MIPS R-format Instructions

op	rs	rt	rd	shamt	funct
6 bits	5 bits	5 bits	5 bits	5 bits	6 bits

- Instruction fields
  - op: operation code (opcode)
  - rs: first source register number
  - rt: second source register number
  - rd: destination register number
  - shamt: shift amount (00000 for now)
  - funct: function code (extends opcode)

R-format Example

op	rs	rt	rd	shamt	funct
6 bits	5 bits	5 bits	5 bits	5 bits	6 bits

```
add $t0, $s1, $s2
```

special	\$s1	\$s2	\$t0	0	add
0	17	18	8	0	32
000000	10001	10010	01000	00000	100000

$00000010001100100100000000100000_2 = 02324020_{16}$

MIPS I-format Instructions

op	rs	rt	constant or address
6 bits	5 bits	5 bits	16 bits

- Immediate arithmetic and load/store instructions
  - rt: destination or source register number
  - Constant: -2^{15} to $+2^{15} - 1$
  - Address: offset added to base address in rs
- *Design Principle 4:* Good design demands good compromises
  - Different formats complicate decoding, but allow 32-bit instructions uniformly
  - Keep formats as similar as possible

Branch Addressing

- Branch instructions specify
  - Opcode, two registers, target address
- Most branch targets are near branch
  - Forward or backward

op	rs	rt	constant or address
6 bits	5 bits	5 bits	16 bits

- PC-relative addressing
  - Target address = PC + offset × 4
  - PC already incremented by 4 by this time

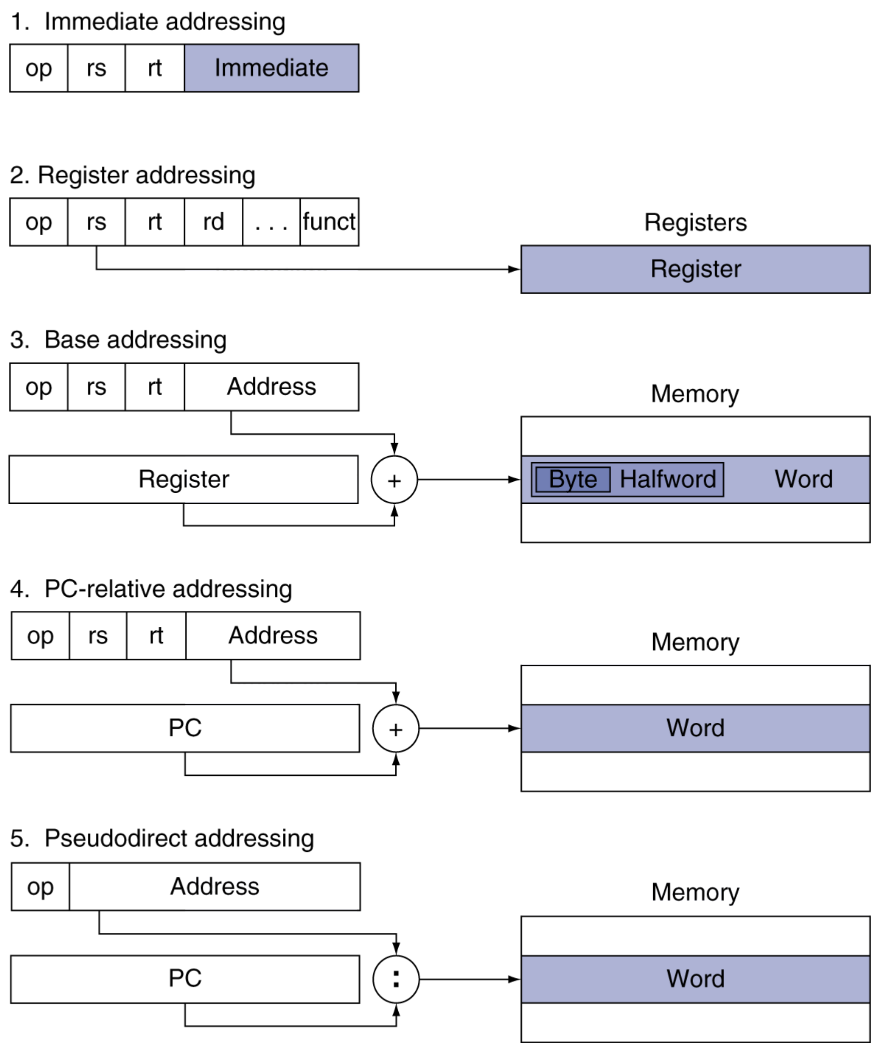
Jump Addressing

- Jump (`j` and `jal`) targets could be anywhere in text segment
  - Encode full address in instruction

op	address
6 bits	26 bits

- (Pseudo)Direct jump addressing
  - Target address = $PC_{31\ldots28}$: (address × 4)

Addressing Mode Summary



Synchronization

- Two processors sharing an area of memory
  - P1 writes, then P2 reads
  - Data race if P1 and P2 don't synchronize
    - Result depends of order of accesses
- Hardware support required
  - Atomic read/write memory operation
  - No other access to the location allowed between the read and write
- Could be a single instruction
  - E.g., atomic swap of register ↔ memory
  - Or an atomic pair of instructions

Synchronization in MIPS

- Load linked: `ll rt, offset(rs)`
- Store conditional: `sc rt, offset(rs)`
  - Succeeds if location not changed since the `ll`
    - Returns 1 in rt
  - Fails if location is changed
    - Returns 0 in rt
- Example: atomic swap (to test/set lock variable)

```
try: add $t0,$zero,$s4 ;copy exchange value
     ll  $t1,0($s1)    ;load linked
     sc  $t0,0($s1)    ;store conditional
     beq $t0,$zero,try ;branch store fails
     add $s4,$zero,$t1 ;put load value in $s4
```

ARM & MIPS Similarities

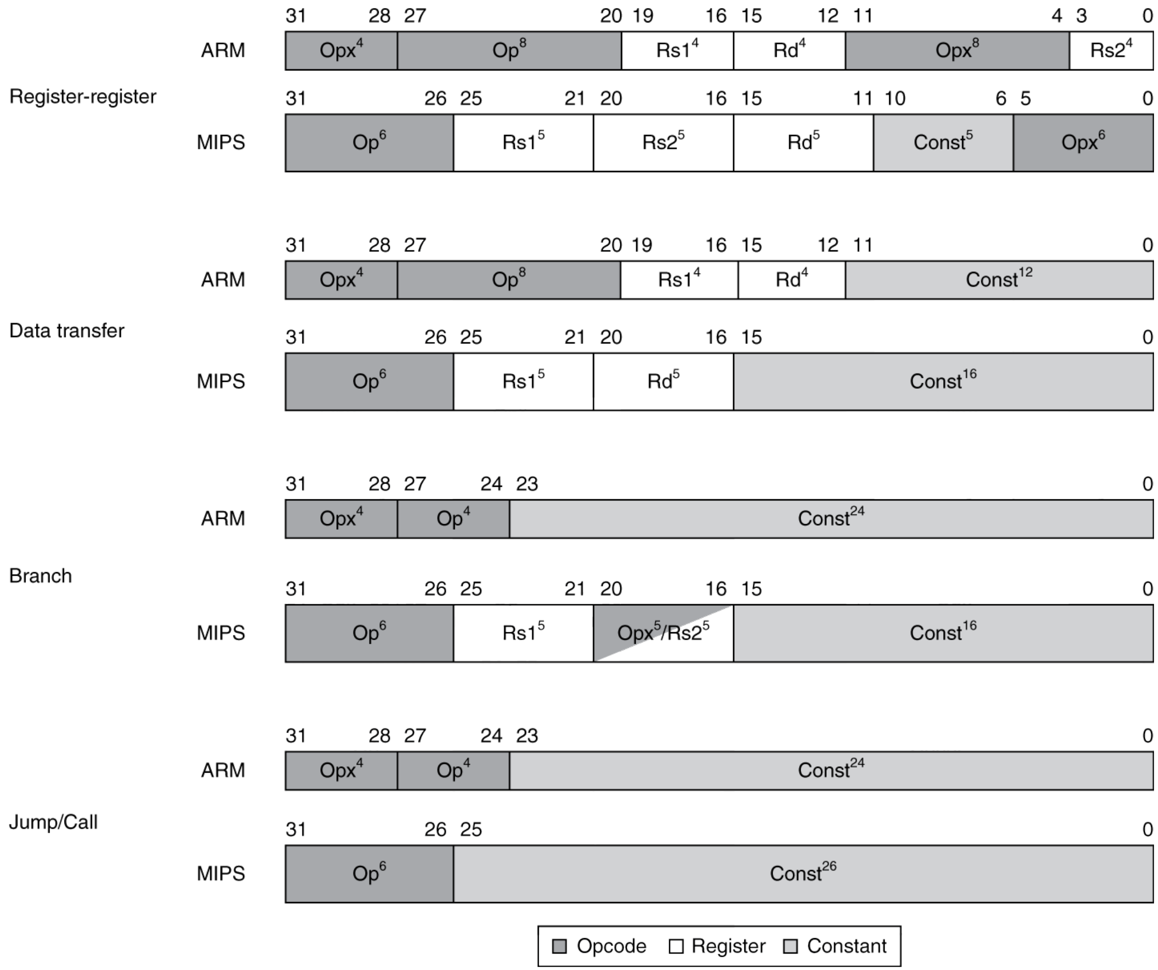
- ARM: the most popular embedded core
- Similar basic set of instructions to MIPS

