

# quollr: An R Package for Visualizing 2D Models in High Dimensional Space

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**Abstract** An abstract of less than 150 words.

```
#library(quollr)
library(readr)
library(ggplot2)
library(dplyr)
library(ggbeeswarm)
library(Rtsne)
library(umap)
library(phateR)
library(reticulate)
library(rsample)

set.seed(20230531)

use_python("~/miniforge3/envs/pcamp_env/bin/python")
use_condaenv("pcamp_env")

reticulate::source_python(paste0(here::here(), "/scripts/function_scripts/Fit_PacMAP_code.py"))
reticulate::source_python(paste0(here::here(), "/scripts/function_scripts/Fit_TriMAP_code.py"))
```

## 1 Introduction

## 2 Methodology

### Usage

- dependancies

```
library(tools)
package_dependencies("quollr")
```

- basic example

### Compute hexagonal bin configurations

```
num_bins_x <- calculate_effective_x_bins(.data = s_curve_noise_umap, x = "UMAP1", cell_area = 1)
num_bins_x
```

```
#> [1] 6
```

```
shape_val <- calculate_effective_shape_value(.data = s_curve_noise_umap, x = "UMAP1", y = "UMAP2")
shape_val
```

```
#> [1] 2.019414
```

```
num_bins_y <- calculate_effective_y_bins(.data = s_curve_noise_umap, x = "UMAP1", y = "UMAP2", shape_val = 1.8330)
num_bins_y
```

```
#> [1] 12
```

## Generate full hex grid

Generating full hexagonal grid contains main three steps:

1. Generate all the hexagonal bin centroids

Steps:

- First compute hex grid bound values along the x and y axis and generate the all the points within the hex box

```
cell_area <- 1
cell_diameter <- sqrt(2 * cell_area / sqrt(3))

hex_size <- cell_diameter/2

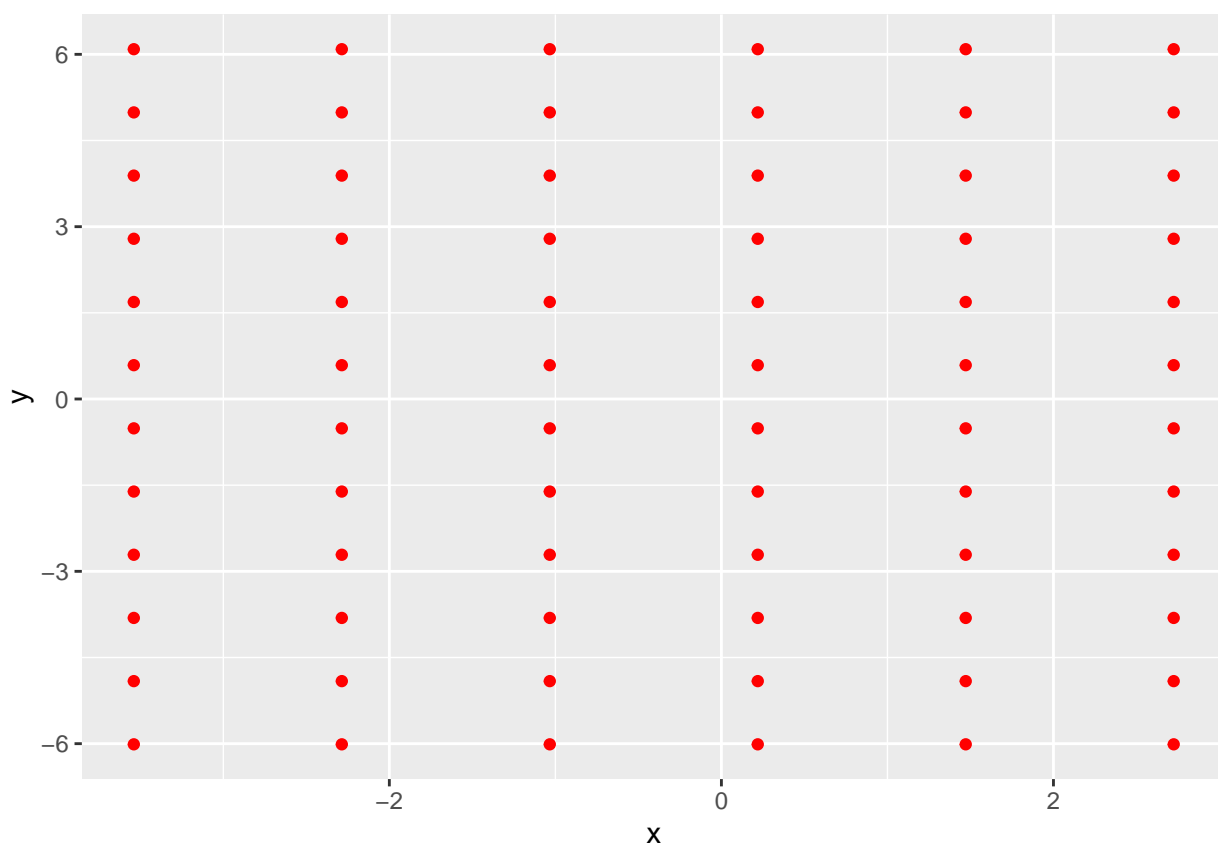
buffer_size <- hex_size/2

x_bounds <- seq(min(s_curve_noise_umap[["UMAP1"]]) - buffer_size,
                max(s_curve_noise_umap[["UMAP1"]]) + buffer_size, length.out = num_bins_x)

y_bounds <- seq(min(s_curve_noise_umap[["UMAP2"]]) - buffer_size,
                max(s_curve_noise_umap[["UMAP2"]]) + buffer_size, length.out = num_bins_y)

box_points <- expand.grid(x = x_bounds, y = y_bounds)

ggplot() +
  geom_point(data = box_points, aes(x = x, y = y), color = "red")
```

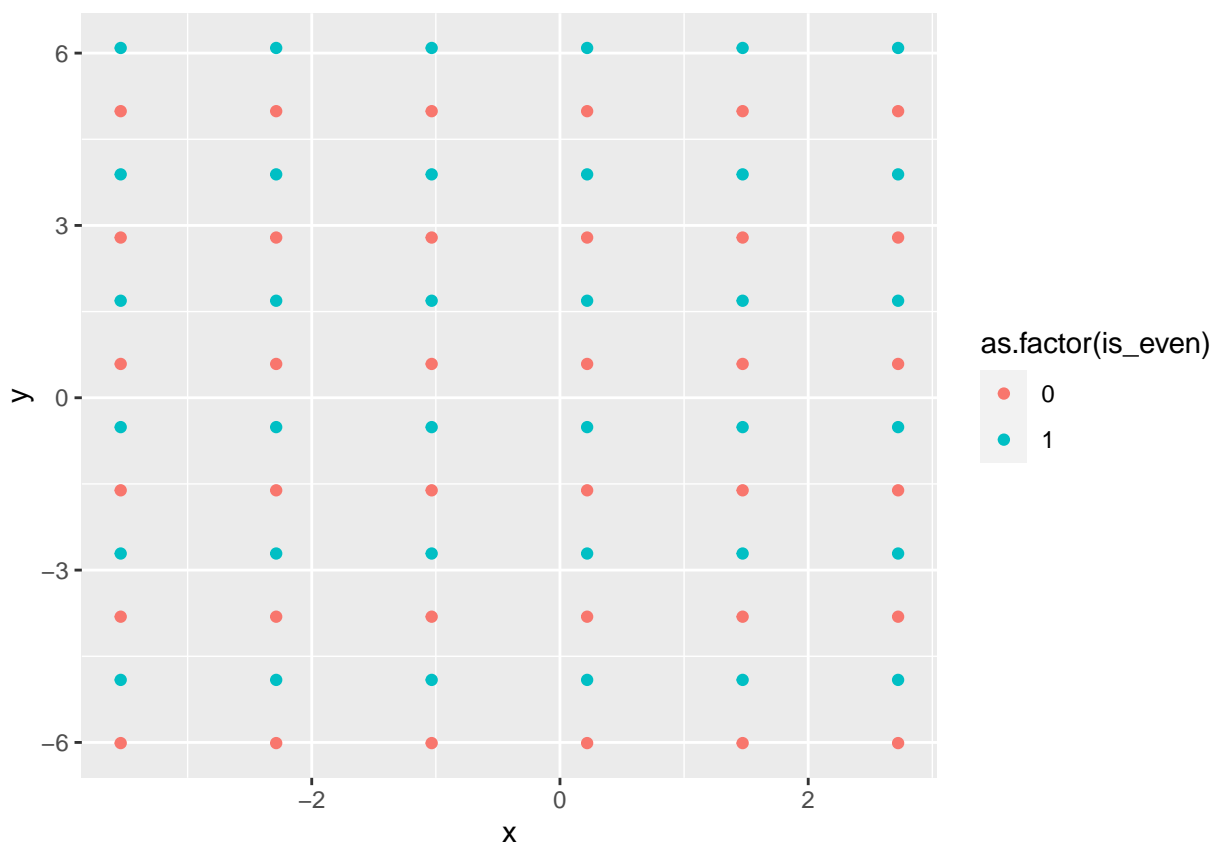


- Second for each x-value, find which y values are in the even row

```
box_points <- box_points |>
  dplyr::arrange(x) |>
  dplyr::group_by(x) |>
  dplyr::group_modify(~ generate_even_y(.x)) |>
```

```
tibble::as_tibble()

ggplot() +
  geom_point(data = box_points,
            aes(x = x, y = y, colour = as.factor(is_even)))
```

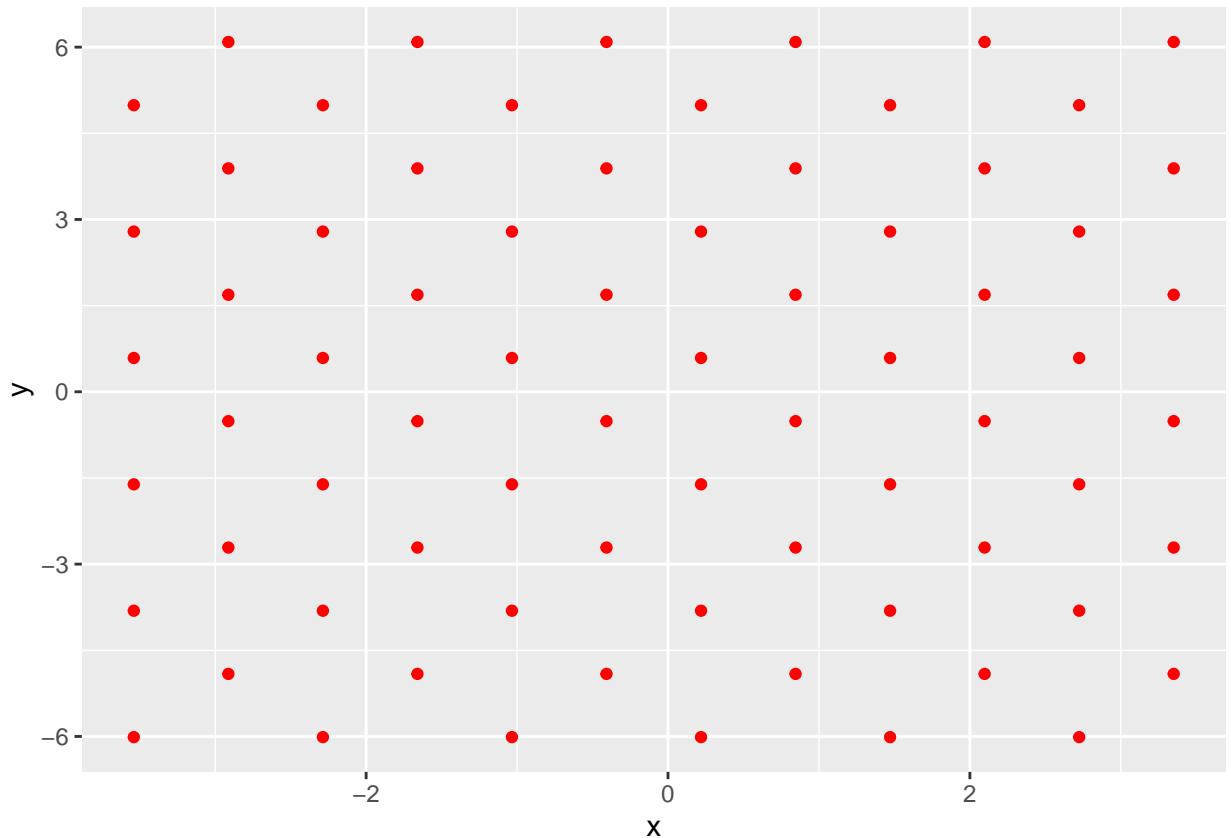


- Then, shift the x values of the even rows

```
## Shift for even values in x-axis
x_shift <- unique(box_points$x)[2] - unique(box_points$x)[1]

box_points$x <- box_points$x + x_shift/2 * ifelse(box_points$is_even == 1, 1, 0)

ggplot() +
  geom_point(data = box_points, aes(x = x, y = y), color = "red")
```



```
all_centroids_df <- generate_full_grid_centroids(nldr_df = s_curve_noise_umap,
  x = "UMAP1", y = "UMAP2",
  num_bins_x = num_bins_x,
  num_bins_y = num_bins_y,
  buffer_size = NA, hex_size = NA)
```

```
glimpse(all_centroids_df)
```

```
#> Rows: 72
#> Columns: 2
#> $ x <dbl> -3.5390002, -2.9126765, -3.5390002, -2.9126765, -3.5390002, -2.91267~
#> $ y <dbl> -6.0111830, -4.9111506, -3.8111182, -2.7110858, -1.6110534, -0.51102~
```

