SSN COLLEGE OF ENGINEERING
RECORD SHEET

Sheet No.....

OPEN BOOK ASSIGNMENT

UMA 1478 Probability and Statistics

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1)

S_1	S_2	S_3	S_4
15	12	14	10
17	10	9	14
14	13	7	13
11	17	10	15
	14	8	12
		7	

H_0: There is no significant difference between the samples.

X_1	X_1^2	X_2	X_2^2	X_3	X_3^2	X_4	X_4^2
15	225	12	144	14	196	10	100
17	289	10	100	9	81	14	196
14	196	13	169	7	49	13	169
11	121	17	289	10	100	15	225
		14	196	8	64	12	144
				7	49		
57	831	66	898	55	539	64	834

K = no. of samples = 4 , No. of observations = 20 = N

$$T = \sum_{i=1}^{4} x_i^{\circ} = 57 + 66 + 55 + 64$$
$$= 242$$

correction factor $= \frac{T^2}{N} = \frac{(242)^2}{20} = 2928.2$

$$TSS = \sum x_i^{\circ 2} - \frac{T^2}{N}$$

$$= 3102 - 2928.2$$

$$= 173.8$$

$$SSC = \frac{(\sum x_1)^2}{n_1} + \frac{(\sum x_2)^2}{n_2} + \frac{(\sum x_3)^2}{n_3} + \frac{(\sum x_4)^2}{n_4} - \frac{T^2}{N}$$

$$= \frac{57^2}{4} + \frac{66^2}{5} + \frac{55^2}{6} + \frac{64^2}{5} - 2928.2$$

$$= 812.25 + 871.2 + 504.166 + 819.2 - 2928.2$$

$$= 78.616$$

$$SSE = TSS - SSC$$
$$= 173.8 - 78.616$$
$$= 95.184$$

$$MSC = \frac{SSC}{K-1} = \frac{78.616}{3} = 26.205$$

$$MSE = \frac{SSE}{N-K} = \frac{95.184}{16} = 5.949$$

ANOVA Table.

Source of Variation	Sum of Squares	Degree of freedom	Mean of Squares	Fraction.
Between Samples	SSC = 78.61	$\gamma_1 = 3$	MSC = ~~26.536~~ 26.205	F_c = ~~1.282~~ ~~4.4094~~ 4.4049
within Samples	SSE = ~~383.42~~ 95.184	$\gamma_2 = 16$	MSE = ~~20.7~~ 5.949	

$F_T = 3/16 = 5.29$ in 1% table.

$F_T > F_C$

$\therefore H_0$ is true.

2.

Doctor \ Treatment	1	2	3	4
A	10	14	19	20
B	11	15	17	21
C	9	12	16	19
D	8	13	17	20

H_0 - No significant difference between treatments

H_1 - No significant difference between doctors

Treatment

	Doctor	1	2	3	4					
		x_1	x_2	x_3	x_4	x_1^2	x_2^2	x_3^2	x_4^2	
y_1	A	10	14	19	20	100	196	361	400	$(\Sigma y_1 = 63)$
y_2	B	11	15	17	21	121	225	289	441	$(\Sigma y_2 = 64)$
y_3	C	9	12	16	19	81	144	256	361	$(\Sigma y_3 = 56)$
y_4	D	8	13	17	20	64	169	289	400	$(\Sigma y_4 = 58)$
		38	54	69	80	366	734	1195	1602	

$N = 4+4+4+4 = 16$

$K = 4$

$T = \Sigma x_i = 38 + 54 + 69 + 80 = 241$

Correc. factor $= \frac{T^2}{N} = \frac{(241)^2}{16} = 3630.0625$

$TSS = \Sigma x_i^2 - \frac{T^2}{N}$

$= 1602 + 1195 + 734 + 366 - 3630.0625$

$= 266.94$

$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} + \frac{(\Sigma x_4)^2}{n_4} - \frac{T^2}{N}$

$= \frac{(38)^2}{4} + \frac{(54)^2}{4} + \frac{(69)^2}{4} + \frac{(80)^2}{4} - 3630.0625$