	ARM	MIPS
Date announced	1985	1985
Instruction size	32 bits	32 bits
Address space	32-bit flat	32-bit flat
Data alignment	Aligned	Aligned
Data addressing modes	9	3
Registers	15 × 32-bit	31 × 32-bit
Input/output	Memory mapped	Memory mapped

Compare and Branch in ARM

- Uses condition codes for result of an arithmetic/logical instruction
  - Negative, zero, carry, overflow
  - Compare instructions to set condition codes without keeping the result
- Each instruction can be conditional
  - Top 4 bits of instruction word: condition value
  - Can avoid branches over single instructions

Instruction Encoding



The Intel x86 ISA

- Evolution with backward compatibility
  - 8080 (1974): 8-bit microprocessor
    - Accumulator, plus 3 index-register pairs
  - 8086 (1978): 16-bit extension to 8080
    - Complex instruction set (CISC)
  - 8087 (1980): floating-point coprocessor
    - Adds FP instructions and register stack
  - 80286 (1982): 24-bit addresses, MMU
    - Segmented memory mapping and protection
  - 80386 (1985): 32-bit extension (now IA-32)
    - Additional addressing modes and operations
    - Paged memory mapping as well as segments

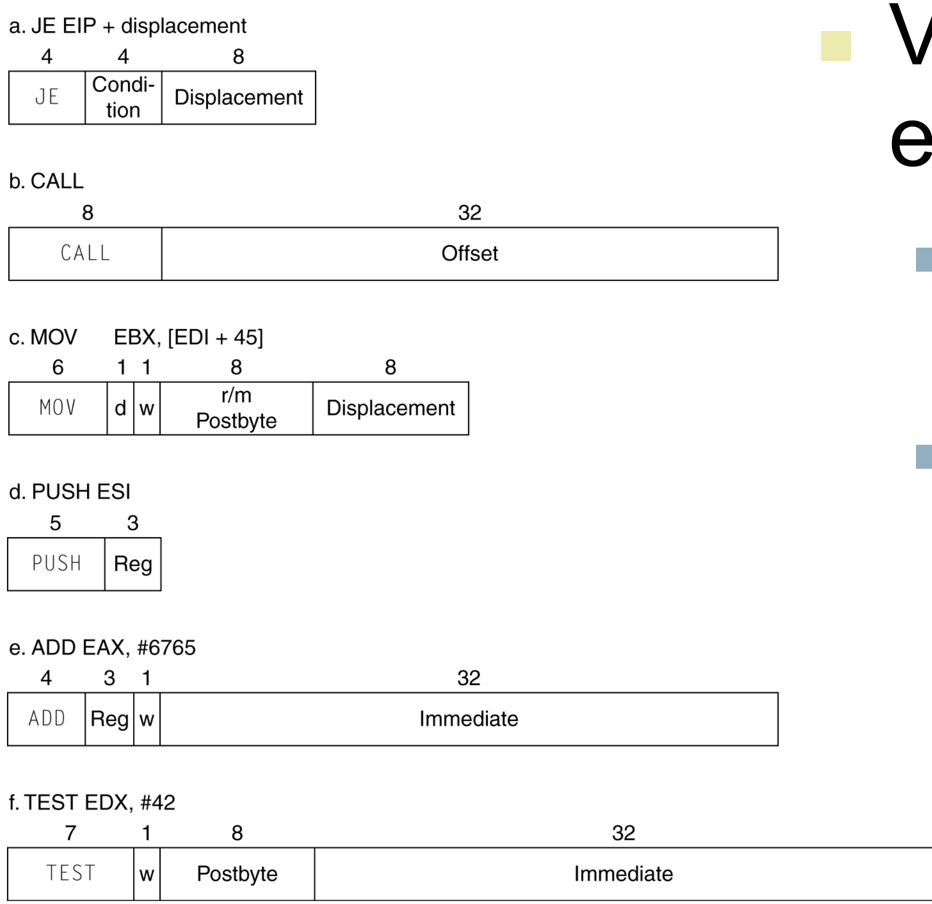
The Intel x86 ISA

- Further evolution…
  - i486 (1989): pipelined, on-chip caches and FPU
    - Compatible competitors: AMD, Cyrix, …
  - Pentium (1993): superscalar, 64-bit datapath
    - Later versions added MMX (Multi-Media eXtension) instructions
    - The infamous FDIV bug
  - Pentium Pro (1995), Pentium II (1997)
    - New microarchitecture (see Colwell, *The Pentium Chronicles*)
  - Pentium III (1999)
    - Added SSE (Streaming SIMD Extensions) and associated registers
  - Pentium 4 (2001)
    - New microarchitecture
    - Added SSE2 instructions

The Intel x86 ISA

- And further…
  - AMD64 (2003): extended architecture to 64 bits
  - EM64T – Extended Memory 64 Technology (2004)
    - AMD64 adopted by Intel (with refinements)
    - Added SSE3 instructions
  - Intel Core (2006)
    - Added SSE4 instructions, virtual machine support
  - AMD64 (announced 2007): SSE5 instructions
    - Intel declined to follow, instead…
  - Advanced Vector Extension (announced 2008)
    - Longer SSE registers, more instructions
- If Intel didn't extend with compatibility, its competitors would!
  - Technical elegance ≠ market success

