EQUIVALENCE OF DFA AND NFA

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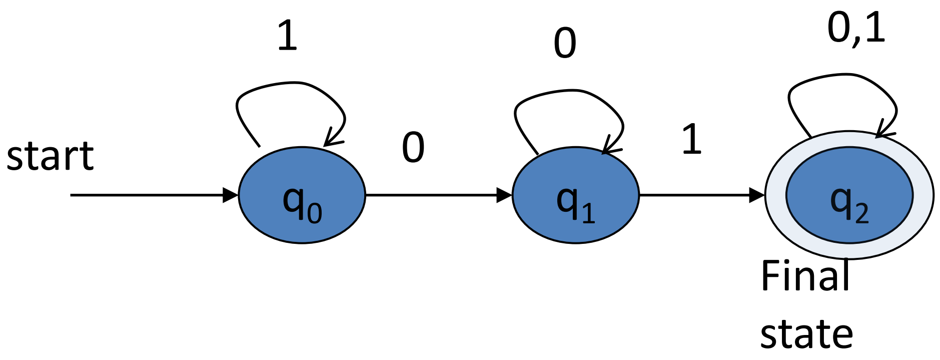
AP/CSE

LEARNING OBJECTIVE

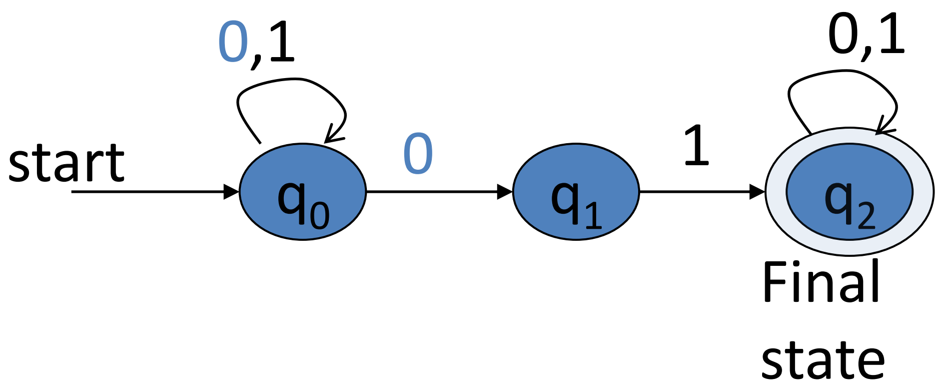
- To construct finite automata for any given pattern
  - Equivalence of DFA and NFA

INTRODUCTION

DFA



NFA



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DFA → NFA

NFA ⇏ DFA

EQUIVALENCE OF DFA AND NFA

- As every DFA is an NFA, the class of languages accepted by NFA's includes the class of languages accepted by DFA's.
- DFA can simulate NFA.
- For every NFA, there exist an equivalent DFA.

DFA → NFA

NFA → DFA
Convert

EQUIVALENCE OF DFA AND NFA

States	Inputs	
	0	1
$\rightarrow * q_0$	$\{q_0\}$	$\{q_1\}$
q_1	$\{q_1\}$	$\{q_0, q_1\}$

NFA → DFA

$M \rightarrow M'$

$M = (Q, \Sigma, S, \delta, F)$

$Q \rightarrow \{q_0, q_1\}$

$\Sigma \rightarrow \{0, 1\}$

$S \rightarrow q_0$

$F \rightarrow \{q_0\}$

$\delta \rightarrow$ transition table

$M' = (Q', \Sigma, S', F', \delta')$

$Q' \rightarrow$? —

$\Sigma = \{0, 1\}$

$S' = \underline{[q_0]}$ → DFA

$F' \rightarrow$? —

$\delta' \rightarrow$? —

2^Q

[ ]