## 2. Generate hexagonal coordinates

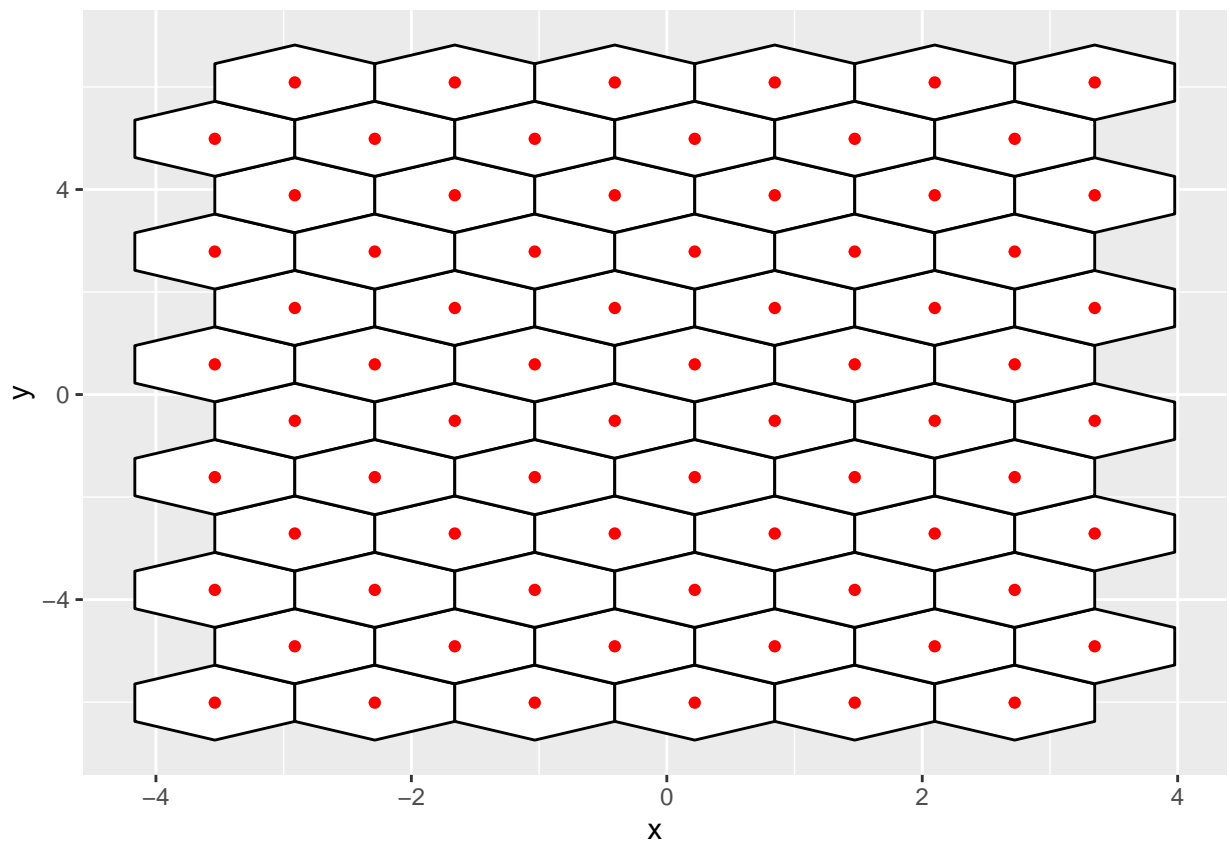
Steps: - Compute horizontal width of the hexagon

- Compute vertical width of the hexagon and multiply by a factor for overlapping ( $\sqrt{3}/2 * 1.15$ )
- Obtain hexagon polygon coordinates
- Obtain the number of hexagons in the full grid
- Generate the coordinates for the hexagons

```
hex_grid <- gen_hex_coordinates(all_centroids_df)
glimpse(hex_grid)
```

```
#> Rows: 432
#> Columns: 3
#> $ x <dbl> -2.912676, -2.912676, -3.539000, -4.165324, -4.165324, -3.539000, ~
#> $ y <dbl> -5.645998, -6.376368, -6.741553, -6.376368, -5.645998, -5.280813, ~
#> $ id <int> 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4~
```

```
ggplot(data = hex_grid, aes(x = x, y = y)) + geom_polygon(fill = "white", color = "black", aes(group = id)) +
  geom_point(data = all_centroids_df, aes(x = x, y = y), color = "red")
```



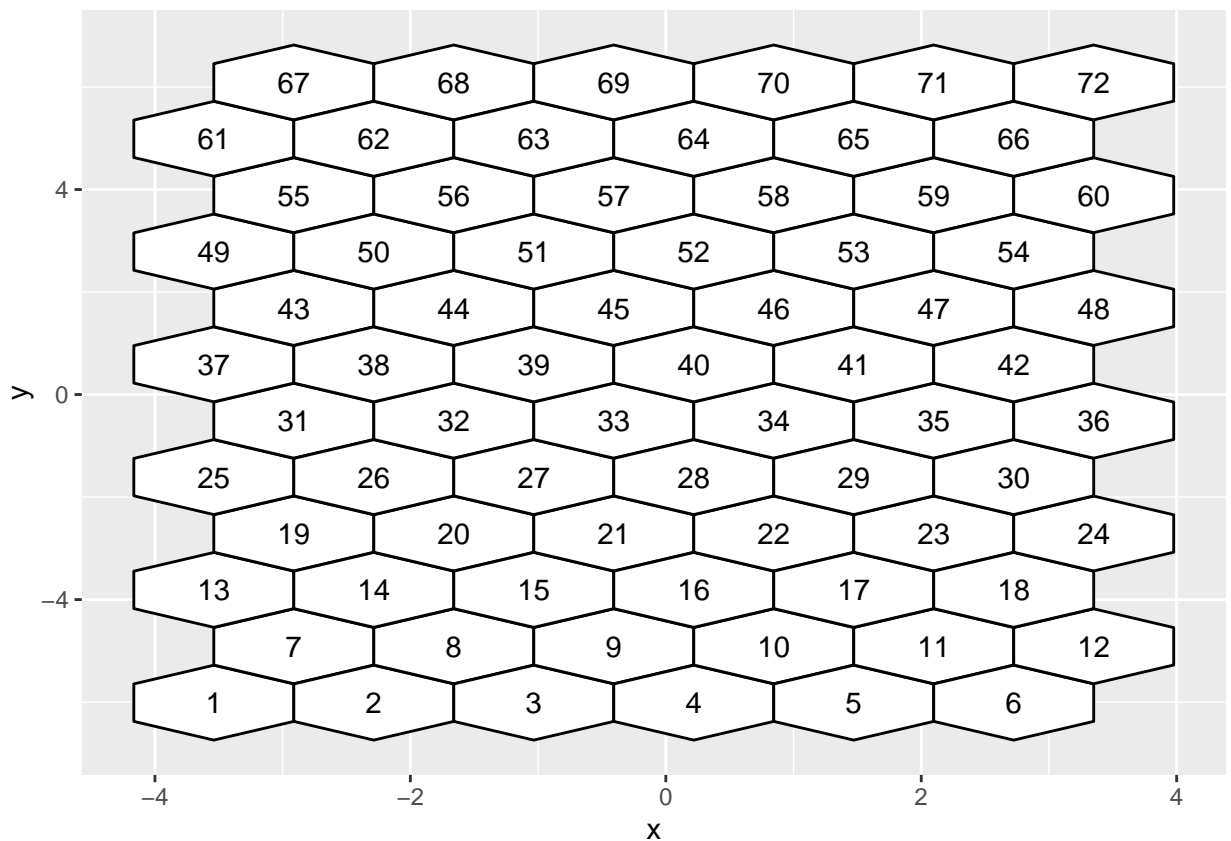
### 3. Map hexagonal IDs

Steps:

- Filter the data set with specific y value
- Order the x values for a specific y value
- Repeat the process for all unique y values

```
full_grid_with_hexbin_id <- map_hexbin_id(all_centroids_df)
```

```
ggplot(data = hex_grid, aes(x = x, y = y)) + geom_polygon(fill = "white", color = "black", aes(group = id)) +  
  geom_text(data = full_grid_with_hexbin_id, aes(x = c_x, y = c_y, label = hexID))
```



#### 4. Map polygon IDs

Steps:

- Filter specific hexagon
- Filter specific polygon
- Check the selected hexagonal centroid exists within the polygon
- if so assign that id to centroid, if not check until find the polygon which contains the centroid

```
full_grid_with_polygon_id <- map_polygon_id(full_grid_with_hexbin_id, hex_grid)
```

#### 4. Assign data into hexagons

- Compute distances between nlcr coordinates and hex bin centroids
- Find the hexagonal centroid that have the minimum distance

```
s_curve_noise_umap_with_id <- assign_data(s_curve_noise_umap, full_grid_with_hexbin_id)
```

#### 5. Compute standardized counts

- Compute number of data points within each hexagon
- Compute standardise count by dividing the counts by the maximum

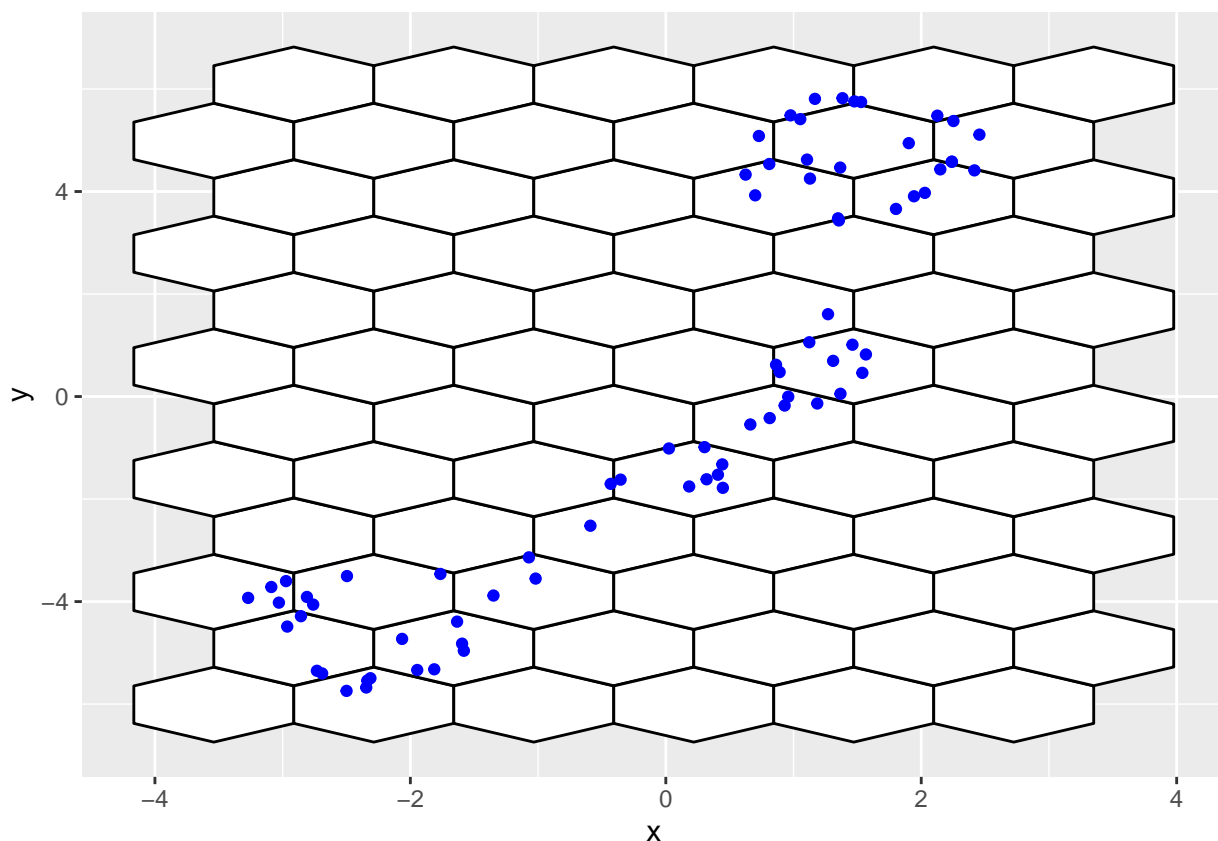
```
df_with_std_counts <- compute_std_counts(nldr_df = s_curve_noise_umap_with_id)
```

#### 6. Extract full grid info

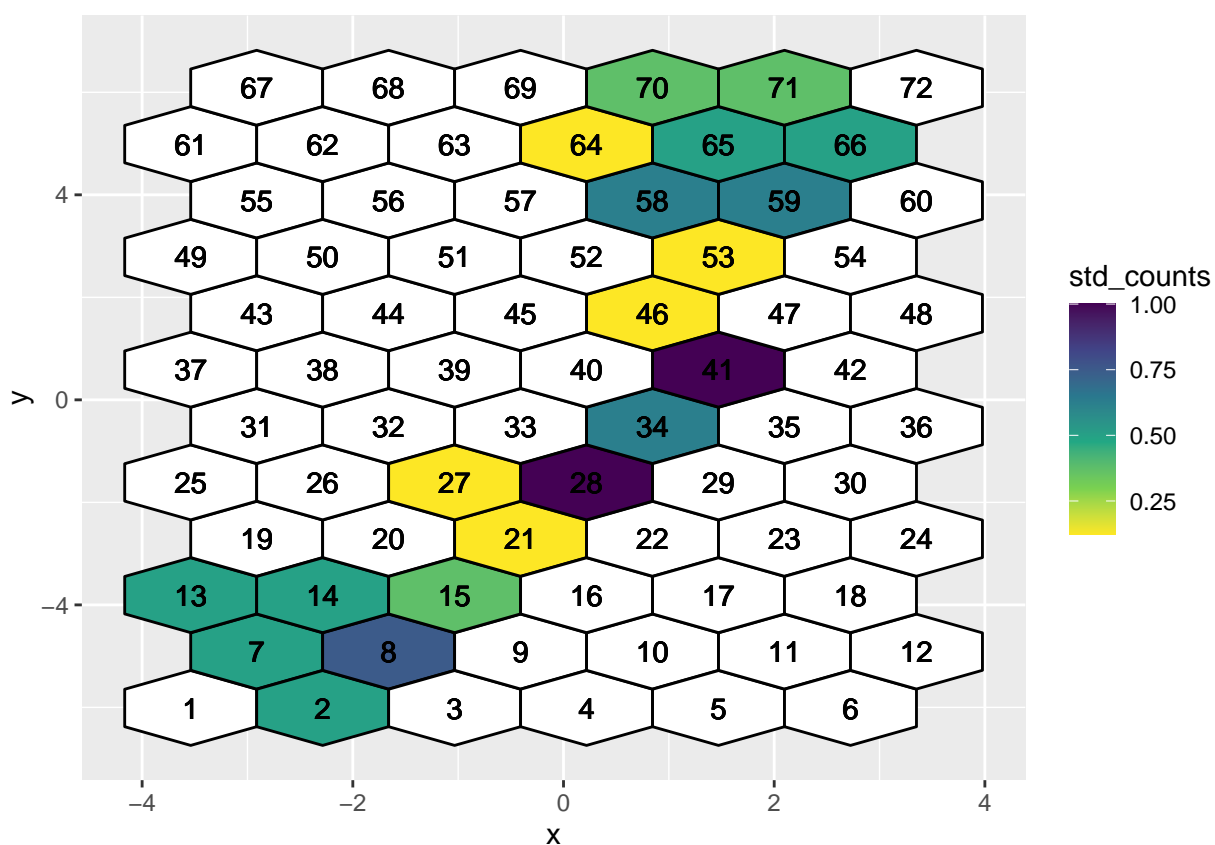
- Assign standardize counts for hex bins
- Join with the hexagonal coordinates

```
hex_full_count_df <- generate_full_grid_info(full_grid_with_polygon_id, df_with_std_counts, hex_grid)
```

```
ggplot(data = hex_grid, aes(x = x, y = y)) + geom_polygon(fill = "white", color = "black", aes(group = id)) +  
  geom_point(data = s_curve_noise_umap, aes(x = UMAP1, y = UMAP2), color = "blue")
```



```
ggplot(data = hex_full_count_df, aes(x = x, y = y)) +
  geom_polygon(color = "black", aes(group = polygon_id, fill = std_counts)) +
  geom_text(aes(x = c_x, y = c_y, label = hexID)) +
  scale_fill_viridis_c(direction = -1, na.value = "#ffffff")
```