$= 250.1875$

$$SSR = \frac{(\Sigma y_1)^2}{n_1} + \frac{(\Sigma y_2)^2}{n_2} + \frac{(\Sigma y_3)^2}{n_3} + \frac{(\Sigma y_4)^2}{n_4} - \frac{T^2}{N}$$

$$= \frac{63^2}{4} + \frac{64^2}{4} + \frac{56^2}{4} + \frac{58^2}{4} - 3630.0625$$

$$= 11.1875$$

$$SSE = TSS - (SSC + SSR)$$

$$= 266.94 - (250.1875 + 11.1875)$$

$$= 5.565$$

$$MSC = \frac{SSC}{C-1} = \frac{250.1875}{3} = 85.39583$$

$$MSR = \frac{SSR}{R-1} = \frac{11.1875}{3} = 3.72916$$

$$MSE = \frac{SSE}{(r-1)(c-1)} = \frac{5.565}{3(3)} = 0.61833$$

$$F_C = \frac{MSC}{MSE} = \frac{85.39583}{0.61833} = 138.1072$$

$$F_R = \frac{MSR}{MSE} = \frac{3.72916}{0.61833} = 5.4548$$

ANOVA Table

Source of Variation	Sum of Squares	Degree of freedom	Mean of Squares	F ratio
Between columns	SSC = 250.19	γ_1 C-1 = 3	MSC = 85.396	F_C = 138.1072
Between rows	SSR = 11.19	γ_2 R-1 = 3	MSR = 3.73	F_R = 5.4548
Error or residual	SSE = 5.565	γ (R-1)(C-1) = 9	MSE = 0.6183	
Total	TSS = 266.94			

$F_{TC}(\gamma_1, \gamma)$ = 8.81 at 5%.

$F_C > F_{TC}$ ∴ H_0 is rejected. There is difference between treatments

$F_{TR}(\gamma_2, \gamma)$ = 8.81 at 5%.

$F_R < F_{TR}$ ∴ H_1 is accepted. There is no difference between doctors

3.

	Engine 1	Engine 2	Engine 3
Day 1	$B_1(16)$	$B_2(17)$	$B_3(20)$
Day 2	$B_2(16)$	$B_3(21)$	$B_1(15)$
Day 3	$B_3(15)$	$B_1(12)$	$B_2(13)$

H_0 - no difference between days

H_1 - no difference between engine

H_2 - no difference between burns

code by -16

	Engine 1	Engine 2	Engine 3	T_i
Day 1	$B_1(0)$	$B_2(1)$	$B_3(4)$	5
Day 2	$B_2(0)$	$B_3(5)$	$B_1(-1)$	4
Day 3	$B_3(-1)$	$B_1(-4)$	$B_2(-3)$	-8

	X_1	X_2	X_3	T_i	X_1^2	X_2^2	X_3^2
Y_1	0	-4	-1	-5	0	16	1
Y_2	0	1	-3	-2	0	1	9
Y_3	-1	5	4	8	1	25	16
ΣX_i	-1	2	0		1	42	26

$N = 9, \quad K = 3$

$T = \Sigma X_i = -1 + 2 + 0 = 1$

Correction factor $= \frac{T^2}{N} = \frac{1^2}{9} = \frac{1}{9} = 0.111$

TSS: $\Sigma x_i^2 - \frac{T^2}{N}$

$= 1+4+26 - \frac{1}{9}$

$= 66.888$

SSC: $\frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \frac{T^2}{N}$

$= \frac{(-1)^2}{3} + \frac{2^2}{3} + \frac{0^2}{3} - \frac{1}{9}$

$= \frac{1}{3} + \frac{4}{3} - \frac{1}{9}$

$= \frac{3+12-1}{9} = 14/9 = 1.555$

SST: $\frac{(\Sigma Y_1)^2}{n_1} + \frac{(\Sigma Y_2)^2}{n_2} + \frac{(\Sigma Y_3)^2}{n_3} - \frac{T^2}{N}$

$= \frac{(-5)^2}{3} + \frac{(-2)^2}{3} + \frac{8^2}{3} - \frac{1}{9}$

$= \frac{(25+4+64)3-1}{9}$

$= \frac{278}{9} = 30.888$

SSR: $\frac{(\Sigma T_1)^2}{n_1} + \frac{(\Sigma T_2)^2}{n_2} + \frac{(\Sigma T_3)^2}{n_3} - \frac{T^2}{N}$