$\underline{0, 1}$

$Q \times \Sigma \rightarrow 2^Q$

EQUIVALENCE OF DFA AND NFA

δ

States	Inputs	
	0	1
$* \rightarrow q_0$	$\{q_0\}$	$\{q_1\}$
q_1	$\{q_1\}$	$\{q_0, q_1\}$

NFA

DFA $\delta'([q_0], 0) = [q_0] \rightarrow$ DFA as $\delta(q_0, 0) = \{q_0\}$

$\delta'([q_0], 1) = [q_1] \rightarrow$ as $\delta(q_0, 1) = \{q_1\}$ NFA

$\delta'([q_1], 0) = [q_1] \rightarrow$ S.S DFA $\delta(q_1, 0) = \{q_1\}$

$\delta'([q_1], 1) = [q_0 q_1] \rightarrow$ S.S DFA $\delta(q_1, 1) = \{q_0, q_1\} \rightarrow$ NFA

δ'	0	1
$* \rightarrow [q_0]$	$[q_0]$	$[q_1]$
$[q_1]$	$[q_1]$	$[q_0 q_1]$
$* [q_0 q_1]$	$[q_0 q_1]$	$[q_0 q_1]$

$\delta'([q_0 q_1], 0) = [q_0 q_1]$

$\delta(\{q_0, q_1\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$

$= \{q_0\} \cup \{q_1\} = \{q_0, q_1\}$

$\delta'([q_0 q_1], 1) = [q_0 q_1]$

$\delta(\{q_0, q_1\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1) = \{q_1\} \cup \{q_0, q_1\}$

$= \{q_0, q_1\}$

EQUIVALENCE OF DFA AND NFA

States	Inputs	
	0	1
$\rightarrow * q_0$	$\{q_0\}$	$\{q_1\}$
q_1	$\{q_1\}$	$\{q_0, q_1\}$

NFA

δ'

$\rightarrow *$ A

B

$*$ C

States	Inputs	
	0	1
$[q_0]$	$[q_0]$	$[q_1]$
$[q_1]$	$[q_1]$	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_1]$

DFA

$M' \rightarrow Q' = \{[q_0], [q_1], [q_0 q_1]\}$

$\Sigma = \{0, 1\}$ $\quad S' \rightarrow [q_0]$

$F' = \{[q_0], [q_0 q_1]\}$

DFA → NFA

NFA → DFA

NFA ⇄ DFA

EQUIVALENCE OF DFA AND NFA

Theorem

- For every NFA, there exists a DFA which simulates the behavior of NFA. If L is the set accepted by NFA, then there exists a DFA which also accepts L.

DFA $\rightarrow$ NFA

NFA There exists DFA

PROOF

- Let M= (Q, Σ, q_0, F, δ) be NFA accepting L we construct DFA M^1 = ($Q^{|}$, Σ, $q_0^{|}$, $F^{|}$, $\delta^{|}$), where

- $Q^{|} = 2^{Q}$ (power set of Q) (any state in $Q^{|}$ is denoted by [q_1, q_2, ...q_i] where q_l, q_2, ... $q_i \in Q$)
- $q_0^{|} = [q_0]$
- $F^{|}$ is set of final states.

PROOF CONT...

- As M (NFA) starts with initial state q_0. $q_0^{|}$ is defined as $[q_0]$.
- In $M^{|}$ (DFA) the final state ($F^{|}$) can be subset of *Q* containing all final states of F.
- Now we define

DFA

$\delta^{|}([q_1, q_2, \ldots\ldots q_i], a) = \delta(q_1, a) \cup \delta(q_2, a) \cup \ldots\ldots \delta(q_i, a)$

NFA

equivalently,

$\delta^{|}([q_1, q_2, \ldots. q_i], a) = [p_1, p_2, \ldots. p_j]$

if and only if

$\delta(\{q_1, q_2, \ldots. q_i\}, a) = \{p_1, p_2, \ldots. p_j\}$

PROOF BY INDUCTION

Input string x

$\delta^{|}([q_0], x) = [p_1, p_2, \ldots. p_j]$ DFA

if and only if

$\delta(q_0, x) = \{p_1, p_2, \ldots. p_j\}$ NFA

Basis

- The result is trivial if string length is 0 i.e., $|x| = 0$
- since $q_0^{|} = [q_0]$. x must be ε

ε

$q_0^{|}$ $\quad$ $\boxed{q_0}$

PROOF BY INDUCTION

Induction

- Suppose the hypothesis is true for inputs of length m.
- Let xa be a string of length m +1 with a in Σ.

