LANGUAGES OF PDA

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AP/CSE

LEARNING OBJECTIVE

- To Design pushdown automata for any CFL (K3)
  - To understand the languages of PDA

ACCEPTANCE BY FINAL STATE

- Acceptance by final state
  - Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA.
  - The language accepted by final state is denoted by $L_F(M)$

$$L_F(M) = \{w \mid (q_0, w, z_0) \vdash^* (p, \varepsilon, \gamma) \text{ for some } p \text{ in } F \text{ and } \gamma \text{ in } \Gamma^*\}$$

PALINDROME L_{WWR}

1. $\delta(q_0,0, Z)=\{(q_0,0Z)\}$
2. $\delta(q_0,1, Z)=\{(q_0,1Z)\}$

First symbol push on stack

3. $\delta(q_0,0, 0)=\{(q_0,00)\}$
4. $\delta(q_0,0, 1)=\{(q_0,01)\}$
5. $\delta(q_0,1, 0)=\{(q_0,10)\}$
6. $\delta(q_0,1, 1)=\{(q_0,11)\}$

Grow the stack by pushing new symbols on top of old (w-part)

7. $\delta(q_0, \varepsilon, 0)=\{(q_1, 0)\}$
8. $\delta(q_0, \varepsilon, 1)=\{(q_1, 1)\}$
9. $\delta(q_0, \varepsilon, Z)=\{(q_1, Z)\}$

Switch to popping mode (boundary between w and w^R)

10. $\delta(q_1,0, 0)=\{(q_1, \varepsilon)\}$
11. $\delta(q_1,1, 1)=\{(q_1, \varepsilon)\}$

Shrink the stack by popping matching symbols (w^R-part)

12. $\delta(q_1, \varepsilon, Z)=\{(q_2, Z)\}$

Reach Final State

ACCEPTANCE BY EMPTY STACK

- Acceptance by empty stack
  - Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA.
  - The language accepted by empty stack is denoted by $L_E(M)$

$$L_E(M) = \{w \mid (q_0, w, z_0) \vdash^* (p, \varepsilon, \varepsilon) \text{ for some } p \text{ in } Q\}$$

PALINDROME L_{WWR}

1. $\delta(q_0,0, Z)=\{(q_0,0Z)\}$
2. $\delta(q_0,1, Z)=\{(q_0,1Z)\}$

First symbol push on stack

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Grow the stack by pushing new symbols on top of old (w-part)

7. $\delta(q_0, \varepsilon, 0)=\{(q_1, 0)\}$
8. $\delta(q_0, \varepsilon, 1)=\{(q_1, 1)\}$
9. $\delta(q_0, \varepsilon, Z)=\{(q_1, Z)\}$

Switch to popping mode (boundary between w and w^R)

10. $\delta(q_1,0, 0)=\{(q_1, \varepsilon)\}$
11. $\delta(q_1,1, 1)=\{(q_1, \varepsilon)\}$

Shrink the stack by popping matching symbols (w^R-part)

12. $\delta(q_1, \varepsilon, Z)=\{(q_1, \varepsilon)\}$

Empty the stack

ACCEPTANCE BY FINAL STATE AND EMPTY STACK

- Acceptance by final state and empty stack
  - Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA.
  - The language accepted by empty stack and final state is denoted $L(M)$

$$L(M) = \{w \mid (q_0, w, z_0) \mid\!\!-\!\!-^* (p, \varepsilon, \varepsilon) \text{ for some } p \text{ in } F\}$$

PALINDROME L_{WWR}

1. $\delta(q_0, 0, Z) = \{(q_0, 0Z)\}$
2. $\delta(q_0, 1, Z) = \{(q_0, 1Z)\}$

First symbol push on stack

3. $\delta(q_0, 0, 0) = \{(q_0, 00)\}$
4. $\delta(q_0, 0, 1) = \{(q_0, 01)\}$
5. $\delta(q_0, 1, 0) = \{(q_0, 10)\}$
6. $\delta(q_0, 1, 1) = \{(q_0, 11)\}$

Grow the stack by pushing new symbols on top of old (w-part)

7. $\delta(q_0, \varepsilon, 0) = \{(q_1, 0)\}$
8. $\delta(q_0, \varepsilon, 1) = \{(q_1, 1)\}$
9. $\delta(q_0, \varepsilon, Z) = \{(q_1, Z)\}$

