SSN COLLEGE OF ENGINEERING
Department of Mathematics

UMA 1477 & UMA1478

Expectation & MGF

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Expectations

Let X be a discrete random variable with probability mass function p(x).

Then $E(X)=\sum_x xp(x) or \sum_i x_i p(x_i)$

For Discrete Random Variable (r^th^ moment about origin)

$$E(X^r)=\sum x^r p(x)=\mu_r'$$

Put r =1, Mean = $\mu_1'=E(X)=\sum_x xp(x)$

Put r=2, $\mu_2'=E(X^2)=\sum_x x^2 p(x)$

$$\therefore Var(X)=\mu_2=\mu_2'-\mu_1'^2=E(X^2)-(E(X))^2$$

(rth moment about mean or r^th^ central moments)

$$\mu_r=E(X-E(X))^r=\sum(x-\bar{x})^r p(x)$$

$$r=2, \mu_2=\sum(x-\bar{x})^2 p(x)$$

$$r=3, \mu_3=\sum(x-\bar{x})^3 p(x)$$

For Continuous Random Variable

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

$$E(X^r) = \mu_r' = \int_{-\infty}^{\infty} x^r f(x) dx$$

$$Mean\, \mu_1' = \int_{-\infty}^{\infty} x f(x) dx$$

$$\mu_2' = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$Var(X) = \mu_2 = \mu_2' - \mu_1'^2 = E(X^2) - (E(X))^2 = V(X) = \sigma_x^2$$

$$S.D(X) = \sqrt{\sigma_x^2} = \sigma_x$$

Note:

If X is a random variable

E(aX+b) = aE(X)+b, E(a) = a

V(aX+b) $=a^2$Var(X)

Moment Generating Function (M.g.f)

M.g.f of a r.v X is

$$M_x(t) = E(e^{tX}) = \begin{cases} \sum e^{tx} p(x) & if X is discrete \\ \int_{-\infty}^{\infty} e^{tx} f(x) dx & if X is continous \end{cases}$$

$$M_X(t) = E(e^{tX}) = E\left[1 + \frac{tX}{1!} + \frac{(tX)^2}{2!} + \cdots\right] = 1 + tE(X) + \frac{t^2 E(X^2)}{2!} + \cdots + \frac{t^r E(X^r)}{r!} +$$

$$M_X(t) = 1 + \frac{t\mu_1'}{1!} + \frac{t^2 \mu_2'}{2!} + \cdots \frac{t^r \mu_r'}{r!} + \cdots = \sum_{r=0}^{\infty} \frac{t^r \mu_r'}{r!}$$

μ_r' = rth moment = coeff of $\frac{t^r}{r!}$ in $M_x(t)$

Properties :

1. $\mu_r' = \left[\dfrac{d^r}{dt^r} M_x(t)\right]_{t=0}$, (Differentiating $M_x(t)$ w.r.to t and putting t=0).
2. $M_{CX}(t) = M_X(ct)$
   LHS = $M_{CX}(t)$ = $E[e^{tCX}]$
   RHS=$M_X(Ct)$ =$E[e^{tCX}]$
   LHS = RHS
3. The moment generating function of the sum of a given number of independent random variables is equal to the product of their respective moment generating functions.
   i.e. $M_{X1+X2+X3+...+Xn}(t) = M_{X1}(t).M_{X2}(t)..... M_{Xn}(t)$.
   proof:
   By definition $M_{X1+X2+X3+...+Xn}(t) = E[e^{t(X_1+X_2+X_3+...+X_n)}]$
   $= E[e^{tX_1}. e^{tX_2}....... e^{tX_n}]$
   $= E[e^{tX_1}].E[e^{tX_2}]......E[e^{tX_n}]$.
   (Since $X_1, X_2....X_n$ are independent).
   $= M_{X1}(t).M_{X2}(t)...M_{Xn}(t)$
4. MGF is affected by the change of origin
5. MGF is affected by the change of scale