Assignment - I

UNIT I - Baye's Theorem**-** Discrete and continuous random variables – Moments – Moment generating functions – Binomial, Poisson, Geometric, Uniform, Exponential, Gamma and Normal distributions, Function of random variable.

PART - A

1. State Baye's Theorem.
2. If A and B are independent then show that $\bar{A}$ and $\bar{B}$ are also.
3. List the limitations of Poisson distribution.
4. If the MGF of a random variable is $(0.3e^t + 0.7)^8$, what is the MGF of $3X + 2$ and $E(X)$.
5. The mean of Binomial distribution is 20 and standard deviation is 4. Identify the parameters of the distribution.
6. If $f(x) = ke^{-x}, x \geq 0$ is the pdf of a random variable, then find k.
7. Find the probability distribution of the total number of heads obtained in four tosses of an unbiased coin.
8. Test whether $f(x) = \begin{cases} |x|, -1 \leq x \leq 1 \\ 0, else \end{cases}$ is a pdf of a random variable.
9. If a random variable X has the MGF $M_X(t) = \frac{2}{2-t}$ then find the mean and variance.
10. Write the properties of M.G.F.
11. If $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \cap B) = 0.15$, find $P(A/\bar{B})$.
12. State and prove the memoryless property of Geometric Distribution.
13. The probability of a candidate can pass in an examination is 0.6.
    a) What is the probability that he will pass in third trial?
    b) What is the probability that if he pass before third trail?
14. The number of hardware failures of a computer system in a week of operations has the following p.d.f, Evaluate the mean of the number of failures in a week.

No.of failures	0	1	2	3	4	5	6
Probability	.18	.28	.25	.18	.06	.04	.01

15. The CDF of a continuous random variable is given by $F(x) = \{1 - e^{-\frac{x}{5}}, x > 0$. Find $P(X > 2)$.

15. Find the r^th moment of a random variable X with p.d.f $f(x) = x(2-x), 0 < x < 2$.

PART - B

1. At a certain university, 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3:2 in favour of women. If a student is selected at random from among all those over six feet tall, what is the probability that the student is a woman?
2. A factory production line is manufacturing bolts using three machines, A, B and C. Of the total output, machine A is responsible for 25%, machine B for 35% and machine C

for the rest. It is known from previous experience with the machines that 5% of the output from machine A is defective, 4% from machine B and 2% from machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from (a) machine A (b) machine B (c) machine C?

3. There are 4 candidates for the office of the highway commissioner; the respective probabilities that they will be selected are 0.3,0.2,0.4 and 0.1 and the probabilities for a project's approval are 0.35,.85,.45 and 0.15, depending on which of the 4 candidates is selected. What is the probability of the project getting approved?
4. The probability that a student passes a certain exam is 0.9 given that he studied. The probability that he passes the exam without studying is 0.2. Assume that the probability passed the exam, what is the probability that he studied?
5. Three urns contains 3 white, 1 red and 1 blue balls; 2 white , 3 red and 4 blue balls; 1 white , 3 red and 2 blue balls respectively. One urn is chosen at random and from it 2 balls are drawn at random. If they are found to be 1 red and 1 blue ball, what is the probability that the first urn was chosen?
6. A bag contains 5 balls of unknown colors. A ball is drawn at random from it and is found to be white. Find the probability that bag contains only white ball.
7. If the probability mass function of a random variable X is given by $P(X = x) = kx^3, x = 1,2,3,4$ find (i) value of k (ii) $P(\frac{1}{2} < X < \frac{5}{2} / X > 1)$ (iii) c.d.f of X.
8. Two dice are thrown 120 times. Find the average number of times in which the number of the first die exceeds the number in the second die.
9. State and prove memoryless property of Geometric distribution.
10. If $f(x) = x e^{-x^2}, x \geq 0$ then find mean and variance of X.
11. The probability density function of a random variable X is given by
12.

$$f_X(x) = \begin{cases} x, & 0 < x < 1 \\ k(2-x), & 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}.$$

(1) Find the value of 'k'. (2) Find $P(0.2 < x < 1.2)$

(3) What is $P[0.5 < x < 1.5 / x \geq 1]$ (4) Find the distribution function of $f(x)$.

13. The C.D.F of a random variable X

$$F(x) = \begin{cases} 0, x < 0 \\ x^2, 0 \leq x < \frac{1}{2} \\ 1 - \frac{3}{25}(3-x)^2, \frac{1}{2} \leq x < 3 \\ 1, x \geq 3 \end{cases}$$, find the p.d.f, $P(|X| \leq 1)$; $P(\frac{1}{3} \leq X < 4)$

14. A continuous random variable X has the p.d.f $f(x) = Ce^{-|x|}, -\infty < x < \infty$, find the MGF of X.
15. Give the MGF of Poisson distribution and hence find its mean and variance.
16. Calculate the MGF of Geometric distribution and hence find its mean and variance.
17. A car hire firm has 2 cars. The number of demands for a car on each day is distributed as Poisson variate with mean 0.5. Calculate the proportion of days on which(i) neither car is used (ii) Some demand is refused.

18. If m things are distributed among a men and b women, show that the probability that the number of things received by men is odd is $\frac{1}{2}\left[\frac{(b+1)^m - (b-a)^m}{(b+a)^m}\right]$ (Hint : P(a thing is received by men) = $p = \frac{a}{a+b}$.
19. Explain the MGF of Normal distribution and hence find its mean and variance.
20. The marks obtained by a number of students for a certain subject is assumed to be normally distributed with mean 65 and standard deviation 5.If 3 students are taken at random from this set Identify the probability that exactly 2 of them will have marks over 70?
21. If X and Y are independent random variables following N(8,2) and $N(12,4\sqrt{3})$ respectively, find the value of λ such that $P[2X - Y \leq 2\lambda] = P[X + 2Y \geq \lambda]$.
22. VLSI chips, essential to the running condition of a computer system, fail in accordance with a Poisson distribution with the rate of one chip in about 5 weeks .if there are two spare chips on hand and if a new supply will arrive in 8 weeks. Evaluate the probability that during the next 8 weeks the system will be down for a week or more, owing to a lack of chips?
23. In a test on 2000 electric bulbs, it was found that the life of a Philips bulbs was normally distributed with an average of 2400 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for (i) more than 2150 hours (ii) less than 1950 hours.
24. An irregular 6-faced die is thrown and the expectation that in 10 throws it will give 5 even numbers is twice the expectation that it will give 4 even numbers. How many times in 10,000 sets of 10 throws would you expect to give no even number?
25. At least one half of an airplane's engines are required to function in order for it to operate. If each engine functions independently with probability of failure q, for what values of q is a 2-engine plane to be preferred for operation to a 4-engine plane?