Solved Problems – Baye's Theorem

Posted on August 1, 2017 / Under Probability & Statistics

Q1. There are three bags. First bag contains 1 white, 2 red and 3 green balls. Second bag contains 2 white, 3 red and 1 green balls. Third bag contains 3 white, 1 red and 2 green bals.
A bag is chosen at random and 2 balls are drawn from it. These are found to be 1 white and 1 red. Find the probability that the balls so drawn came from the second bag.

Solution:
Let A1 be the event of choosing first bag
Let A2 be the event of choosing second bag
Let A3 be the event of choosing third bag

Let E be the event of drawing 1 white and 1 red ball.
Therefore, we have to find out $P(A_2/E)$

By Baye's theorem,

$$\begin{aligned} P(A_2/E) &= \frac{P(A_2)P(E/A_2)}{\sum_{i=1}^{3} P(A_i)P(E/A_i)} \\ &= \frac{\frac{1}{3}x\frac{2}{5}}{\frac{1}{3}x\frac{2}{15} + \frac{1}{3}x\frac{2}{5} + \frac{1}{3}x\frac{1}{5}} \\ &= \frac{6}{11} \end{aligned}$$

Q2. Three machines A,B, and C produce respectively 60%, 30% and 10% of the total number of items of a factory. The percentages of defective output of these machines are 2%, 3% and 4% respectively. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine C.

Solution:
Let A,B and C stand for the events of selection of an item from machines A,B and C.
Therefore, P(A) = 60/100 = 0.6, P(B) = 0.3, P(C) = 0.1

Let E be the event of selecting defective item.
P(E/A) = 0.02, P(E/B) = 0.03, P(E/C) = 0.04

We have to find P(C/E)

By Baye's theorem,

$$P(C/E) = \frac{P(C).P(E/C)}{P(A).P(E/A) + P(B)P(E/B) + P(C)P(E/C)}$$

=0.16

Q3. In a bolt manufacturing factory, there are four machines A,B,C and D, manufacturing 20%, 15%, 25% and 40% respectively of the total production. Out of these 5%, 4%, 3% and 2% are defective. If a bolt is drawn at random, what is the probability that it is defective. If it is defective, what is the probability that it is manufactured by A or D ?

Solution:
Let A1, A2, A3, A4 be the events of selecting a bolt manufactured by machines A,B,C and D.
P(A1) = 0.2
P(A2) = 0.15

Search Topics

To search type and hit enter

Topics

Latest Notes

P(A3) = 0.25
P(A4) = 0.4
Let E be the event of selecting a defective bolt.

Given P(E/A1) = 0.05, P(E/A2) = 0.04, P(E/A3) = 0.03, P(E/A4) = 0.02

P(E) = 0.2(0.05) + 0.15(0.04) + 0.25(0.03) + 0.4(0.02) = 0.0315

We have to find P(A1 U A4/E)

$$\begin{aligned} P((A1 \cup A4)/E) &= P(A1/E) + P(A4/E) \\ &= \frac{0.2(0.05)}{0.0315} + \frac{0.4(0.02)}{0.0315} \\ &= 0.3175 + 0.254 \\ &= 0.5715 \end{aligned}$$

Q4. In a certain college, 4% of the boys and 1% of girls are taller than 1.8m. Further, 60% of the students are girls. If a student is selected at random and is found to be taller than 1.8m, what is the probability that the student is a girl ?

Solution:
Let A1 be the event of selecting a girl and A2 be the event of selecting a boy. Let E be the event where a student is taller than 1.8m.

P(A1) = 0.60; P(A2) = 0.40
P(E/A1) = 0.01; P(E/A2) = 0.04

$$\begin{aligned} P(A1/E) &= \frac{P(A1).P(E/A1)}{P(A1).P(E/A1) + P(A2).P(E/A2)} \\ &= \frac{0.6 \times 0.01}{0.6 \times 0.01 + 0.4 \times 0.04} \\ &= \frac{0.006}{0.006 + 0.016} \\ &= 0.2727 \end{aligned}$$

Q5: The chance that a doctor will diagnose a disease correctly is 60%. The chance that a patient will die after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. If a patient dies, what is the chance that his disease was correctly diagnosed?

Solution:
Let A1 be the event of correct diagnosis
Let A2 be the event of wrong diagnosis
Let E be the event that a patient dies.

Given, P(A1) = 0.6; P(A2) = 0.4
P(E/A1) = 0.4; P(E/A2) = 0.7

$$P(A1/E) = \frac{P(A1)P(E/A1)}{P(A1)P(E/A1) + P(A2)P(E/A2)} = 0.4615$$

Q5. A bag contains 3 coins, out of which, one has both faces heads. Other two coins are normal and fair. A coin is chosen at random from the bag and tossed 4 times in succession. If head turns up each time, what is the probability that it is the two headed coin.

Solution:
Let A1 be the event of selecting 2 headed coin
Let A2 and A3 be the events of selecting remaining fair coins
Let E be the event of getting 4 heads in succession.

P(A1) = P(A2) = P(A3) = 1/3
P(E/A1) = 1; P(E/A2) = P(E/A3) = 1/2 . 1/2 . 1/2 . 1/2 = 1/16

$$P(A1/E) = \frac{1/3 \quad . \quad 1}{1/3.(1+1/16+1/16)} = \frac{1/18}{16} = \frac{16}{18} = \frac{8}{9}$$

Q6. Three typists A,B,C typed 50%, 30% and 20% pages of a book. The percentage of wrongly typed pages by them is 3,4,5 respectively. If a page is selected from the book at random, what is the probability that it is wrongly typed and is typed by A ?

Solution:
P(A1) = 0.5, P(A2) = 0.3, P(A3) = 0.2
P(E/A1) = 0.03, P(E/A2) = 0.04, P(E/A3) 0.05

P(E) = 0.5×0.3 + 0.3×0.4 + 0.2×0.05
=0.015 + 0.012 + 0.010
=0.037

$$P(A1/E) = \frac{P(A1).P(E/A1)}{P(E)} = \frac{0.015}{0.037}$$

Q7. A tea set has 4 sets of cups and saucers. Two of these sets are of one colour and the other two sets are of different colours (A total of 3 colours). If the cups are placed randomly on saucers, what is the probability that no cup is on a saucer of same colour ?

Solution:
Let 2 sets (having same colour) be of colour C1 and the other 2 sets be of colours C2 and C3.
Let saucers be kept in C1C1C2C3 order.
Then the different possibilities in which cups can be kept are:

C1C1C2C3
C1C1C3C2
C1C2C1C3
C1C3C1C2
C1C2C3C1
C1C3C2C1
C2C1C1C3
C3C1C1C2
C2C1C3C1
C3C1C2C1
C2C3C1C1
C3C2C1C1

There are a total of 12 possibilities, of which, only last 2 are favourable to the event "no cup is on a saucer of same colour".

Therefore, the required probability = 2/12 = 1/6

« Baye's Theorem

Covid 19 morbidity counts follow Benford's Law ? »

WRITTEN BY TESSY C

Lecturer and Research Scholar in Mathematics.

- Opal Wesolick on Building a Robotic Arm using Arduino
- Justin Jose on Mosquitto MQTT Publish – Subscribe from PHP
- mohan on Mosquitto MQTT Publish – Subscribe from PHP