Switch to popping mode (boundary between w and w^R)

10. $\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$
11. $\delta(q_1, 1, 1) = \{(q_1, \varepsilon)\}$

Shrink the stack by popping matching symbols (w^R-part)

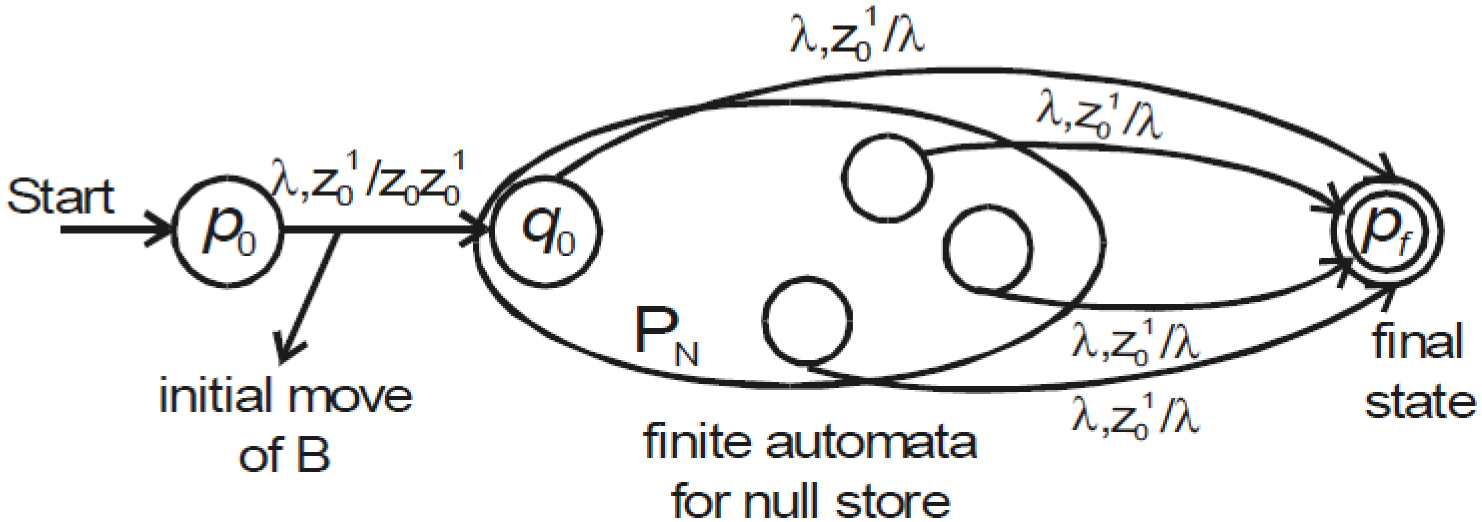
12. $\delta(q_1, \varepsilon, Z) = \{(\mathbf{q_2}, \varepsilon)\}$

Reach Final State and empty the stack

EMPTY STACK TO FINAL STATE

- Theorem

  If A = (Q, Σ, Γ, δ, q_0, z_0, F) is a PDA accepting L by empty stack, we can find a PDA B=($Q^|$, Σ, $Γ^|$, $δ_B$, p_0, $z_0{}^|$, $F^|$) which accepts L by final state i.e. L = L(A) = L(B)



EMPTY STACK TO FINAL STATE

$$R_1 : \delta_B(p_0, \lambda, z_0^{\,1}) = \{(q_0, z_0 z_0^{\,1})\}$$

$$R_2 : \delta_B(q, a, z) = \delta(q, a, z) \text{ for all } q \text{ in } Q,\ a \text{ in } \Sigma \text{ or } \lambda \text{ and } z \text{ in } \Gamma$$

$$R_3 : \delta_B(q, \lambda, z_0^{\,1}) = \{(p_f, \lambda)\}$$

By theorem

$$(q, x, \alpha) \overset{*}{\vdash} (p, y, \beta) \Rightarrow (q, xw, \alpha\gamma) \overset{*}{\vdash} (p, yw, \beta\gamma)$$

we get

$$(q_0, w, z_0 z_0^{\,1}) \overset{*}{\underset{A}{\vdash}} (q, \lambda, z_0^{\,1})$$

