PUSH DOWN AUTOMATA

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AP/CSE

LEARNING OBJECTIVE

- To Design pushdown automata for any CFL (K3)
  - To understand what is PDA

$\delta(q, a) = p$

INTRODUCTION

- The regular languages →the finite automaton.
- Context free language → push down automata.
- Finite automata cannot recognize all languages. Because some languages are not regular.
- Finite automata have strictly finite memories, whereas recognition of context free language may require storing an unbounded amount of information.
- Push down automata is a machine similar to finite automata that will accept context free languages, except more powerful.

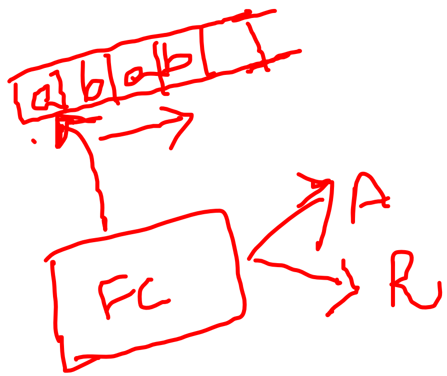
EXAMPLE

- $L=\{a^n b^n : n \geq 0\}$ CFL CFG
- $L = \{ww^R : w \in \{a,b\}^*\}$

pumping lemma for RL

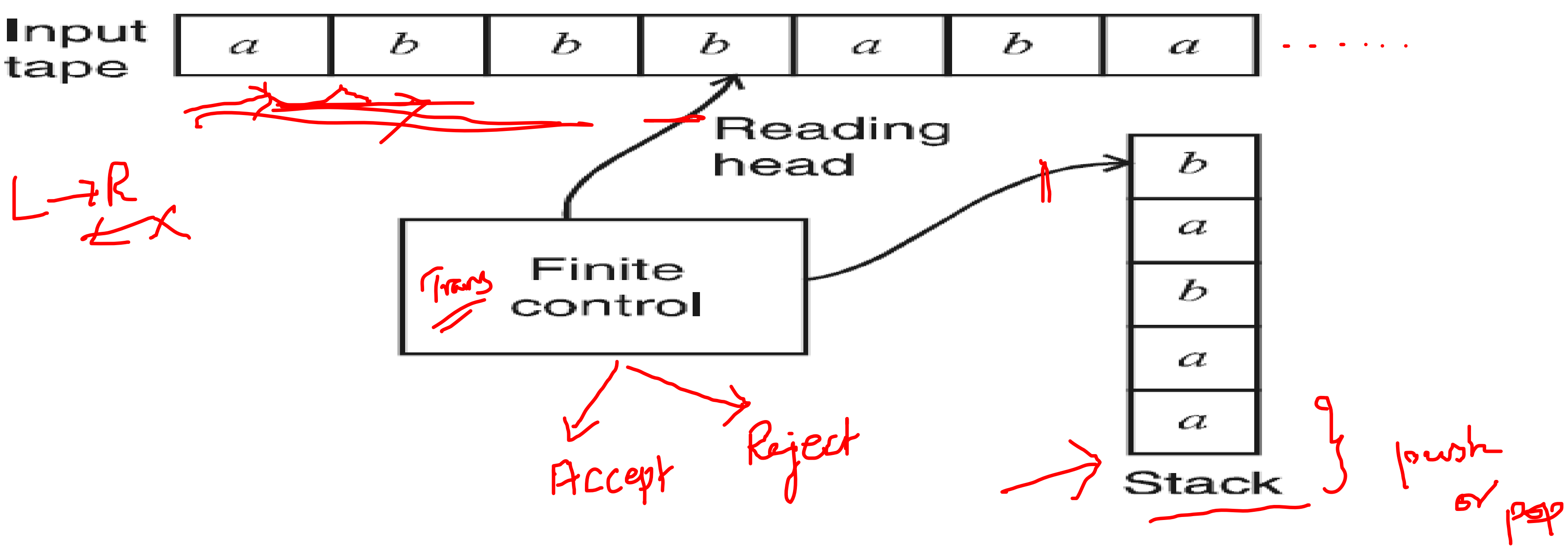
CFL

CFL → PDA



PUSH DOWN AUTOMATA

- Finite automaton with control of both an input tape and a stack (or) Last in-first out (Lifo) list.