x86 Instruction Encoding



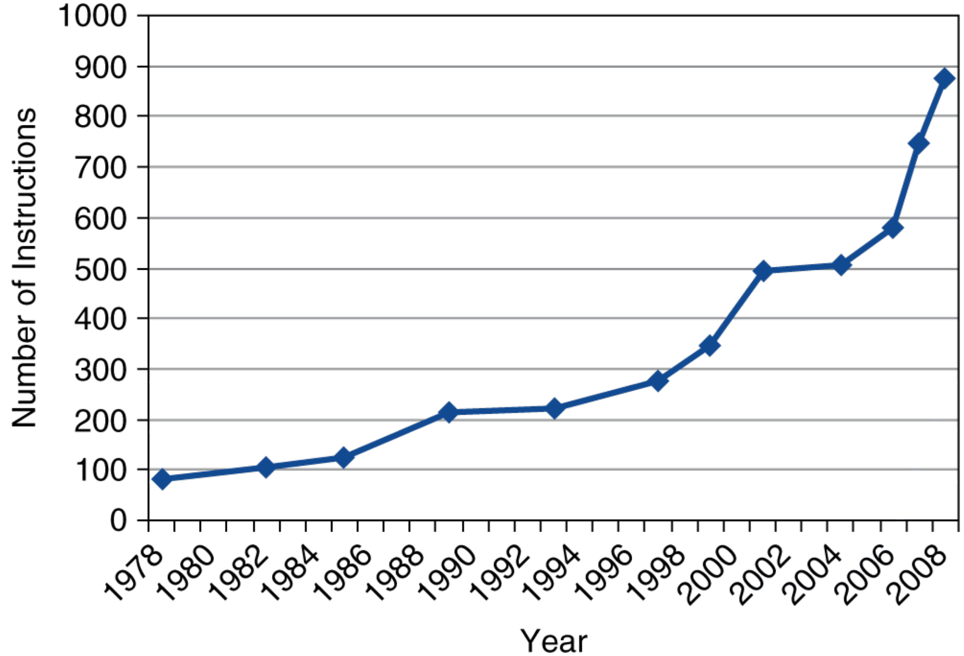
- Variable length encoding
  - Postfix bytes specify addressing mode
  - Prefix bytes modify operation
    - Operand length, repetition, locking, …

Fallacies

- Powerful instruction ⇒ higher performance
  - Fewer instructions required
  - But complex instructions are hard to implement
    - May slow down all instructions, including simple ones
  - Compilers are good at making fast code from simple instructions
- Use assembly code for high performance
  - But modern compilers are better at dealing with modern processors
  - More lines of code ⇒ more errors and less productivity

Fallacies

- Backward compatibility ⇒ instruction set doesn't change
  - But they do accrete more instructions



x86 instruction set

Concluding Remarks

- Measure MIPS instruction executions in benchmark programs
  - Consider making the common case fast
  - Consider compromises

Instruction class	MIPS examples	SPEC2006 Int	SPEC2006 FP
Arithmetic	`add, sub, addi`	16%	48%
Data transfer	`lw, sw, lb, lbu, lh, lhu, sb, lui`	35%	36%
Logical	`and, or, nor, andi, ori, sll, srl`	12%	4%
Cond. Branch	`beq, bne, slt, slti, sltiu`	34%	8%
Jump	`j, jr, jal`	2%	0%

Chapter 3

Arithmetic for Computers

Arithmetic for Computers

- Operations on integers
  - Addition and subtraction
  - Multiplication and division
  - Dealing with overflow
- Floating-point real numbers
  - Representation and operations

Integer Addition

- Example: 7 + 6

		(0)	(0)	(1)	(1)	(0)	(Carries)
. . .		0	0	0	1	1	1
. . .		0	0	0	1	1	0
. . .	(0)	0	(0) 0	(0) 1	(1) 1	(1) 0	(0) 1

- Overflow if result out of range
  - Adding +ve and –ve operands, no overflow
  - Adding two +ve operands
    - Overflow if result sign is 1
  - Adding two –ve operands
    - Overflow if result sign is 0

Integer Subtraction

- Add negation of second operand
- Example: 7 – 6 = 7 + (–6)

+7:	0000 0000 … 0000 0111
–6:	1111 1111 … 1111 1010
+1:	0000 0000 … 0000 0001

- Overflow if result out of range
  - Subtracting two +ve or two –ve operands, no overflow
  - Subtracting +ve from –ve operand
    - Overflow if result sign is 0
  - Subtracting –ve from +ve operand
    - Overflow if result sign is 1

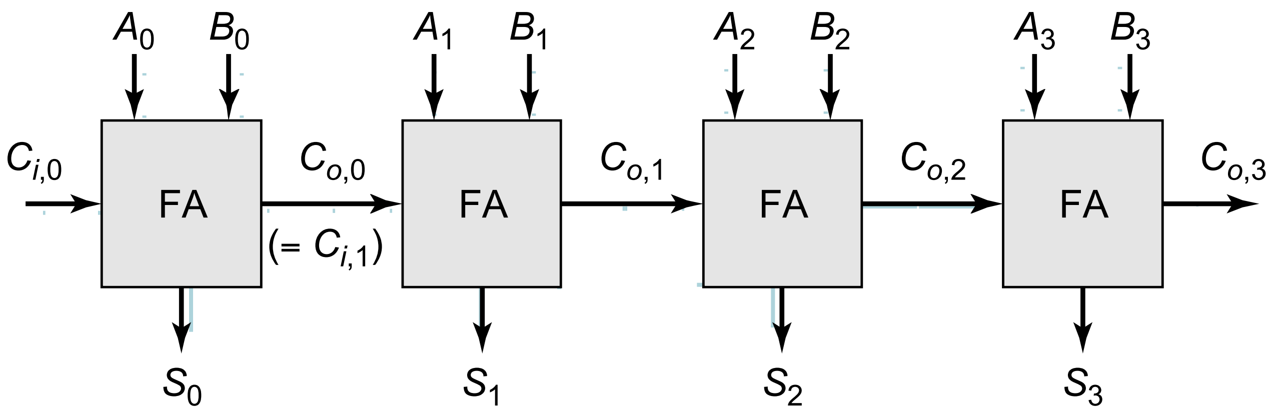
Dealing with Overflow

- Some languages (e.g., C) ignore overflow
  - Use MIPS `addu`, `addui`, `subu` instructions
- Other languages (e.g., Ada, Fortran) require raising an exception
  - Use MIPS `add`, `addi`, `sub` instructions
  - On overflow, invoke exception handler
    - Save PC in exception program counter (EPC) register
    - Jump to predefined handler address
    - `mfc0` (move from coprocessor reg) instruction can retrieve EPC value, to return after corrective action

Other Adders

- BASICS of ADDING LOGIC
  - Carry-out
  - Sum Generation
- Ripple Add
- Carry Bypass
- Carry Select
- Carry Lookahead

The Ripple-Carry Adder



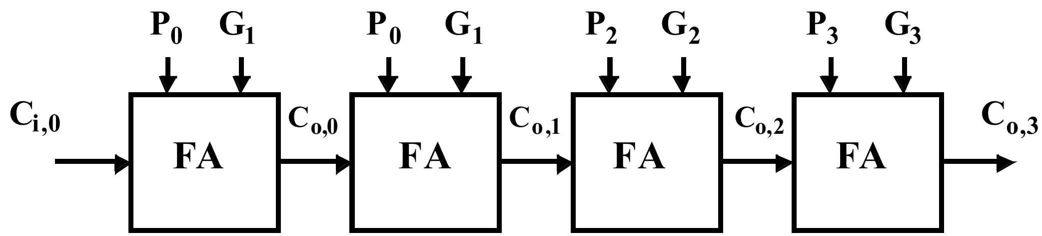
Worst case delay linear with the number of bits

$$t_d = O(N)$$

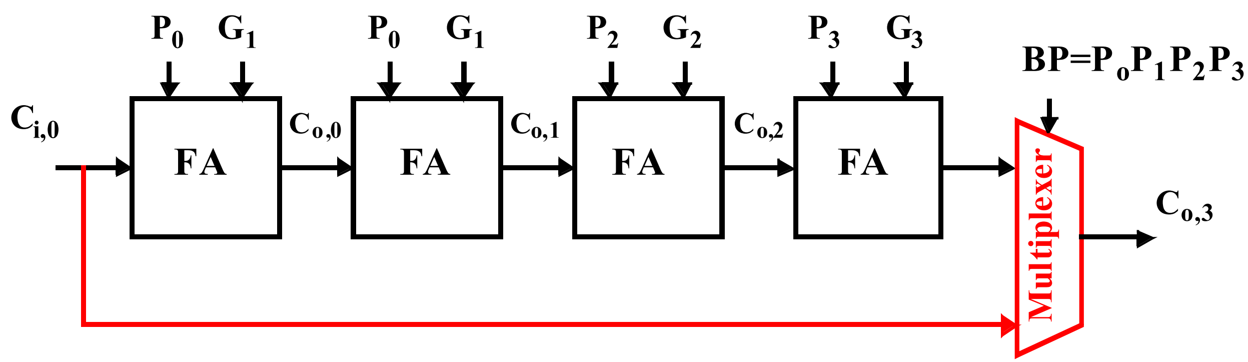
$$t_{adder} = (N-1)t_{carry} + t_{sum}$$

