DETERMINISTIC FINITE AUTOMATA

Dr. A. Beulah

AP/CSE

LEARNING OBJECTIVE

- To construct finite automata for any given pattern and find its equivalent regular expressions
  - To understand what is Regular Expression

FINITE AUTOMATA

- The FA is a mathematical model of a system, with discrete inputs and outputs and a finite number states and a set of transitions from state to state that occurs on input symbols from alphabet Σ.
- The FA is classified as:
  - Deterministic Finite Automata (DFA)
  - Non Deterministic Finite Automata (NFA)

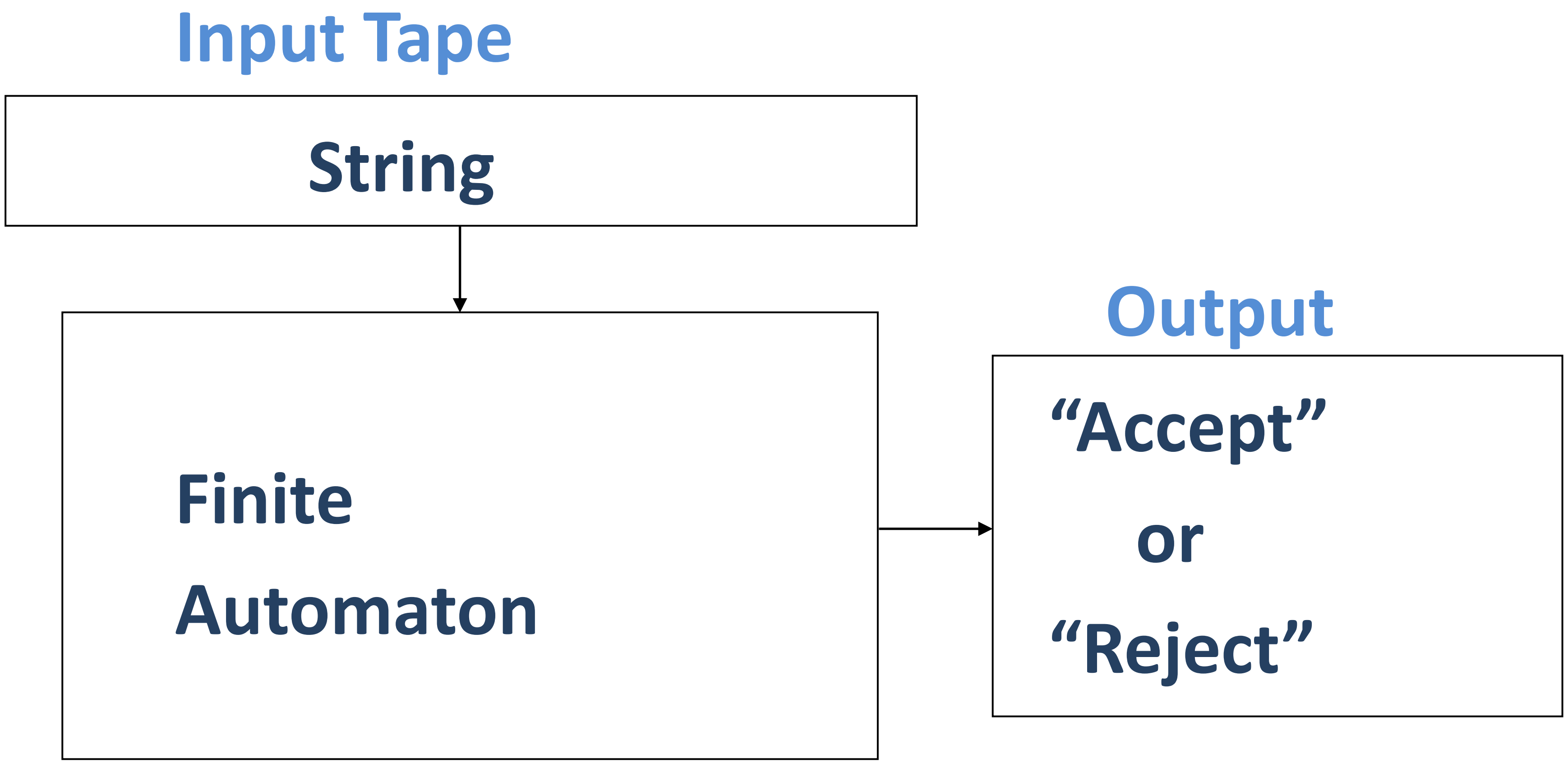
$\{q_0, q_1, \ldots q_n\}$

$\Sigma = \{0, 1\}$

exactly 1 transition

0, or more transition

FINITE AUTOMATA CONT…



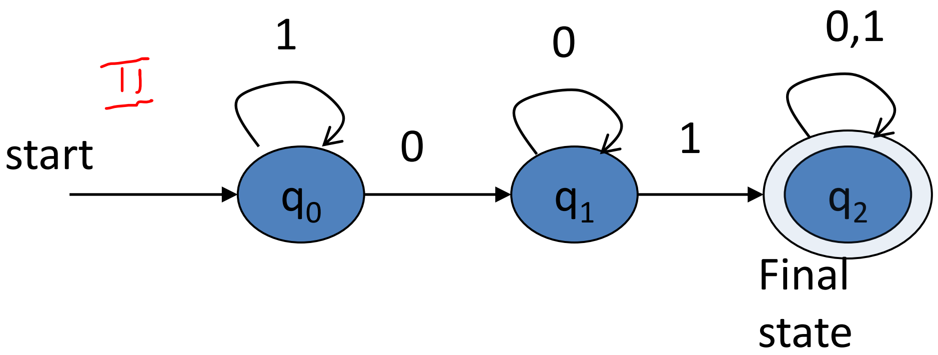
DFA SPECIFICATION

- A Deterministic Finite Automata (DFA) is a 5-tuple $(Q, \Sigma, S, F, \delta)$ where
  - Q is a finite set of states
  - Σ is an alphabet   or set of input symbols
  - S: $q_0 \in Q$ is the initial state