  Then $\delta'([q_0], xa) = \delta'(\delta'([q_0],x), a)$
- By induction hypothesis

  $$\delta'([q_0], x) = [p_1,p_2.............p_j]$$
- if and only if

  $$\delta(q_0, x) = \{p_1,p_2.............p_j\}$$

$m+1$

$|x| = m$

$|xa| = m+1$

$\delta(q_0, x)$

$\delta(xa$

$\delta(q_0, xa) = \delta(\delta(q_0, x), a)$

$\delta' a\ [\ ]$

$\{\ \}$

PROOF BY INDUCTION

- By definition of $\delta^{|}$

  $\delta^{|}([p_1,p_2..............p_j], a) = [r_1,r_2..............r_k]$
- if and only if

  $\delta(\{p_1,p_2..............p_j\}, a) = \{r_1,r_2..............r_k\}$
- Thus

  $\delta^{|}([q_0], xa) = [r_1,r_2..............r_k]$
- if and only if

  $\delta(q_0, xa) = \{r_1,r_2..............r_k\}$
- which establishes the inductive hypothesis.
- Thus L(M) = L(M1)

EXAMPLE

- Construct a DFA for the given NFA.

$$M=(\{q0, q1\},\{0,1\}, \delta,q0,\{q1\})$$

States	Inputs	
	0	1
δ $\rightarrow q_0$	$\{q_0,q_1\}$	$\{q_1\}$
$*q_1$	ϕ	$\{q_0,q_1\}$

$M=(Q, \Sigma, \delta, S, F)$

$Q=\{q_0, q_1\}$

$\Sigma=\{0,1\}$

$S \rightarrow q_0$ $F=\{q_1\}$

$M'=(Q', \Sigma, \delta', S', F')$

$Q' =$ ____

$\Sigma=\{0,1\}$

$\delta' - ?$

$S' \rightarrow [q_0]$

$F' - ?$

DFA

$\delta'([q_0],0) = [q_0 q_1]$

$\delta'([q_0],1) = [q_1]$

$\delta'([q_0q_1],0) = [q_0q_1]$

$\delta'([q_0q_1],1) = [q_0q_1]$

$\delta(q_0,0)=\{q_0,q_1\}$

$\delta(q_0,1)=\{q_1\}$

$\delta(\{q_0,q_1\},0)$

$=\delta(q_0,0) \cup \delta(q_1,0)$

$=\{q_0,q_1\} \cup \{\phi\}$

$=\{q_0,q_1\}$

$\delta(\{q_0,q_1\},1)$

$=\{q_0q_1\}$

δ'	0	1
$\rightarrow[q_0]$	$[q_0q_1]$	$[q_1]$
$*[q_0q_1]$	$[q_0q_1]$	$[q_0q_1]$
$*[q_1]$	ϕ	$[q_1]$