Goal: Make the fastest possible carry path circuit

Carry-Bypass Adder

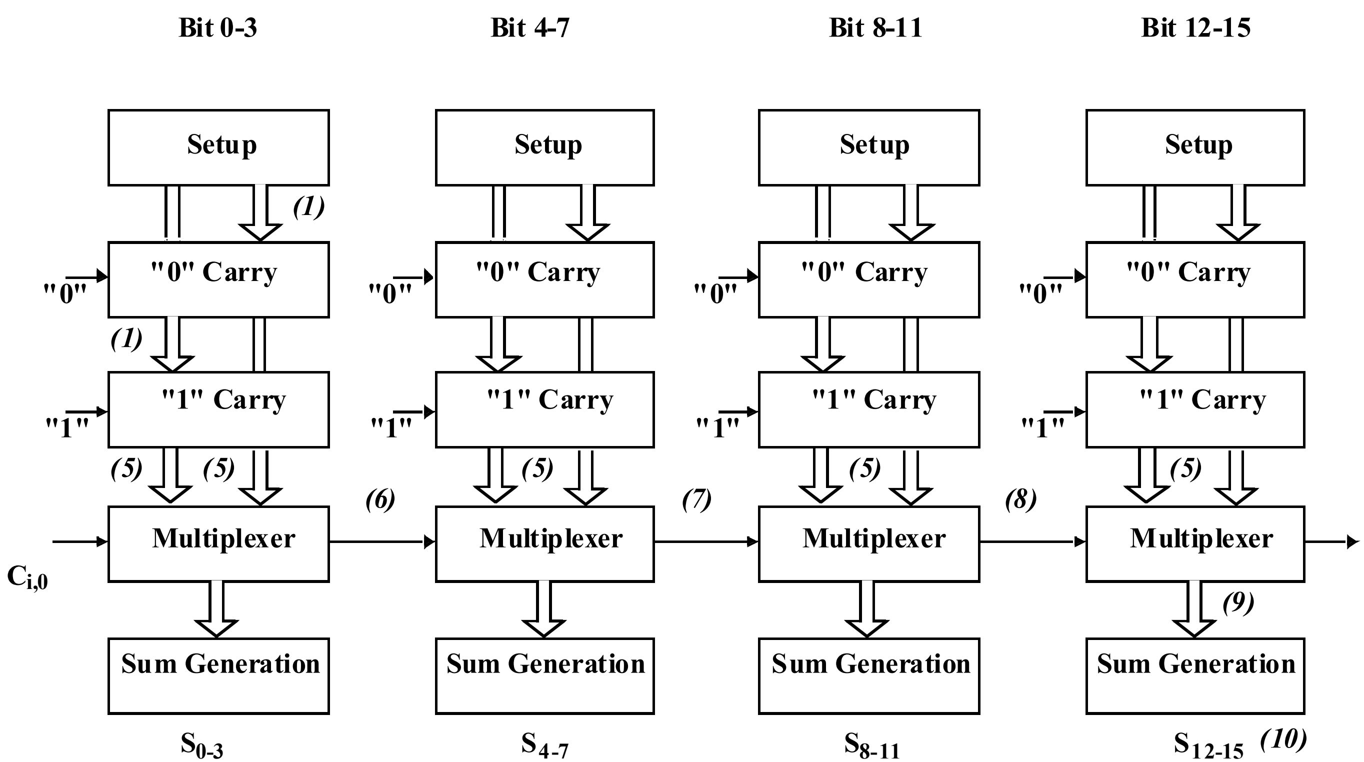


Also called Carry-Skip



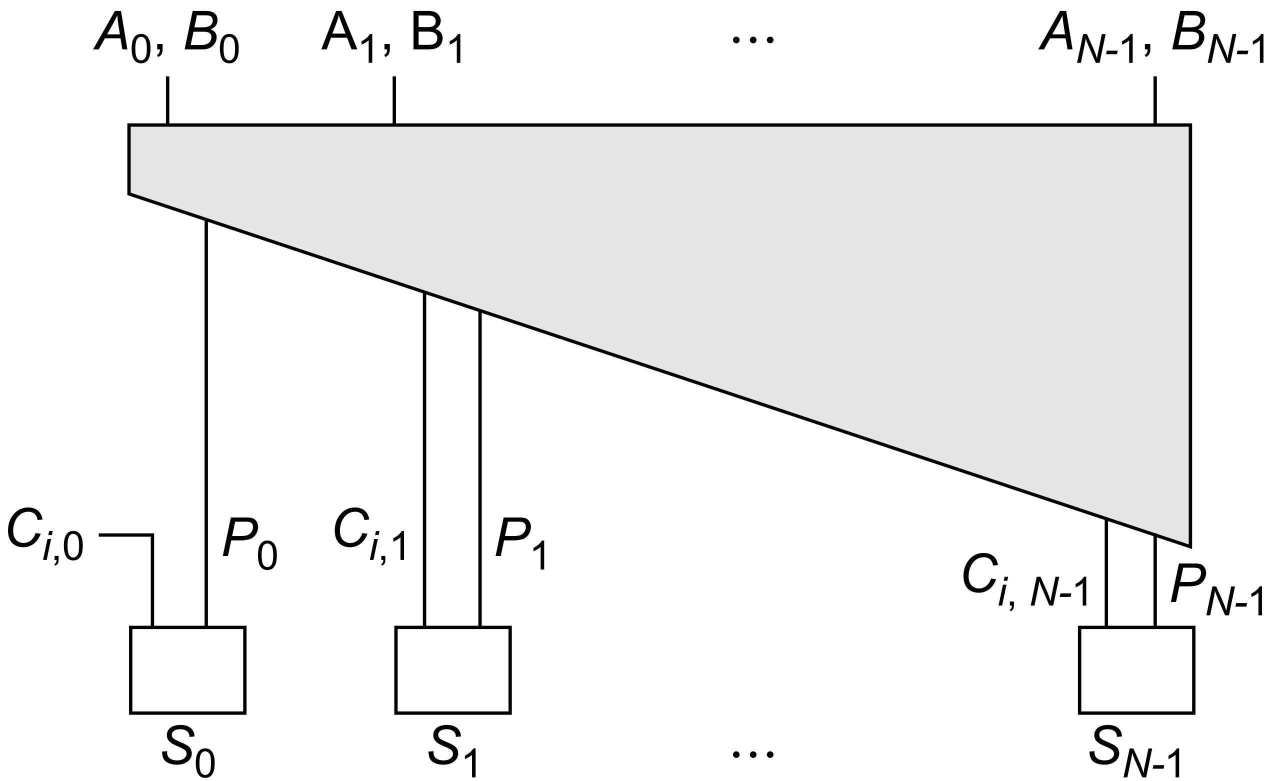
Idea: If (P0 and P1 and P2 and P3 = 1) then $C_{o3} = C_0$, else "kill" or "generate".