COMPARE FA AND PDA

- $\delta(p, a) = q$ → M is in state p, then on reading 'a' from input tape go to state q.
- $\delta(p, \varepsilon) = q$ → M is in state p, goes to state q, without consuming input.

DFA : $Q \times \Sigma \rightarrow Q$

NFA : $Q \times \Sigma \rightarrow 2^Q$

ε-NFA → $Q \times \{\Sigma \cup \{\varepsilon\}\} \rightarrow 2^Q$

$S \rightarrow AB$

$A \rightarrow BS \mid b$

$B \rightarrow SA \mid a$

GNF

COMPARE FA AND PDA

- $\delta(p, a, \beta)=\{(q, \gamma)\}$ → M is in state p, the symbol read from input tape is 'a',and β is on top of stack, goes to state q, and replace β by γ on top of stack.
- $\delta(s, a, \varepsilon)= \{(s, a)\}$ → M is in state s, reads 'a', remains in state s and push a onto stack (e-empty stack).
- $\delta(s, c, \varepsilon) = \{(f, \varepsilon)\}$ → if read 'c' in state s and stack is empty, goes to final state f and nothing to push onto stack.
- $\delta(s, \varepsilon, \varepsilon) = \{(f, \varepsilon)\}$
- PDA's are non-deterministic.

$\delta(q_1, a, b) = \{(p, X)\}$

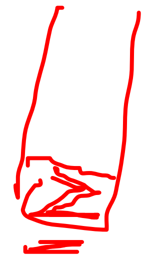
$\delta($c/s, i/p, top sta$) = ($n/s, newstack up$)$

Repla

$Q \times \Sigma \to 2^Q$

DEFINITION

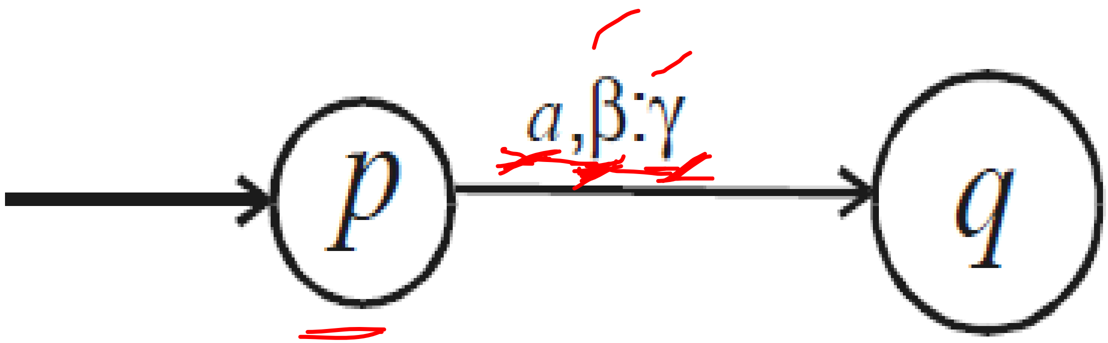
- $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$, where
  - Q is a finite set of states.
  - Σ is finite set of alphabet
  - Γ is finite set of stack alphabet
  - $q_0 \in Q$ is the start state (or) initial state
  - z_0 in Γ is a particular stack symbol called start symbol. $Z_0 \in \Gamma$
  - $F \subseteq Q$ is the set of final (or) favorable states.
  - $\delta \rightarrow Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma^* \rightarrow 2^{Q \times \Gamma^*}$

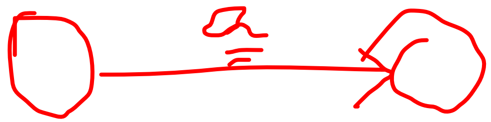


$Q \times \Sigma \cup \{\varepsilon\} \times \Gamma^* \rightarrow 2^{Q \times \Gamma^*}$

TRANSITION DIAGRAM

- $\delta(p, a, \beta) = \{(q, \gamma)\}$





INSTANTANEOUS DESCRIPTION (ID)