$= \frac{5^2}{3} + \frac{4^2}{3} + \frac{(-8)^2}{3} - \frac{1}{9}$

$= \frac{(25+16+64)3-1}{9} = 34.888$

$SSE = TSS - (SSC + SSR + SST)$

$= 68.888 - (1.555 + 30.888 + 34.888)$

$= 1.557$

$MSC = \frac{SSC}{C-1} = \frac{1.555}{2} = 0.7775$

$MST = \frac{SST}{R-1} = \frac{30.888}{2} = 15.444$

$MSE = \frac{SSE}{(n-1)(n-2)} = \frac{1.557}{(3-1)(3-2)} = \frac{1.557}{2} = 0.7785$

$MSR = \frac{SSR}{n-1} = \frac{34.888}{3-1} = \frac{34.888}{2} = 17.444$

Source of Variation	Sum of Squares	Degrees of freedom	Mean Sum of Squares	F Ratio
Between days	SSR = ~~30.888~~ 34.888	3-1=2	MSR = 17.444	$F_R = 22.43$
Between engines	SSC = 1.555	3-1=2	MSC = 0.7775	$F_C = 1$
Between buses	SST = 30.888	3-1=2	MST = 15.444	$F_T = 19.86$
Error	SSE = 1.557	(3-1)(3-2) = 2	MSE = 0.7785	

$F_{0.05}(2,2) = 19 = F_{Tab}$

$F_R > F_{Tab}$ $\therefore$ H_0 Rejected.

Difference in days

$F_C < F_{Tab}$ $\therefore$ H_1 accepted.

No difference in Eng ing

$F_T > F_{Tab}$ $\therefore$ H_2 rejected

There is difference between burners

4. 2^2 factorial method.

Treatment combination	Blocks I	II	III
1	12	19	10
K	15	20	16
P	24	16	17
KP	24	17	29

code by -15

	Treatment combo	Blocks I	II	III		X_1^2	X_2^2	X_3^4
Y_1	1	-3	4	-5	-4	9	16	25
Y_2	K	0	5	1	6	0	25	1
Y_3	P	9	1	2	12	81	1	4
Y_4	KP	9	2	14	25	81	4	196
		X_1	X_2	X_3				
		15	12	12		171	46	226

$T = \Sigma X_i = 15 + 12 + 12 = 39$ $\quad N = 12,\ K = 3$

Correction factor: $\frac{T^2}{N} = \frac{39^2}{12} = 126.75$

$TSS = \Sigma X_i^2 - T^2/N$

$= 171 + 46 + 226 - 126.75$

$= \cancel{208.25}\ 316.25$

$$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \frac{T^2}{N}$$

$$= \frac{15^2}{4} + \frac{12^2}{4} + \frac{12^2}{4} - 126.75$$

$= 1.5$

$$SSR = \frac{(\Sigma y_1)^2}{n_1} + \frac{(\Sigma y_2)^2}{n_2} + \frac{(\Sigma y_3)^2}{n_3} + \frac{(\Sigma y_4)^2}{n_4} - \frac{T^2}{N}$$

$$= \frac{(-4)^2}{3} + \frac{6^2}{3} + \frac{12^2}{3} + \frac{25^2}{3} - 126.75$$

$= 146.9166$

$SSE = TSS - (SSR + SSC)$

$= 316.25 - (146.9166 + 1.5)$

$= 167.8334$

$[K] = [ab] + [a] + [b]$.

$[K] = [Kp] + [k] + - [p] - [1]$

$= 25 + 6 - 12 - (-4)$

$= 23$

$[p] = [Kp] + [p] - [k] - [1]$

$= 25 + 12 - 6 - (-4)$

$= 35$

$[Kp] = [Kp] - [K] - [p] + [1]$

$= 25 - 12 - 6 + (-4)$

$[Kp] = 3$

$S_K = \frac{[K]^2}{4r} = \frac{(23)^2}{4(4)} = \cancel{44.0833}\ 33.0625$

$S_p = \frac{[p]^2}{4r} = \frac{(35)^2}{4(4)} = \cancel{102.0833}\ 76.5625$

$S_{Kp} = \frac{Kp^2}{4r} = \frac{3^2}{4(4)} = \cancel{10.0833}\ \cancel{0.5625}\ 0.5625$

Anova table.

