Assignment 1 – Propositional Logic

1. Find the following propositional formulas are valid, satisfiable or unsatisfiable?

| | Valid | Satisfiable | Unsatisfiable |
|---|---|---|---|
| $A$ | | | |
| $A \vee B$ | | | |
| $A \vee \neg A$ | | | |
| $A \wedge \neg A$ | | | |
| $A \rightarrow \neg A$ | | | |
| $A \rightarrow B$ | | | |
| $A \rightarrow (B \rightarrow A)$ | | | |
| $A \rightarrow (A \rightarrow B)$ | | | |
| $A \leftrightarrow \neg A$ | | | |

2. Which of the following statements are true?

| | | | | Y/N |
|---|---|---|---|---|
| If | $F$ is valid, | then | $F$ is satisfiable | |
| If | $F$ is satisfiable, | then | $\neg F$ is satisfiable | |
| If | $F$ is valid, | then | $\neg F$ is satisfiable | |
| If | $F$ is unsatisfiable, | dann | $\neg F$ is valid | |

3. Convert the following formulas into DNF and CNF form

$$(p \rightarrow q) \rightarrow (\neg r \wedge q)$$

$$((\neg A \rightarrow B) \vee ((A \wedge \neg C) \leftrightarrow B))$$

4. Find the DNF and CNF for the formula F given in the truth table.

| p | q | r | F |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 |

5. Check the following horn formula are satisfiable or unsatisfiable

1. $(p_5 \rightarrow p_{11}) \wedge (p_2 \wedge p_3 \wedge p_5 \rightarrow p_{13}) \wedge (T \rightarrow p_5) \wedge (p_5 \wedge p_{11} \rightarrow \perp)$
2. $(T \rightarrow q) \wedge (T \rightarrow s) \wedge (w \rightarrow \perp) \wedge (p \wedge q \wedge s \rightarrow \perp) \wedge (v \rightarrow s) \wedge (T \rightarrow r) \wedge (r \rightarrow p)$
3. $(T \rightarrow q) \wedge (T \rightarrow s) \wedge (w \rightarrow \perp) \wedge (p \wedge q \wedge s \rightarrow v) \wedge (v \rightarrow s) \wedge (T \rightarrow r) \wedge (r \rightarrow p)$
4. $\neg b \wedge (\neg a \vee b \vee \neg c) \wedge a \wedge (\neg a \vee c)$ Hint: Convert to Horn formula

6. Prove that the following formula is unsatisfiable using resolution

$$F = \{\{A, B, \neg C\}, \{\neg A\}, \{A, B, C\}, \{A, \neg B\}\}$$

7. Prove R using resolution
   (P -> Q) -> R
   (P -> P) -> R
   (R -> S) -> ⌐(S -> Q)

8. Check whether the following entailment is true or false

$$\{(\neg(p \wedge q)), (p \wedge q)\} \models (p \leftrightarrow q)?$$

9. Check the last formula is the consequence of the 1st two using resolution

$$P \rightarrow Q$$
$$\neg P \rightarrow R$$
$$\neg Q \rightarrow \neg R$$