- An ID is a triple (q, w, γ) where
  - q is the current state
  - w is the remaining input
  - γ is the stack contents.
- P = (Q, Σ, Γ, δ, q_0, z_0 ,F)
- The instantaneous descriptions of pushdown automata is such that

  (q1, aw, bx) |– (q2, w, yx) is possible if and only if (q2, y)∈δ (q, a, b)

$\delta(q_0, 1001) = \delta(q_1, 001)$

$\delta(q, a, b) = \{(p, \cancel{X})\}$

$(q, aba, bYZ) \vdash (p, ba, XYZ)$

INSTANTANEOUS DESCRIPTION (ID)

- $(q, x, \alpha) \vdash^{*} (q_1, y, \beta)$ represents n moves,

  we write $(q, x, \alpha) \vdash^{n} (q_1, y, \beta)$

- In particular $(q, x, \alpha) \vdash^{0} (q, x, \alpha)$.

TWO TYPES OF TRANSITIONS

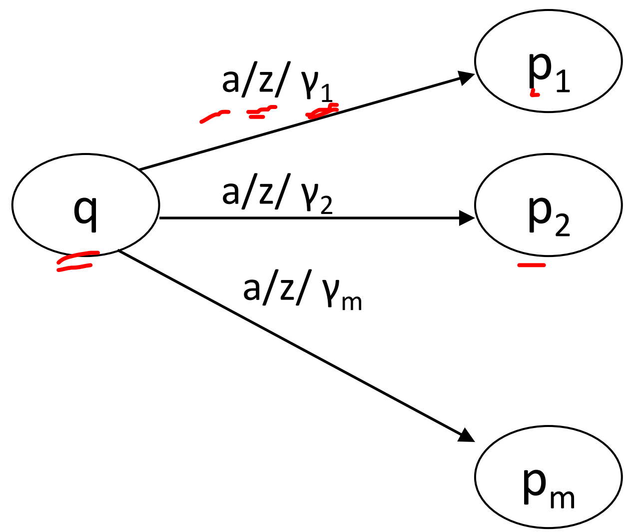
- $\delta(q, a, z) = \{(p_1, \gamma_1), \ldots\ldots\ldots (p_m, \gamma_m)\}$

  q and p_i, $1 \le i \le m$ are states,

  $a \in \Sigma$

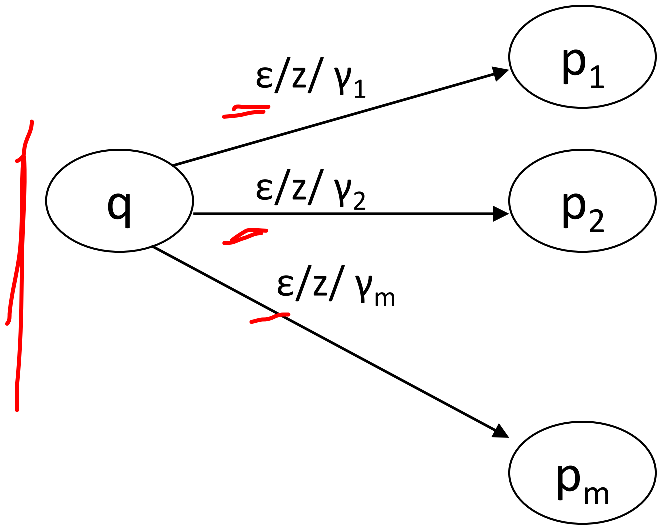
  $z \in \Gamma$

  $\gamma_i \in \Gamma^*$ $1 \le i \le m$,



TWO TYPES OF TRANSITIONS

- $\delta(q, \varepsilon, z) = \{(p_1, \gamma_1)\ (p_2, \gamma_2), \ldots\ldots.. (p_m, \gamma_m)\}$



PALINDROME L_{WWR}

- Q = {*q0, q1, q2*} = *No. of states*
- Σ = {0, 1} = Input symbol alphabet
- Γ = {0, 1, *z*}
- Start state = q_0
- Start stack symbol = z
- Final state = $\{q_2\}$

