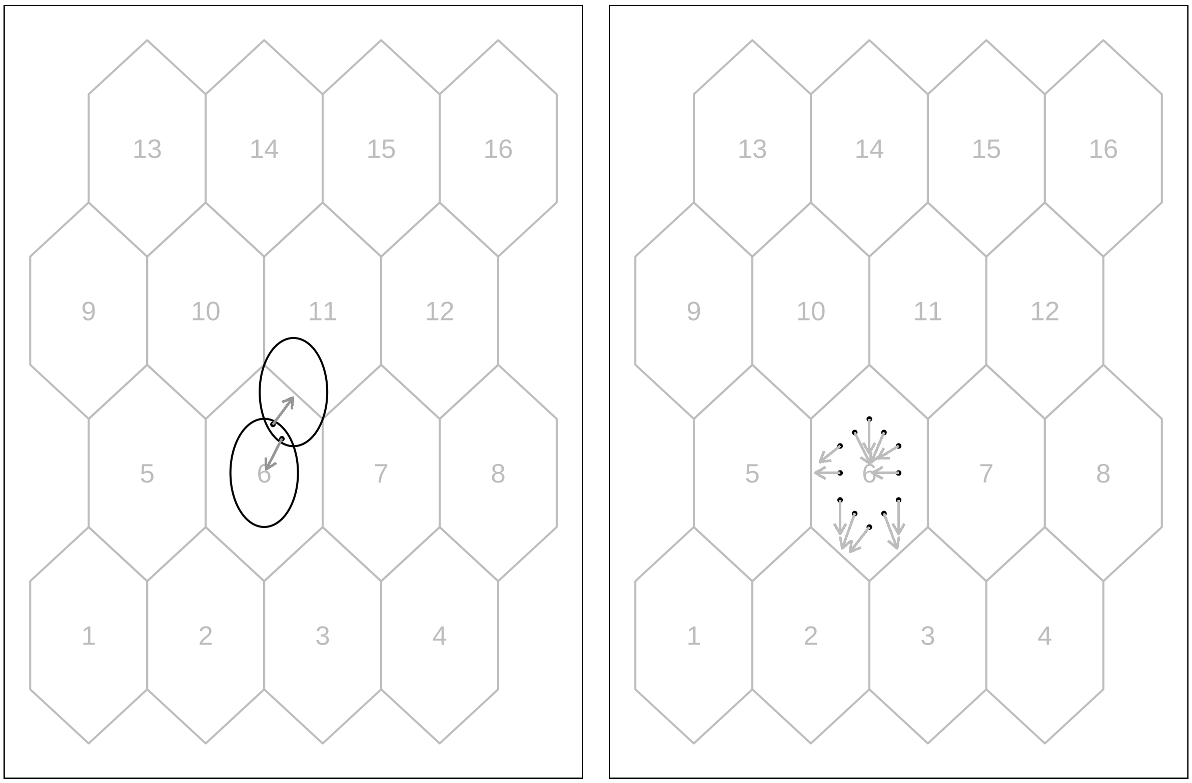

Figure 2: Binning the data. Points are assigned to the nearest centroid. If a point is equidistant from multiple centroids, assigned to the lowest centroid.