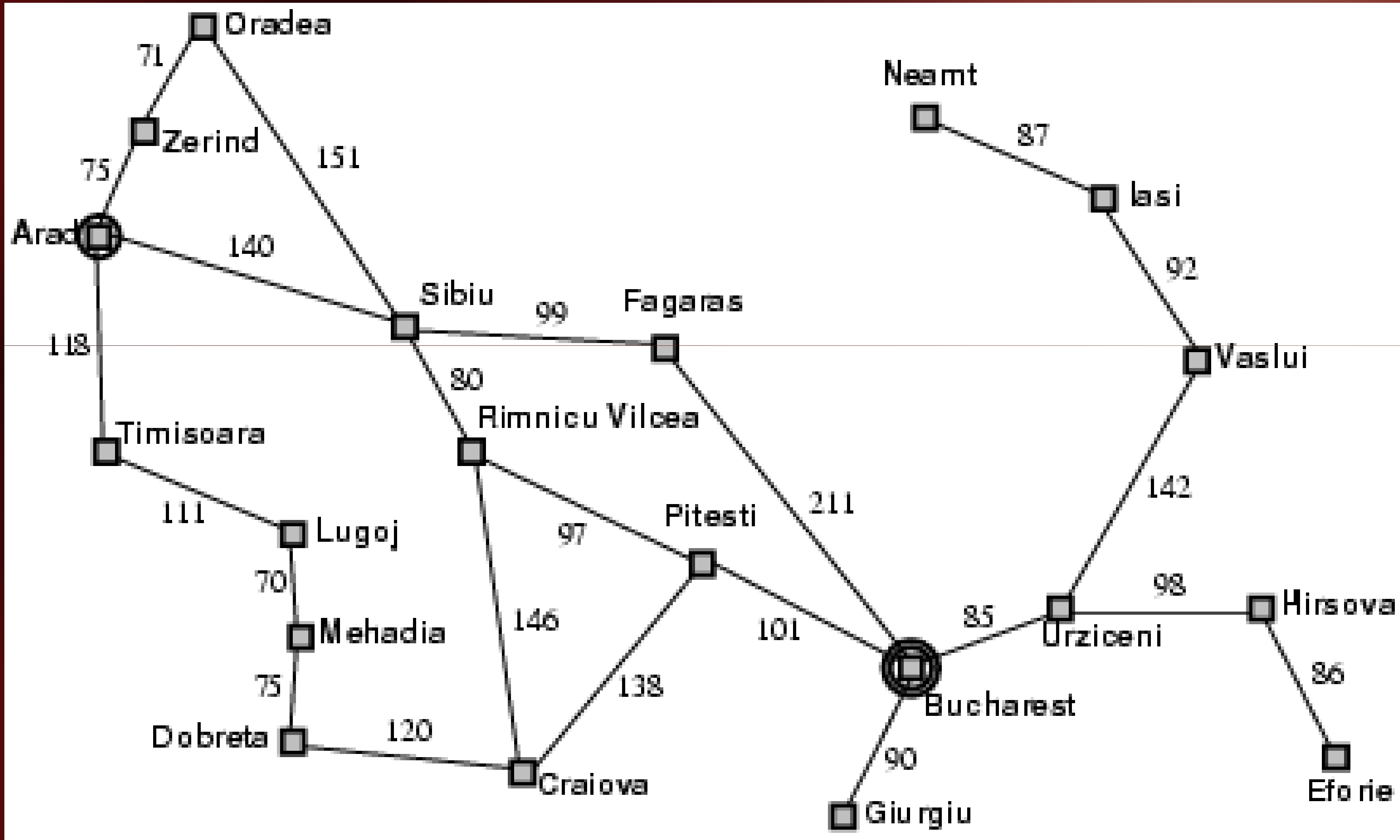
Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
-
- Formulate goal:
  - be in Bucharest

- Formulate problem:
  - states: various cities
  - actions: drive between cities
  -
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest
  -

Example: Romania



Setting the scene…

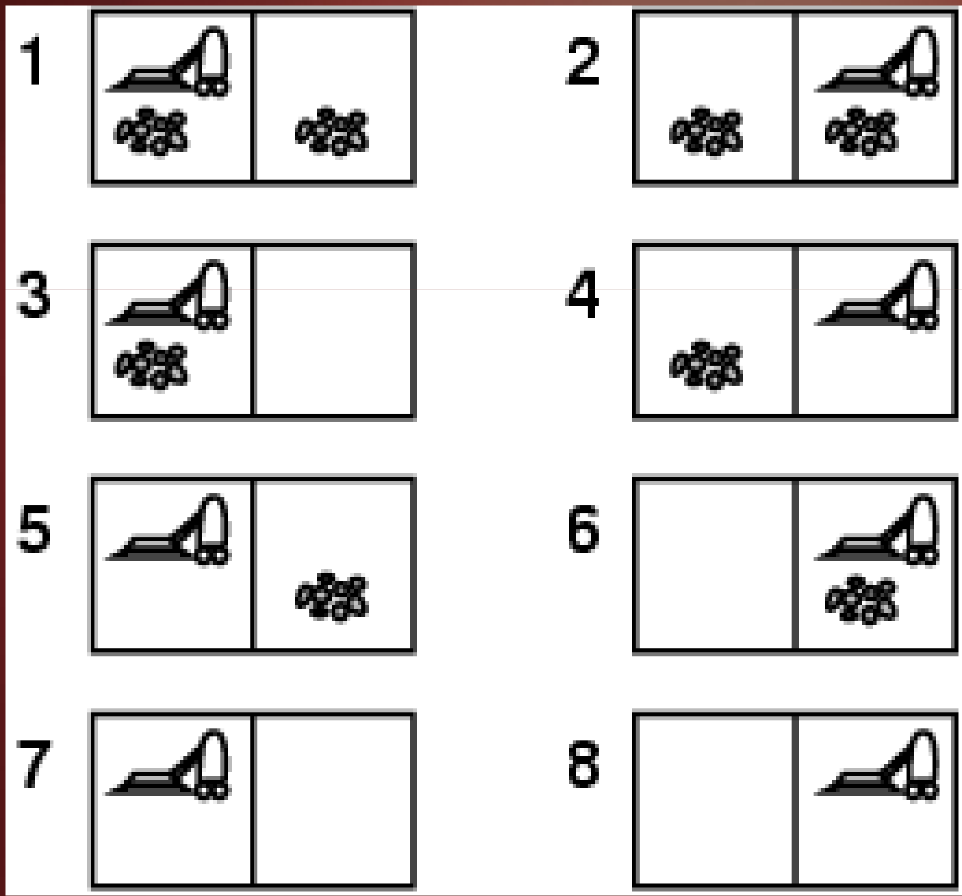
- Problem solving by searching
  - Tree of possible actions sequences.

- Knowledge is Power!
  - States
  - State transfers

Example: Vacuum World

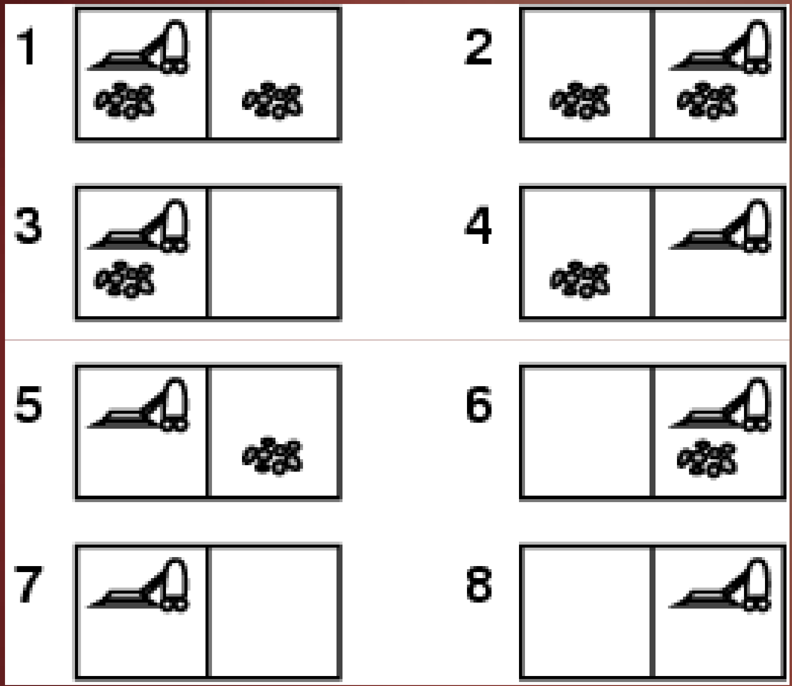
- Single-state Problem:
  - You know all.