  - $F \subseteq Q$ is a set of accepting states (or final states)
  - $\delta: Q \times \Sigma \rightarrow Q$ is a transition function

(cs, i/p = n/state

EXAMPLE

DFA for strings containing 01



II

$L = \{01, 001, \cdots 00101 \ldots\}$

What makes this DFA deterministic?

$Q \times \Sigma \rightarrow Q$
cs i/p → n/s.

- $Q = \{q_0, q_1, q_2\}$
- $\Sigma = \{0,1\}$
- $S = q_0$ $\in Q$
- $F = \{q_2\}$
- Transition table

III set of transitions

$\delta(q_0, 0) = q_1$
$\delta(q_0, 1) = q_0$
$\delta(q_1, 0) = q_1$
$\delta(q_1, 1) = q_2$
$\delta(q_2, 0) = q_2$
$\delta(q_2, 1) = q_2$

I

symbols

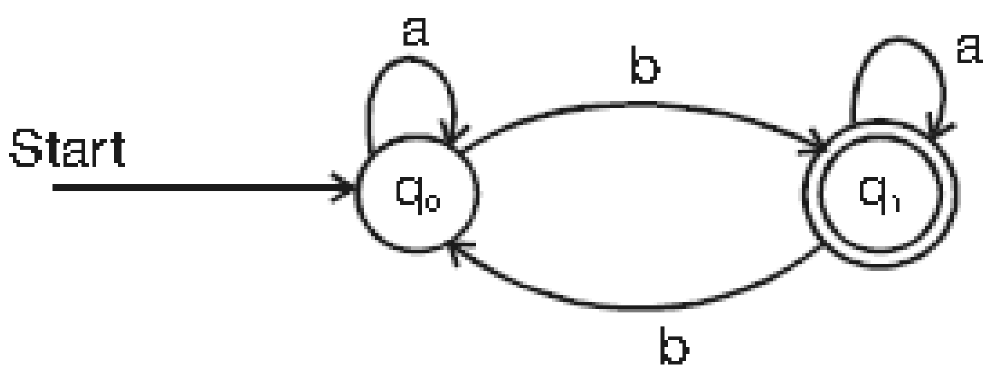
δ	**0**	**1**
→ **q_0**	q_1	q_0
q_1	q_1	q_2
***q_2**	q_2	q_2

states

TRANSITION DIAGRAM OF DFA

- It is a directed graph whose vertices corresponds to states of DFA. The edges are the transitions from one state to another

- In the transition diagram, start state s is represented by → and the final states are represented by double circle.