Sum of variation	Degree of freedom	Sum of squares	Mean squares	F
K	1	$S_K = 44.0833$	44.0833	0.26266×6 $F_K = 1.5759$
P	1	$S_P = 102.0833$	102.0833	0.60824×6 $F_P = 3.64944$
Kp	1	$S_{Kp} = 10.0833$	10.0833	0.06008×6 $F_{Kp} = 0.36048$
E	6 (12-3-4-1)	167.8334	$\frac{167.8334}{6} = 27.972$	

Anova Table

Sum of variation	Degree of freedom	Sum of Squares	Mean Sum of Squares	F ratio
K	1	$S_K = 33.0625$	33.0625	$F_K = 1.1819$
P	1	$S_P = 76.5625$	76.5625	$F_P = 2.7373$
Kp	1	$S_{KP} = 0.5625$	0.5625	$F_{KP} =$ 0.0201
E	N-r-CH 6	$SSE = 167.8334$	27.972	

5. 12 samples of 5 lamps

$\bar{X}$	120	127	152	157	160	134	137	123	140	144	120	127
R	30	44	60	34	38	35	45	62	39	50	35	41

$$\bar{\bar{X}} = \frac{1}{N}\Sigma\bar{X}_i = \frac{1}{12}(120+127+\cdots 120+127)$$