**Buffer size** When generating hexagonal bins in R, a buffer is often included to ensure that the data points are evenly distributed within the bins and to prevent edge effects. The buffer helps in two main ways:

1. **Preventing Edge Effects:** Without a buffer, the outermost data points might fall near the boundary of the hexagonal grid, leading to incomplete bins or uneven distribution of data. By adding a buffer, you create a margin around the outer edges of the grid, ensuring that all data points are fully enclosed within the bins.
2. **Ensuring Even Distribution:** The buffer allows for a smoother transition between adjacent bins. This helps in cases where data points are not perfectly aligned with the grid lines, ensuring that each data point is assigned to the nearest bin without bias towards any specific direction.

Overall, including a buffer when generating hexagonal bins helps to produce more accurate and robust binning results, particularly when dealing with real-world data that may have irregular distributions or boundary effects.

## Construct the 2D model with different options

### Construct the high-D model with different options

```
## To generate a data set with high-D and 2D training data
df_all <- training_data |> dplyr::select(-ID) |>
  dplyr::bind_cols(s_curve_noise_umap_with_id)

## To generate averaged high-D data

df_bin <- avg_highD_data(.data = df_all, column_start_text = "x") ## Need to pass ID column name
```

### Generate the triangular mesh

```
df_bin_centroids <- hex_full_count_df[complete.cases(hex_full_count_df[["std_counts"]]), ] |>
  dplyr::select("c_x", "c_y", "hexID", "std_counts") |>
  dplyr::distinct() |>
  dplyr::rename(c("x" = "c_x", "y" = "c_y"))
```

```
df_bin_centroids
```

```
#> # A tibble: 20 x 4
#>       x      y hexID std_counts
#>   <dbl> <dbl> <int>    <dbl>
#> 1 -2.91 -4.91     7      0.5
#> 2 -3.54 -3.81    13      0.5
#> 3 -2.29 -6.01     2      0.5
#> 4 -1.66 -4.91     8      0.75
#> 5 -2.29 -3.81    14      0.5
#> 6 -1.03 -3.81    15     0.375
#> 7 -0.407 -2.71    21     0.125
#> 8 -1.03 -1.61    27     0.125
#> 9  0.219 -1.61    28      1
#> 10  0.845 -0.511   34     0.625
#> 11  0.845  1.69   46     0.125
#> 12  0.845  3.89   58     0.625
#> 13  0.219  4.99   64     0.125
#> 14  0.845  6.09   70     0.375
#> 15  1.47  0.589   41      1
#> 16  1.47  2.79   53     0.125
#> 17  2.10  3.89   59     0.625
#> 18  1.47  4.99   65      0.5
#> 19  2.10  6.09   71     0.375
#> 20  2.72  4.99   66      0.5
```

```
tr1_object <- triangulate_bin_centroids(df_bin_centroids, x, y)
tr_from_to_df <- generate_edge_info(triangular_object = tr1_object)
```



## Compute parameter defaults

**Shift the hexagonal grid origin** If shift\_x happen to the positive direction of x it should input as a positive value, if not other way If shift\_y happen to the positive direction of y it should input as a positive value, if not other way

1. Assign shift along the x and y axis (limited the amount should less than the cell\_diameter)
2. Generate bounds with shift origin

```
all_centroids_df_shift <- extract_coord_of_shifted_hex_grid(nldr_df = s_curve_noise_umap,
  x = "UMAP1", y = "UMAP2",
  num_bins_x = num_bins_x,
  num_bins_y = num_bins_y,
  shift_x = 0.2690002, shift_y = 0.271183,
  buffer_size = NA, hex_size = NA)
```

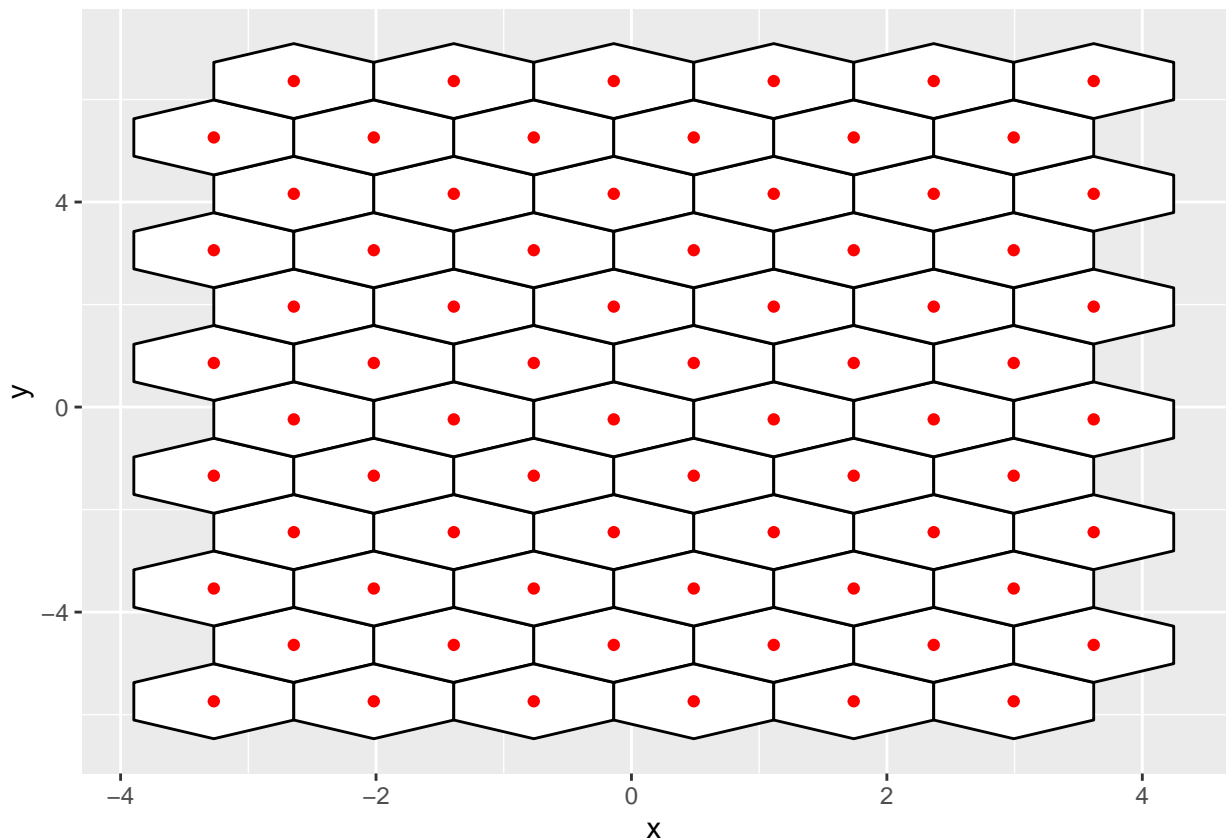
```
glimpse(all_centroids_df_shift)
```

```
#> Rows: 72
#> Columns: 2
#> $ x <dbl> -3.2700000, -2.6436763, -3.2700000, -2.6436763, -3.2700000, -2.64367~
#> $ y <dbl> -5.7400000, -4.6399676, -3.5399352, -2.4399028, -1.3398704, -0.23983~
```

```
hex_grid <- gen_hex_coordinates(all_centroids_df_shift)
glimpse(hex_grid)
```

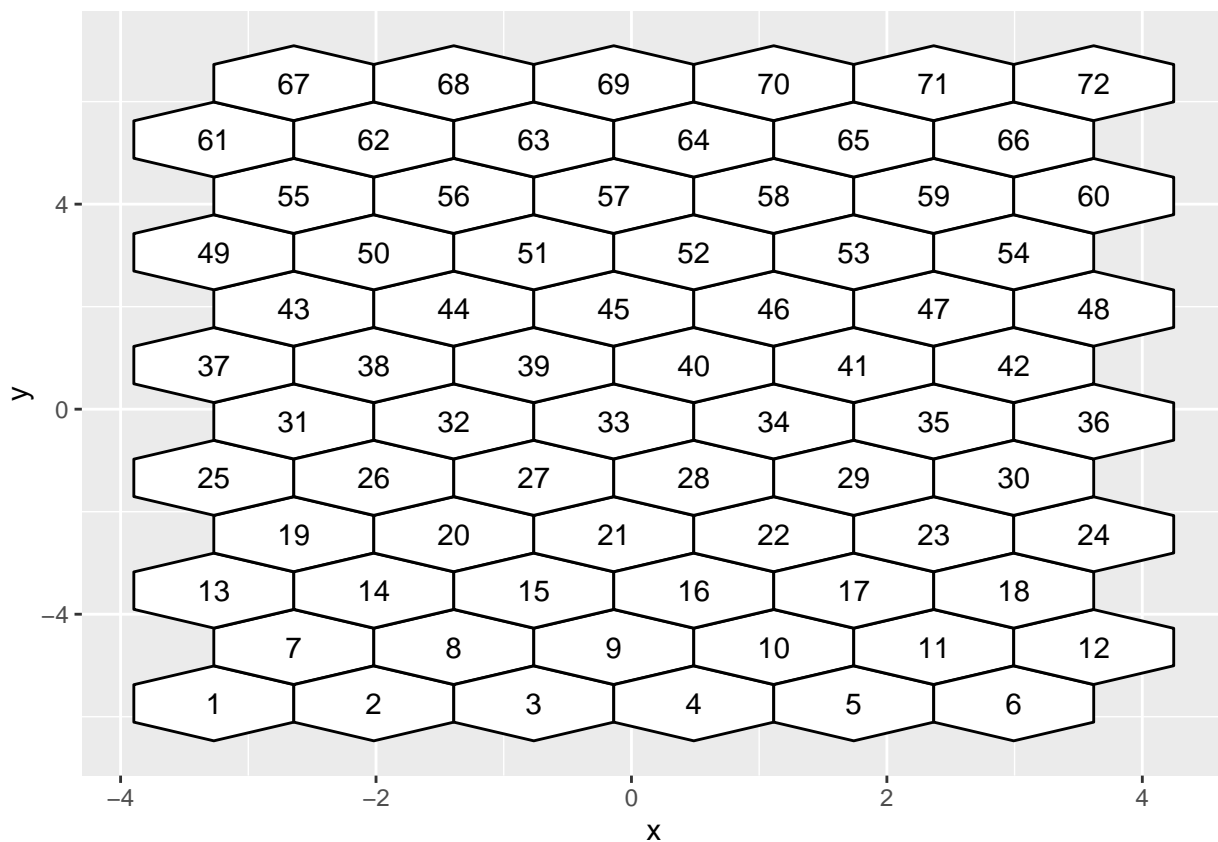
```
#> Rows: 432
#> Columns: 3
#> $ x <dbl> -2.643676, -2.643676, -3.270000, -3.896324, -3.896324, -3.270000, ~~
#> $ y <dbl> -5.3748152, -6.1051848, -6.4703696, -6.1051848, -5.3748152, -5.0096~
#> $ id <int> 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4~
```

```
ggplot(data = hex_grid, aes(x = x, y = y)) + geom_polygon(fill = "white", color = "black", aes(group = id)) +
  geom_point(data = all_centroids_df_shift, aes(x = x, y = y), color = "red")
```



```
full_grid_with_hexbin_id <- map_hexbin_id(all_centroids_df_shift)

ggplot(data = hex_grid, aes(x = x, y = y)) + geom_polygon(fill = "white", color = "black", aes(group = id)) +
  geom_text(data = full_grid_with_hexbin_id, aes(x = c_x, y = c_y, label = hexID))
```