EXAMPLE

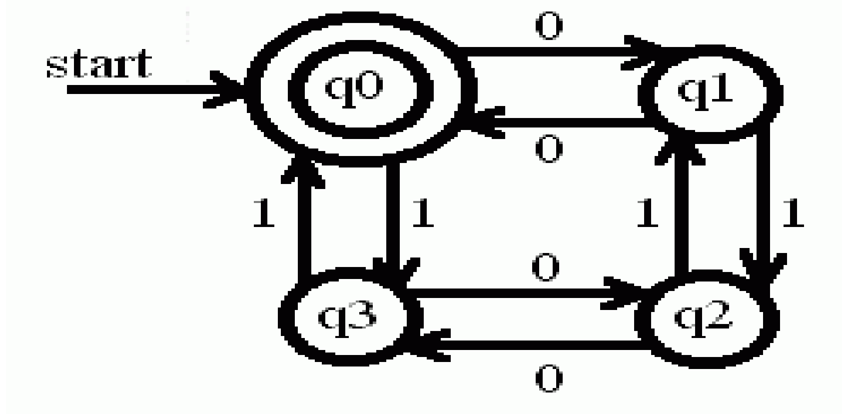
- The DFA for the above transition is represented as: S, *F* ,δ) where

  $Q = \{q_0, q_1, q_2, q_3\}$

  $\Sigma = \{0,1\}$

  $S = q_0 \rightarrow$ Start State

  $F = \{q_0\}$



110101

$\delta(q_0, 1) = q_3$

$q_0 \xrightarrow{1} q_3 \xrightarrow{1} q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3 \xrightarrow{1} q_0$

δ → Transition Table??

L = ?. L = {00, 11, 0101, 0110, ... 1100, 0011, 1010, ... }

even no 0's and 1's.

000000, 1111...

EXAMPLE

- Suppose 110101 is input to M, check the validity of the input.
- Finite automata is in start state and reads from left most.

$\delta\ (q_0, 1) = q_3$

$\delta\ (q_3, 1) = q_0$ (Reader reads next symbols)

$\delta\ (q_0, 0) = q_1$ (Reader moves one position right)

$\delta\ (q_1, 1) = q_2$

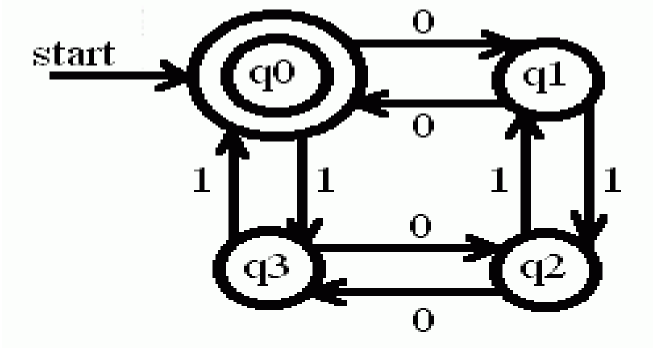
$\delta\ (q_2, 0) = q_3$

$\delta\ (q_3, 1) = q_0$

since q_0 is a final state, the given string is accepted.

$$q0 \xrightarrow{1} q3 \xrightarrow{1} q0 \xrightarrow{0} q1 \xrightarrow{1} q2 \xrightarrow{0} q3 \xrightarrow{1} q0$$

EXAMPLE



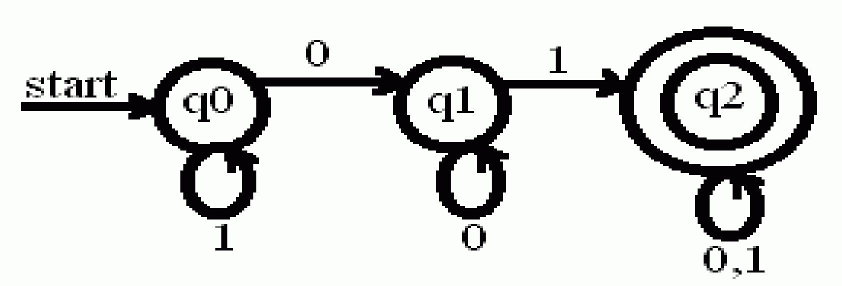
- L=? $\{00, 11, 0101 \dots\}$ Even no of 0's & 1's
- 1010
- 1101

$\delta(q_0, 1010) = \delta(q_3, 010)$
$= \delta(q_2, 10)$
$= \delta(q_1, 0)$
$= q_0 \in F$ ∴ The string is accepted

1101

$\delta(q_0, 1101) = \delta(q_3, 101)$
$= \delta(q_0, 01)$
$= \delta(q_1, 1)$
$= q_2$
$q_2 \notin F$ ∴ Rejected.