- Start in #5
  - Solution? *[Right, Clean]*



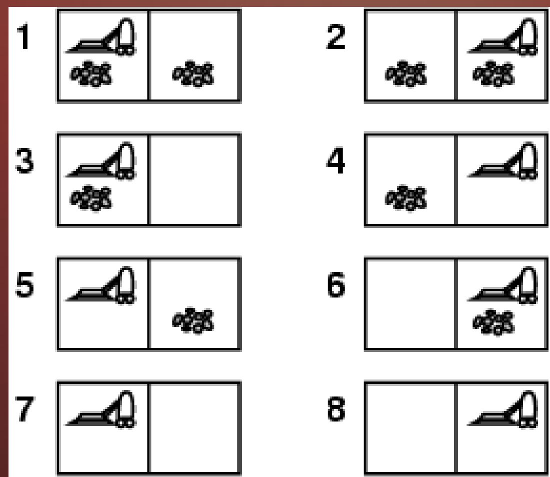
Example: vacuum world

- Multiple State Problem
  - Sensorless
- Start in {*1,2,3,4,5,6,7,8*}
  - Solution?
    [Right, Clean, Left, Clean]



Example: vacuum world

- Contingency
  - Nondeterminism: *Cleaning* may dirty a clean carpet.
  - Partially observable: Location, dirt at current location.
  - Percept: *[L, Clean],* i.e., start in #5 or #7
    Solution?
    *[Right, **if** dirt **then** Clean]*



Single-state problem formulation

A problem is defined by four items:

1. initial state e.g., "at Arad"

2. actions or successor function *S(x)* = set of action–state pairs
   - e.g., *S(Arad) = { <Arad → Zerind, Zerind>, <Arad → Timisoara, Timisoara>, … }*
   -

3. goal test, can be
   - explicit, e.g., *x* = "at Bucharest"
   - implicit, e.g., *Checkmate(x)*
   -

4. path cost (additive)
   - e.g., sum of distances, number of actions executed, etc.
   - *c(x,a,y)* is the step cost, assumed to be ≥ 0
   -

- A solution is a sequence of actions leading from the initial state to a goal state

Selecting a state space

- Real world is absurdly complex
  → state space must be abstracted for problem solving

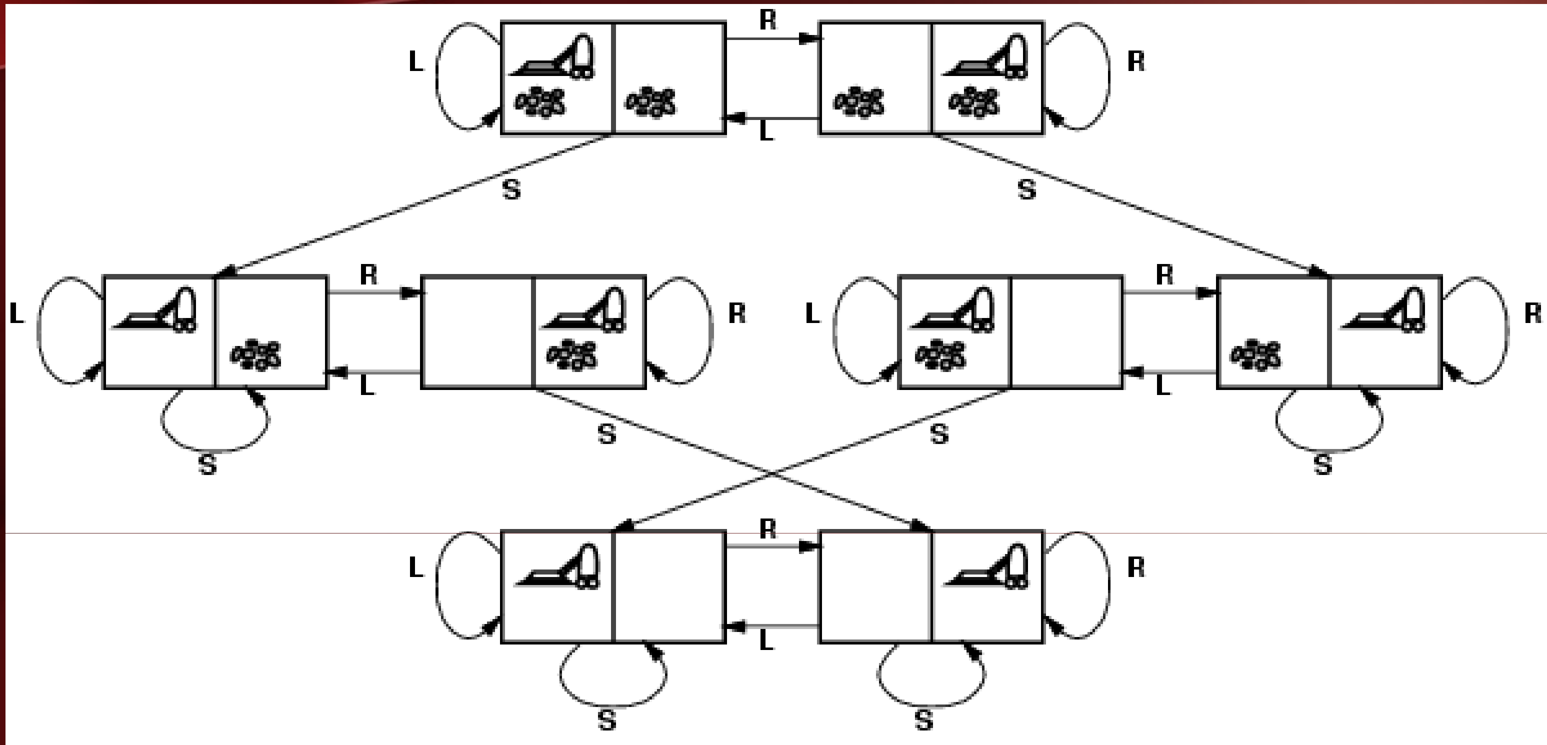
- (Abstract) state = set of real states

- (Abstract) action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.

- For guaranteed realizability, any real state "in Arad“ must get to some real state "in Zerind"
-

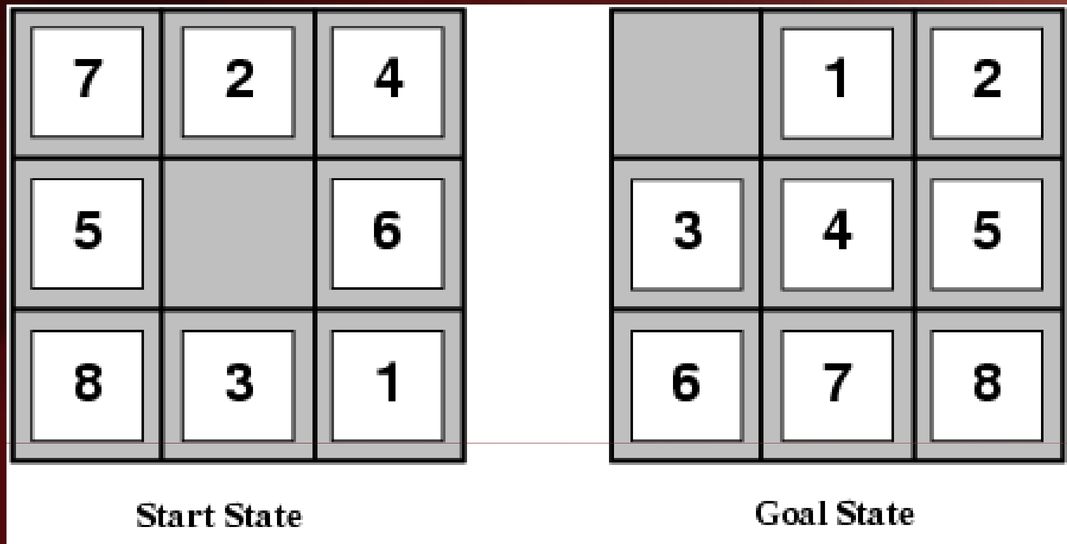
- (Abstract) solution =
  - set of real paths that are solutions in the real world
  -
- Each abstract action should be "easier" than the original problem

Vacuum world state space graph



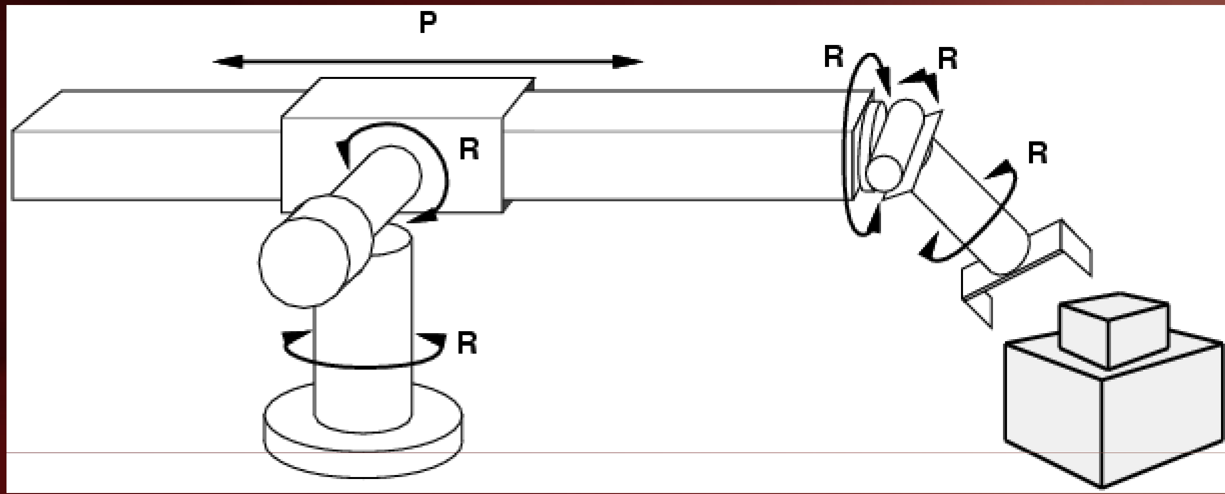
- States? Dirt and robot location
- Actions? *Left*, *Right*, *Clean*
- Goal test? No dirt at all locations
- Path cost? 1 per action