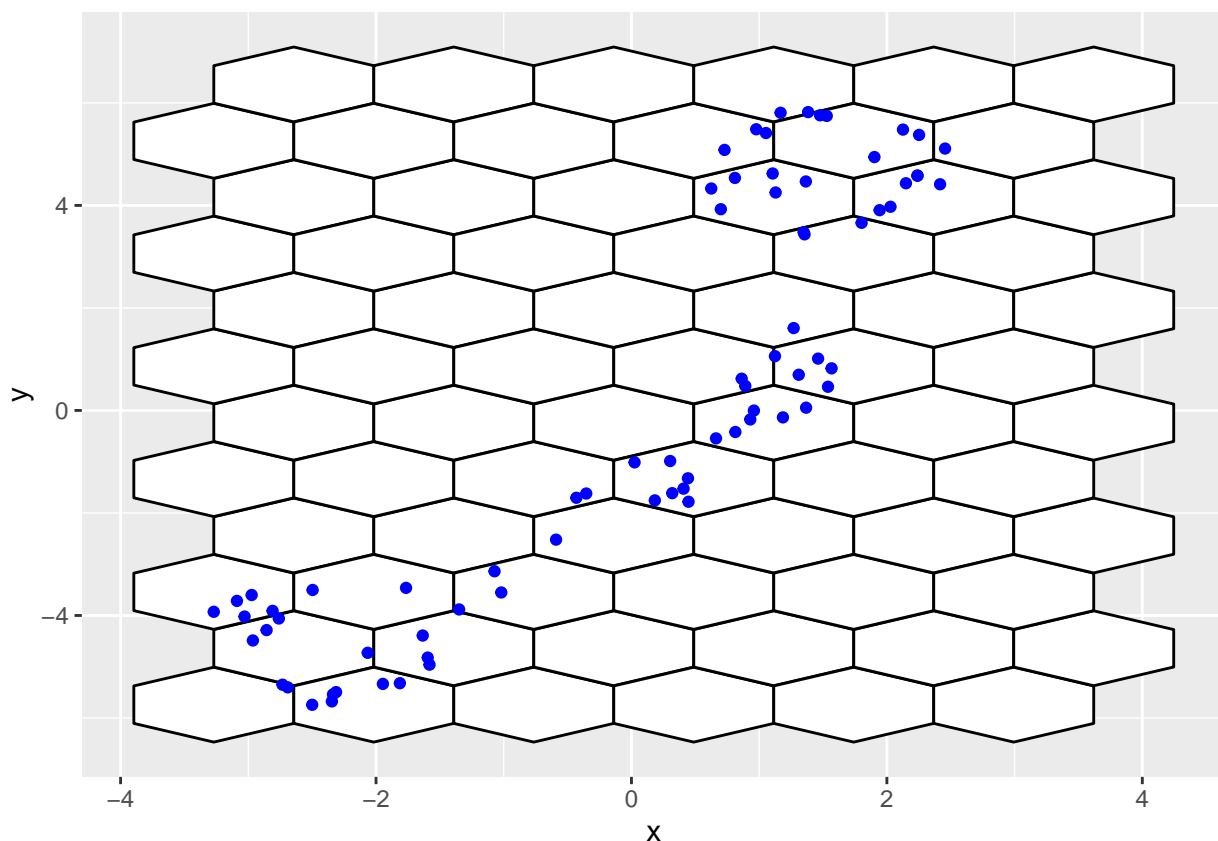
```
full_grid_with_polygon_id <- map_polygon_id(full_grid_with_hexbin_id, hex_grid)

s_curve_noise_umap_with_id <- assign_data(s_curve_noise_umap, full_grid_with_hexbin_id)

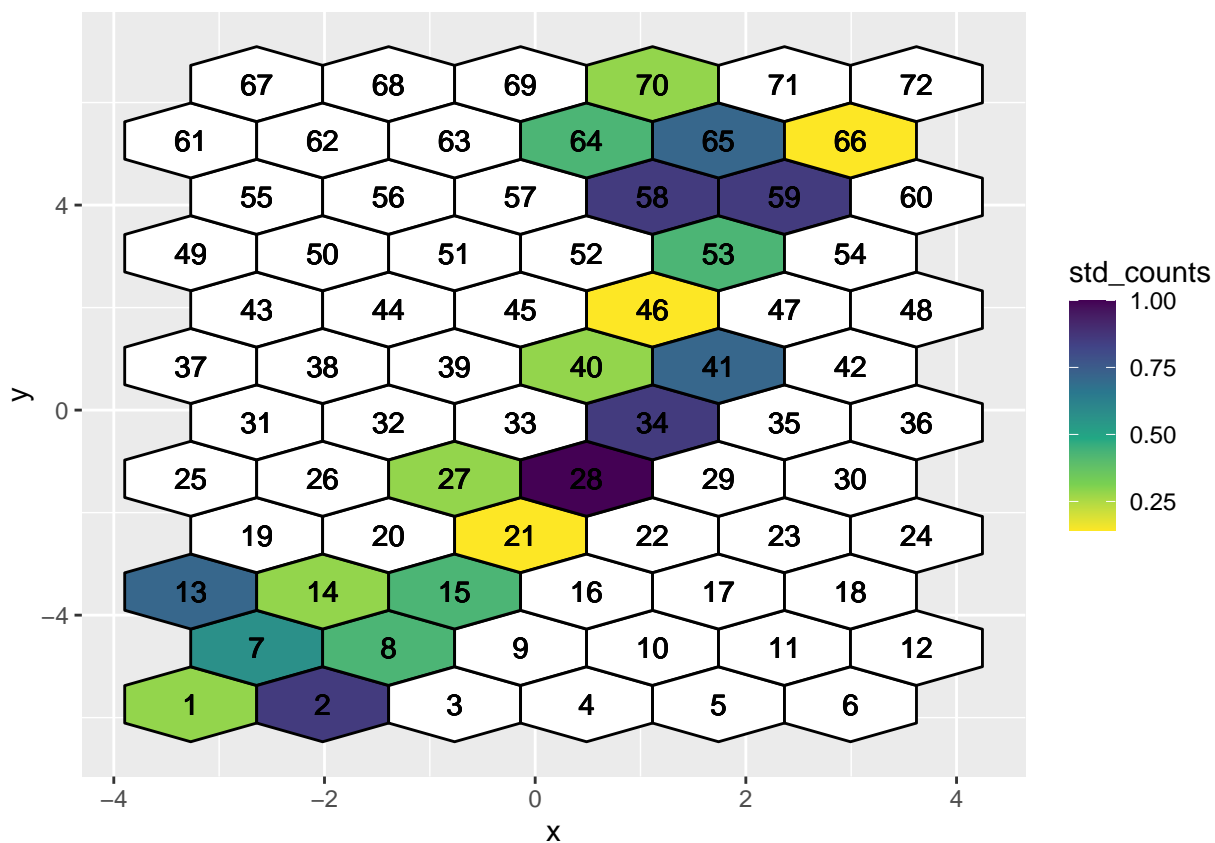
df_with_std_counts <- compute_std_counts(nldr_df = s_curve_noise_umap_with_id)

hex_full_count_df <- generate_full_grid_info(full_grid_with_polygon_id, df_with_std_counts, hex_grid)

ggplot(data = hex_grid, aes(x = x, y = y)) + geom_polygon(fill = "white", color = "black", aes(group = id)) +
  geom_point(data = s_curve_noise_umap, aes(x = UMAP1, y = UMAP2), color = "blue")
```



```
ggplot(data = hex_full_count_df, aes(x = x, y = y)) +
  geom_polygon(color = "black", aes(group = polygon_id, fill = std_counts)) +
  geom_text(aes(x = c_x, y = c_y, label = hexID)) +
  scale_fill_viridis_c(direction = -1, na.value = "#ffffff")
```



```
df_bin_centroids <- hex_full_count_df[complete.cases(hex_full_count_df[["std_counts"]]), ] |>
```

```

dplyr::select("c_x", "c_y", "hexID", "std_counts") |>
dplyr::distinct() |>
dplyr::rename(c("x" = "c_x", "y" = "c_y"))

df_bin_centroids

#> # A tibble: 21 x 4
#>       x      y hexID std_counts
#>   <dbl> <dbl> <int>      <dbl>
#> 1 -3.27 -5.74     1      0.286
#> 2 -2.64 -4.64     7      0.571
#> 3 -3.27 -3.54    13      0.714
#> 4 -2.02 -5.74     2      0.857
#> 5 -1.39 -4.64     8      0.429
#> 6 -2.02 -3.54    14      0.286
#> 7 -0.765 -3.54    15      0.429
#> 8 -0.138 -2.44    21      0.143
#> 9 -0.765 -1.34    27      0.286
#> 10  0.488 -1.34    28       1
#> # i 11 more rows

tr1_object <- triangulate_bin_centroids(df_bin_centroids, x, y)
tr_from_to_df <- generate_edge_info(triangular_object = tr1_object)

bin_centroids_shift <- ggplot(data = hex_full_count_df, aes(x = c_x, y = c_y)) +
  geom_point(color = "#bdbdbd") +
  geom_point(data = shifted_hex_coord_df, aes(x = c_x, y = c_y), color = "#feb24c") +
  coord_cartesian(xlim = c(-5, 8), ylim = c(-10, 10)) +
  theme_void() +
  theme(legend.position="none", legend.direction="horizontal", plot.title = element_text(size = 7, hjust = 0.5, vjust = 1),
        axis.title.x = element_blank(), axis.title.y = element_blank(),
        axis.text.x = element_blank(), axis.ticks.x = element_blank(),
        axis.text.y = element_blank(), axis.ticks.y = element_blank(),
        panel.grid.major = element_blank(), panel.grid.minor = element_blank(), #change legend key width
        legend.title = element_text(size=8), #change legend title font size
        legend.text = element_text(size=6)) +
  guides(fill = guide_colourbar(title = "Standardized count")) +
  annotate(geom = 'text', label = "a", x = -Inf, y = Inf, hjust = -0.3, vjust = 1, size = 3)

hex_grid_shift <- ggplot(data = shifted_hex_coord_df, aes(x = x, y = y)) +
  geom_polygon(fill = NA, color = "#feb24c", aes(group = polygon_id)) +
  geom_polygon(data = hex_full_count_df, aes(x = x, y = y, group = polygon_id),
              fill = NA, color = "#bdbdbd") +
  coord_cartesian(xlim = c(-5, 8), ylim = c(-10, 10)) +
  theme_void() +
  theme(legend.position="none", legend.direction="horizontal", plot.title = element_text(size = 7, hjust = 0.5, vjust = 1),
        axis.title.x = element_blank(), axis.title.y = element_blank(),
        axis.text.x = element_blank(), axis.ticks.x = element_blank(),
        axis.text.y = element_blank(), axis.ticks.y = element_blank(),
        panel.grid.major = element_blank(), panel.grid.minor = element_blank(), #change legend key width
        legend.title = element_text(size=8), #change legend title font size
        legend.text = element_text(size=6)) +
  guides(fill = guide_colourbar(title = "Standardized count")) +
  annotate(geom = 'text', label = "b", x = -Inf, y = Inf, hjust = -0.3, vjust = 1, size = 3)

## Before shift
before_shift_plot <- ggplot(data = hex_full_count_df, aes(x = x, y = y)) +
  geom_polygon(color = "black", aes(group = polygon_id, fill = std_counts)) +
  geom_text(aes(x = c_x, y = c_y, label = hexID), size = 2) +
  scale_fill_viridis_c(direction = -1, na.value = "#ffffff", option = "C") +
  coord_equal() +
  theme_void() +
  theme(legend.position="bottom", legend.direction="horizontal", plot.title = element_text(size = 7, hjust = 0.5, vjust = 1),
        axis.title.x = element_blank(), axis.title.y = element_blank(),

```

```

    axis.text.x = element_blank(), axis.ticks.x = element_blank(),
    axis.text.y = element_blank(), axis.ticks.y = element_blank(),
    panel.grid.major = element_blank(), panel.grid.minor = element_blank(), #change legend key width
    legend.title = element_text(size=8), #change legend title font size
    legend.text = element_text(size=6)) +
  guides(fill = guide_colourbar(title = "Standardized count")) +
  annotate(geom = 'text', label = "a", x = -Inf, y = Inf, hjust = -0.3, vjust = 1, size = 3)

## After shift
after_shift_plot <- ggplot(data = shifted_hex_coord_df, aes(x = x, y = y)) +
  geom_polygon(color = "black", aes(group = polygon_id, fill = std_counts)) +
  geom_text(aes(x = c_x, y = c_y, label = hexID), size = 2) +
  scale_fill_viridis_c(direction = -1, na.value = "#ffffff", option = "C") +
  coord_equal() +
  theme_void() +
  theme(legend.position="none", legend.direction="horizontal", plot.title = element_text(size = 7, hjust = 0.5, vjust = 1),
    axis.title.x = element_blank(), axis.title.y = element_blank(),
    axis.text.x = element_blank(), axis.ticks.x = element_blank(),
    axis.text.y = element_blank(), axis.ticks.y = element_blank(),
    panel.grid.major = element_blank(), panel.grid.minor = element_blank(), #change legend key width
    legend.title = element_text(size=8), #change legend title font size
    legend.text = element_text(size=6)) +
  guides(fill = guide_colourbar(title = "Standardized count")) +
  annotate(geom = 'text', label = "b", x = -Inf, y = Inf, hjust = -0.3, vjust = 1, size = 3)

```

### Benchmark value to remove the low-density hexagons

#### Benchmark value to remove the long edges

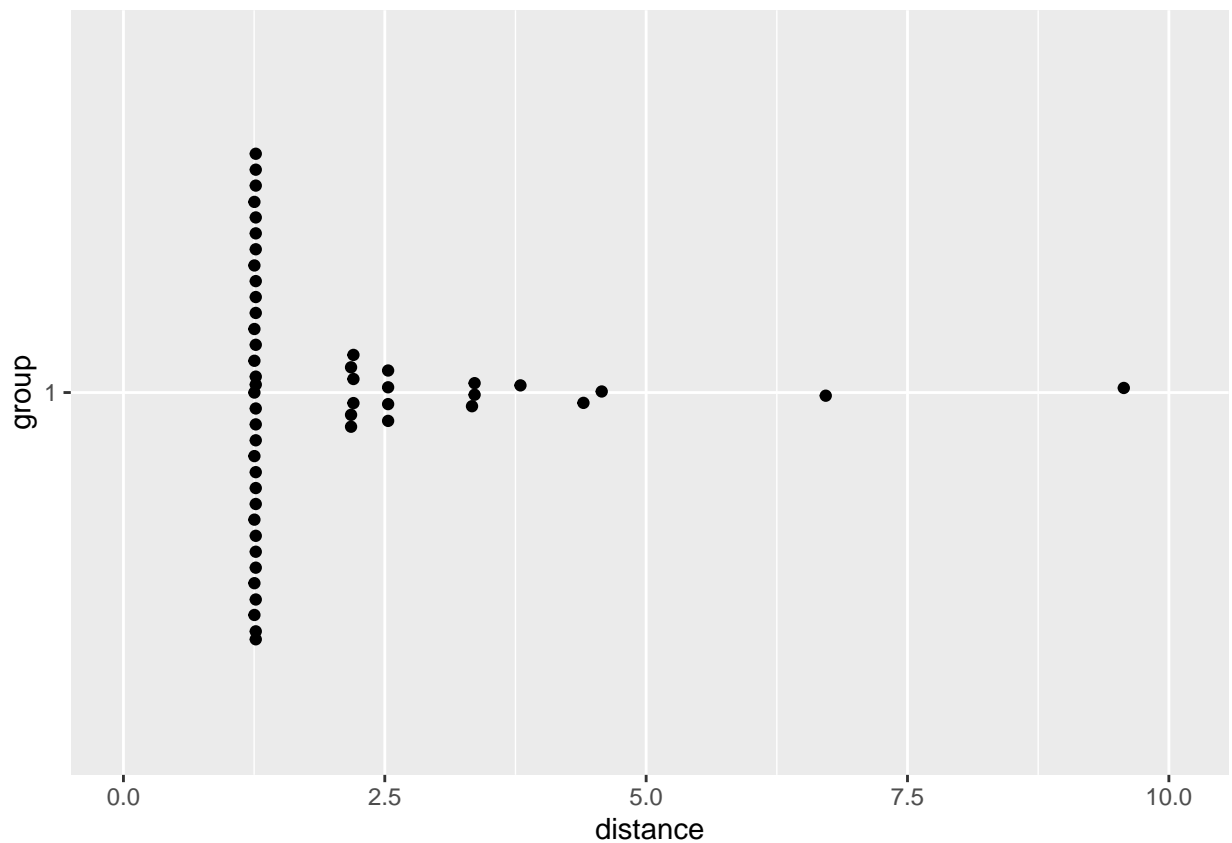
```

## Compute 2D distances
distance <- cal_2d_dist(.data = tr_from_to_df)

## To plot the distribution of distance
plot_dist <- function(distance_df){
  distance_df$group <- "1"
  dist_plot <- ggplot(distance_df, aes(x = group, y = distance)) +
    geom_quasirandom()+
    ylim(0, max(unlist(distance_df$distance))+ 0.5) + coord_flip()
  return(dist_plot)
}

plot_dist(distance)