EXAMPLE



- L=? L={01, 101, 1101, 0100...}
- 1010
- 11011

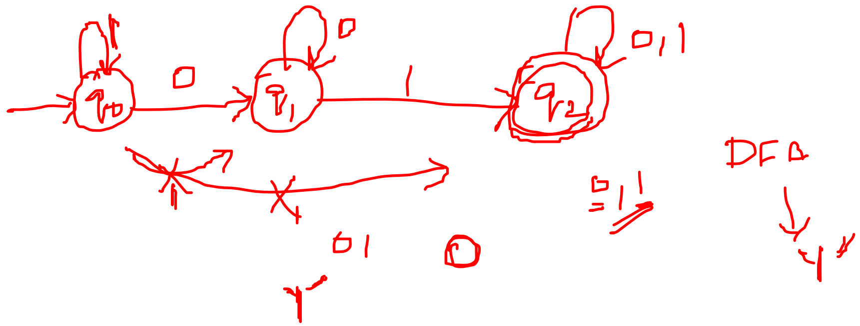
$\delta(q_0, 1010) = \delta(q_0, 010)$
$= \delta(q_1, 10)$
$= \delta(q_2, 0)$
$= q_2 \in F$ $\therefore$ The string is accepted

$\delta(q_0, 11011) = \delta(q_0, 1011)$
$= \delta(q_0, 011)$
$= \delta(q_1, 11)$
$= \delta(q_2, 1)$
$= q_2 \in F$ $\therefore$ The string is accepted

DFA CONSTRUCTION

- Construct a DFA which accepts strings containing 01

L = {01, 101, 1101, 0101, 001, 010, . . . }

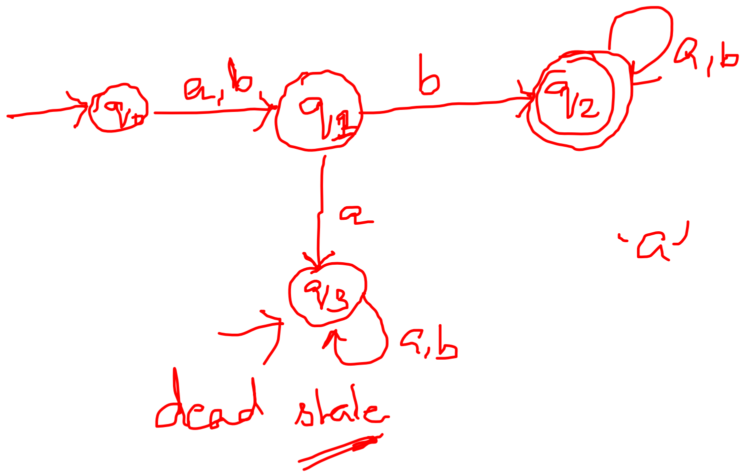


δ	0	1
→q_0	q_1	q_0
q_1	q_1	q_2
*q_2	q_2	q_2

DFA CONSTRUCTION

- DFA accepting a set of strings over Σ={a, b} in which the second symbol from left-hand side is always ‘b’