Example: The 8-puzzle



- States? Locations of tiles
- Actions? Move blank left, right, up, down
- Goal test? Given
- Path cost? 1 per move
-

Example: robotic assembly



- States? real-valued coordinates of robot joint angles parts of the object to be assembled
- Actions? continuous motions of robot joints
- Goal test? complete assembly
- Path cost? time to execute

Example Cryptarithmatic

```
  FORTY       Solution:   29786          F=2,
+   TEN                  + 850           O=9,
+   TEN                  + 850           R=7,
-------                  -----           etc.
  SIXTY                   31486
```

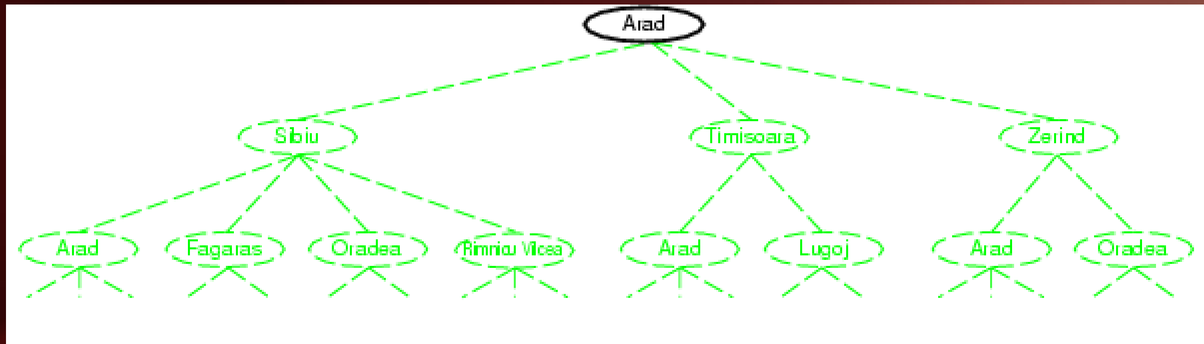
- States? A cryptharithmetic puzzle w/ some letters replaced with digits.
-
- Actions? Replacing a letter with an unused digit.
- Goal test? Puzzle contains only digits.
- Path cost? ZERO. All solutions equally valid.

Tree search algorithms

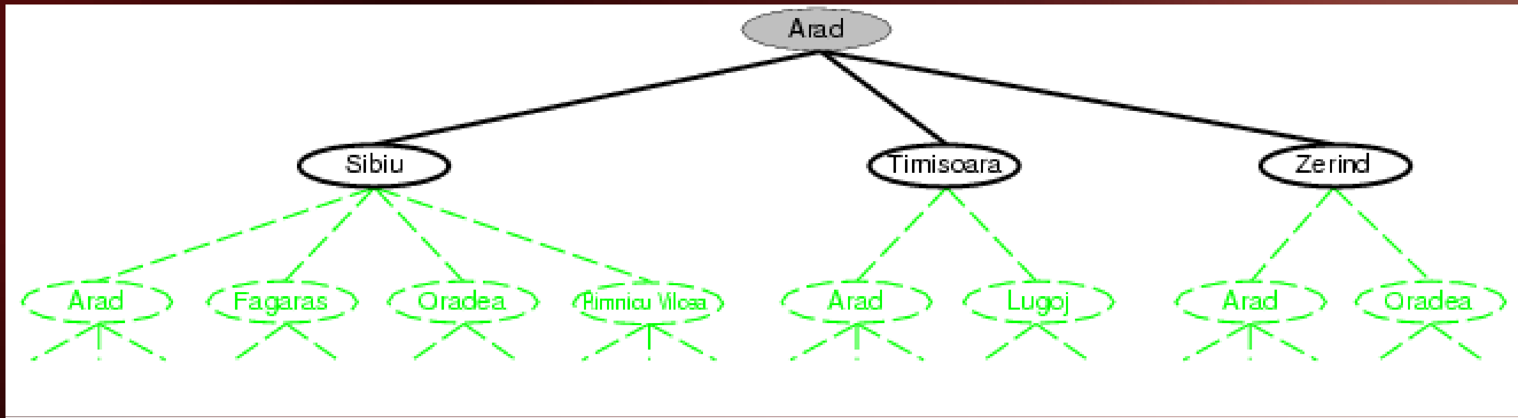
- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (a.k.a.~expanding states)
  -

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

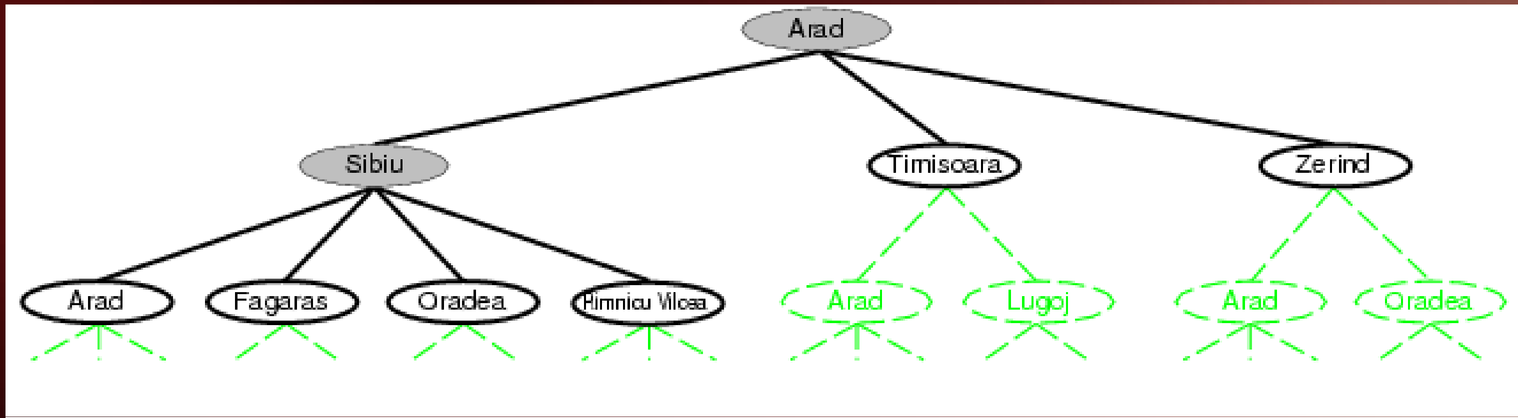
Tree search example



Tree search example

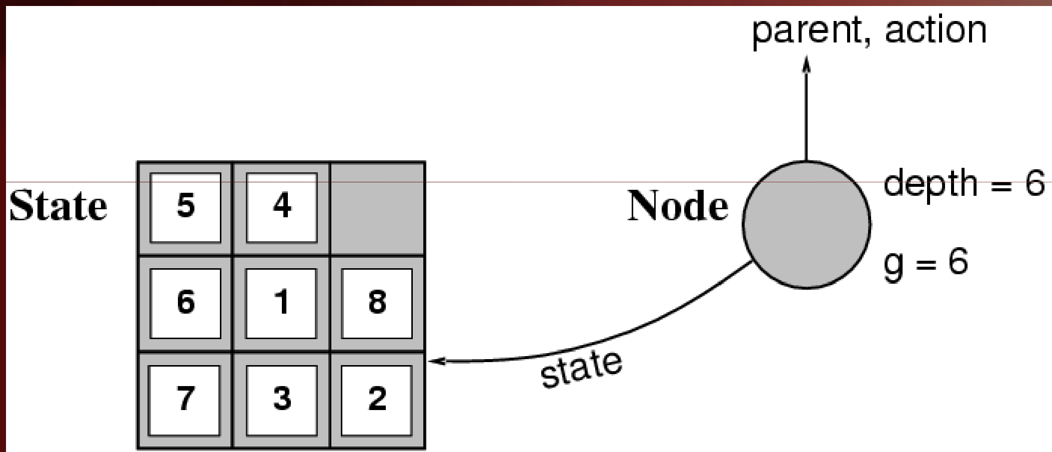


Tree search example



Implementation: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost *g(x)*, depth



- The `Expand` function creates new nodes, filling in the various fields, using the `SuccessorFn` of the problem to create the corresponding states.
-

Implementation: general tree search

function TREE-SEARCH(*problem, fringe*) **returns** a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
  loop do
    if *fringe* is empty **then return** failure
    node ← REMOVE-FRONT(*fringe*)
    if GOAL-TEST[*problem*](STATE[*node*]) **then return** SOLUTION(*node*)
    fringe ← INSERTALL(EXPAND(*node, problem*), *fringe*)

---

function EXPAND(*node, problem*) **returns** a set of nodes
  successors ← the empty set
  for each *action, result* **in** SUCCESSOR-FN[*problem*](STATE[*node*]) **do**
    s ← a new NODE
    PARENT-NODE[*s*] ← *node*; ACTION[*s*] ← *action*; STATE[*s*] ← *result*
    PATH-COST[*s*] ← PATH-COST[*node*] + STEP-COST(*node, action, s*)
    DEPTH[*s*] ← DEPTH[*node*] + 1
    add *s* to *successors*
  return *successors*