Linear Carry Select



$$t_{add} = t_{setup} + \left(\frac{N}{M}\right) t_{carry} + M t_{mux} + t_{sum}$$

LookAhead - Basic Idea



$$C_{o,k} = f(A_k, B_k, C_{o,k-1}) = G_k + P_k C_{o,k-1}$$

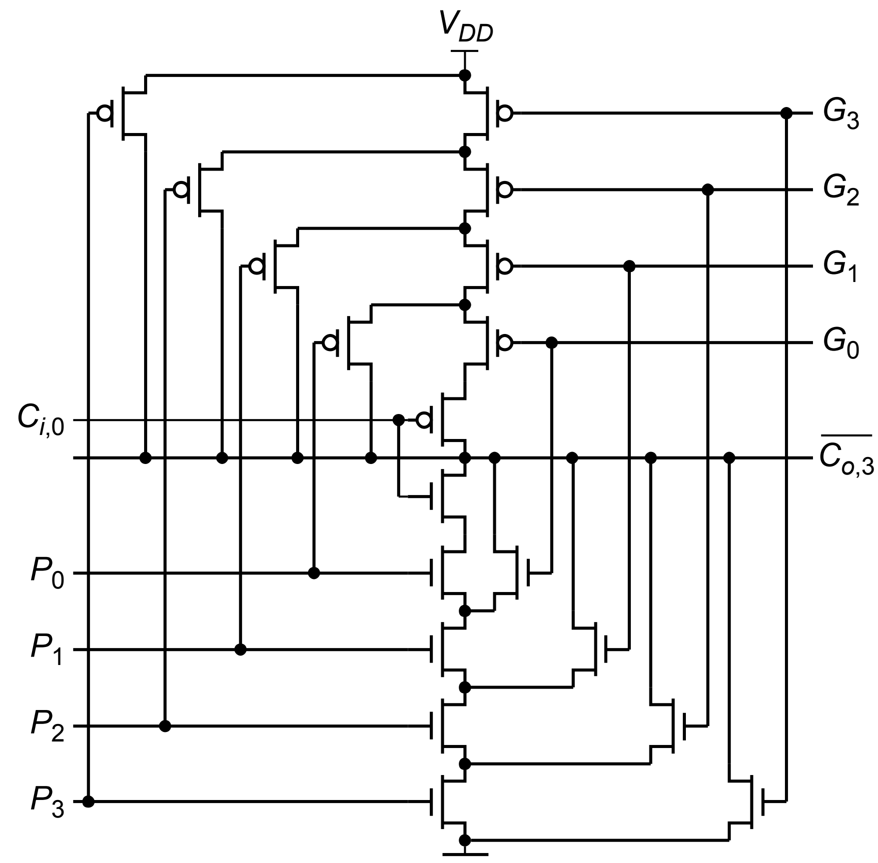
Look-Ahead: Topology

Expanding Lookahead equations:

$$C_{o,k} = G_k + P_k(G_{k-1} + P_{k-1}C_{o,k-2})$$

All the way:

$$C_{o,k} = G_k + P_k(G_{k-1} + P_{k-1}(\cdots + P_1(G_0 + P_0 C_{i,0})))$$



Carry Lookahead Trees

$$C_{o,0} = G_0 + P_0 C_{i,0}$$

$$C_{o,1} = G_1 + P_1 G_0 + P_1 P_0 C_{i,0}$$

$$C_{o,2} = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_{i,0}$$

$$= (G_2 + P_2 G_1) + (P_2 P_1)(G_0 + P_0 C_{i,0}) = G_{2:1} + P_{2:1} C_{o,0}$$

Can continue building the tree hierarchically.

Multiplication

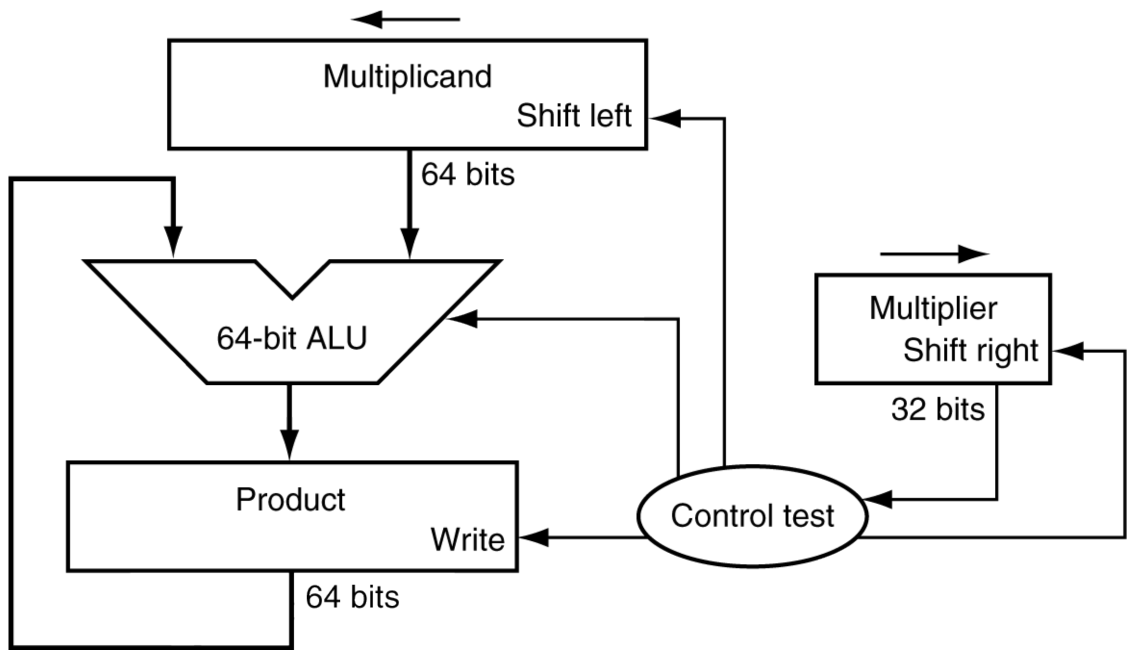
- Start with long-multiplication approach