L={ab, bb, abb, aba, abaa, bbaa ....   a/b

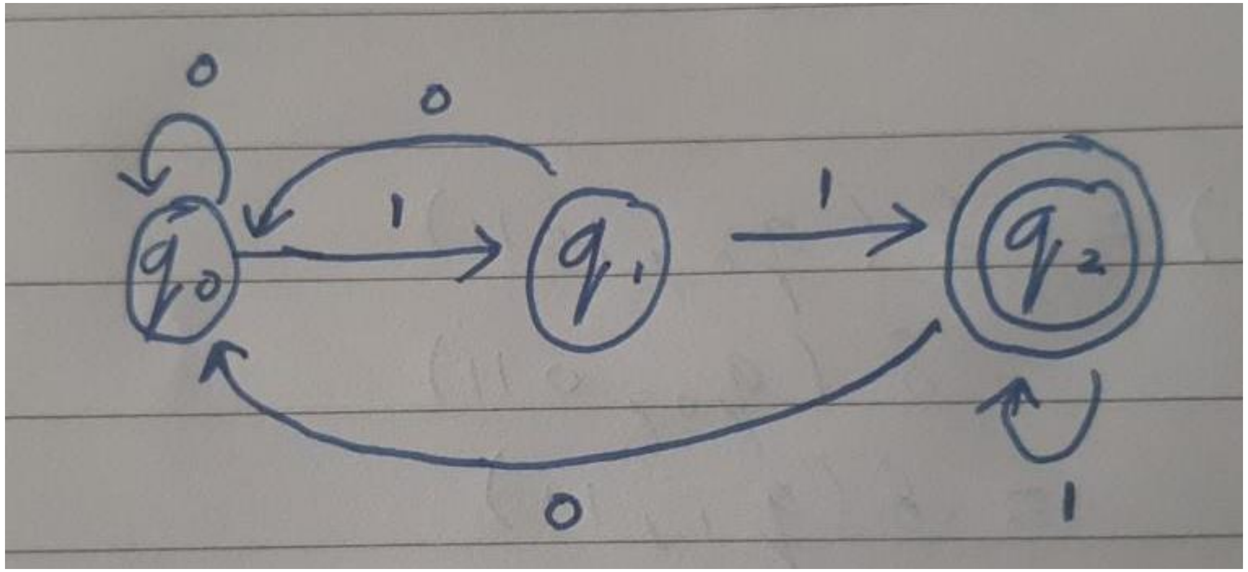


EXAMPLE

- Design a DFA that accepts input strings of **0's** and **1's** that end with **11**

L=? L={11, 011, 111, 1011, 0011. . . . .}

Transition Diagram?



EXAMPLE

- Design a DFA that accepts strings over $\Sigma = \{0, 1\}$ with three consecutive 0's.

L=?

Transition Diagram?

PROPERTIES OF TRANSITION FUNCTION (δ)

1. ***$\delta(q, \varepsilon) = q$***

This means the state of the system can be changed only by an input symbol else remains in original state.

2. For all strings *w and input symbol a*

$$\delta(q, aw) = \delta(\delta(q, a), w)$$

$\hat{\delta}(q, w)$

similarly $\delta(q, wa) = \delta(\delta(q,w), a)$

3. The transition function δ can be extended to $\bar{\delta}$ (or) $\hat{\delta}$ that operates on states and strings (as opposed to states and symbols)

   Basis : $\bar{\delta}(q, \varepsilon) = q$

   Induction : $\bar{\delta}(q, xa) = \delta(\bar{\delta}(q, x), a)$

LANGUAGE OF A DFA

- A DFA A accepts string *w* if there is a path from q_0 to an accepting (or final) state that is labeled by *w*

- *i.e.,* $L(A) = \{\, w \mid \hat{\delta}(q_0, w) \in F \,\}$

L(D)

$q_2 \in F$

- *I.e., L(A) = all strings that lead to a final state from* q_0

Automata

DFA

Def

5Q

δ

δ TF

$\delta: Q \times \Sigma \to Q$

ETF

$\hat{\delta}(q, wa)$

$= \delta(\hat{\delta}(q, w), a)$

L(A)

L(

- - - -

TEST YOUR KNOWLEDGE

1. Construct a DFA which contains a substring aabb
2. Construct a DFA to accept a strings of even number of zeros over $\Sigma = \{0,1\}$
3. Construct a DFA to accepting strings containing odd number of b's over $\Sigma = \{a,b\}$

SUMMARY

- Introduction to Finite Automata
- Definition of DFA
- Transition diagram, transition function and properties of transition function

LEARNING OUTCOME

On successful completion of this topic, the student will be able to:

- Understand the concepts of DFA (K3)

REFERENCE

- Hopcroft J.E., Motwani R. and Ullman J.D, “Introduction to Automata Theory, Languages and Computations”, Second Edition, Pearson Education, 2008