Search strategies

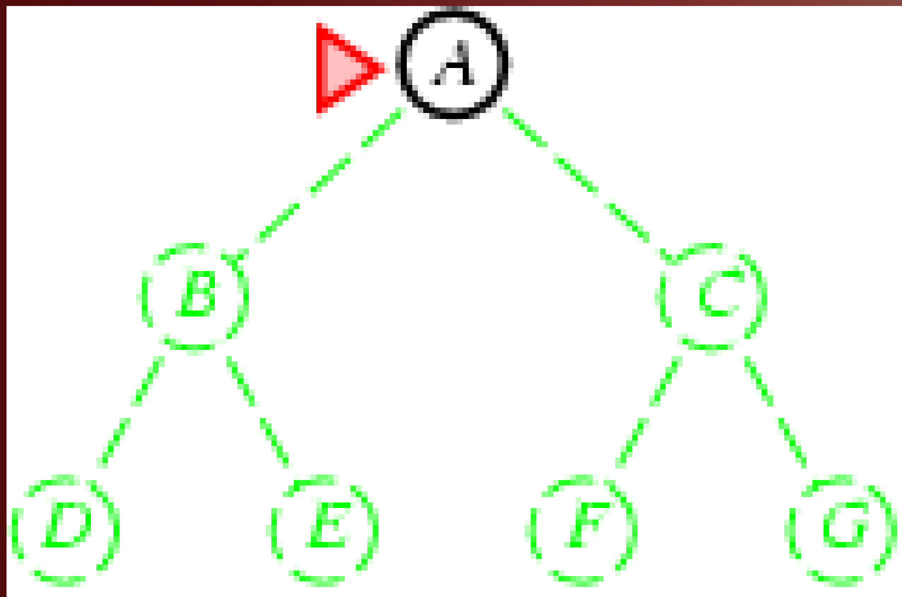
- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
  -
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)
  -

Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Depth-limited search
  - Iterative deepening search

Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end
  -



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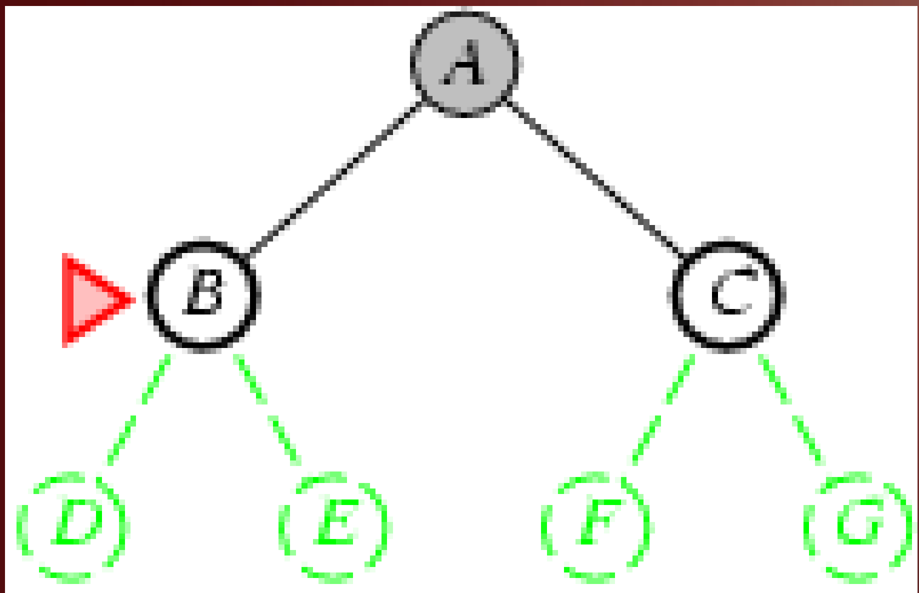

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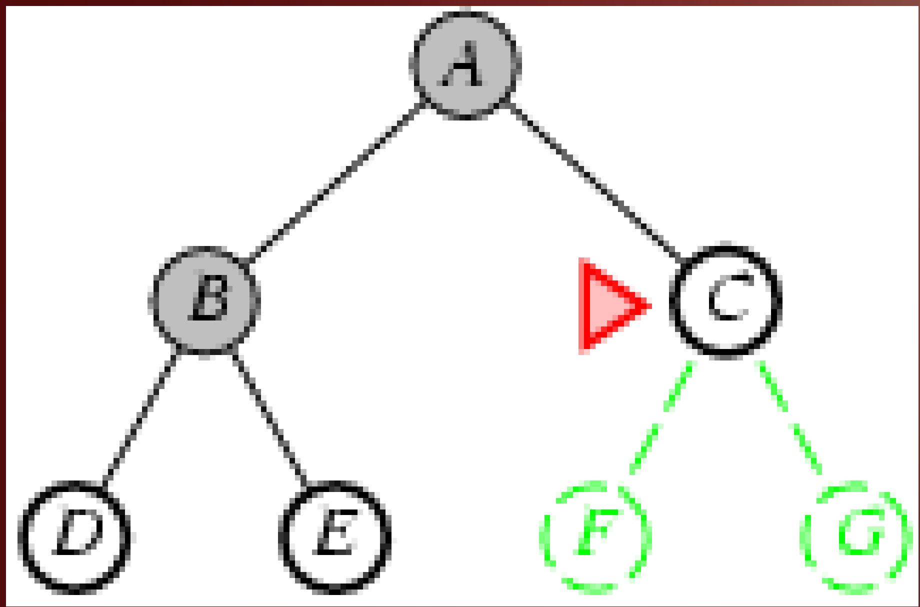

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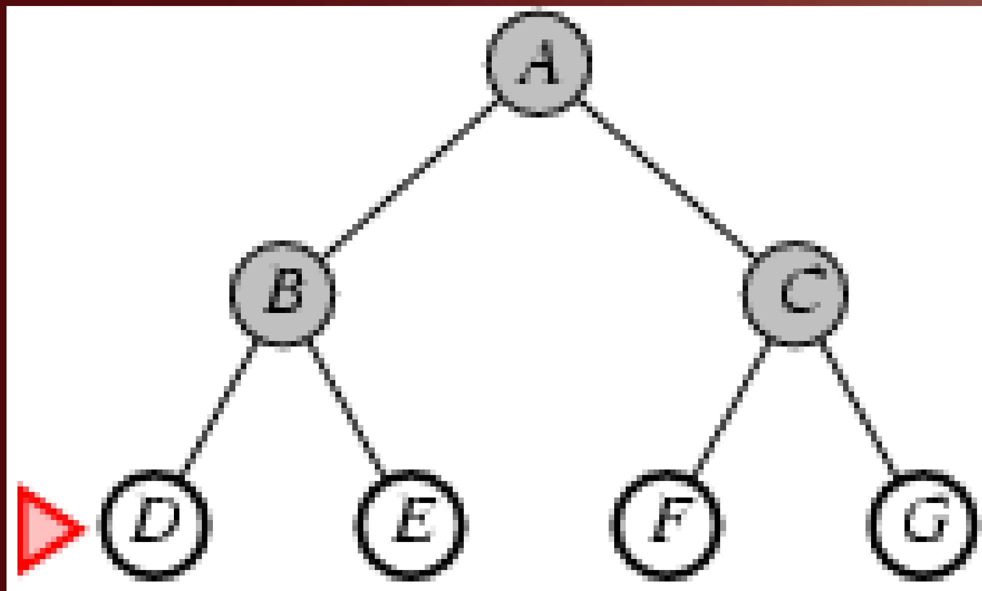