```



```
benchmark <- find_benchmark_value(.data = distance, distance_col = "distance")
```

### Model function

### Predict 2D embeddings

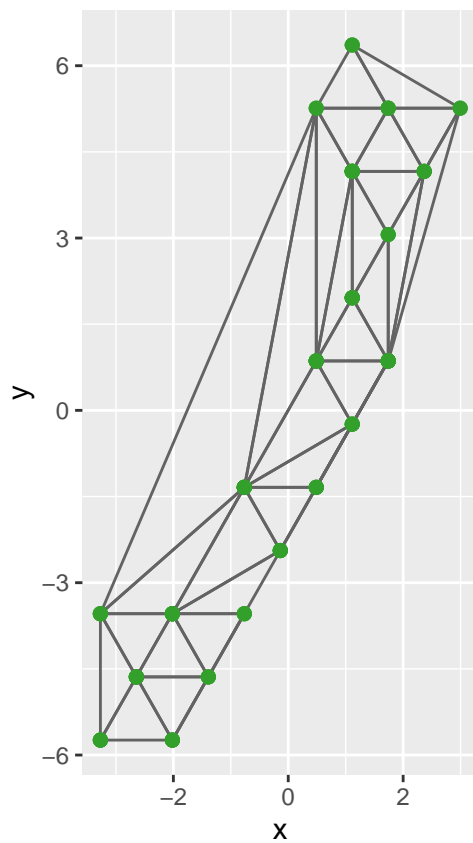
### Compute residuals

### Visualizations

#### geom\_trimesh

```
trimesh <- ggplot(df_bin_centroids, aes(x = x, y = y)) +  
  geom_point(size = 0.1) +  
  geom_trimesh() +  
  coord_equal()
```

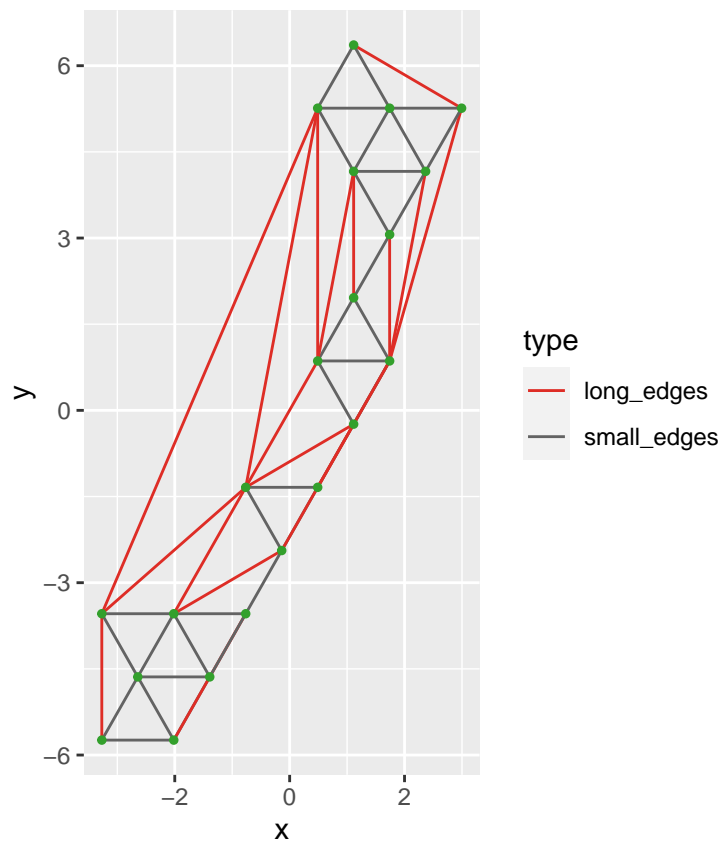
```
trimesh
```



### coloured\_long\_edges

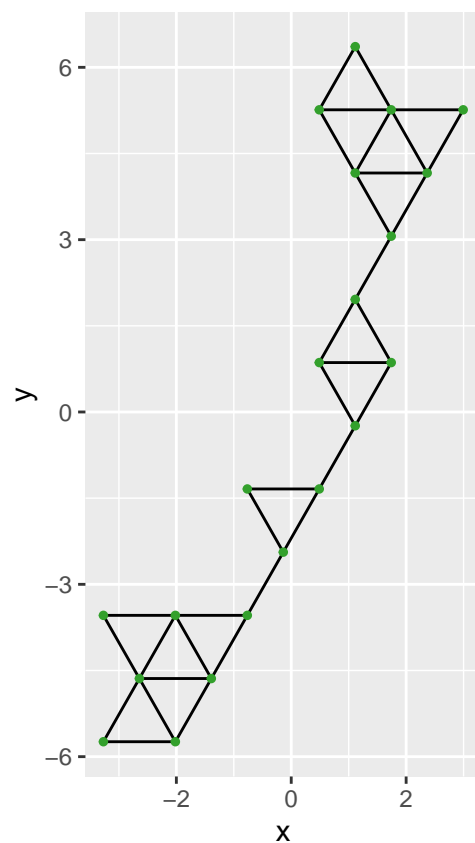
```
trimesh_gr <- colour_long_edges(.data = distance, benchmark_value = benchmark,  
                                triangular_object = tr1_object, distance_col = distance)
```

```
trimesh_gr
```



### remove long edges

```
trimesh_removed <- remove_long_edges(.data = distance, benchmark_value = benchmark,  
                                     triangular_object = tr1_object, distance_col = distance)  
trimesh_removed
```





**show\_langevitour**

```
tour1 <- show_langevitour(df_all, df_bin, df_bin_centroids, benchmark_value = benchmark,  
                          distance = distance, distance_col = "distance")  
tour1
```

## Tests

### 3 Application

```

medlea_df <- read_csv("data/medlea_dataset.csv")
names(medlea_df)[2:(NCOL(medlea_df) - 1)] <- paste0("x", 1:(NCOL(medlea_df) - 2))

medlea_df <- medlea_df |> ## Since only contains zeros
  select(-x10)

#medlea_df[,2:(NCOL(medlea_df) - 1)] <- scale(medlea_df[,2:(NCOL(medlea_df) - 1)])

calculate_pca <- function(feature_dataset, num_pcs){
  pcaY_cal <- prcomp(feature_dataset, center = TRUE, scale = TRUE)
  PCAresults <- data.frame(pcaY_cal$x[, 1:num_pcs])
  summary_pca <- summary(pcaY_cal)
  var_explained_df <- data.frame(PC= paste0("PC",1:50),
                                var_explained=(pcaY_cal$sdev[1:50])^2/sum((pcaY_cal$sdev[1:50])^2))
  return(list(prcomp_out = pcaY_cal,pca_components = PCAresults, summary = summary_pca, var_explained_pca = var_
)
features <- medlea_df[,2:(NCOL(medlea_df) - 1)]
pca_ref_calc <- calculate_pca(features, 8)
pca_ref_calc$summary

#> Importance of components:
#>
#>      PC1      PC2      PC3      PC4      PC5      PC6      PC7
#> Standard deviation  3.1691 3.0609 2.7226 1.87967 1.71219 1.34192 1.27525
#> Proportion of Variance 0.1969 0.1837 0.1453 0.06928 0.05748 0.03531 0.03189
#> Cumulative Proportion 0.1969 0.3806 0.5260 0.59526 0.65274 0.68805 0.71993
#>      PC8      PC9      PC10     PC11     PC12     PC13     PC14
#> Standard deviation  1.16992 1.13465 1.06628 1.03279 0.97899 0.96264 0.9528
#> Proportion of Variance 0.02684 0.02524 0.02229 0.02091 0.01879 0.01817 0.0178
#> Cumulative Proportion 0.74677 0.77202 0.79431 0.81522 0.83402 0.85219 0.8700
#>      PC15     PC16     PC17     PC18     PC19     PC20     PC21
#> Standard deviation  0.9116 0.9090 0.79750 0.76725 0.72414 0.65310 0.61052
#> Proportion of Variance 0.0163 0.0162 0.01247 0.01154 0.01028 0.00836 0.00731
#> Cumulative Proportion 0.8863 0.9025 0.91496 0.92650 0.93678 0.94514 0.95245
#>      PC22     PC23     PC24     PC25     PC26     PC27     PC28
#> Standard deviation  0.6019 0.55399 0.52293 0.46638 0.41959 0.3976 0.34697
#> Proportion of Variance 0.0071 0.00602 0.00536 0.00426 0.00345 0.0031 0.00236
#> Cumulative Proportion 0.9596 0.96557 0.97093 0.97520 0.97865 0.9818 0.98411
#>      PC29     PC30     PC31     PC32     PC33     PC34     PC35
#> Standard deviation  0.33415 0.30618 0.29237 0.28458 0.26033 0.25420 0.22792
#> Proportion of Variance 0.00219 0.00184 0.00168 0.00159 0.00133 0.00127 0.00102
#> Cumulative Proportion 0.98630 0.98814 0.98982 0.99140 0.99273 0.99400 0.99502
#>      PC36     PC37     PC38     PC39     PC40     PC41     PC42
#> Standard deviation  0.21644 0.20437 0.19127 0.1744 0.15586 0.15252 0.12519
#> Proportion of Variance 0.00092 0.00082 0.00072 0.0006 0.00048 0.00046 0.00031
#> Cumulative Proportion 0.99594 0.99676 0.99747 0.9981 0.99855 0.99900 0.99931
#>      PC43     PC44     PC45     PC46     PC47     PC48     PC49
#> Standard deviation  0.10485 0.08598 0.08008 0.06491 0.04841 0.04094 0.03791
#> Proportion of Variance 0.00022 0.00014 0.00013 0.00008 0.00005 0.00003 0.00003
#> Cumulative Proportion 0.99952 0.99967 0.99980 0.99988 0.99992 0.99996 0.99999
#>      PC50     PC51
#> Standard deviation  0.02347 0.01421
#> Proportion of Variance 0.00001 0.00000
#> Cumulative Proportion 1.00000 1.00000

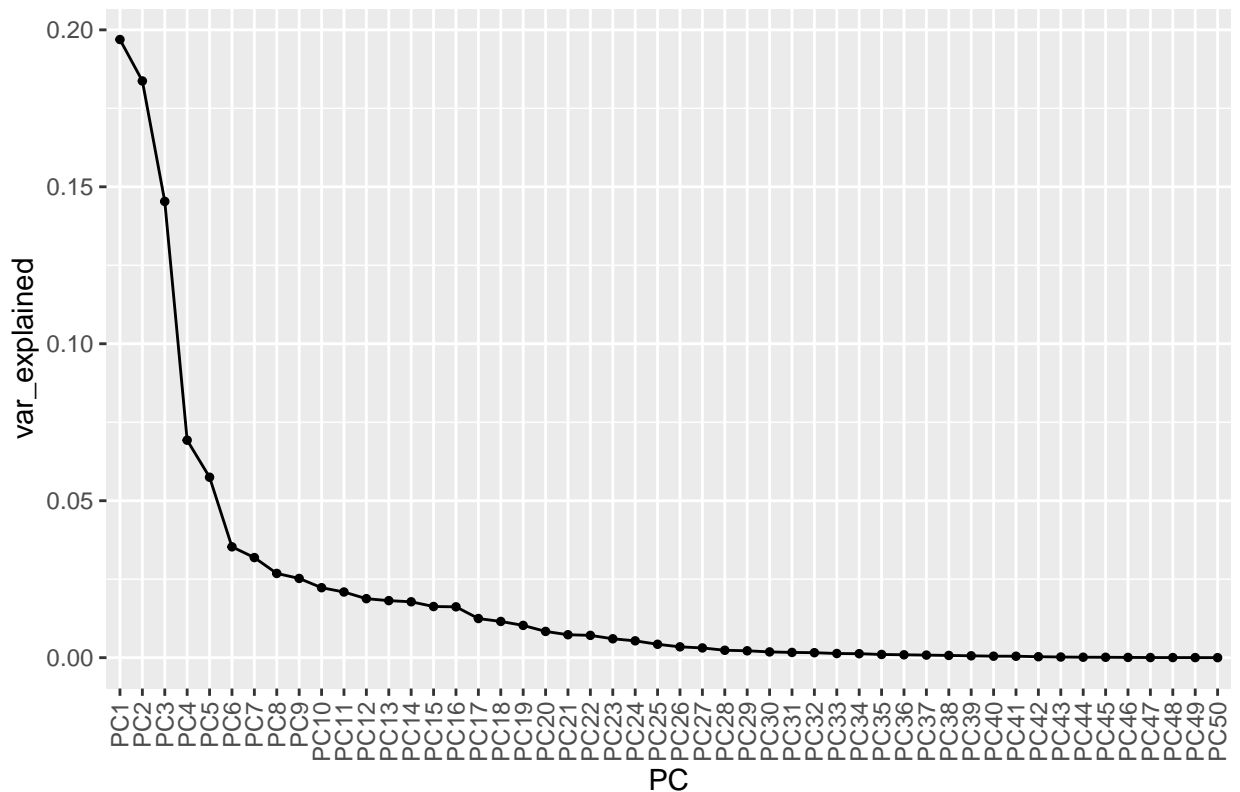
var_explained_df <- pca_ref_calc$var_explained_pca
data_pca <- pca_ref_calc$pca_components |>
  mutate(ID = 1:NROW(pca_ref_calc$pca_components),
         shape_label = medlea_df$Shape_label)

var_explained_df |>