$\delta'([q_1],1) = [q_1]$

$\delta'([q_1],0) = \phi$

$\delta(q_1,1) = \{q_1\}$

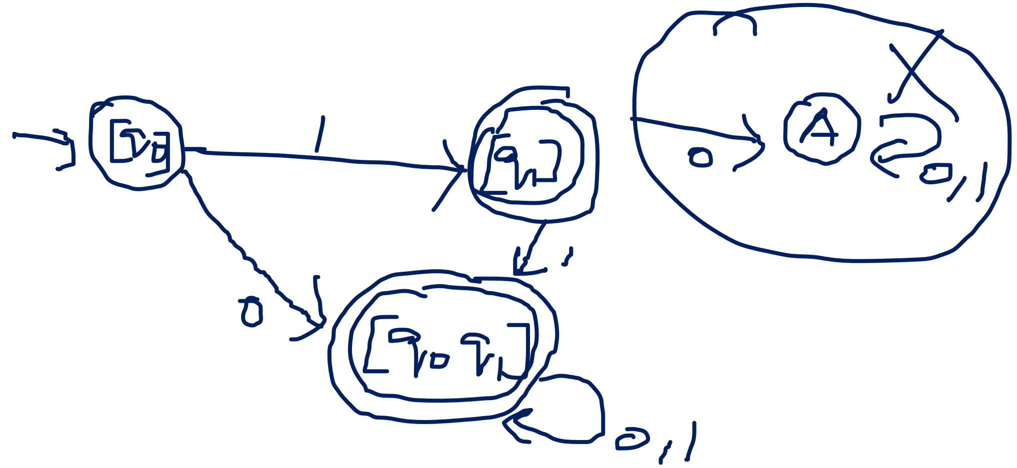
ϕ

EXAMPLE CONT…

- Construct a DFA for the given NFA.

$$M=(\{q0, q1\},\{0,1\}, \delta,q0,\{q1\})$$

States	Inputs	
	0	1
q_0	$\{q_0,q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_0,q_1\}$



States	Inputs	
	0	1
$\rightarrow[q_0]$	$[q_0\ \ q_1]$	$[q_1]$
$*[q_1]$	$\underline{\phi}$	$[q_0\ \ q_1]$
$*[q_0\ \ q_1]$	$[q_0\ \ q_1]$	$[q_0\ q_1]$

EXAMPLE

- Construct a DFA for the given NFA

M = ({q0, q1, q2}, {0, 1}, δ, q0, {q2})

States	Inputs	
	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_2\}$
q_1	$\{q_0\}$	$\{q_1\}$
$*q_2$	ϕ	$\{q_0, q_1\}$

EXAMPLE CONT..

- Construct a DFA for the given NFA

M = ({q0, q1, q2}, {0, 1}, δ, q0, {q2})

States	Inputs	
	0	1
q_0	$\{q_0, q_1\}$	$\{q_2\}$
q_1	$\{q_0\}$	$\{q_1\}$
q_2	ϕ	$\{q_0, q_1\}$

EXAMPLE CONT…

- Construct a DFA for the given NFA

M = ({*q*0, *q*1, *q*2}, {*0, 1*}, δ, *q*0, {*q*2})

States	Inputs	
	0	1
q_0	$\{q_0, q_1\}$	$\{q_2\}$
q_1	$\{q_0\}$	$\{q_1\}$
q_2	ϕ	$\{q_0, q_1\}$

States	Inputs	
	a	b
$[q_0]$	$[q_0, q_1]$	$[q_2]$
$[q_2]$	ϕ	$[q_0\ q_1]$
$[q_0 q_1]$	$[q_0 q_1]$	$[q_1\ q_2]$
$[q_1\ q_2]$	$[q_0]$	$[q_0\ q_1]$

TEST YOUR KNOWLEDGE

- Construct the equivalent DFA following NFA given in transition table:

	inputs	
states	0	1
→p	{p,q}	{p}
q	{r}	{r}
r	{s}	-
*s	{s}	{s}

DFA

States	Inputs	
	0	1
q_0	$\{q_0, q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_0, q_1\}$

SUMMARY

- Equivalence of DFA and NFA
- How to convert NFA to a DFA

LEARNING OUTCOME

On successful completion of this topic, the student will be able to:

- Equivalence of NFA and DFA (K3)

REFERENCE

- Hopcroft J.E., Motwani R. and Ullman J.D, “Introduction to Automata Theory, Languages and Computations”, Second Edition, Pearson Education, 2008