Properties of breadth-first search

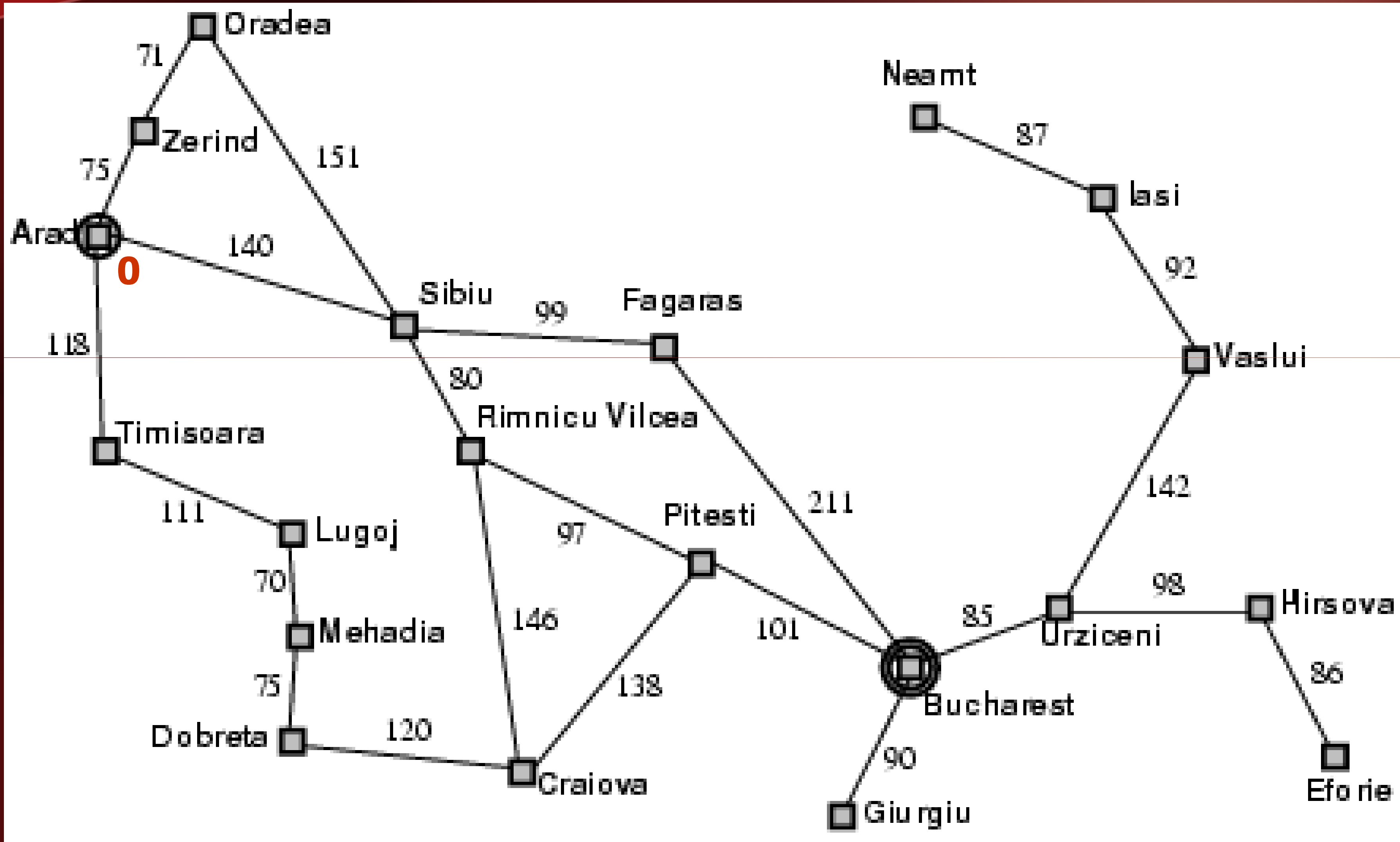
- Complete? Yes (if *b* is finite)
- Time? $1+b+b^2+b^3+\ldots +b^d + b(b^d-1) = O(b^{d+1})$
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost ± 1 per step)

Depth	Nodes	Time	Memory
0	1	1 millisecond	100 bytes
2	111	.1 seconds	11 kilobytes
4	11,111	11 seconds	1 megabyte
6	10^6	18 minutes	111 megabytes
8	10^8	31 hours	11 gigabytes
10	10^{10}	128 days	1 terabyte
12	10^{12}	35 years	111 terabytes
14	10^{14}	3500 years	11,111 terabytes

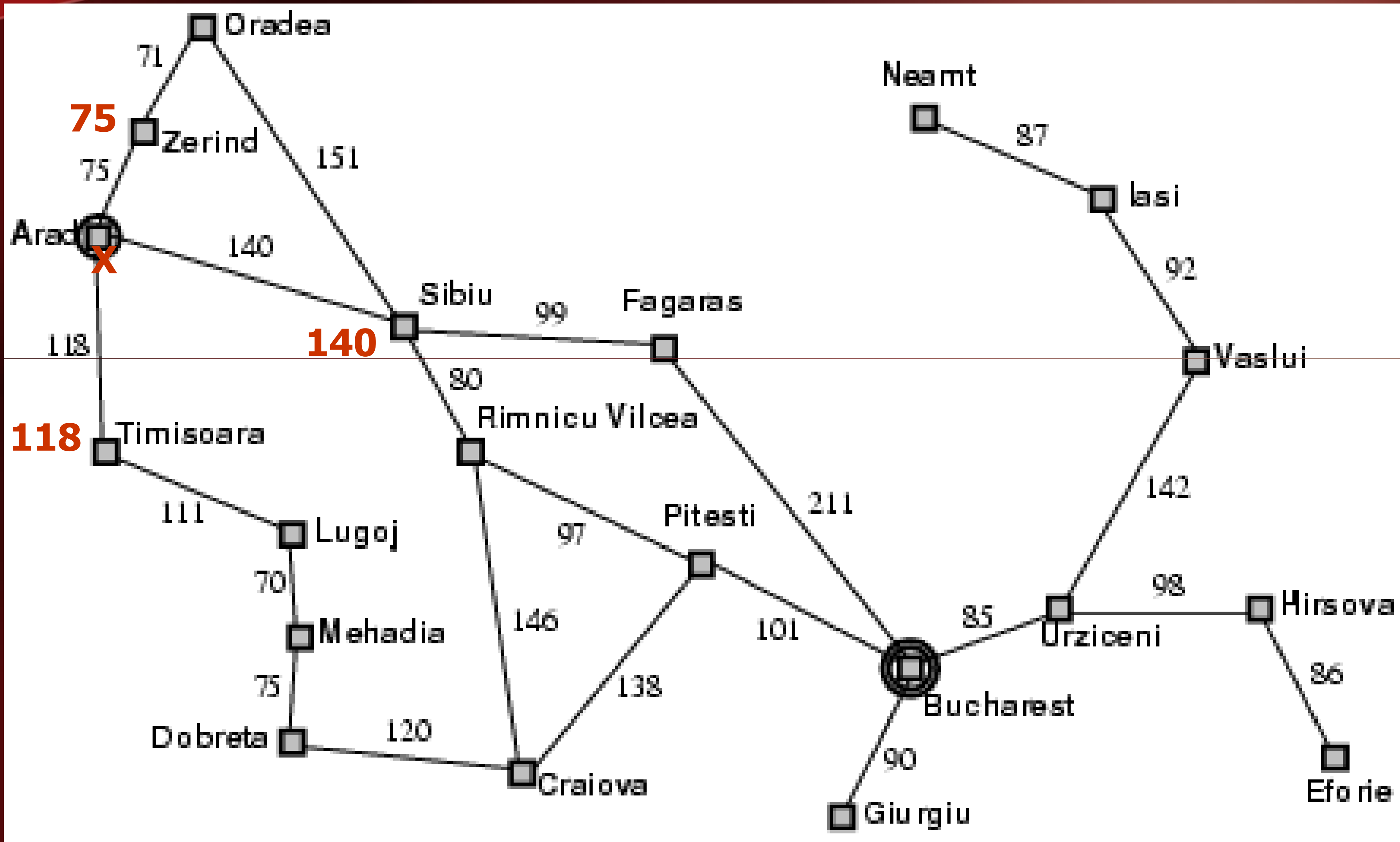
Uniform-cost search

- Expand least-cost unexpanded node
- Implementation:
  - *fringe* = queue ordered by path cost
- Equivalent to breadth-first if step costs all equal