```

```
ggplot(aes(x = PC,y = var_explained, group = 1))+
  geom_point(size=1)+
  geom_line()+
  labs(title="Scree plot: PCA on scaled data") +
  scale_x_discrete(limits = paste0(rep("PC", 50), 1:50)) +
  theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1))
```

Scree plot: PCA on scaled data



```
data_split <- initial_split(data_pca)
training_data <- training(data_split) |>
  arrange(ID)
test_data <- testing(data_split) |>
  arrange(ID)

UMAP_fit <- umap(training_data |> dplyr::select(-c(ID, shape_label)), n_neighbors = 37, n_components = 2)

UMAP_data <- UMAP_fit$layout |>
  as.data.frame()
names(UMAP_data)[1:(ncol(UMAP_data))] <- paste0(rep("UMAP", (ncol(UMAP_data))), 1:(ncol(UMAP_data)))

UMAP_data <- UMAP_data |>
  mutate(ID = training_data$id)

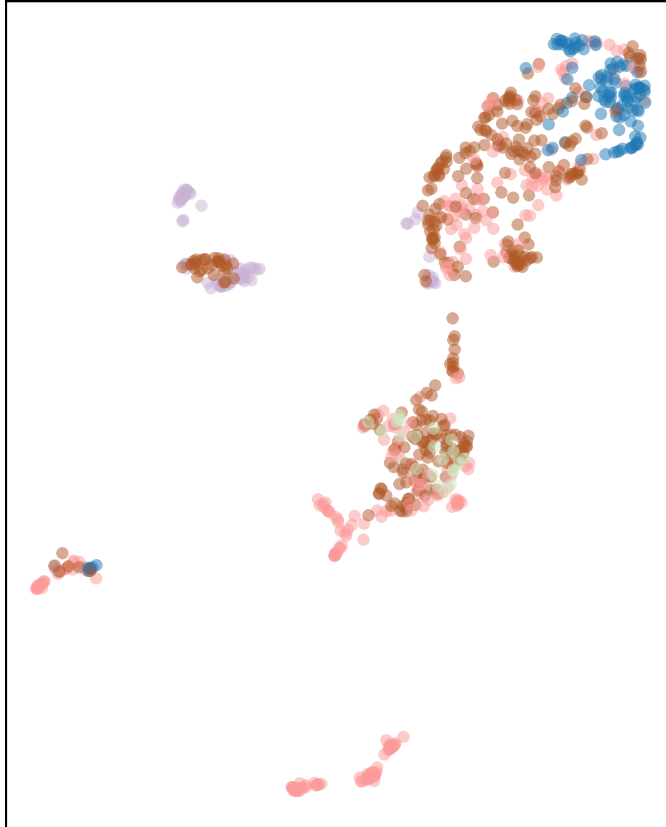
UMAP_data_with_label <- UMAP_data |>
  mutate(shape_label = training_data$shape_label)

UMAP_data_with_label |>
  ggplot(aes(x = UMAP1,
    y = UMAP2, color = shape_label))+
  geom_point(alpha=0.5) +
  coord_equal() +
  theme(plot.title = element_text(hjust = 0.5, size = 18, face = "bold")) + #ggtitle("(a)") +
  theme_linedraw() +
  theme(legend.position = "none", plot.title = element_text(size = 7, hjust = 0.5, vjust = -0.5),
    axis.title.x = element_blank(), axis.title.y = element_blank(),
```

```

axis.text.x = element_blank(), axis.ticks.x = element_blank(),
axis.text.y = element_blank(), axis.ticks.y = element_blank(),
panel.grid.major = element_blank(), panel.grid.minor = element_blank(), #change legend key width
legend.title = element_text(size=5), #change legend title font size
legend.text = element_text(size=4),
legend.key.height = unit(0.25, 'cm'),
legend.key.width = unit(0.25, 'cm')) +
scale_color_manual(values=c("#b15928", "#1f78b4", "#cab2d6", "#cceb5", "#fb9a99", "#e31a1c", "#6a3d9a", "#ff7f00"))

```



```

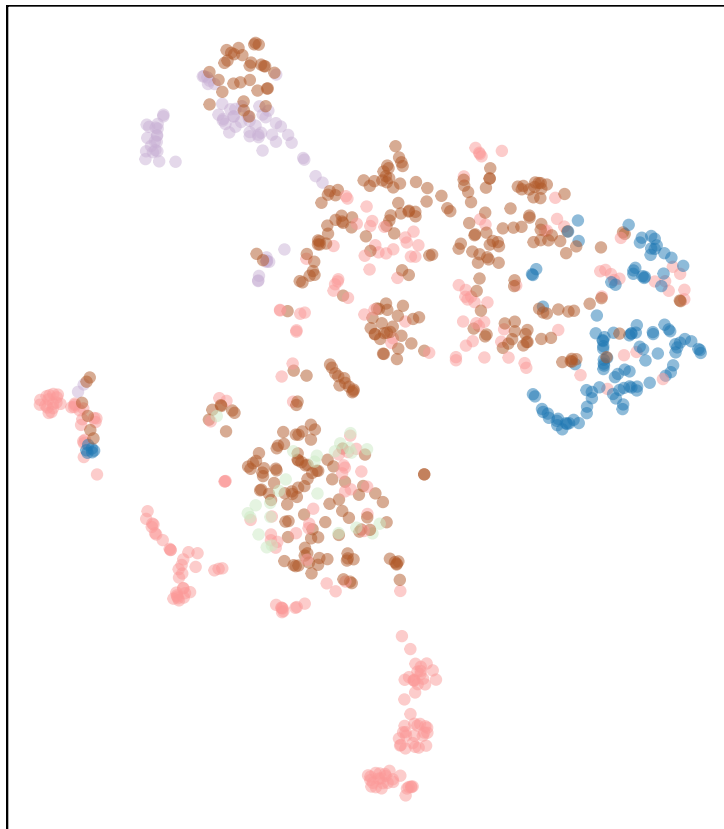
tSNE_data <- Fit_tSNE(training_data |> dplyr::select(-c(ID, shape_label)), opt_perplexity = calculate_effective

tSNE_data <- tSNE_data |>
  select(-ID) |>
  mutate(ID = training_data$ID)

tSNE_data_with_label <- tSNE_data |>
  mutate(shape_label = training_data$shape_label)

tSNE_data_with_label |>
  ggplot(aes(x = tSNE1,
             y = tSNE2, color = shape_label))+
  geom_point(alpha=0.5) +
  coord_equal() +
  theme(plot.title = element_text(hjust = 0.5, size = 18, face = "bold")) + #ggtitle("(a)") +
  theme_linedraw() +
  theme(legend.position = "none", plot.title = element_text(size = 7, hjust = 0.5, vjust = -0.5),
        axis.title.x = element_blank(), axis.title.y = element_blank(),
        axis.text.x = element_blank(), axis.ticks.x = element_blank(),
        axis.text.y = element_blank(), axis.ticks.y = element_blank(),
        panel.grid.major = element_blank(), panel.grid.minor = element_blank(), #change legend key width
        legend.title = element_text(size=5), #change legend title font size
        legend.text = element_text(size=4),
        legend.key.height = unit(0.25, 'cm'),
        legend.key.width = unit(0.25, 'cm')) +
  scale_color_manual(values=c("#b15928", "#1f78b4", "#cab2d6", "#cceb5", "#fb9a99", "#e31a1c", "#6a3d9a", "#ff7f00"))

```



```
PHATE_data <- Fit_PHATE(training_data |> dplyr::select(-c(ID, shape_label)), knn = 5, with_seed = 20240110)
```

```
#> Calculating PHATE...
#> Running PHATE on 824 observations and 8 variables.
#> Calculating graph and diffusion operator...
#> Calculating KNN search...
#> Calculating affinities...
#> Calculated graph and diffusion operator in 0.01 seconds.
#> Calculating optimal t...
#> Automatically selected t = 22
#> Calculated optimal t in 0.39 seconds.
#> Calculating diffusion potential...
#> Calculated diffusion potential in 0.32 seconds.
#> Calculating metric MDS...
#> Calculated metric MDS in 8.83 seconds.
#> Calculated PHATE in 9.55 seconds.
```

```
PHATE_data <- PHATE_data |>
  select(PHATE1, PHATE2)
PHATE_data <- PHATE_data |>
  mutate(ID = training_data$ID)
```

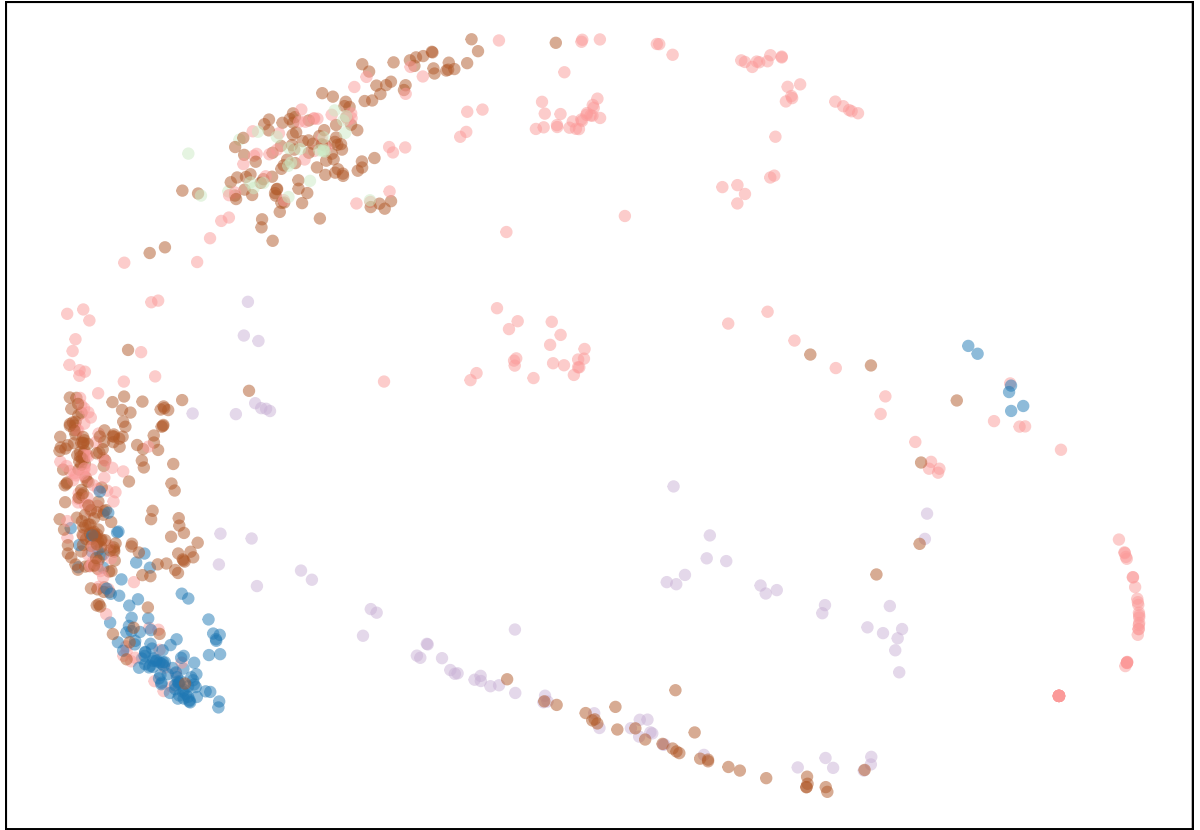
```
PHATE_data_with_label <- PHATE_data |>
  mutate(shape_label = training_data$shape_label)
```

```
PHATE_data_with_label |>
  ggplot(aes(x = PHATE1,
             y = PHATE2, color = shape_label))+
  geom_point(alpha=0.5) +
  coord_equal() +
  theme(plot.title = element_text(hjust = 0.5, size = 18, face = "bold")) + #ggtitle("(a)") +
  theme_linedraw() +
  theme(legend.position = "none", plot.title = element_text(size = 7, hjust = 0.5, vjust = -0.5),
        axis.title.x = element_blank(), axis.title.y = element_blank(),
        axis.text.x = element_blank(), axis.ticks.x = element_blank(),
```

```

axis.text.y = element_blank(), axis.ticks.y = element_blank(),
panel.grid.major = element_blank(), panel.grid.minor = element_blank(), #change legend key width
legend.title = element_text(size=5), #change legend title font size
legend.text = element_text(size=4),
legend.key.height = unit(0.25, 'cm'),
legend.key.width = unit(0.25, 'cm')) +
scale_color_manual(values=c("#b15928", "#1f78b4", "#cab2d6", "#cceb5", "#fb9a99", "#e31a1c", "#6a3d9a", "#ff7f00"))