$$\bar{\bar{X}} = 136.75$$

$$\bar{R} = \frac{1}{N}\Sigma R_i = \frac{1}{12}(30+44+60+\cdots+35+41)$$

$$\bar{R} = 42.75$$

Control limit for $\bar{X}$ chart

central line : $\bar{\bar{X}} = 136.75$

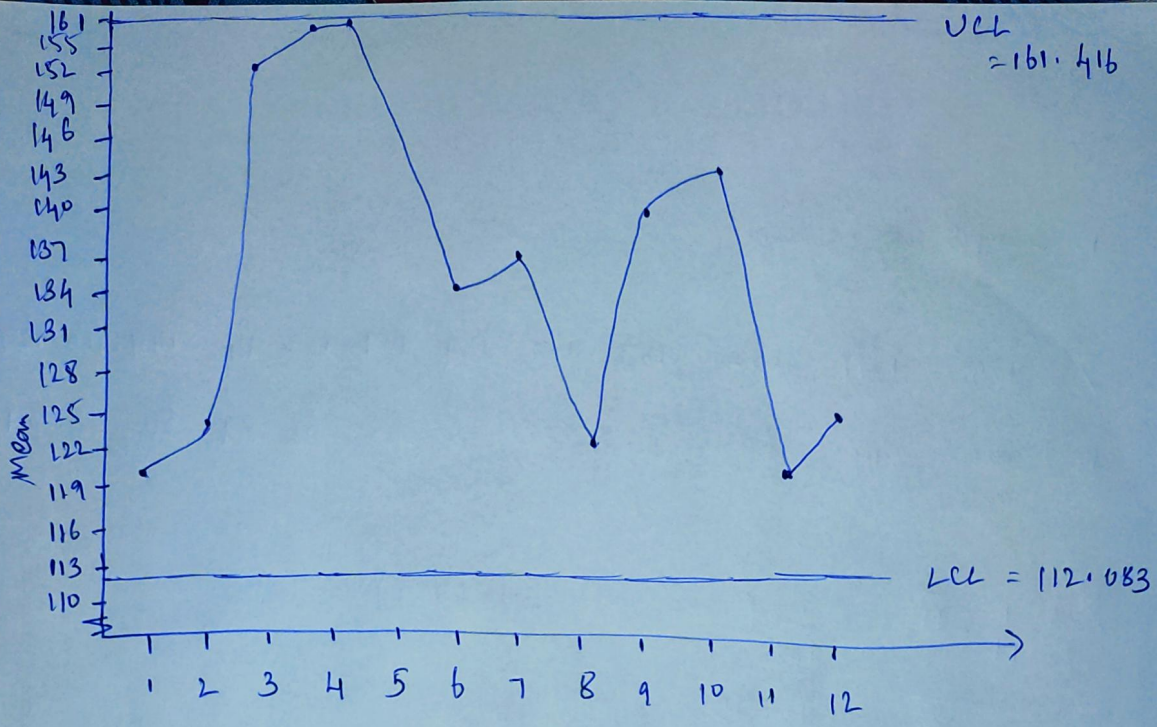
$A_2 = 0.577$

$$LCL = \bar{\bar{X}} - A_2\bar{R} = 136.75 - (0.577)(42.75)$$
$$= 112.08325$$

$$UCL = \bar{\bar{X}} + A_2\bar{R} = 136.75 + 0.577(42.75)$$
$$= 161.41675$$

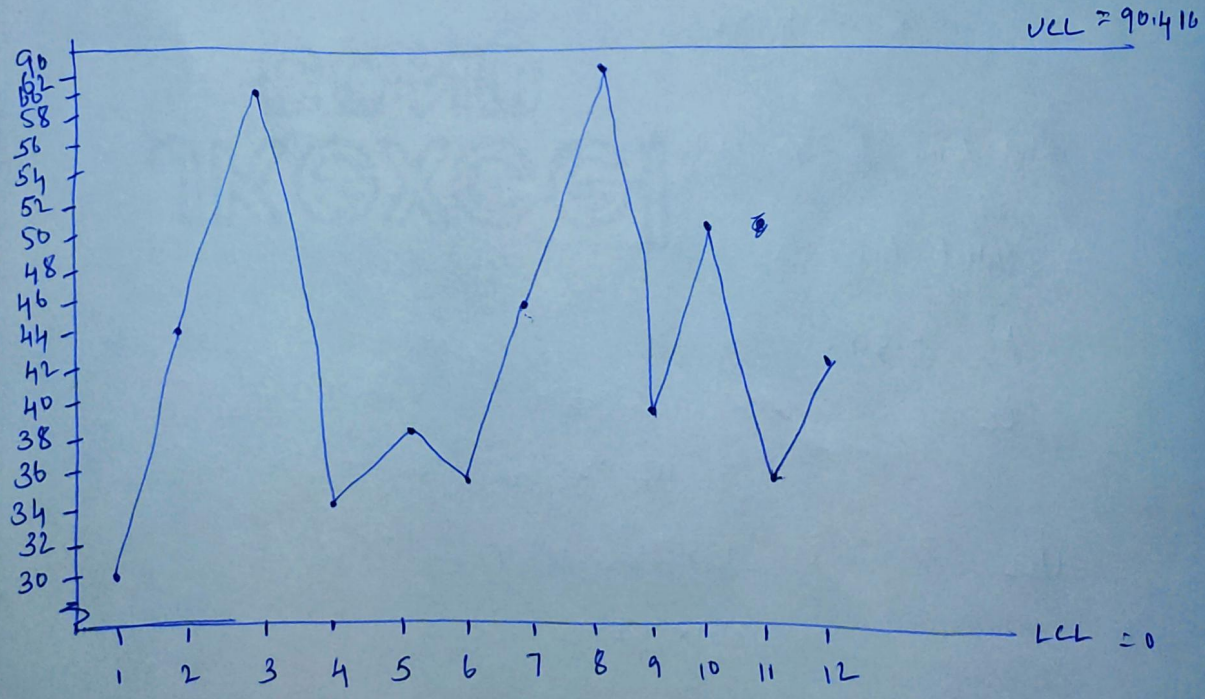
$CL = \bar{R} = 42.75$

$LCL = D_3\bar{R} = 0$ $\qquad$ $UCL = D_4\bar{R} = (2.115)(42.75)$
$= 90.41625$



Sample No

Mean in control



Range in control

6. 15 samples of 4 boxes.

Sample No.	Weight of boxes				Mean	Range
1	10	10.2	11.3	12.4	10.975	2.4
2	10.3	10.9	10.7	11.7	10.9	1.4
3	11.5	10.7	11.4	12.4	11.05	1.7
4	11	11.1	10.7	11.4	11.05	0.7
5	11.3	11.6	11.9	12.1	11.725	0.8
6	10.7	11.4	10.7	11.0	10.95	0.7
7	11.3	11.4	10.7	11.0	11.025	1.1
8	12.3	12.1	12.7	10.7	11.95	2.0
9	11.0	13.1	13.1	12.4	12.4	2.1
10	11.3	12.1	10.7	11.5	11.4	1.4
11	12.5	11.9	11.8	11.3	11.875	1.2
12	11.9	12.1	11.6	11.4	11.75	0.7
13	12.1	11.1	12.1	11.7	11.75	1.0
14	11.9	12.1	13.1	12.0	12.275	1.2
15	10.6	11.9	11.7	12.1	11.575	1.5

$$\bar{\bar{X}} = \frac{1}{N}\sum \bar{X}_i = \frac{173.1}{15} = 11.54$$

$$\bar{R} = \frac{1}{N}\sum R_i = \frac{19.9}{15} = 1.3266$$

Central limit for $\bar{x}$ chart.