multiplicand

multiplier

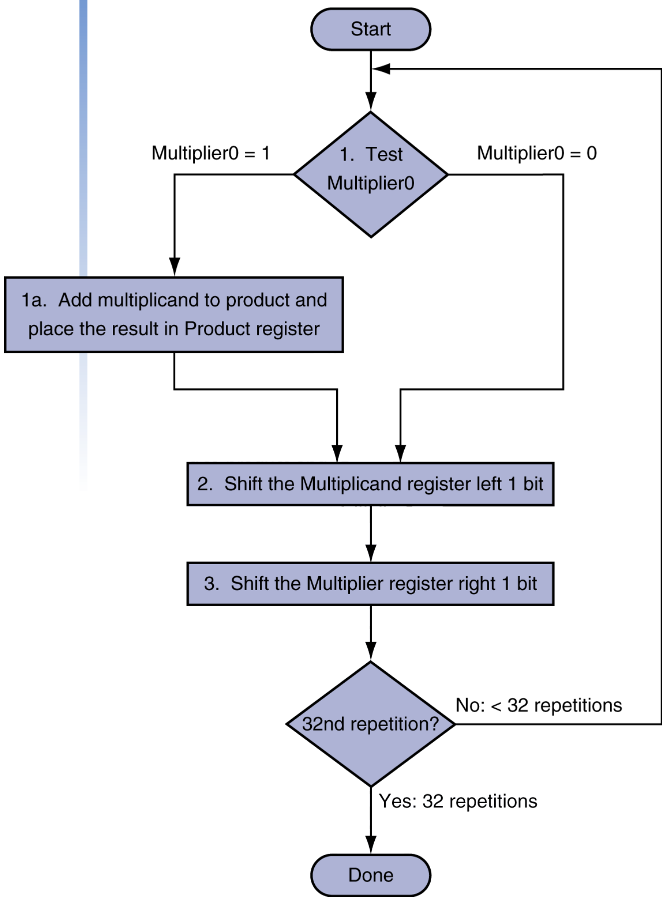
product

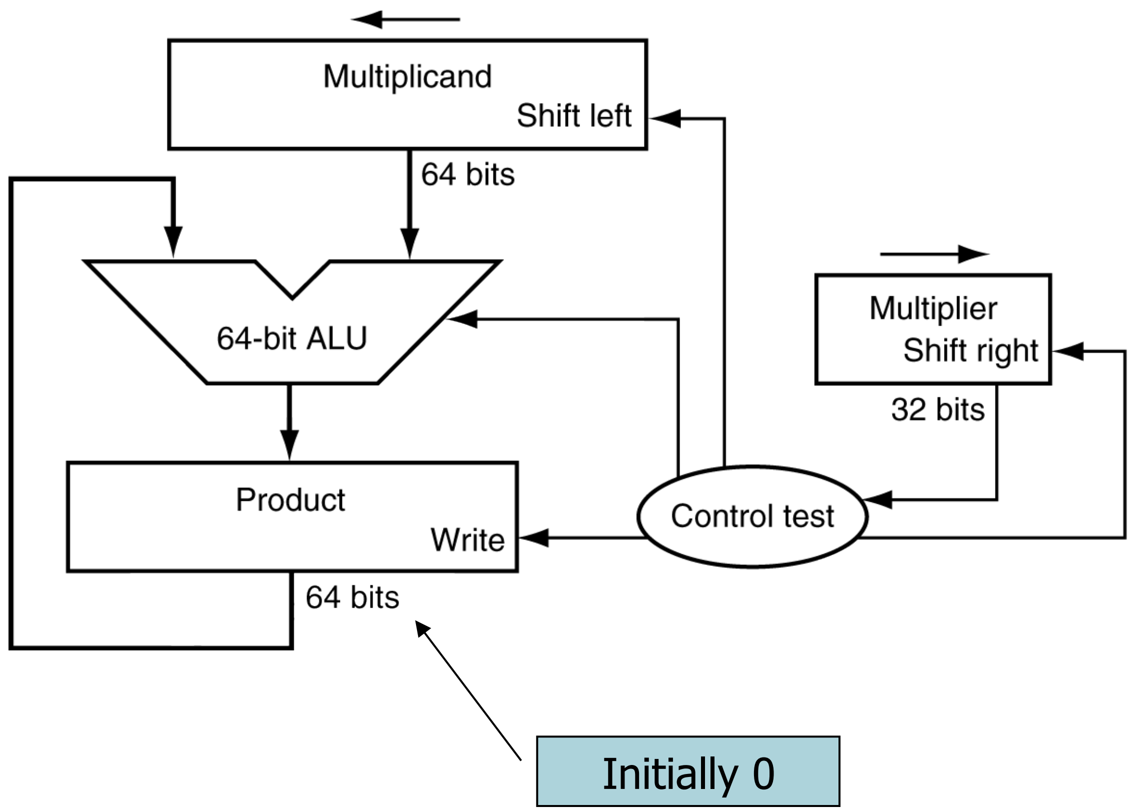
```
          1000
    ×     1001
    ----------
          1000
         0000
        0000
       1000
    ----------
       1001000
```



Length of product is the sum of operand lengths

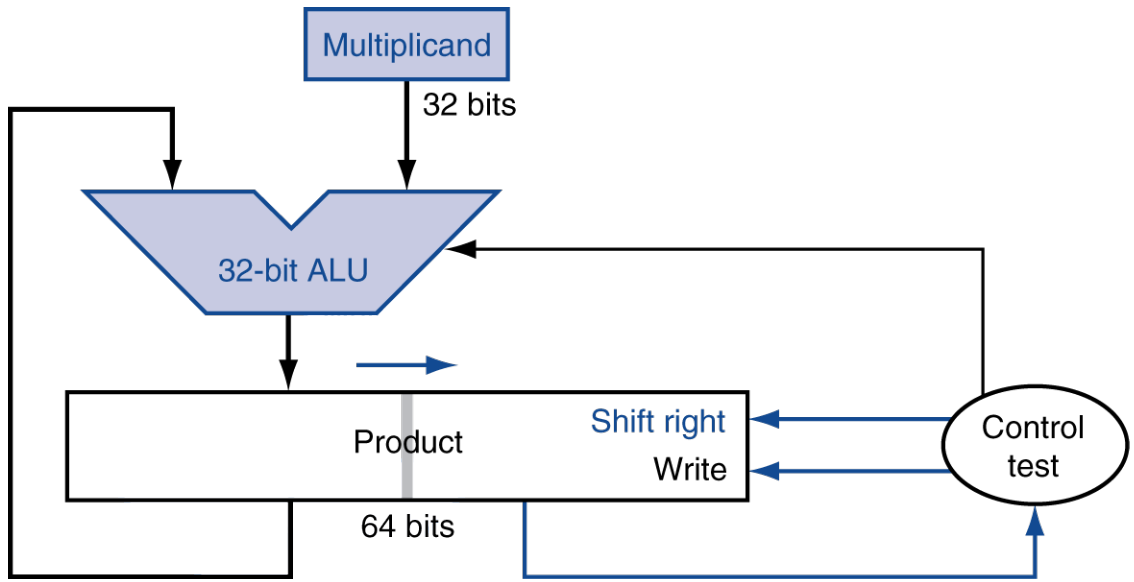
Multiplication Hardware





Optimized Multiplier (ignore)

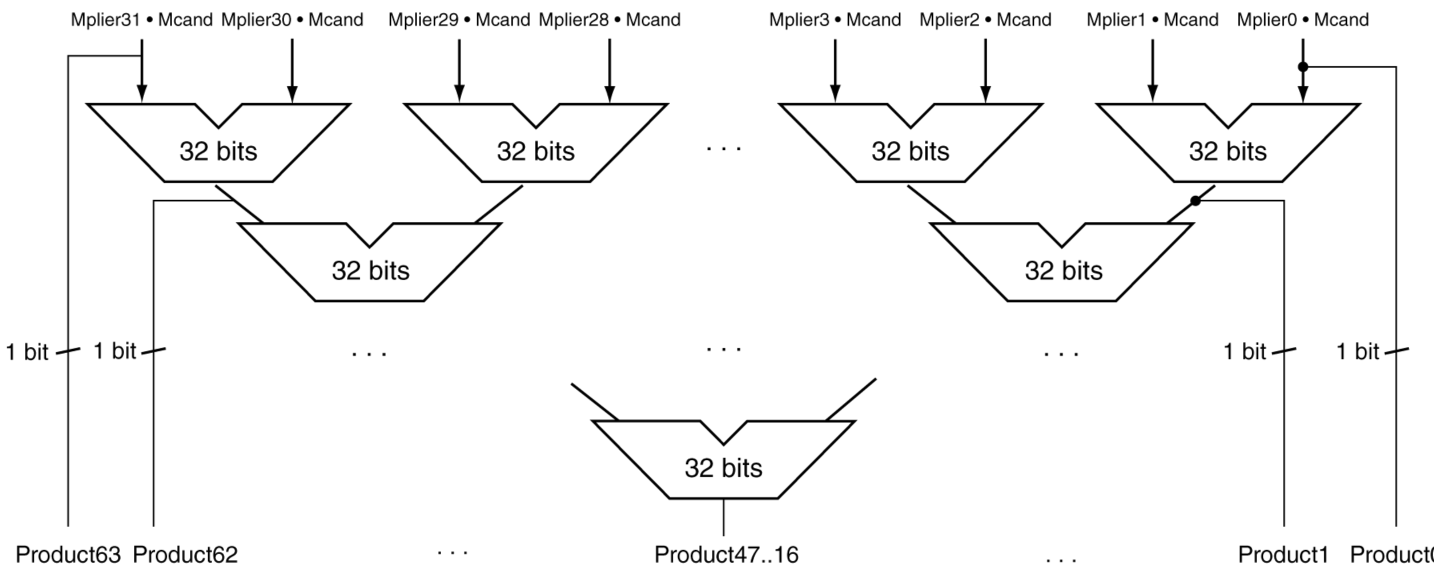
- Perform steps in parallel: add/shift



- One cycle per partial-product addition
  - That's ok, if frequency of multiplications is low