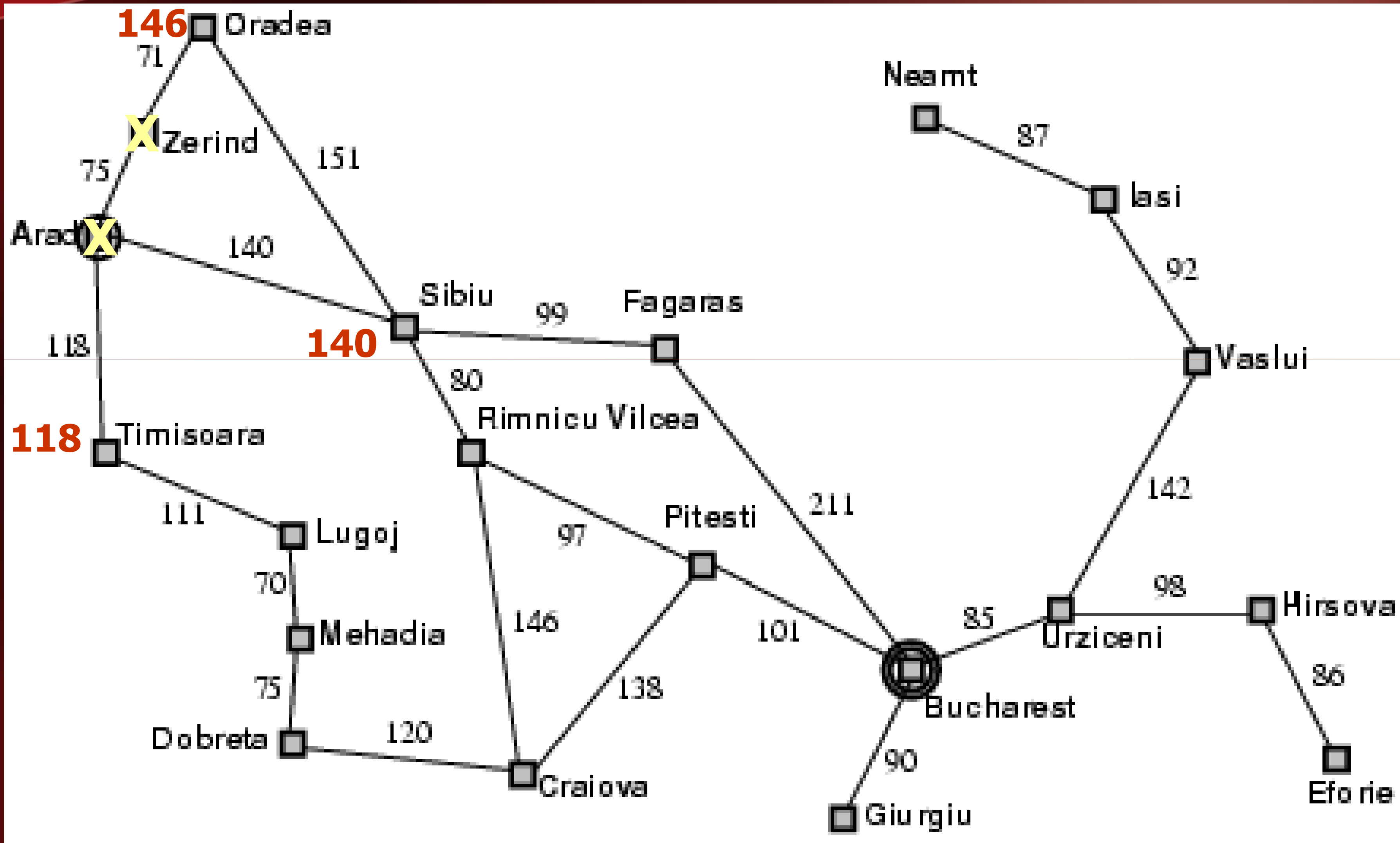
Uniform-cost search Sample



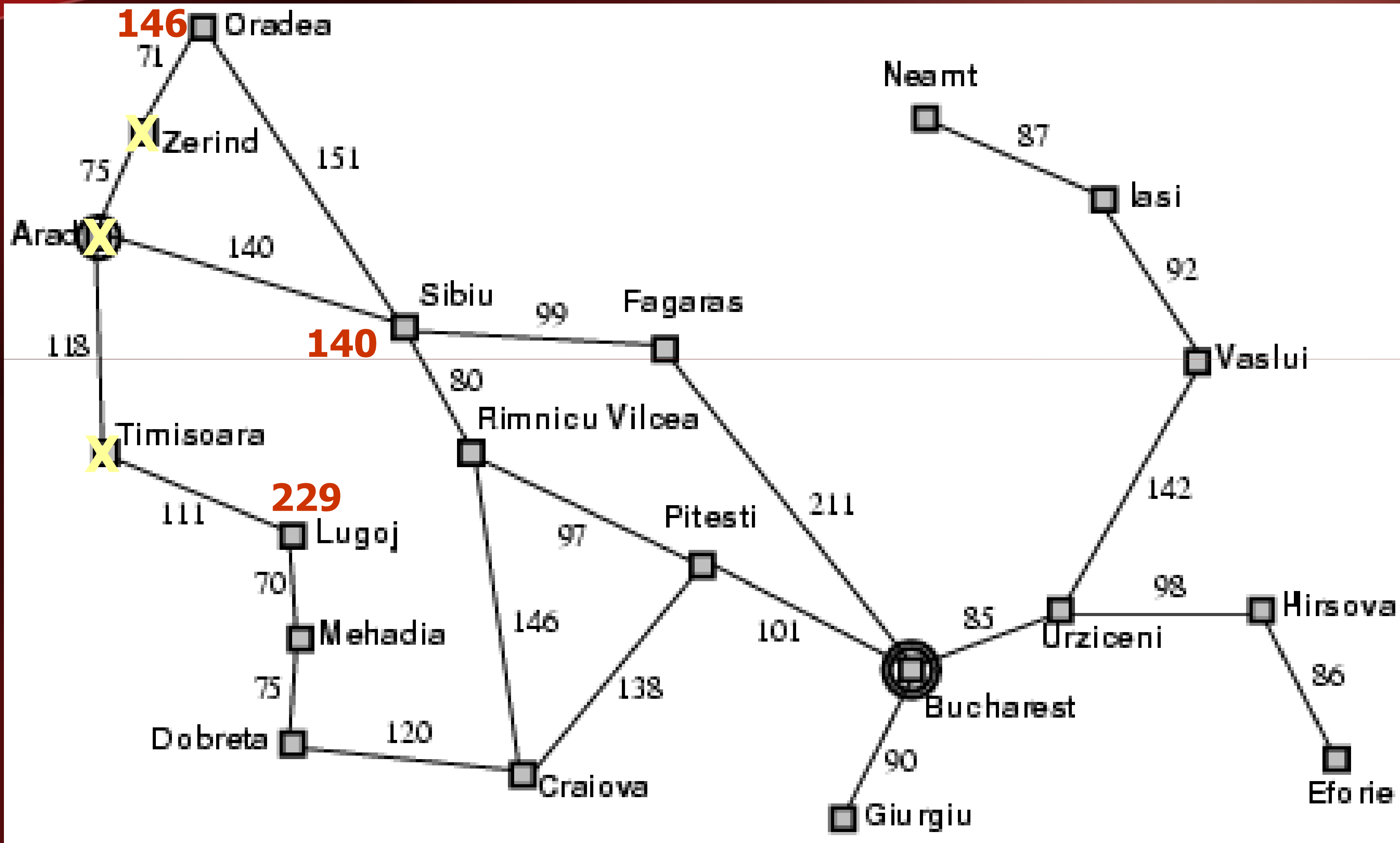
Uniform-cost search Sample



Uniform-cost search Sample



Uniform-cost search Sample

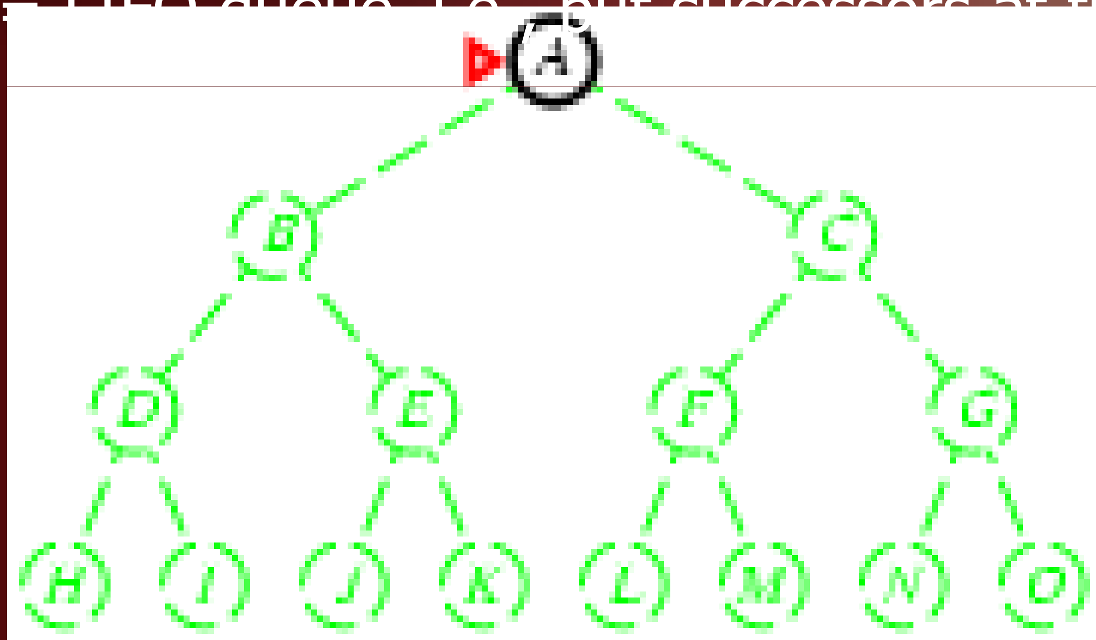


Uniform-cost search

- Complete? Yes
- Time? # of nodes with $g \leq$ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$ where C^* is the cost of the optimal solution
- Space? # of nodes with $g \leq$ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$
- Optimal? Yes

Depth-first search

- Expand deepest unexpanded node
-
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front
  -



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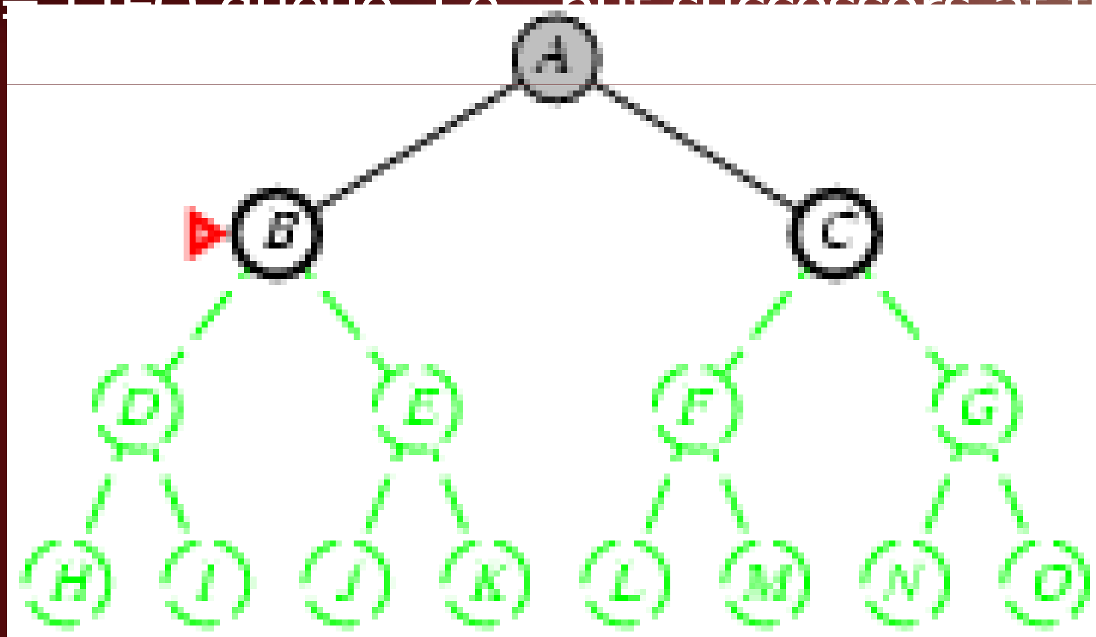