$CL = \bar{x} = 11.54$

$LCL = \bar{x} - A_2\bar{R}$

A_2 for $\bar{x}$ chart, 4 size -- 0.729

$LCL = 11.54 - 0.729(1.3266) = 10.573$

$UCL = \bar{x} + A_2\bar{R}$

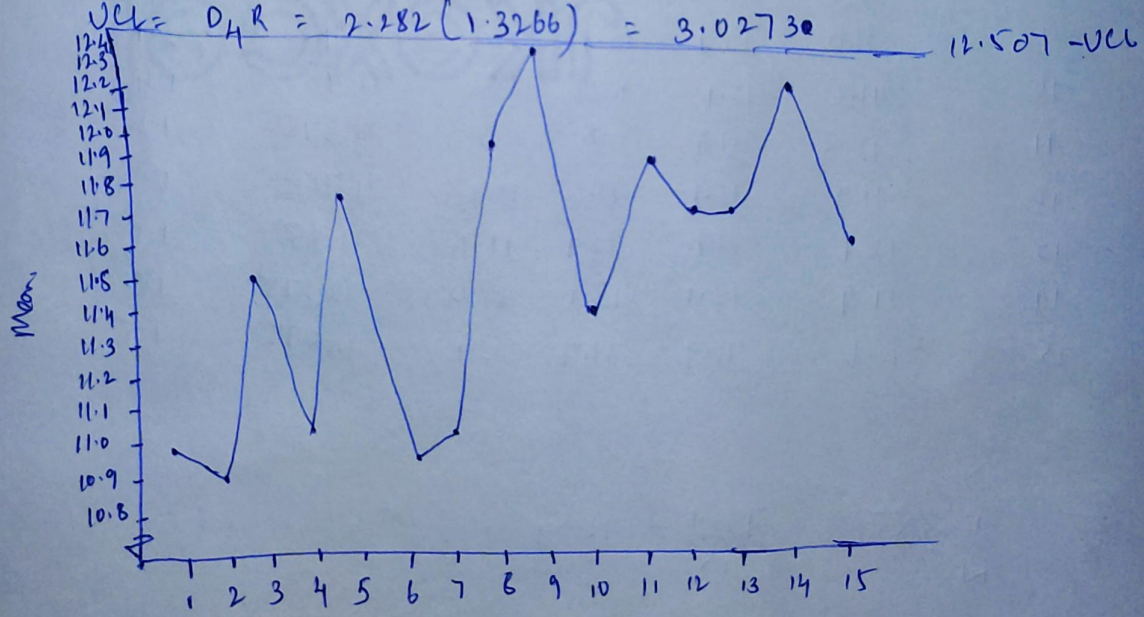
$= 11.54 + 0.729(1.3266) = 12.507$

Central limit for $\bar{R}$ chart

$CL = \bar{R} = 1.3266$ $\qquad D_3 = 0$ (size 4)