```



```

tem_dir <- tempdir()

Fit_TriMAP_data(training_data |> dplyr::select(-c(ID, shape_label)), tem_dir)

path <- file.path(tem_dir, "df_2_without_class.csv")
path2 <- file.path(tem_dir, "dataset_3_TriMAP_values.csv")

Fit_TriMAP(as.integer(2), as.integer(5), as.integer(4), as.integer(3), path, path2)

TriMAP_data <- read_csv(path2)
TriMAP_data <- TriMAP_data |>
  mutate(ID = training_data$ID)

TriMAP_data_with_label <- TriMAP_data |>
  mutate(shape_label = training_data$shape_label)

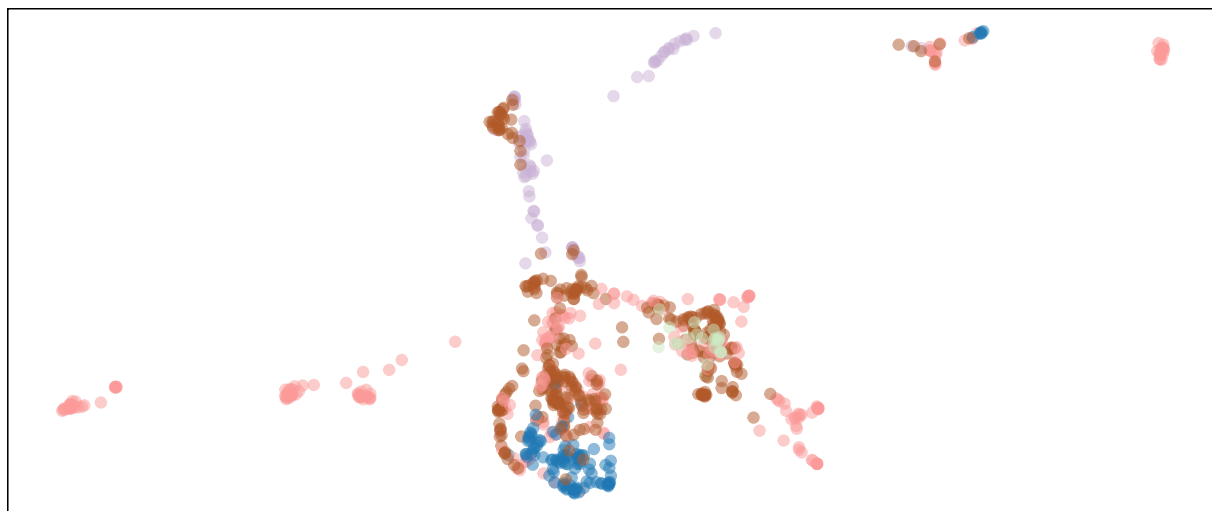
TriMAP_data_with_label |>
  ggplot(aes(x = TriMAP1,
             y = TriMAP2, color = shape_label))+
  geom_point(alpha=0.5) +
  coord_equal() +
  theme(plot.title = element_text(hjust = 0.5, size = 18, face = "bold")) + #ggtitle("(a)") +
  theme_linedraw() +
  theme(legend.position = "none", plot.title = element_text(size = 7, hjust = 0.5, vjust = -0.5),
        axis.title.x = element_blank(), axis.title.y = element_blank(),
        axis.text.x = element_blank(), axis.ticks.x = element_blank(),
        axis.text.y = element_blank(), axis.ticks.y = element_blank(),

```

```

    panel.grid.major = element_blank(), panel.grid.minor = element_blank(), #change legend key width
    legend.title = element_text(size=5), #change legend title font size
    legend.text = element_text(size=4),
    legend.key.height = unit(0.25, 'cm'),
    legend.key.width = unit(0.25, 'cm')) +
scale_color_manual(values=c("#b15928", "#1f78b4", "#cab2d6", "#cceb5", "#fb9a99", "#e31a1c", "#6a3d9a", "#ff7f00"))

```



```

tem_dir <- tempdir()

Fit_PacMAP_data(training_data |> dplyr::select(-c(ID, shape_label)), tem_dir)

path <- file.path(tem_dir, "df_2_without_class.csv")
path2 <- file.path(tem_dir, "dataset_3_PaCMAP_values.csv")

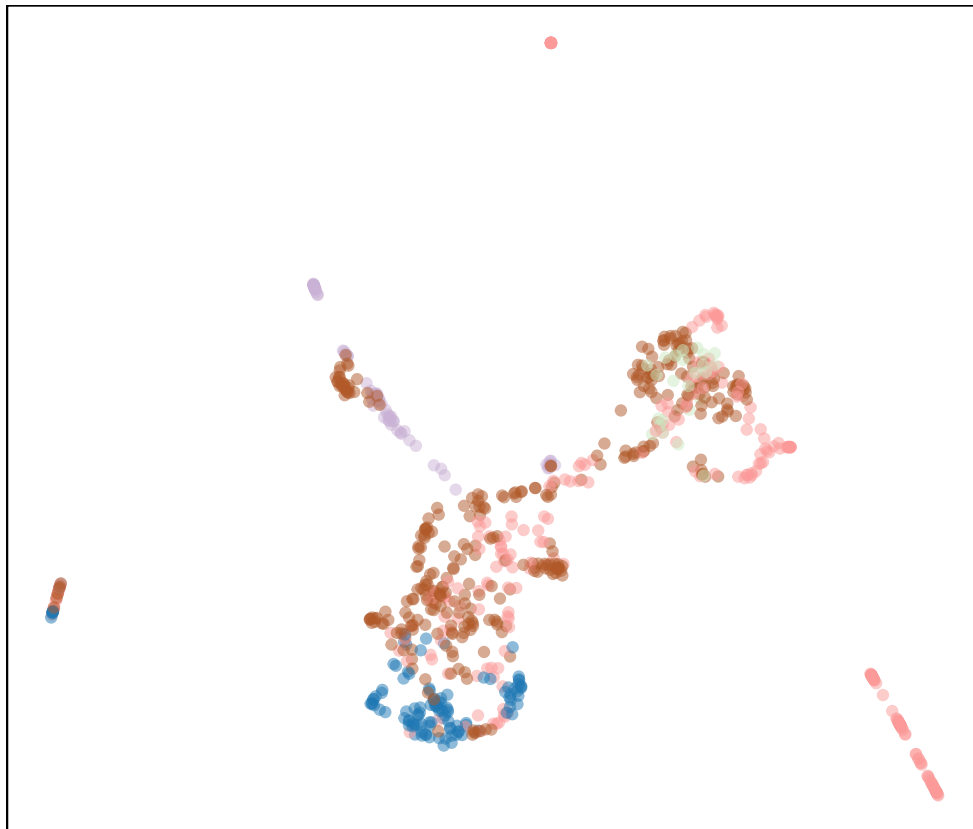
Fit_PaCMAP(as.integer(2), as.integer(10), "random", 0.9, as.integer(2), path, path2)

PaCMAP_data <- read_csv(path2)
PaCMAP_data <- PaCMAP_data |>
  mutate(ID = training_data$ID)

PaCMAP_data_with_label <- PaCMAP_data |>
  mutate(shape_label = training_data$shape_label)

PaCMAP_data_with_label |>
  ggplot(aes(x = PaCMAP1,
             y = PaCMAP2, color = shape_label))+
  geom_point(alpha=0.5) +
  coord_equal() +
  theme(plot.title = element_text(hjust = 0.5, size = 18, face = "bold")) + #ggtitle("(a)") +
  theme_linedraw() +
  theme(legend.position = "none", plot.title = element_text(size = 7, hjust = 0.5, vjust = -0.5),
        axis.title.x = element_blank(), axis.title.y = element_blank(),
        axis.text.x = element_blank(), axis.ticks.x = element_blank(),
        axis.text.y = element_blank(), axis.ticks.y = element_blank(),
        panel.grid.major = element_blank(), panel.grid.minor = element_blank(), #change legend key width
        legend.title = element_text(size=5), #change legend title font size
        legend.text = element_text(size=4),
        legend.key.height = unit(0.25, 'cm'),
        legend.key.width = unit(0.25, 'cm')) +
scale_color_manual(values=c("#b15928", "#1f78b4", "#cab2d6", "#cceb5", "#fb9a99", "#e31a1c", "#6a3d9a", "#ff7f00"))

```



```
num_bins_x <- calculate_effective_x_bins(.data = tSNE_data, x = "tSNE1", cell_area = 1)
num_bins_x <- 13
```

```
shape_val <- calculate_effective_shape_value(.data = tSNE_data, x = "tSNE1", y = "tSNE2")
shape_val
```

```
#> [1] 1.13882
```

```
num_bins_y <- calculate_effective_y_bins(.data = tSNE_data, x = "tSNE1", y = "tSNE2", shape_val = 0.8417289, num_
num_bins_y
```

```
#> [1] 14
```

```
all_centroids_df <- generate_full_grid_centroids(nldr_df = tSNE_data,
                                                x = "tSNE1", y = "tSNE2",
                                                num_bins_x = num_bins_x,
                                                num_bins_y = num_bins_y,
                                                buffer_size = NA, hex_size = NA)
```

```
hex_grid <- gen_hex_coordinates(all_centroids_df)
```

```
full_grid_with_hexbin_id <- map_hexbin_id(all_centroids_df)
```

```
full_grid_with_polygon_id <- map_polygon_id(full_grid_with_hexbin_id, hex_grid)
```

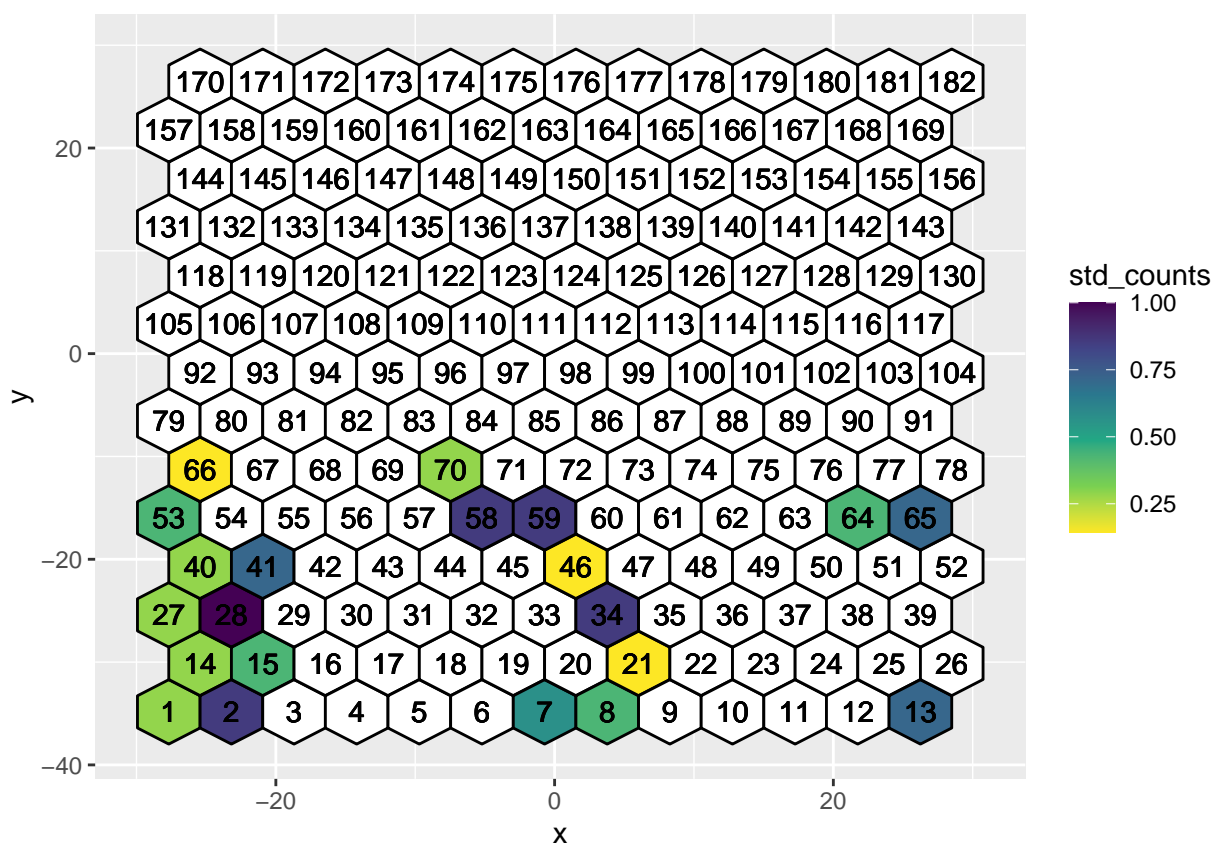
```
tSNE_data_with_id <- assign_data(tSNE_data, full_grid_with_hexbin_id)
```

```
df_with_std_counts <- compute_std_counts(nldr_df = s_curve_noise_umap_with_id)
```

