UCS1524 – Logic Programming

Refinements of Resolution

Session Meta Data

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Session Objectives

- Understanding the concept of refinements of resolution in first order logic (FOL)
- Learning various refinements of resolution namely, P-resolution and N-Resolution, linear resolution, unit resolution, input resolution, SLD and LUSH resolutions

Session Outcomes

- At the end of this session, participants will be able to
  - apply different types of refinements of resolution in FOL.

Agenda

- Refinements of resolution
- Strategies & Restrictions
- P resolution & N resolution
- Linear resolution
- Input resolution
- Unit resolution
- SLD resolution
- LUSH resolution

Refinements of Resolution

- There are many possibilities to find two resolvable clauses for producing new resolvents.
- Among this huge number of possible resolution steps, only a few might lead to the derivation of the empty clause.
- Possibilities of improving the efficiency of the resolution algorithm is known as *refinements of resolution.*
- *Refinements of resolution are possible with Strategies and Constraints*

Strategies

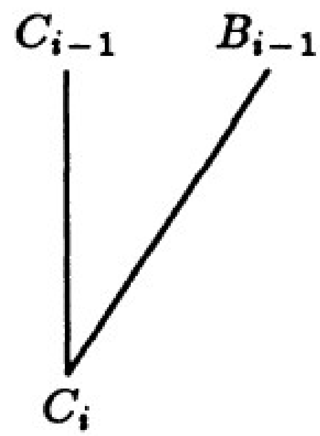
- Strategies are just heuristic rules which prescribe the order through which the search space has to be explored.
- Hence, the *size of the search space is not affected by a strategy.*
- *But for a* clever strategy, there is some hope that only a small portion of the space has to be searched until a solution (a derivation of the empty clause) is found.
- In the worst case, the entire space has to be searched.
- Example:
  - *unit preference strategy: whenever possible,* resolution steps are performed when one of the parent clauses is a *unit, i.e.* consists of one literal only.

Restrictions

- Strategies are theoretical, when combined with restrictions steps will be reduced
- The resolution restrictions simply forbid certain resolution steps if the clauses involved do not have a certain syntactic form, depending on the type of restriction.
- Therefore, the number of possible choices for the next resolution step is smaller as compared to the general case.
- Two types of restrictions
  - *P-restriction or P-resolution : requires that at least one of* the parent clauses has to be positive, i.e., consists of positive literals only.
  - *N-restriction or N-resolution : requires that at least one parent* clause is negative.

Linear resolution

The empty clause is *linearly resolvable* from a clause set F, *based on* a clause $C \in F$, if there is a sequence of clauses $(C_0, C_1, \ldots, C_n)$ such that $C_0 = C$, $C_n = \square$, and for $i = 1, 2, \ldots, n$,



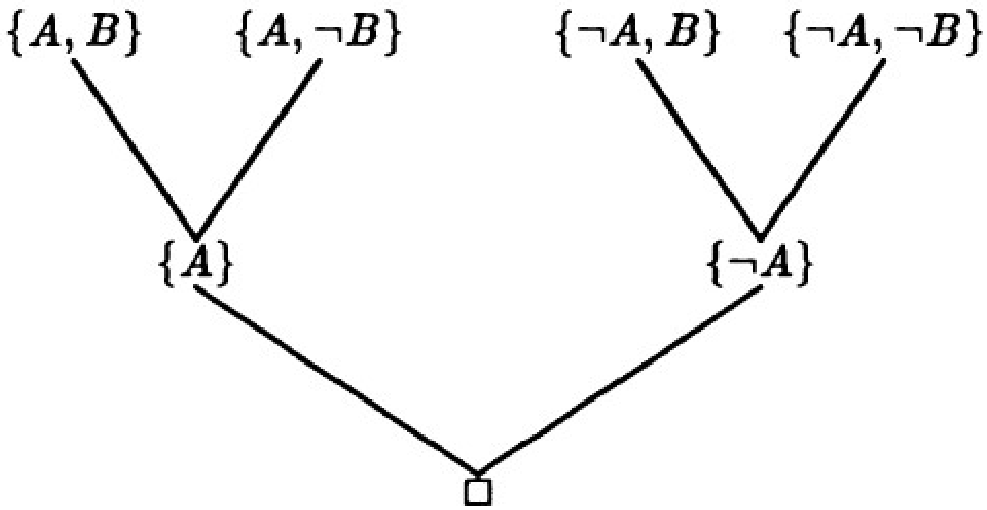
where the clause B_{i-1} (the so-called *side clause*) is either an element of F (i.e. an *input clause*) or $B_{i-1} = C_j$ for some $j < i$.

