SSN College of Engineering, Kalavakkam – 603 110

Department of Mathematics

UNIT –I

Probability & Random Variables

ASSIGNMENT – II

Classes: IV sem IT- A&B, CSE-A&C, BME

Part – A

1. If X has mean 4 and variance 9, while has mean -2 and variance 5 and the two are independent, find $E(XY)$ and $E(XY^2)$.
2. When the two random variables X and Y are said to be independent in discrete and continuous case?
3. X and Y are independent random variables with variance 2 and 3. Find the variance of $3X + 4Y$.
4. If $f(x,y) = kxye^{-(x^2+y^2)}, x \geq 0, y \geq 0$ is the joint pdf, find k?
5. If the distribution function of $(X,Y)$is $F(x,y) = (1-e^{-x})(1-e^{-y})$, $x > 0, y > 0$, find the joint density function.
6. Let X and Y be independent standard normal random variables. Find the p.d.f. of $Z = \frac{X}{Y}$
7. Let X and Y be independent uniform random variables over (0, 1).Find the p.d.f. of $Z = X + Y$
8. Prove that $-1 \leq r(X,Y) \leq 1$.
9. If X and Y are uncorrelated, can you say that they are independent?
10. In a correlation analysis, the equations of the two regression lines are $3x + 12y = 19$ and $3y + 9x = 46$, find the mean value of X and Y.
11. State Central limit theorem for independent and identically distributed random variables.
12. A coin is tossed 10 times. What is the probability of getting 3 or 4 or 5 heads? Use CLT.

PART-B

1. The joint distribution of X_1 and X_2 is given by $f(x_1,x_2) = \frac{x_1+x_2}{21}$, $x_1 = 1,2$ and $x_2 = 1,2$. Find all the marginal and conditional distributions.
2. If the joint pdf of two dimensional random variables (X,Y) is given by $f(x,y) = \begin{cases} k(6-x-y); & 0 < x < 2,\ 2 < y < 4 \\ 0 \quad ; & \text{otherwise,} \end{cases}$
3. The joint pdf of a two dimensional r.v (X,Y) is $f(x,y) = xy^2 + \frac{x^2}{8}$, $0 \leq x \leq 2$, $0 \leq y \leq 1$. Find (i) $P(X > 1/Y < \frac{1}{2})$ (ii)$P(Y < \frac{1}{2}/X > 1)$ (iii) $P(X+Y \leq 1)$ (iv) P(X < Y)
4. The joint density function of random variables X and Y is $f(x,y) = 2, 0 < x < y < 1$, find the marginal and conditional probability density functions. Are X and Y independent?
5. If the joint distribution function of X and Y is given by

$$F(x,y) = \begin{cases} (1-e^{-x})(1-e^{-y}) & \text{for } x>0, y>0 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the marginal densities of X and Y.

(ii) Are X and Y independent.

(iii) Find $P(1 < X < 3,\ 1 < Y < 2)$

6. X and Y are two random variables having joint density function

$$f(x) = \frac{1}{8}(6-x-y),\ 0<x<2,\ 2<y<4$$
$$= 0 \qquad \text{otherwise}$$

Find (i) $P(X < 1 \cap Y < 3)$, (ii) $P(X+Y<3)$ and (iii) $P(X<1/Y<3)$

7. The joint pdf of a two dimensional r.v. (X,Y) is given by $f(x,y) = \begin{cases} 4xye^{-(x^2+y^2)} & , \ x \geq 0, y \geq 0 \\ 0 & , \ \text{otherwise} \end{cases}$. Find the density function of $U = \sqrt{X^2+Y^2}$.

8. If the joint pdf of (X,Y) is given by $f(x,y) = x + y, 0 \leq x, y \leq 1$, find the pdf of $U = XY$.

9. If X and Y are independent random variable with pdf $f(x) = e^{-x}, x \geq 0$ and $f(y) = e^{-y}, y \geq 0$ find the density function of $U = \frac{X}{X+Y}$ and $V = X + Y$. .Are they independent?

10. The joint distribution of (X,Y) is given below. Find $COV(X,Y)$.

X\ Y	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

11. Find the coefficient of correlation for the following heights (in inches) of fathers (X) and their sons (Y):

X:	65	66	67	67	68	69	70	72
Y :	67	68	65	68	72	72	69	71

12. Find the coefficient of correlation and obtain the lines of regression from the data given below:

x:	62	64	65	69	70	71	72	74
y:	126	125	139	145	165	152	180	208

13. If X and Y are independent random variables with means 5 and 10 and standard deviations 2 and 3 respectively. Find $r(U,V)$ where $U = 3X + 4Y$ and $V = 3X - Y$.

14. The joint p.d.f of R.Vs X and Y is given by

$$f(x,y) = \begin{cases} 3(x+y), & 0 \leq x \leq 1, 0 \leq y \leq 1, x+y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) the marginal p.d.f of *X*.

(ii) $P(X+Y<1/2)$
(iii) $r(X,Y)$

15. The following data is collected from 200 students who recently completed higher secondary examination.

	Passed	Failed
Had Tution	144	16
Had no tution	29	11

Based on this information, examine whether there is any association between special coaching and passing the examination.

16. If the regression lines are $2x+3y=26$ and $x+6y=31$, find $\bar{x},\bar{y}$ and the correlation coefficient between *x* and *y*.

17. If X and Y are independent exponential random variables each with parameter 1, find the pdf of $U=X-Y$.

18. Two random variables (X,Y) have joint density function $f(x,y)=2-x-y, 0\le x\le 1$, $0\le y\le 1$. Find the correlation coefficient of (X,Y) and the equations of the regression lines.

19. The life time of a certain brand of an electric bulb may be considered as a random variable with mean 1200 hours and S.D 250 hours. Find the probability, using central limit theorem that the average life time of 60 bulbs exceeds 1250 hours.

20. State Central Limit theorem in Lindberg-Levy's form.

A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using Central limit theorem, with what probability can we assert that the mean of the sample will not differ from $\mu=60$ by more than 4?