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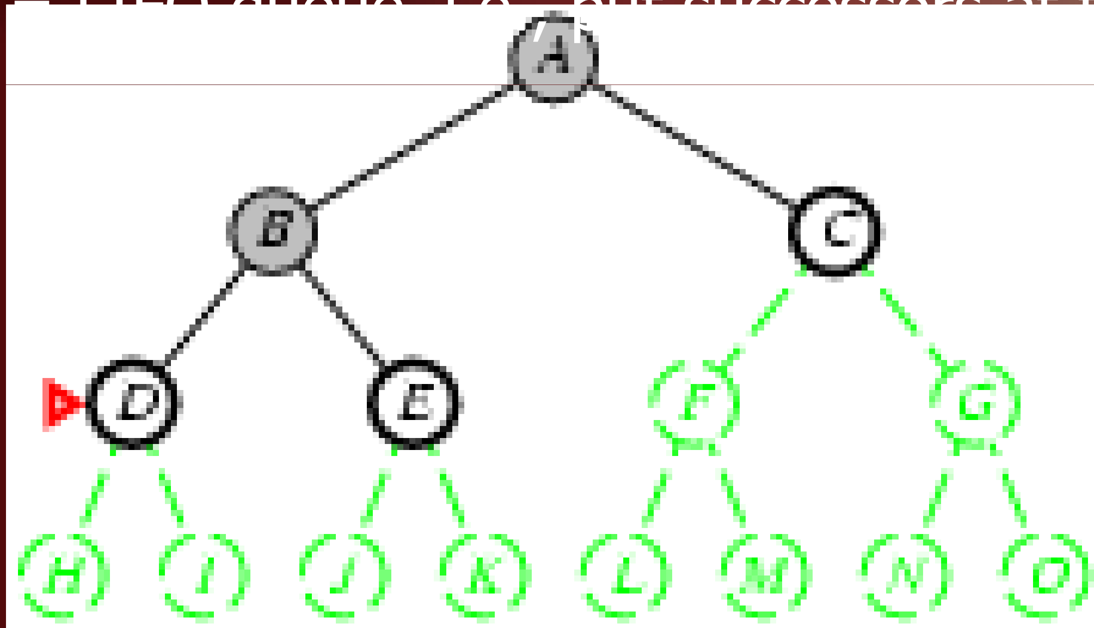

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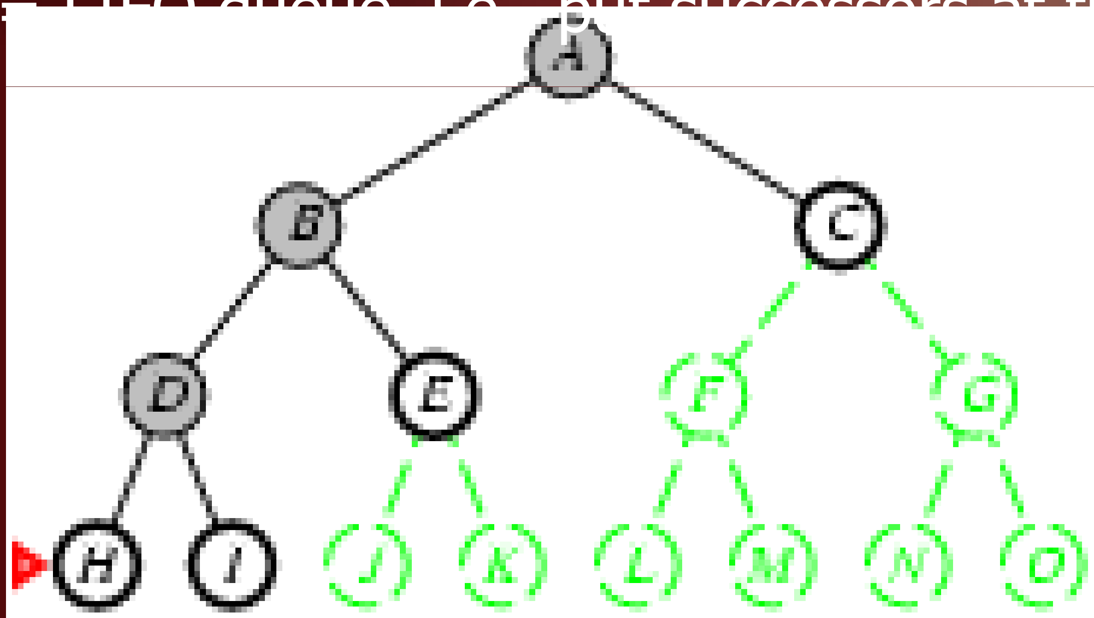

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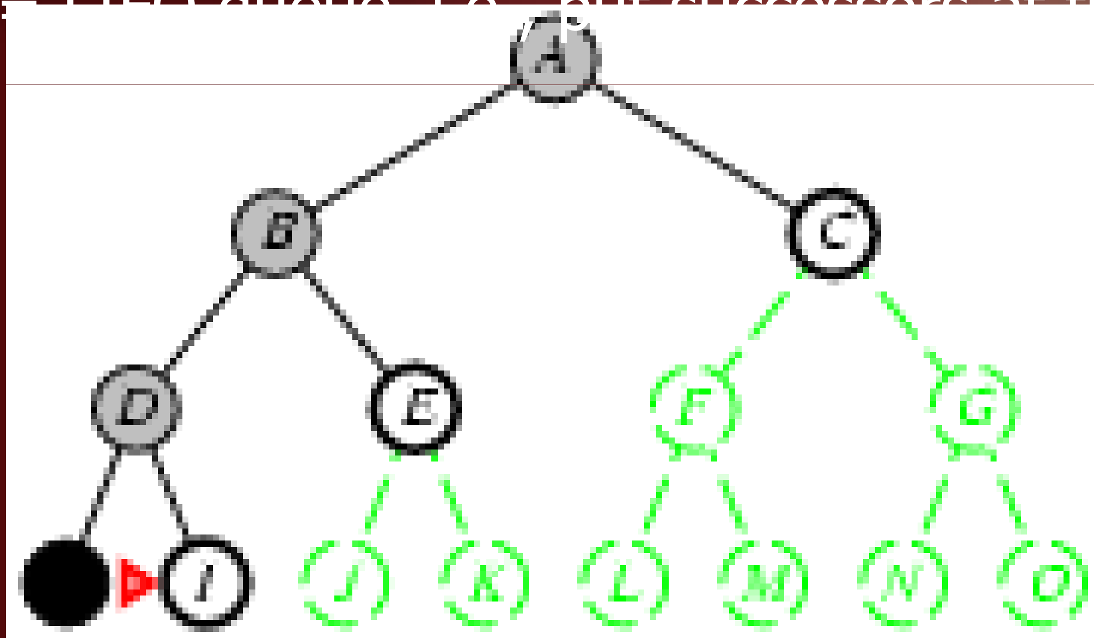
  -



Depth-first search

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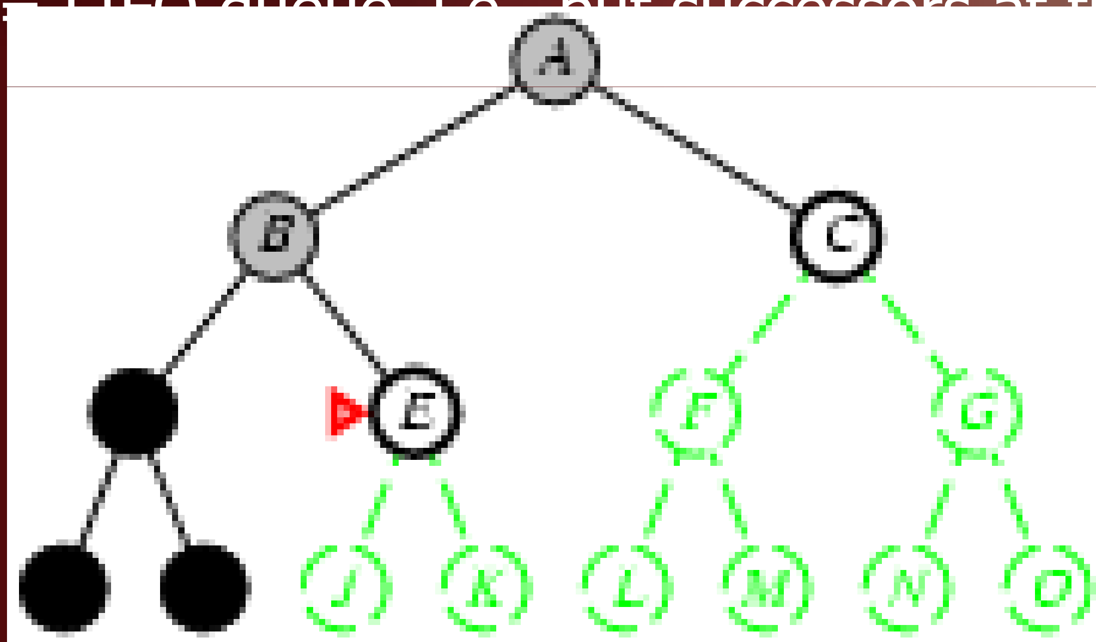
  -