Linear resolution

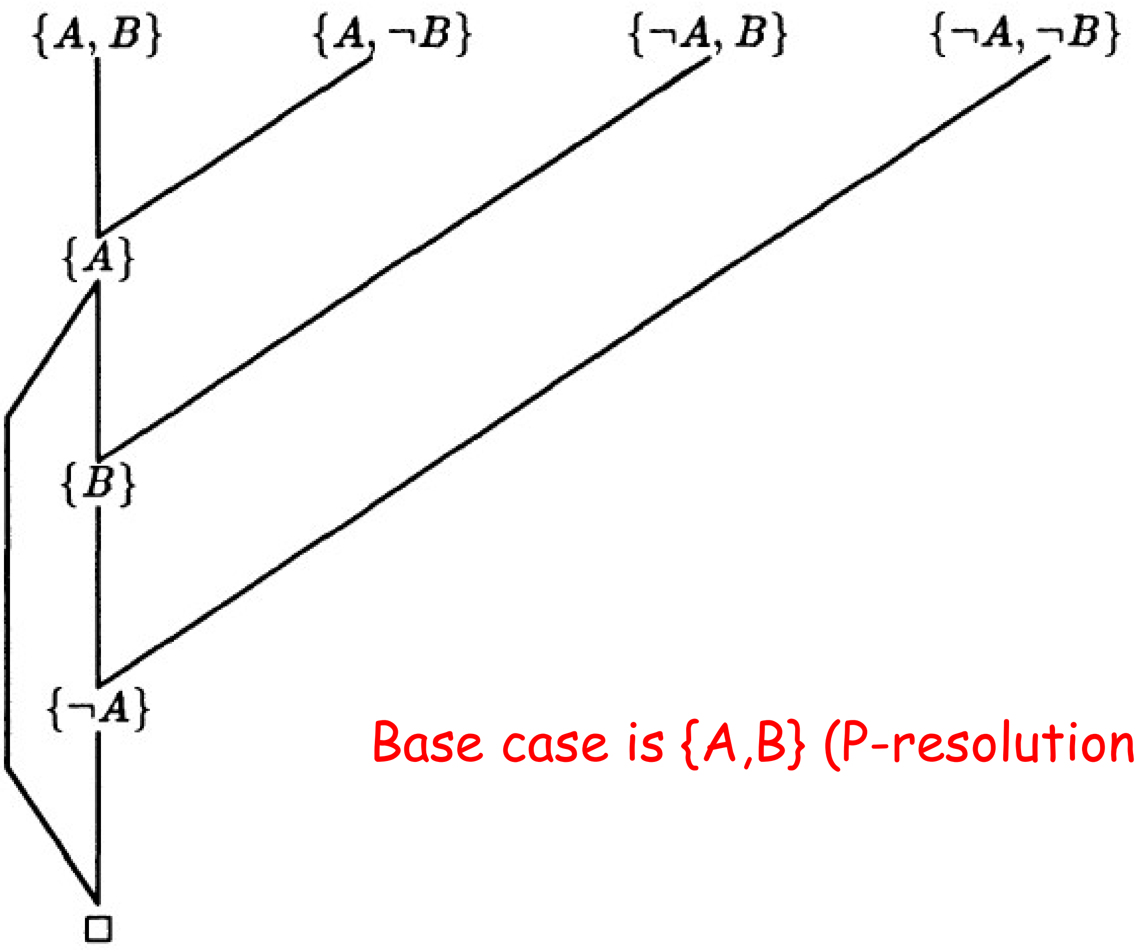
Example: Consider the unsatisfiable clause set

$$F = \{\{A, B\}, \{A, \neg B\}, \{\neg A, B\}, \{\neg A, \neg B\}\}.$$

The usual resolution refutation is given by the following diagram and takes 3 resolution steps.