Faster Multiplier

- Uses multiple adders
  - Cost/performance tradeoff

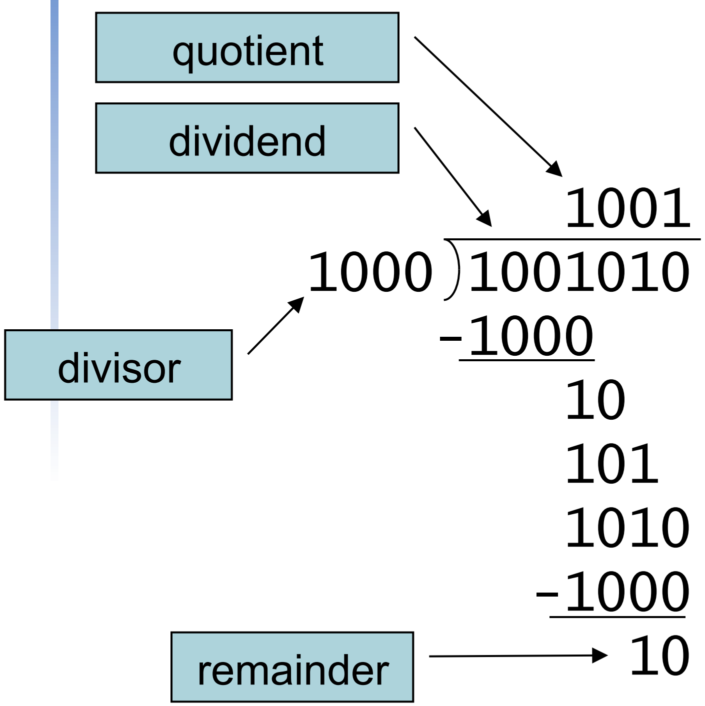


- Can be pipelined
  - Several multiplication performed in parallel

MIPS Multiplication

- Two 32-bit registers for product
  - HI: most-significant 32 bits
  - LO: least-significant 32-bits
- Instructions
  - `mult rs, rt  /  multu rs, rt`
    - 64-bit product in HI/LO
  - `mfhi rd  /  mflo rd`
    - Move from HI/LO to rd
    - Can test HI value to see if product overflows 32 bits
  - `mul rd, rs, rt`
    - Least-significant 32 bits of product –> rd

Division (IGNORE THIS)



n-bit operands yield *n*-bit quotient and remainder

- Check for 0 divisor
- Long division approach
  - If divisor ≤ dividend bits
    - 1 bit in quotient, subtract
  - Otherwise
    - 0 bit in quotient, bring down next dividend bit
- Restoring division
  - Do the subtract, and if remainder goes < 0, add divisor back
- Signed division
  - Divide using absolute values
  - Adjust sign of quotient and remainder as required

Floating Point

- Representation for non-integral numbers
  - Including very small and very large numbers
- Like scientific notation
  - -2.34×10^{56} — normalized
  - $+0.002 \times 10^{-4}$ — not normalized
  - $+987.02 \times 10^{9}$ — not normalized
- In binary
  - $\pm 1.xxxxxxx_2 \times 2^{yyyy}$
- Types `float` and `double` in C

Floating Point Standard

- Defined by IEEE Std 754-1985
- Developed in response to divergence of representations
  - Portability issues for scientific code
- Now almost universally adopted
- Two representations
  - Single precision (32-bit)
  - Double precision (64-bit)

IEEE Floating-Point Format

	single: 8 bits double: 11 bits	single: 23 bits double: 52 bits
S	Exponent	Fraction

$$x = (-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$$

- S: sign bit (0 ⇒ non-negative, 1 ⇒ negative)
- Normalize significand: 1.0 ≤ |significand| < 2.0
  - Always has a leading pre-binary-point 1 bit, so no need to represent it explicitly (hidden bit)
  - Significand is Fraction with the “1.” restored
- Exponent: excess representation: actual exponent + Bias
  - Ensures exponent is unsigned
  - Single: Bias = 127; Double: Bias = 1203

Single-Precision Range

- Exponents 00000000 and 11111111 reserved
- Smallest value
  - Exponent: 00000001
    $\Rightarrow$ actual exponent = 1 – 127 = –126
  - Fraction: 000…00 $\Rightarrow$ significand = 1.0
  - $\pm 1.0 \times 2^{-126} \approx \pm 1.2 \times 10^{-38}$
- Largest value
  - exponent: 11111110
    $\Rightarrow$ actual exponent = 254 – 127 = +127
  - Fraction: 111…11 $\Rightarrow$ significand $\approx$ 2.0
  - $\pm 2.0 \times 2^{+127} \approx \pm 3.4 \times 10^{+38}$

Double-Precision Range

- Exponents 0000…00 and 1111…11 reserved
- Smallest value
  - Exponent: 00000000001
    $\Rightarrow$ actual exponent = 1 – 1023 = –1022
  - Fraction: 000…00 $\Rightarrow$ significand = 1.0
  - $\pm 1.0 \times 2^{-1022} \approx \pm 2.2 \times 10^{-308}$
- Largest value
  - Exponent: 11111111110
    $\Rightarrow$ actual exponent = 2046 – 1023 = +1023
  - Fraction: 111…11 $\Rightarrow$ significand ≈ 2.0
  - $\pm 2.0 \times 2^{+1023} \approx \pm 1.8 \times 10^{+308}$

Floating-Point Precision

- Relative precision
  - all fraction bits are significant
  - Single: approx 2^{-23}
    - Equivalent to $23 \times \log_{10}2 \approx 23 \times 0.3 \approx 6$ decimal digits of precision
  - Double: approx 2^{-52}
    - Equivalent to $52 \times \log_{10}2 \approx 52 \times 0.3 \approx 16$ decimal digits of precision

Floating-Point Example

- Represent –0.75
  - $-0.75 = (-1)^1 \times 1.1_2 \times 2^{-1}$
  - S = 1
  - Fraction = $1000\ldots00_2$
  - Exponent = –1 + Bias
    - Single: –1 + 127 = 126 = 01111110_2
    - Double: –1 + 1023 = 1022 = 01111111110_2
- Single: 1011111101000…00
- Double: 1011111111101000…00

Floating-Point Example

- What number is represented by the single-precision float

  11000000101000…00

Denormal Numbers

- Exponent = 000...0 ⇒ hidden bit is 0

$$x = (-1)^S \times (0 + \text{Fraction}) \times 2^{-\text{Bias}}$$

- Smaller than normal numbers
  - allow for gradual underflow, with diminishing precision
- Denormal with fraction = 000...0

$$x = (-1)^S \times (0 + 0) \times 2^{-\text{Bias}} = \pm 0.0$$

Two representations of 0.0!