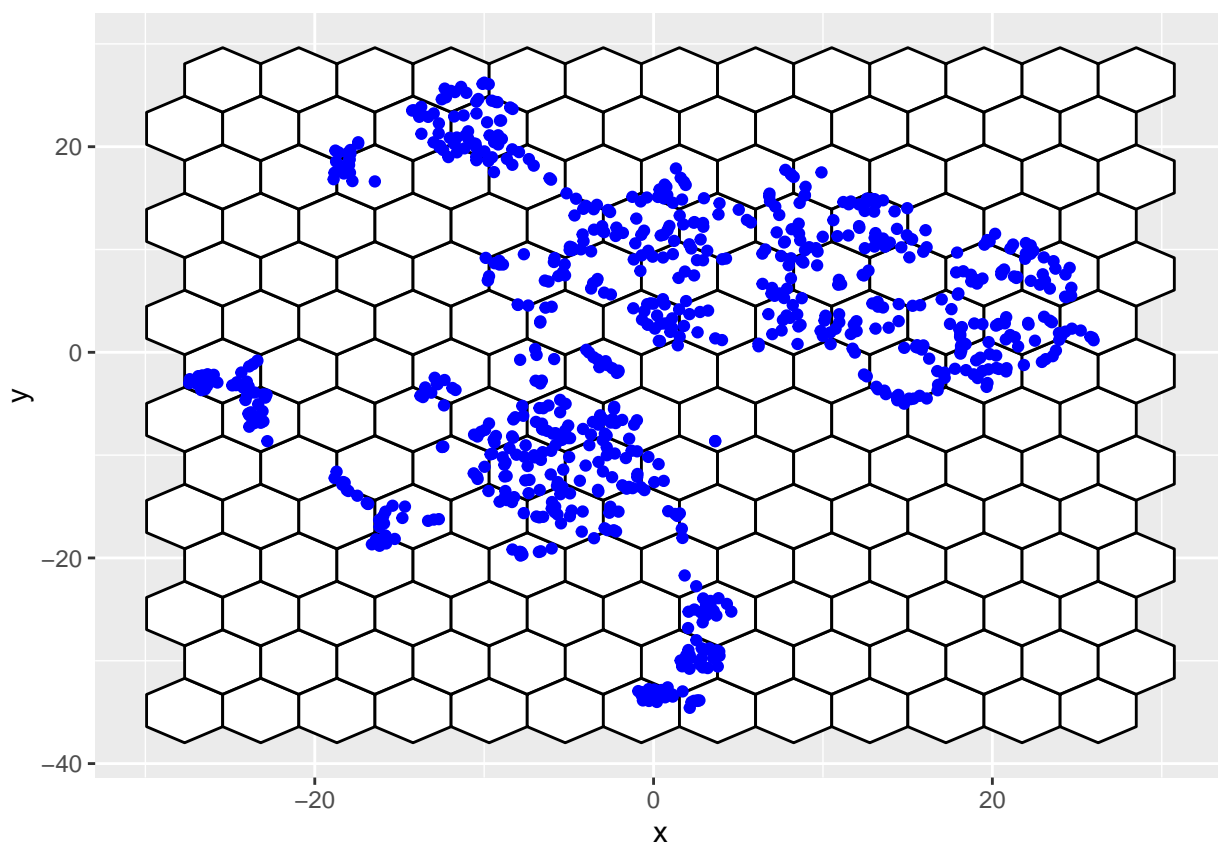
```
hex_full_count_df <- generate_full_grid_info(full_grid_with_polygon_id, df_with_std_counts, hex_grid)
```

```
ggplot(data = hex_full_count_df, aes(x = x, y = y)) +
  geom_polygon(color = "black", aes(group = polygon_id, fill = std_counts)) +
  geom_text(aes(x = c_x, y = c_y, label = hexID)) +
  scale_fill_viridis_c(direction = -1, na.value = "#ffffff")
```





```
ggplot(data = hex_grid, aes(x = x, y = y)) + geom_polygon(fill = "white", color = "black", aes(group = id)) +
  geom_point(data = tSNE_data, aes(x = tSNE1, y = tSNE2), color = "blue")
```



```
df_bin_centroids <- hex_full_count_df[complete.cases(hex_full_count_df[["std_counts"]]), ] |>
  dplyr::select("c_x", "c_y", "hexID", "std_counts") |>
```

```

dplyr::distinct() |>
dplyr::rename(c("x" = "c_x", "y" = "c_y"))

df_bin_centroids

#> # A tibble: 21 x 4
#>       x     y hexID std_counts
#>   <dbl> <dbl> <int>     <dbl>
#> 1 -27.7 -34.8     1      0.286
#> 2 -25.4 -30.1    14      0.286
#> 3 -27.7 -25.4    27      0.286
#> 4 -25.4 -20.7    40      0.286
#> 5 -27.7 -16.0    53      0.429
#> 6 -25.4 -11.3    66      0.143
#> 7 -23.2 -34.8     2      0.857
#> 8 -20.9 -30.1    15      0.429
#> 9 -23.2 -25.4    28       1
#> 10 -20.9 -20.7    41      0.714
#> # i 11 more rows

tr1_object <- triangulate_bin_centroids(df_bin_centroids, x, y)
tr_from_to_df <- generate_edge_info(triangular_object = tr1_object)

## To generate a data set with high-D and 2D training data
df_all <- training_data |> dplyr::select(-c(ID, shape_label)) |>
  dplyr::bind_cols(tSNE_data_with_id)

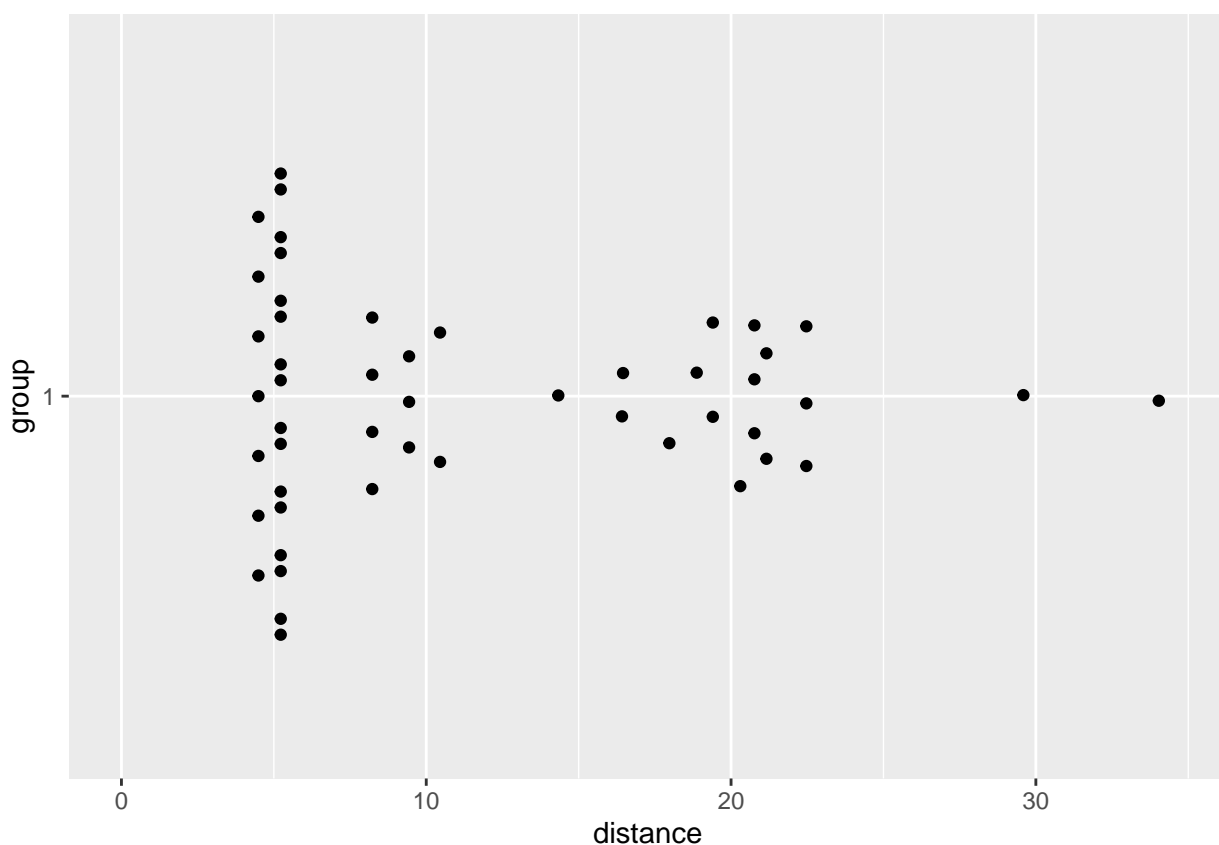
## To generate averaged high-D data

df_bin <- avg_highD_data(.data = df_all, column_start_text = "PC") ## Need to pass ID column name

## Compute 2D distances
distance <- cal_2d_dist(.data = tr_from_to_df)

plot_dist(distance)

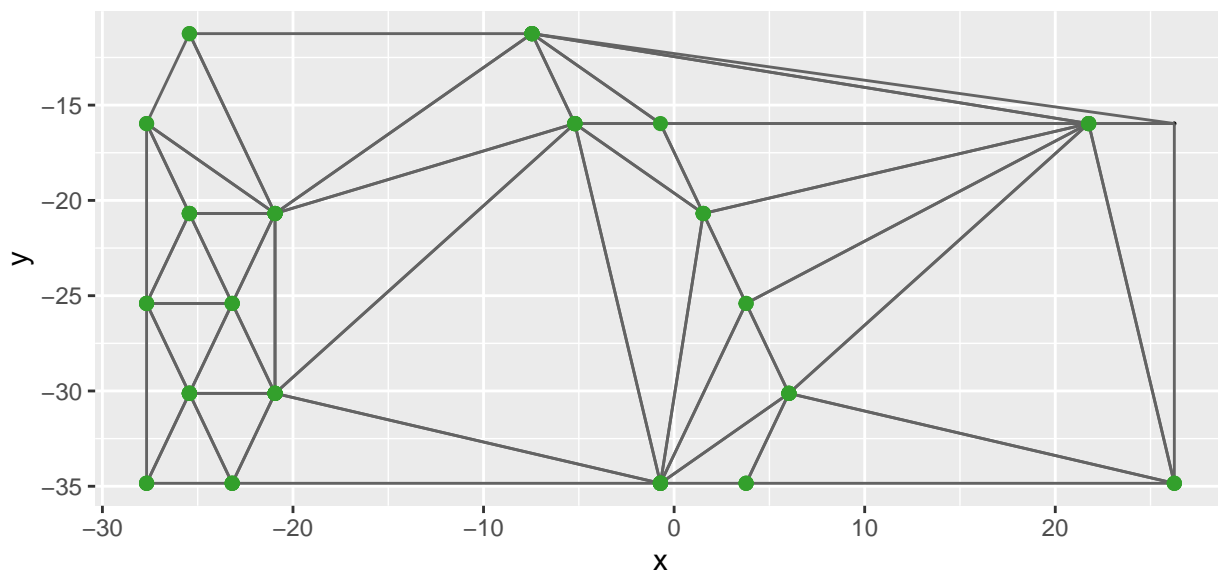
```



```
benchmark <- find_benchmark_value(.data = distance, distance_col = "distance")
```

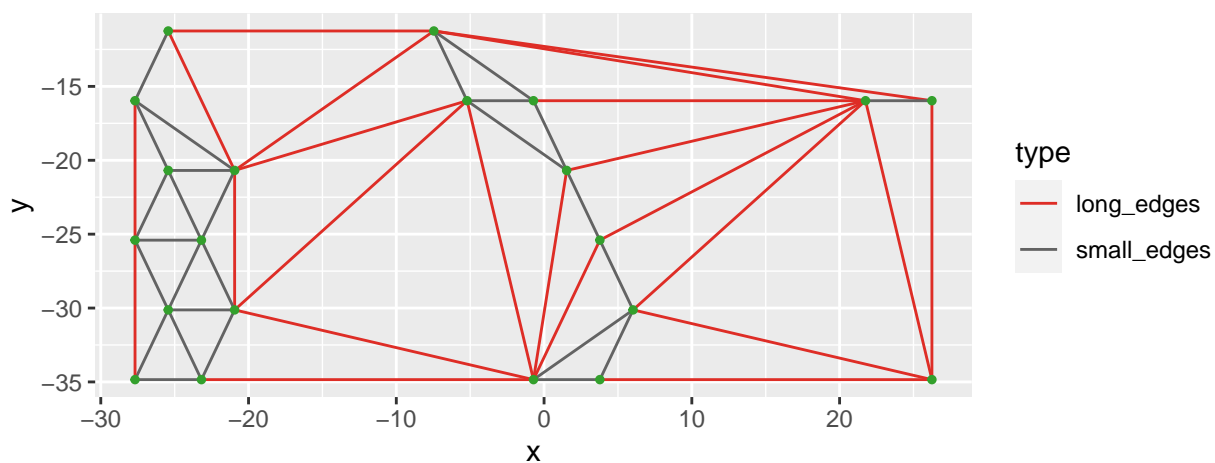
```
trimesh <- ggplot(df_bin_centroids, aes(x = x, y = y)) +  
  geom_point(size = 0.1) +  
  geom_trimesh() +  
  coord_equal()
```

```
trimesh
```



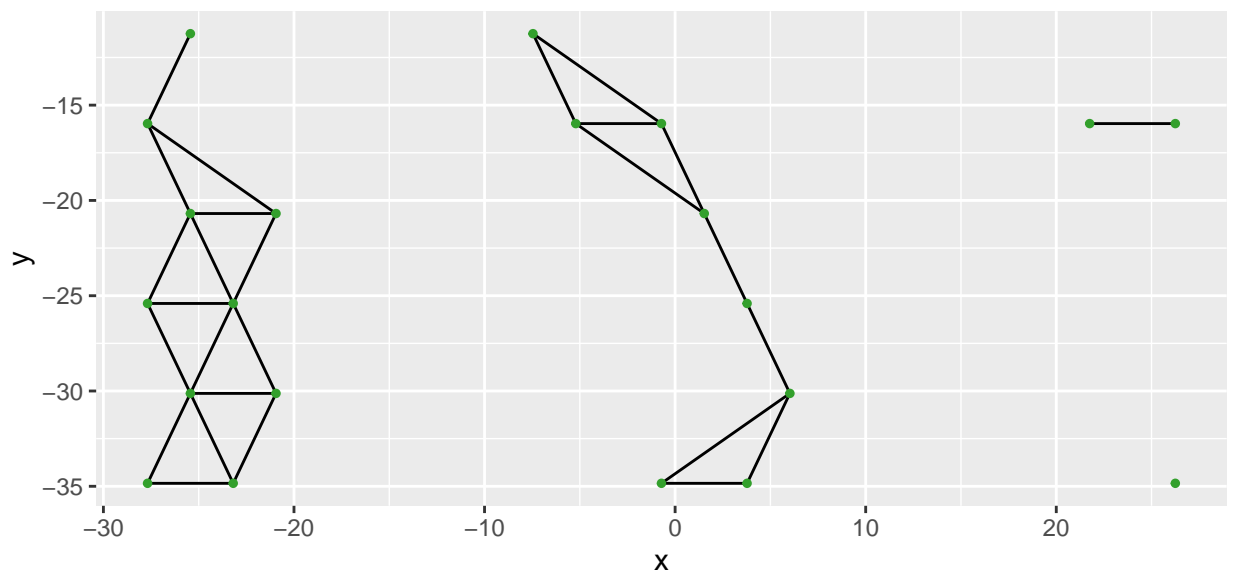
```
trimesh_gr <- colour_long_edges(.data = distance, benchmark_value = benchmark,  
  triangular_object = tr1_object, distance_col = distance)
```

```
trimesh_gr
```



```
trimesh_removed <- remove_long_edges(.data = distance, benchmark_value = benchmark,  
  triangular_object = tr1_object, distance_col = distance)
```

```
trimesh_removed
```



```
tour1 <- show_langevitour(df_all, df_bin, df_bin_centroids, benchmark_value = benchmark,  
  distance = distance, distance_col = "distance", column_start_text = "PC")  
tour1
```



## 4 Conclusion

## 5 Acknowledgements

This article is created using [knitr](#) (Xie 2015) and [rmarkdown](#) (Xie, Allaire, and Golemund 2018) in R with the `rjtools::rjournal_article` template. The source code for reproducing this paper can be found at: <https://github.com/JayaniLakshika/paper-quollr>.

## References

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