$L = \{00, 11, 0110, 1001, 110011 \ldots\}$

$L = \{ww^R \mid w \in (0|1)^*\}$

PALINDROME L_{WWR}

1. $\delta(q_0,0,Z)=\{(q_0,0Z)\}$
2. $\delta(q_0,1,Z)=\{(q_0,1Z)\}$

First symbol push on stack

3. $\delta(q_0,0,0)=\{(q_0,00)\}$
4. $\delta(q_0,0,1)=\{(q_0,01)\}$
5. $\delta(q_0,1,0)=\{(q_0,10)\}$
6. $\delta(q_0,1,1)=\{(q_0,11)\}$

Grow the stack by pushing new symbols on top of old (w-part)

7. $\delta(q_0,\varepsilon,0)=\{(q_1,0)\}$
8. $\delta(q_0,\varepsilon,1)=\{(q_1,1)\}$
9. $\delta(q_0,\varepsilon,Z)=\{(q_1,Z)\}$

Switch to popping mode (boundary between w and w^R)

10. $\delta(q_1,0,0)=\{(q_1,\varepsilon)\}$
11. $\delta(q_1,1,1)=\{(q_1,\varepsilon)\}$

Shrink the stack by popping matching symbols (w^R-part)

12. $\delta(q_1,\varepsilon,Z)=\{(q_2,Z)\}$

Enter acceptance state

EXAMPLE

1. $\delta(q_0, 0, Z) = \{(q_0, 0Z)\}$
2. $\delta(q_0, 1, Z) = \{(q_0, 1Z)\}$
3. $\delta(q_0, 0, 0) = \{(q_0, 00)\}$
4. $\delta(q_0, 0, 1) = \{(q_0, 01)\}$
5. $\delta(q_0, 1, 0) = \{(q_0, 10)\}$
6. $\delta(q_0, 1, 1) = \{(q_0, 11)\}$
7. $\delta(q_0, \varepsilon, 0) = \{(q_1, 0)\}$
8. $\delta(q_0, \varepsilon, 1) = \{(q_1, 1)\}$
9. $\delta(q_0, \varepsilon, Z) = \{(q_1, Z)\}$
10. $\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$
11. $\delta(q_1, 1, 1) = \{(q_1, \varepsilon)\}$
12. $\delta(q_1, \varepsilon, Z) = \{(q_2, Z)\}$

Check 1001

$(q_0, 1001, Z)$

$\vdash (q_0, 001, 1Z)$

$\vdash (q_0, 01, 01Z)$

$\vdash (q_1, 01, 01Z)$

$\vdash (q_1, 1, 1Z)$

$\vdash (q_1, \varepsilon, Z) \vdash (q_2, \varepsilon, Z)$ $q_2 \in F$ ∴ Accepted