$LCL = D_3\bar{R} = 0$ $\qquad D_4 = 2.282$

$UCL = D_4\bar{R} = 2.282(1.3266) = 3.02730$



Mean chart

Mean in control

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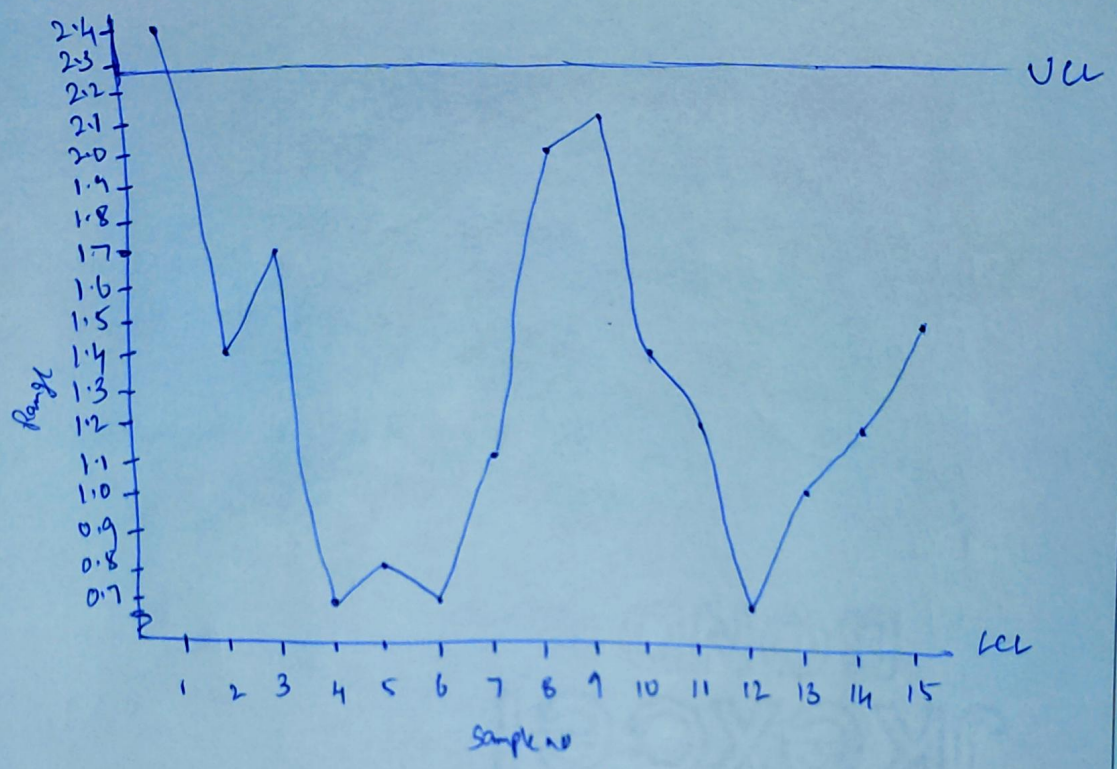


Range chart.

Range not in control.

7. 10 samples each of size 50

Sample	1	2	3	4	5	6	7	8	9	10
np	2	1	1	2	3	5	5	1	2	3

$$n\bar{p} \text{ or } \bar{np} = \frac{\Sigma np}{N} = \frac{2+1+1+2+3+5+5+1+2+3}{10} = 2.5$$

$$\bar{p} = \frac{n\bar{p}}{n} = \frac{2.5}{50} = \frac{5}{100} = 0.05$$

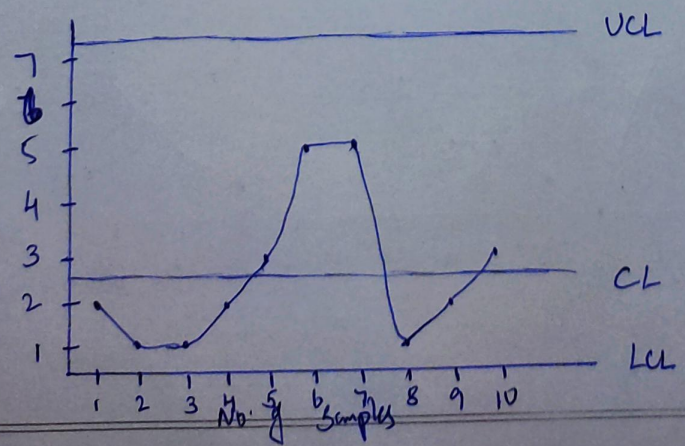
$$CL = n\bar{p} = 2.5$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 2.5 - 3\sqrt{2.5(1-0.05)} = -2.123$$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 2.5 + 3\sqrt{2.5(1-0.05)} = 7.123$$

LCL cannot be negative

$\therefore$ LCL = 0 $\Rightarrow$ y = 0



All points inside the UCL and LCL

$\therefore$ Under control.

8. 15 samples each of size 50
N n

Sample	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defects	2	3	4	2	3	0	1	2	2	3	5	5	1	2	3

$$n\bar{p} \text{ or } \bar{np} = \frac{\Sigma np}{N} = \frac{2+3+4+2+3+0+1+2+2+3+5+5+1+2+3}{15}$$