Linear resolution



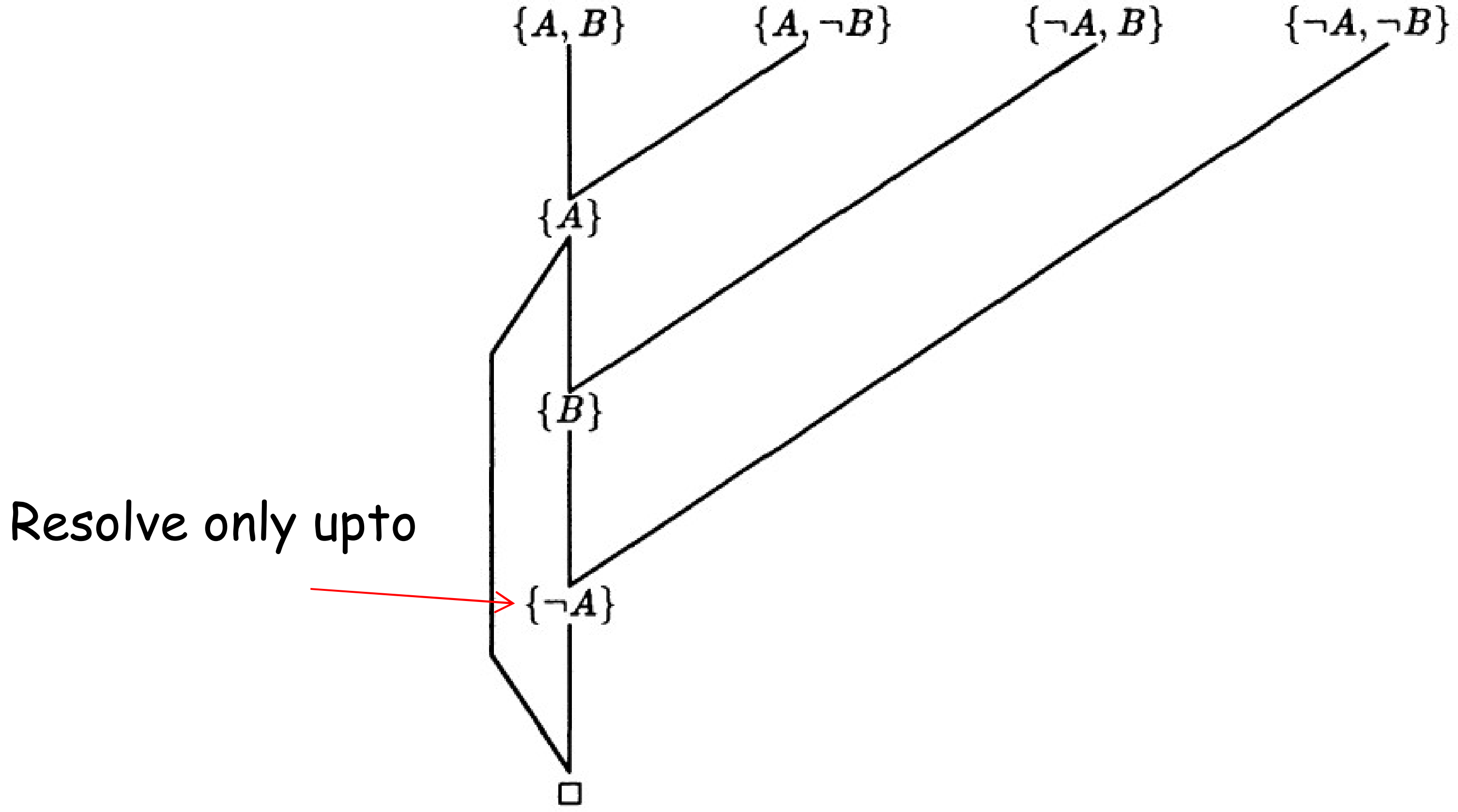
Base case is {A,B} (P-resolution: only positives)

Input Resolution

- The *input-restriction of resolution or input resolution*
  - In each resolution step, one of the parent clauses has to be an "input", i.e. an element of the original clause set F.
  - Input resolution proof necessarily is a linear resolution proof.
  - But in contrast to linear resolution, input resolution is not complete.
  - Example: Consider the unsatisfiable clause set (proved before)
    - The first resolution step produces a clause with a single literal. Each further step produces then, by the input restriction, single element clauses. Therefore, the empty clause is not derivable by input resolution.
    - However, the input resolution is complete when restricted to clause sets which contain only Horn clauses.