- Check 100001

1001

q_0

$q_0 \xrightarrow{\varepsilon} q_1$ ε, Z

q_2

EXAMPLE

- Construct NPDA for

Palindrome L_{wwr}

$L = \{ ww^R \mid w \in (0/1)^* \}$ w Σ

$L = \{00, 11, 1001, 0110, \varepsilon, \ldots\ldots\}$ Σ

1) push w onto the stack

a) $\delta(q_0, 0, Z) = \{(q_0, 0Z)\}$

$\delta(q_0, 1, Z) = \{(q_0, 1Z)\}$

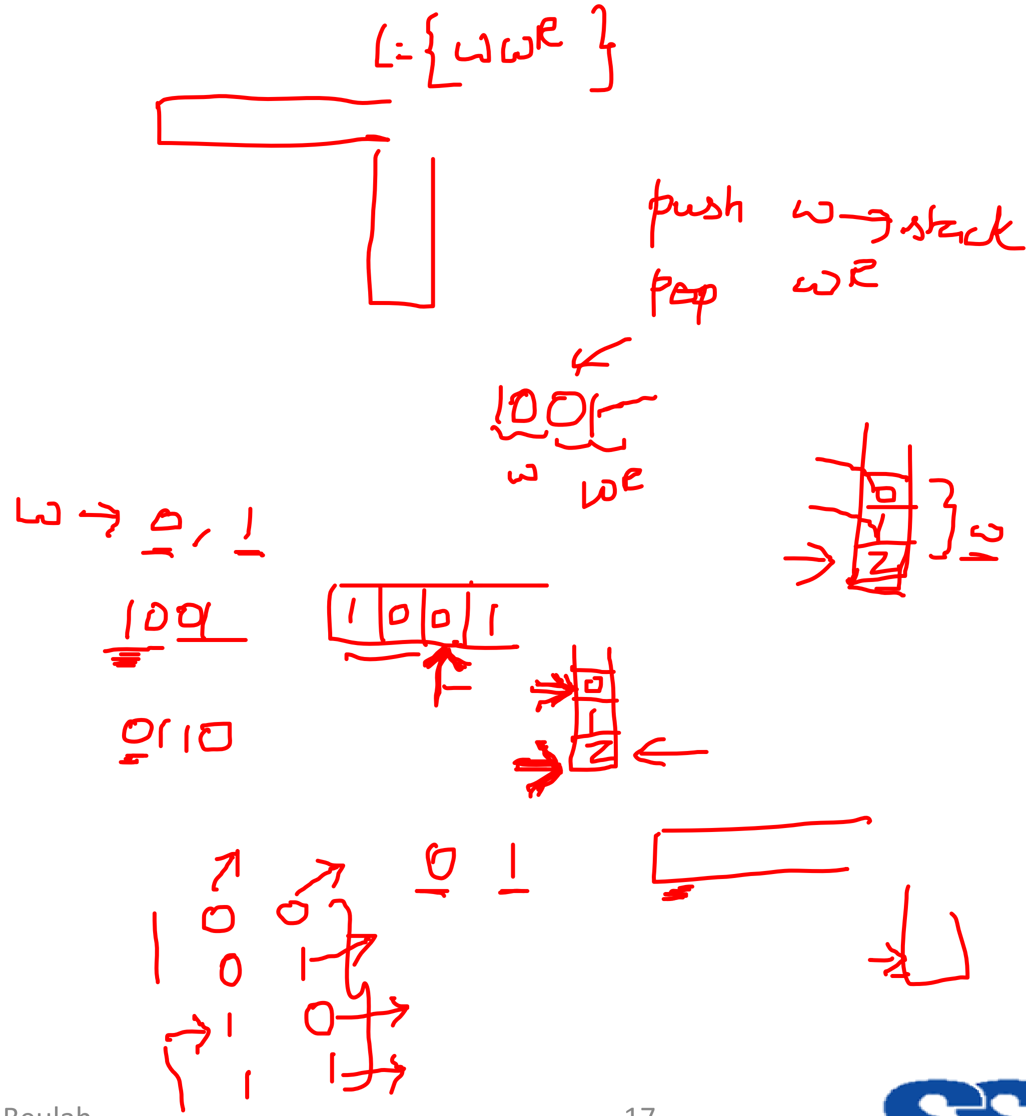
b) push remaining symbols of w onto the stack

$\delta(q_0, 0, 0) = \{(q_0, 00)\}$

$\delta(q_0, 0, 1) = \{(q_0, 01)\}$

$\delta(q_0, 1, 0) = \{(q_0, 10)\}$

$\delta(q_0, 1, 1) = \{(q_0, 11)\}$



(b) emptying the stack

$\delta(q_1, \varepsilon, Z) = \{(q_1, \varepsilon)\}$

(ii) shifting the state to do the pop operation

(i) $\delta(q_0, \varepsilon, 0) = \{(q_1, 0)\}$

$\delta(q_0, \varepsilon, 1) = \{(q_1, 1)\}$

$\delta(q_0, \varepsilon, Z) = \{(q_1, Z)\}$

(iii) pop operation

$\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 1, 1) = \{(q_1, \varepsilon)\}$

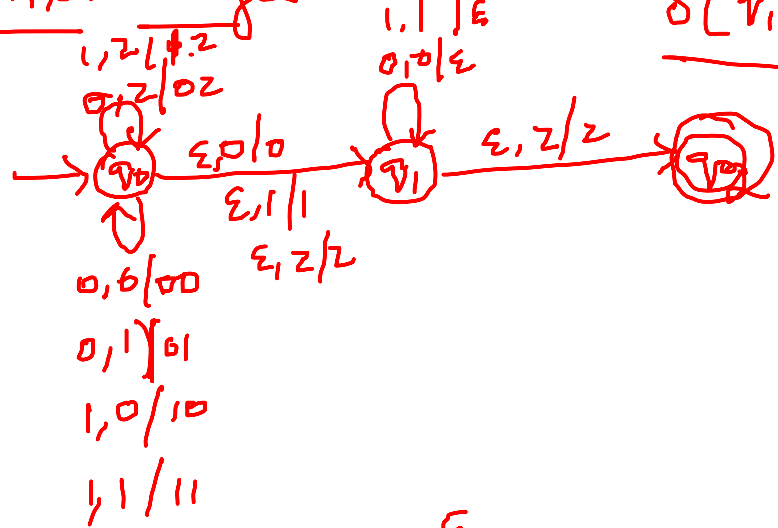
(iv) Reaching final state (a)