$$= \frac{38}{15} = 2.533$$

$$\bar{p} = \frac{n\bar{p}}{n} = \frac{2.533}{50} = 0.05066$$

$CL = n\bar{p} = 2.533$

$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

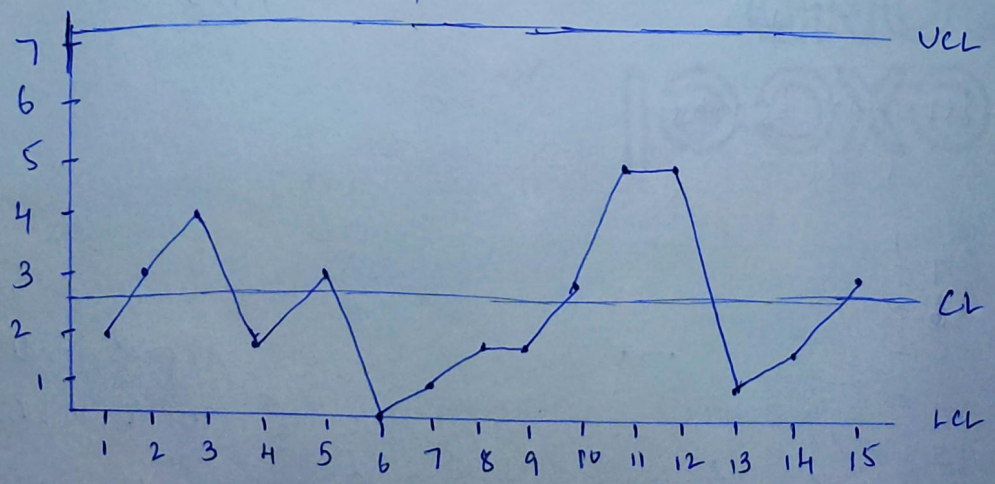
$= 2.533 - 3\sqrt{2.533(1-0.05066)}$

$= -2.1191$ (cannot be -ve $\therefore 0$)

$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$

$= 2.533 + 3\sqrt{2.533(1-0.05066)}$

$= 7.1851$



9.

Roll Number (i)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Number of defects (c)	19	10	8	12	15	22	7	13	18	13	16	14	8	7	6	4	5	6	8	9

$N = 20$

Mean of defectives $\bar{c} = \frac{\Sigma c}{N} = \frac{19+10+\cdots 8+9}{20} = 11 = CL$

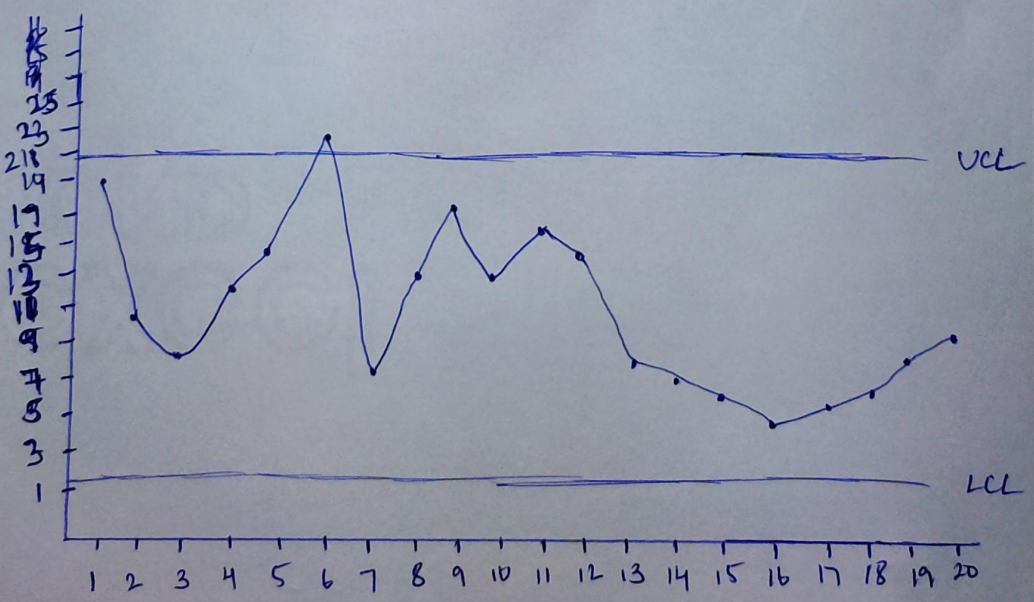
$LCL = \bar{c} - 3\sqrt{\bar{c}}$

$= 11 - 3\sqrt{11} = 1.0501$

$= \cancel{18.2545}$ (can't be negative, ∴ 0)

$UCL = c + 3\sqrt{c}$

$= 11 + 3\sqrt{11} = 20.9498$



Since a point falls outside control line, it is out of control