Input Resolution

$$F = \{\{A, B\}, \{A, \neg B\}, \{\neg A, B\}, \{\neg A, \neg B\}\}$$



Unit Resolution

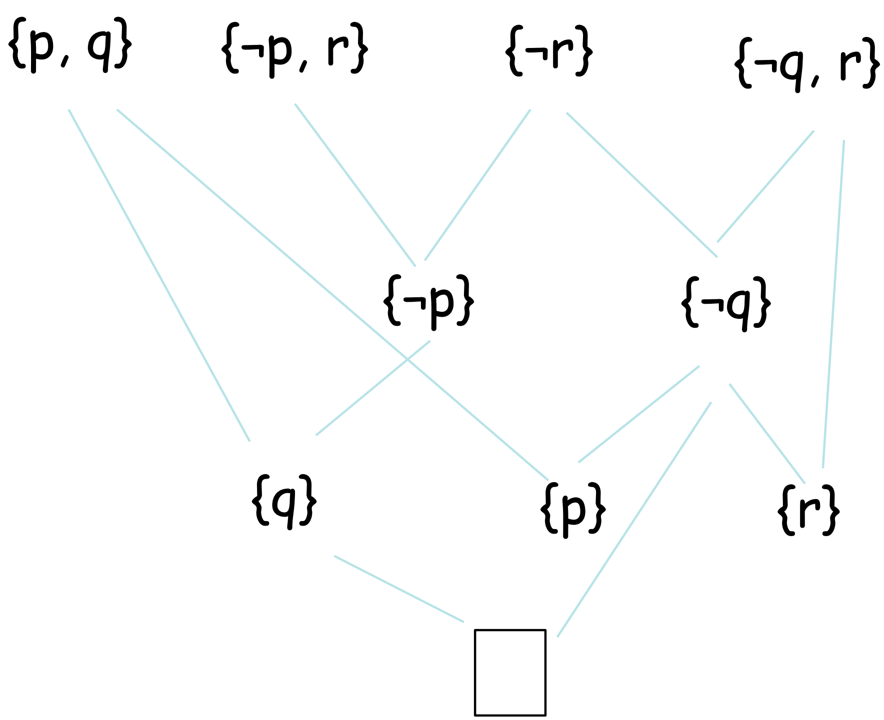
- Unit resolution
  - It is only allowed to produce a resolvent if at least one of the parent clauses is a unit, i.e. contains only a single literal.
  - This resolution restriction has the advantage that the size of the produced resolvents decreases as compared with the parent clauses.
  - Hence, unit resolution is working towards producing the empty clause which has size 0.
  - Unit resolution is also complete for Horn clauses
  - Is this formula complete with unit resolution

$$F = \{\{A, B\}, \{A, \neg B\}, \{\neg A, B\}, \{\neg A, \neg B\}\}$$

Unit Resolution

- Example

  $F=\{\{p,q\}, \{\neg p, r\}, \{\neg q,r\}, \{\neg r\}\}$



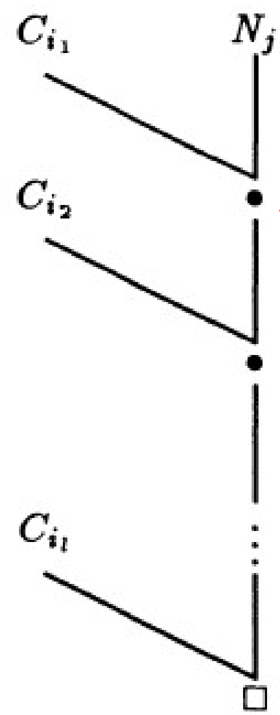
1.	$\{p, q\}$	Premise
2.	$\{\neg p, r\}$	Premise
3.	$\{\neg q, r\}$	Premise
4.	$\{\neg r\}$	Premise
5.	$\{\neg p\}$	2, 4
6.	$\{\neg q\}$	3, 4
7.	$\{q\}$	1, 5
8.	$\{p\}$	1, 6
9.	$\{r\}$	3, 7
10.	$\{\}$	6, 7

SLD Resolution

- *SLD-resolution (SLD = linear resolution with* selection function for definite clauses).
- This restriction is only defined for Horn clauses.
- SLD-resolutions are both input and linear resolutions which have a special form.
- The base clause must be a negative clause (a so-called *goal clause),* and in each resolution step, the side clause must be a non-negative input clause.
- A non-negative Horn clause is also named a *definite clause or a program clause.*

SLD Resolution

For example, let $F = \{C_1, C_2, \ldots, C_n, N_1, \ldots, N_m\}$ be a set of Horn clauses where $C_1, C_2, \ldots, C_n$ are the definite clauses and $N_1, \ldots, N_m$ are the goal clauses. An SLD-resolution of the empty clause must then have the form, for a suitable $j \in \{1, \ldots, m\}$ and for a suitable sequence $i_1, i_2, \ldots, i_l \in \{1, \ldots, n\}$.



intermediate results: can only be negative clauses

SLD-resolutions are always N-resolutions

LUSH Resolution

- SLD : linear resolution with selection function for definite clauses
  - the additional aspect of a *selection function is* mentioned
  - the presence of a selection function (which selects the next definite clause to be resolved with) is treated as combination of SLD-resolution with a special *strategy*

- *LUSH : linear* resolution with unrestricted selection for Horn clauses

Summary

- Refinements of resolution
- Strategies & Restrictions
- P resolution & N resolution
- Linear resolution
- Input resolution
- Unit resolution
- SLD resolution
- LUSH resolution

Check your understanding

- Check whether the given formula is complete and unsatistiable using linear resolution, input resolution, unit resolution and SLD resolution

$$F = \{\{A, B, \neg C\}, \{\neg A\}, \{A, B, C\}, \{A, \neg B\}\}$$