Since null store (or) empty store (δ) is a subset of δ_B

i.e.. $\delta \subset \delta_B$ we have

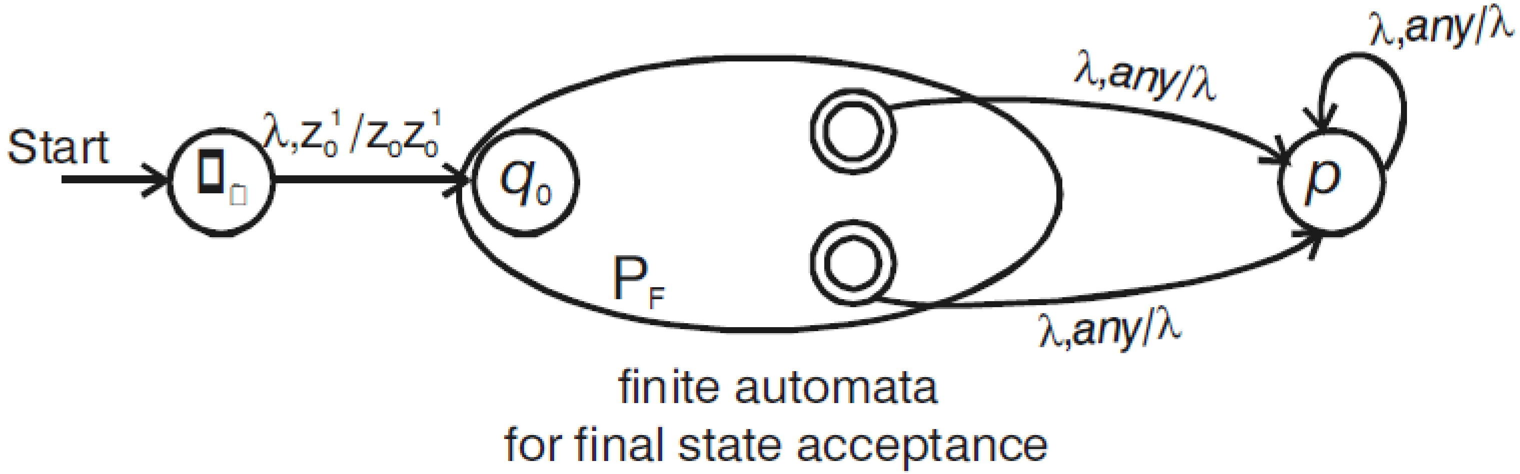
EMPTY STACK TO FINAL STATE

$$
\begin{aligned}
(p_0, w, z_0^{\,1}) \quad &\vdash_{B} (q_0, w, z_0 z_0^{\,1}) \\
&\vdash_{B}^{*} (q, \lambda, z_0^{\,1}) \\
&\vdash_{B} (p_f, \lambda, \lambda)
\end{aligned}
$$

FINAL STATE TO EMPTY STACK

- If $A = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ accepts L by final state, we can find a PDA B, accepting L by empty store.

i.e., L = T(A) = N(B)



FINAL STATE TO EMPTY STACK

$R_1 : \delta_B (p_0, \lambda, z_0^1) = \{(q_0, z_0 z_0^1)\}$

$R_2 : \delta_B(p, \lambda, z) = \{(p,\lambda)\}$ for all $z \in \Gamma \cup \{z_0^1\}$

$R_3 : \delta_B (q, a, z) = \delta(q, a, z)$ for all $a \in z$, $q \in Q$, $z \in \Gamma$.

$R_4 : \delta_B (q, \lambda, z) = \delta(q, \lambda, z) \cup \{(p, \lambda)\}$ for all $z \in \Gamma \cup \{z_0^1\}$ and $q \in F$.

SUMMARY

- Different Languages of PDA
  – Acceptance by Final State
  – Acceptance by Empty Stack
  – From Empty stack to final state
  – From Final state to empty stack

TEST YOUR KNOWLEDGE

REFERENCE

- Hopcroft J.E., Motwani R. and Ullman J.D, “Introduction to Automata Theory, Languages and Computations”, Second Edition, Pearson Education, 2008