SSN COLLEGE OF ENGINEERING
Department of Mathematics

UMA1477 & UMA1478

UNIT- 1 Random Variables

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Random Experiment

An experiment whose outcome is not known in certainty or in advance is known as random experiment.

Example: Tossing a coin, throwing a die.

Event

The set of likelihood of particular outcome.

Example: Tossing a coin and getting a head or tail.

Exhaustive Events

The total number of possible outcomes in any trial is known as exhaustive events.

Example: Throwing a die – six exhaustive events

Sample space

Throwing a single fair die. This experiment may show any number from one to six. The set of all possible outcomes is called sample space.

Mutually exclusive events

Two events are said to be mutually exclusive or incompatible if the happening of one event prevents the happening of the other.

Example: In tossing a coin the events head and tail are mutually exclusive.

Independent Events

Two events are said to be independent if the happening of one event does not affect the happening of the other.

Example: In tossing an unbiased coin the event of getting head in first toss is independent of getting a head or tail in the second, third and subsequent throws.

Conditional Probability

Consider an event A which depends on the event B. The probability of the event A given that the event B has already occurred is denoted as P (A/B) and is called as conditional probability.

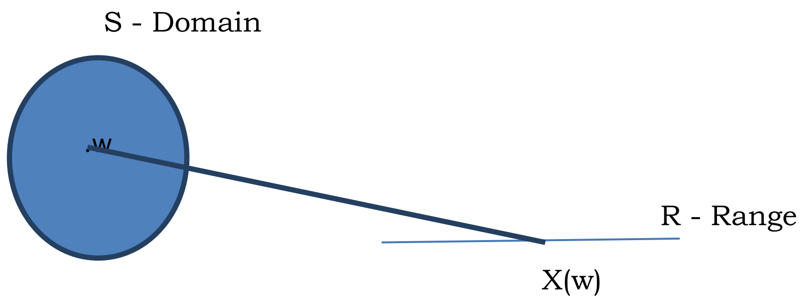
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Elementary theorems

$$P(A \cap \overline{A}) = \phi, \overline{A} is the complement of A$$
$$P(A \cup \overline{A}) = S$$
$$P(\overline{A}) = 1 - P(A)$$
$$P(S \cup \phi) = P(S)$$
$$P(AUB) = P(A) + P(B) - P(A \cap B)$$
$$For any event P(E) \geq 0, \sum p_i = 1$$

One dimensional random variable

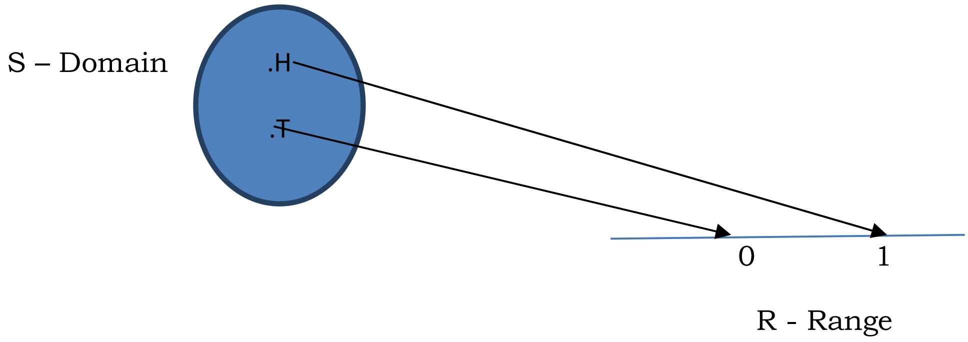
Let S be the sample space associated with a given random experiment. A real valued function defined on S and taking values in $(-\infty, \infty)$ is called a one dimensional random variable.



Example:

In the experiment of tossing a coin once the sample space S= {H, T} where H denotes head and T denotes tail.