$\delta(q_1, \varepsilon, Z) = \{(q_2, Z)\}$

(c) reaching the FS & emptying the stack

$\delta(q_1, \varepsilon, Z) = \{(q_2, \varepsilon)\}$

Transition diagram



ε

$(q_0, \varepsilon, Z) \vdash (q_1, \varepsilon, Z)$

$\vdash (q_2, \varepsilon, Z)$ $q_2 \in F$ ∴ Accepted

0110

$(q_0, 0110, Z) \vdash (q_0, 110, 0Z) \vdash (q_0, 10, 10Z)$

$\vdash (q_1, 10, 10Z) \vdash (q_1, 0, 0Z) \vdash (q_1, \varepsilon, Z)$

$\vdash (q_2, \varepsilon, Z)$ $q_2 \in F$ ∴ Accepted

EXAMPLE

- Construct NPDA for

$L=\{0^n1^n / n>=1\}$

$n>=0$

$\delta(q_0, \epsilon, 0) = \{(q_1, 0)\}$

0011

i. Read 1st symbol

$\delta(q_0, 0, Z) = \{(q_0, 0Z)\}$

ii. Read remaining

$\delta(q_0, 0, 0) = \{(q_0, 00)\}$

iii. Switch state

$\delta(q_0, \epsilon, 0) = \{(q_1, 0)\}$

iv. Pop

$\delta(q_1, 1, 0) = \{(q_1, \epsilon)\}$

v. Final state

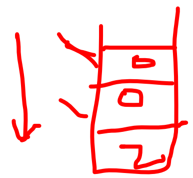
$\delta(q_1, \epsilon, Z) = \{(q_2, Z)\}$

EXAMPLE

$L = \{0^n 1^{2n} \mid n \geq 1\}$

N-PDA $\rightarrow$ emptying the stack

011



LANGUAGES OF PDA

- Acceptance by empty stack
  - Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA.
  - The language accepted by empty stack is denoted by $L_E(M)$

$$L_E(M) = \{w \mid (q_0, w, z_0) \vdash^* (p, \varepsilon, \varepsilon) \text{ for some } p \text{ in } Q\}$$

LANGUAGES OF PDA

- Acceptance by final state
  - Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA.
  - The language accepted by final state is denoted by $L_F(M)$

$$L_F(M) = \{w \mid (q_0, w, z_0) \mid\!\!-^* (p, \varepsilon, \gamma) \text{ for some } p \text{ in } F \text{ and } \gamma \text{ in } \Gamma^*\}$$

$p \in F$

LANGUAGES OF PDA

- Acceptance by final state and empty stack
  - Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA.
  - The language accepted by empty stack and final state is denoted L(M)

$$L(M) = \{w \mid (q_0, w, z_0) \mid\!-^* (p, \varepsilon, \varepsilon) \text{ for some } p \text{ in } F\}$$

$p \in F$

SUMMARY

- Discussion about PDA
- Language of a PDA
- ID for a string/word

TEST YOUR KNOWLEDGE

- What the does the given CFG defines?
  S→aSbS|bSaS|e and w denotes terminal
  a) wwr
  b) wSw
  c) Equal number of a's and b's
  d) None of the mentioned
- A grammar G=(V, T, P, S) is ___________ if every production taken one of the two forms:
  B→aC
  B→a
  a) Ambiguous
  b) Regular
  c) Non Regular
  d) None of the mentioned

REFERENCE

- Hopcroft J.E., Motwani R. and Ullman J.D, “Introduction to Automata Theory, Languages and Computations”, Second Edition, Pearson Education, 2008