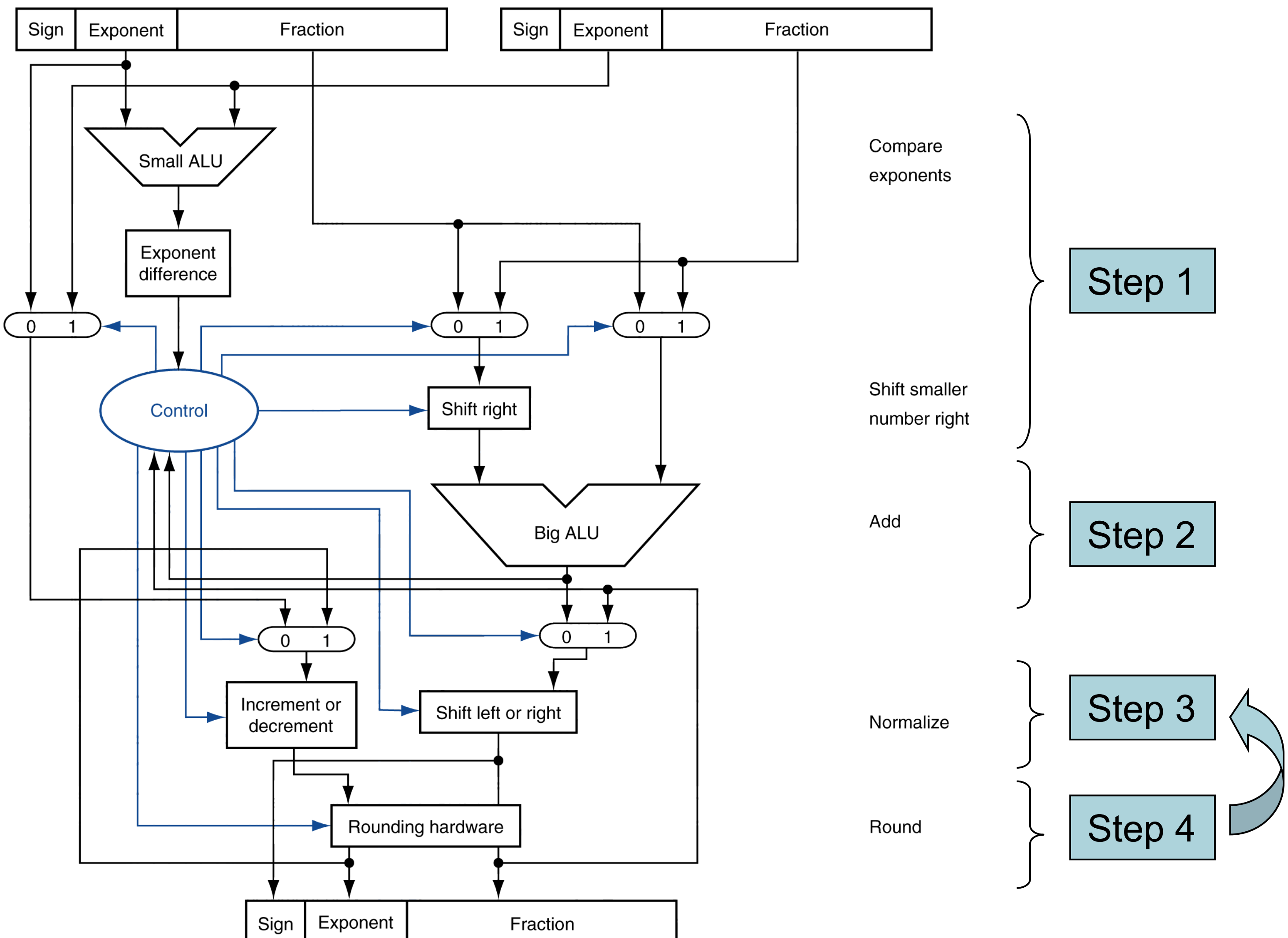
Infinities and NaNs

- Exponent = 111...1, Fraction = 000...0
  - ±Infinity
  - Can be used in subsequent calculations, avoiding need for overflow check
- Exponent = 111...1, Fraction ≠ 000...0
  - Not-a-Number (NaN)
  - Indicates illegal or undefined result
    - e.g., 0.0 / 0.0
  - Can be used in subsequent calculations

Floating-Point Addition (IGNORE)

- Consider a 4-digit decimal example
  - $9.999 \times 10^1 + 1.610 \times 10^{-1}$
- 1. Align decimal points
  - Shift number with smaller exponent
  - $9.999 \times 10^1 + 0.016 \times 10^1$
- 2. Add significands
  - $9.999 \times 10^1 + 0.016 \times 10^1 = 10.015 \times 10^1$
- 3. Normalize result & check for over/underflow
  - 1.0015×10^2
- 4. Round and renormalize if necessary
  - 1.002×10^2

FP Adder Hardware (ignore)



Floating-Point Multiplication (ignore)

- Consider a 4-digit decimal example
  - $1.110 \times 10^{10} \times 9.200 \times 10^{-5}$
- 1. Add exponents
  - For biased exponents, subtract bias from sum
  - New exponent = $10 + -5 = 5$
- 2. Multiply significands
  - $1.110 \times 9.200 = 10.212 \Rightarrow 10.212 \times 10^5$
- 3. Normalize result & check for over/underflow
  - 1.0212×10^6
- 4. Round and renormalize if necessary
  - 1.021×10^6
- 5. Determine sign of result from signs of operands
  - $+1.021 \times 10^6$

FP Arithmetic Hardware (ignore)

- FP multiplier is of similar complexity to FP adder
  - But uses a multiplier for significands instead of an adder
- FP arithmetic hardware usually does
  - Addition, subtraction, multiplication, division, reciprocal, square-root
  - FP ↔ integer conversion
- Operations usually takes several cycles
  - Can be pipelined

FP Instructions in MIPS

- Single-precision arithmetic
  - `add.s`, `sub.s`, `mul.s`, div.s
    - e.g., `add.s $f0, $f1, $f6`
- Double-precision arithmetic
  - `add.d`, `sub.d`, `mul.d`, `div.d`
    - e.g., `mul.d $f4, $f4, $f6`
- Single- and double-precision comparison
  - `c.`*`xx`*`.s`, `c.`*`xx`*`.d` (*xx* is `eq`, `lt`, `le`, …)
  - Sets or clears FP condition-code bit
    - e.g. `c.lt.s $f3, $f4`
- Branch on FP condition code true or false
  - `bc1t`, `bc1f`
    - e.g., `bc1t TargetLabel`

FP Example: °F to °C

- C code:

```
float f2c (float fahr) {
  return ((5.0/9.0)*(fahr - 32.0));
}
```

  - fahr in $f12, result in $f0, literals in global memory space
- Compiled MIPS code:

```
f2c: lwc1   $f16, const5($gp)
     lwc2   $f18, const9($gp)
     div.s  $f16, $f16, $f18
     lwc1   $f18, const32($gp)
     sub.s  $f18, $f12, $f18
     mul.s  $f0,  $f16, $f18
     jr     $ra
```

Accurate Arithmetic

- IEEE Std 754 specifies additional rounding control
  - Extra bits of precision (guard, round, sticky)
  - Choice of rounding modes
  - Allows programmer to fine-tune numerical behavior of a computation
- Not all FP units implement all options
  - Most programming languages and FP libraries just use defaults
- Trade-off between hardware complexity, performance, and market requirements

x86 FP Architecture (ignore)

- Originally based on 8087 FP coprocessor
  - 8 × 80-bit extended-precision registers
  - Used as a push-down stack
  - Registers indexed from TOS: ST(0), ST(1), …
- FP values are 32-bit or 64 in memory
  - Converted on load/store of memory operand
  - Integer operands can also be converted on load/store
- Very difficult to generate and optimize code
  - Result: poor FP performance

x86 FP Instructions (ignore)

Data transfer	Arithmetic	Compare	Transcendental
`FILD mem/ST(i)`	`FIADDP mem/ST(i)`	`FICOMP`	`FPATAN`
`FISTP mem/ST(i)`	`FISUBRP mem/ST(i)`	`FIUCOMP`	`F2XMI`
`FLDPI`	`FIMULP mem/ST(i)`	`FSTSW AX/mem`	`FCOS`
`FLD1`	`FIDIVRP mem/ST(i)`		`FPTAN`
`FLDZ`	`FSQRT`		`FPREM`
	`FABS`		`FPSIN`
	`FRNDINT`		`FYL2X`

- Optional variations
  - I: integer operand
  - P: pop operand from stack
  - R: reverse operand order
  - But not all combinations allowed

Who Cares About FP Accuracy?

- Important for scientific code
  - But for everyday consumer use?
    - “My bank balance is out by 0.0002¢!” ☹
- The Intel Pentium FDIV bug
  - The market expects accuracy
  - See Colwell, *The Pentium Chronicles*

Concluding Remarks

- ISAs support arithmetic
  - Signed and unsigned integers
  - Floating-point approximation to reals
- Bounded range and precision
  - Operations can overflow and underflow
- MIPS ISA
  - Core instructions: 54 most frequently used
    - 100% of SPECINT, 97% of SPECFP
  - Other instructions: less frequent