A random variable X can be defined as $X(w) = \begin{cases} 0 \, if \;\; w \in T \\ 1 \, if \;\; w \in H \end{cases}$



Here X takes two vales such a random variable is called a Bernoulli random variable.

There are two types of random variables. They are,

- Discrete random variable
- Continuous random variable.

Discrete Random Variable

If a random variable takes at most a countable number of values it is called a discrete random variable. In other words, a real valued function defined on a discrete sample space is called a discrete random variable.

Probability mass function

Suppose X is a discrete random variable taking at most a countable number of values x_1, x_2, x_3,…. Each possible outcome x_i is associated with a probability i.e.

P(X=xi) =p (xi) =pi called the probability of xi. The number p (x_i) must satisfy the following conditions.

$$p(x_i) \geq 0$$

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

The function p is called the probability mass function.

Distribution function of a random variable or Cumulative Distribution function

Let X be a discrete random variable taking countable number of values x_1, x_2, x_3…
with associated probabilities $p(x_i) \geq 0$. Then the distribution function of X is given by

$$F(x) = F_X(x) = P(X \leq x) = \sum_{x_i \leq x} p(x_i), -\infty \leq x \leq \infty$$

$$P(X \leq x) = \{w \in S / X(w) \leq x\}$$

Continuous random variable

A random variable X is said to be continuous if it can take all possible values between certain limits i.e., it takes all possible values in a given interval.

Probability density function

If X is continuous random variable such that

$$P(x-\frac{dx}{2} \le X \le x+\frac{dx}{2}) = f(x)dx$$

then f(x) is called the pdf of X provided f(x) satisfied the following conditions.

$$f(x) \ge 0$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Continuous distribution function

If X is a continuous random variable with pdf f(x) then the function

$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)dx, -\infty < x < \infty$ is called the distribution function or cumulative distribution function.

Properties of distribution function

1. $\underset{x\to\infty}{\mathrm{Lt}}\, F(x) = 1, i.e., F(\infty) = 1$
   $\underset{x\to-\infty}{\mathrm{Lt}}\, F(x) = 0, i.e., F(-\infty) = 0$
2. $0 \le F(x) \le 1$
3. If x<y, $F(x) \le F(y)$
4. If X is a discrete random variable taking x_1, x_2, x_3.......
   $x_1 < x_2 <......< x_i$
   $P(X = x_i) = F(x_i) - F(x_{i-1})$
5. $F'(x) = \frac{d}{dx}(F(x)) = f(x),\ f(x) \ge 0$
6. $P(a \le x \le b) = \int_a^b f(x)dx = \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx = P(X \le b) - P(X \le a)$

7. If X is a continuous random variable then

$$P(a < X < b) = P(a \le X < b) = P(a < X \le b) = P(a \le X \le b) = \int_a^b f(x)dx = F(b) - F(a)$$

8. If X is a discrete random variable then

$$P(a < X \le b) = F(b) - F(a)$$

$$P(X \le b) = P(X \le a) + P(a < X \le b)$$

$$\therefore P(a < X \le b) = P(X \le b) - P(X \le a) = F(b) - F(a)$$

9. If X is a discrete random variable then

$$P(a \le X \le b) = P(X = a) + F(b) - F(a)$$

$$Event (a \le X \le b) is the disjo\operatorname{int} union of (a < X \le b) \cup (X = a)$$

$$P(a \le X \le b) = P(a < X \le b) + P(X = a) = F(b) - F(a) + P(X = a)$$

10. If X is a discrete random variable then

$$P(a < X < b) = F(b) - F(a) - P(X = b)$$

$$P(a < X < b) = P(a < X \le b) - P(X = b) = F(b) - F(a) - P(X = b)$$

11. If X is a discrete random variable then

$$P(a \le X < b) = P(X = a) + F(b) - F(a) - P(X = b)$$

$$(a \le X < b) = (X = a) \cup (a < X < b)$$

$$P(a \le X < b) = P(X = a) + P(a < X < b) = P(X = a) + F(b) - F(a) - P(X = b)$$