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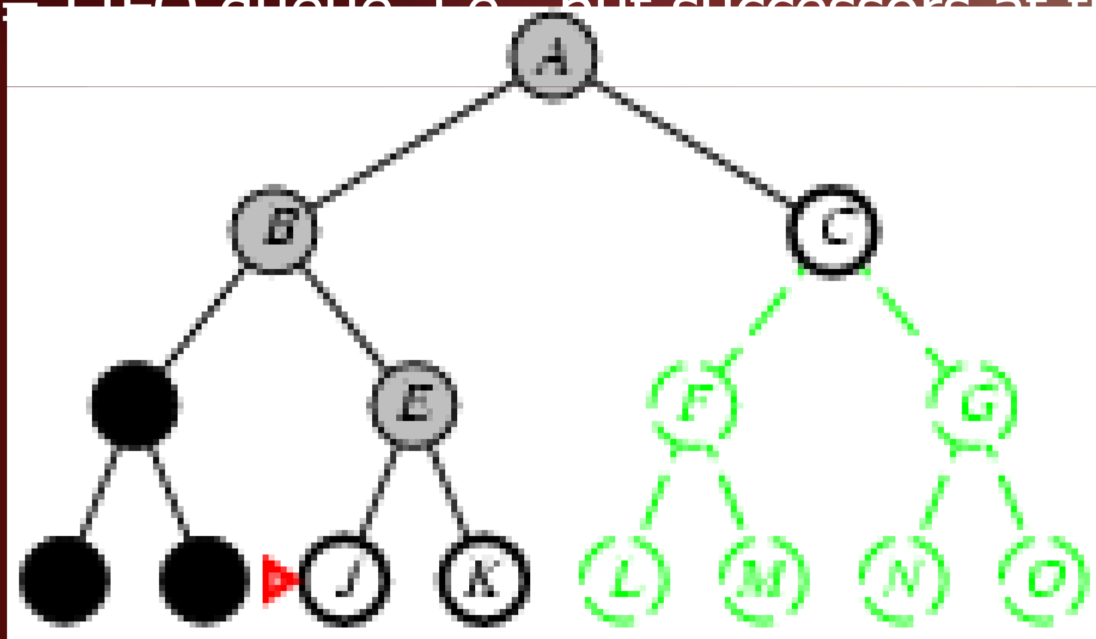

Depth-first search

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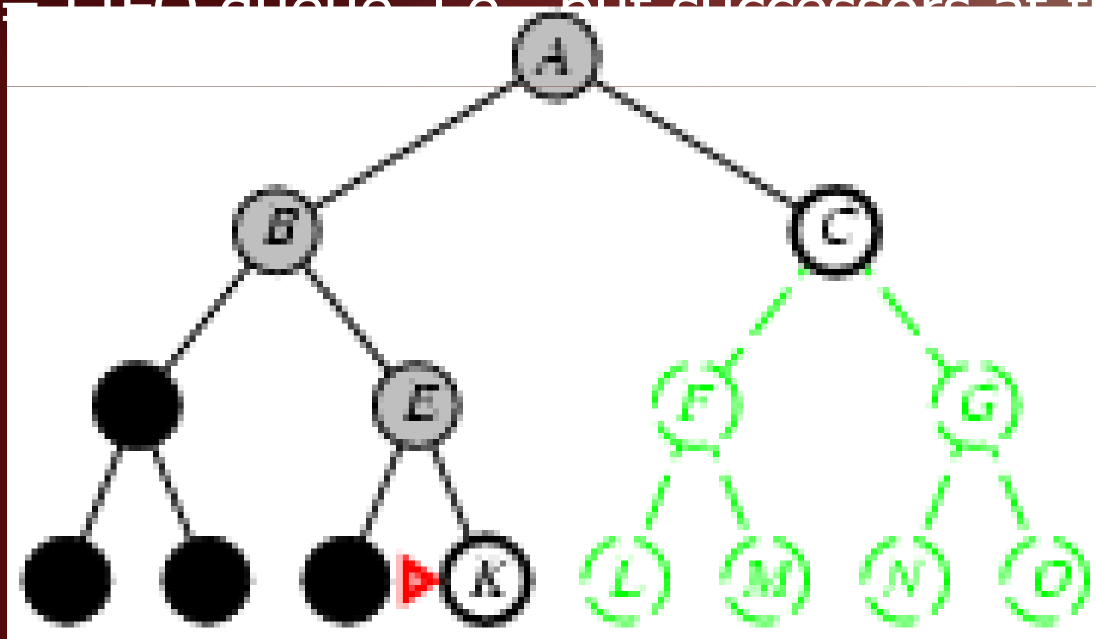

Depth-first search

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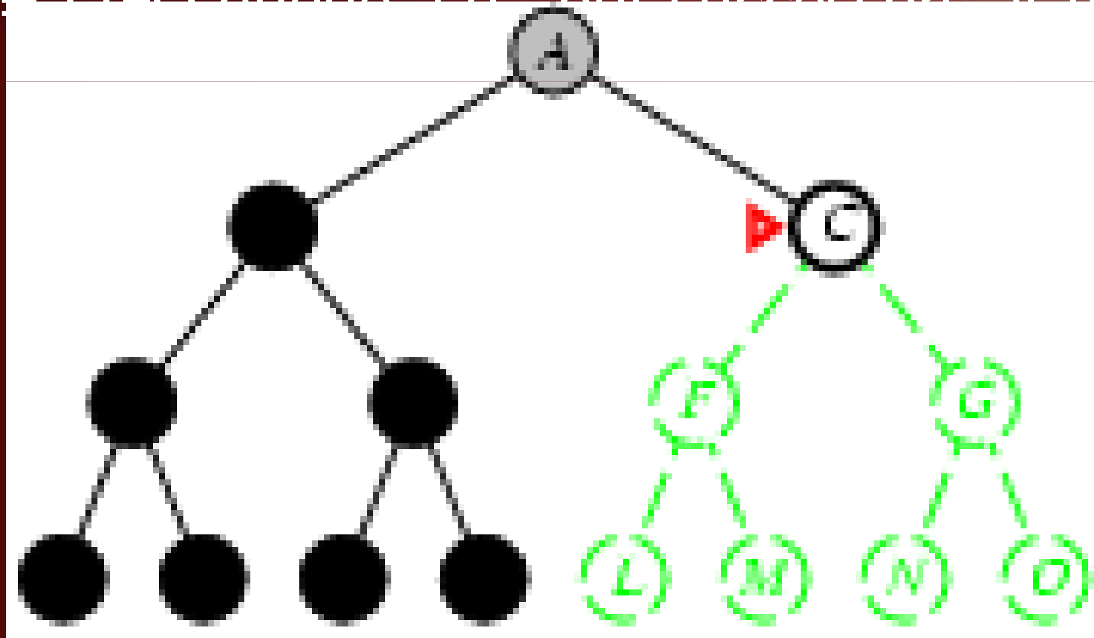

Depth-first search

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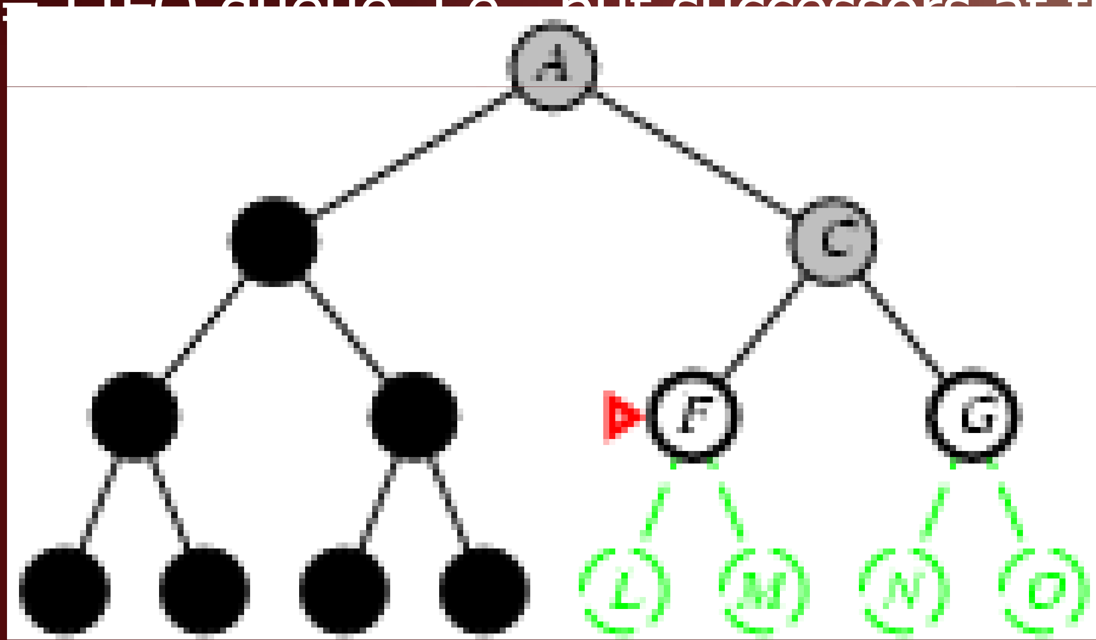

Depth-first search

- Expand deepest unexpanded node
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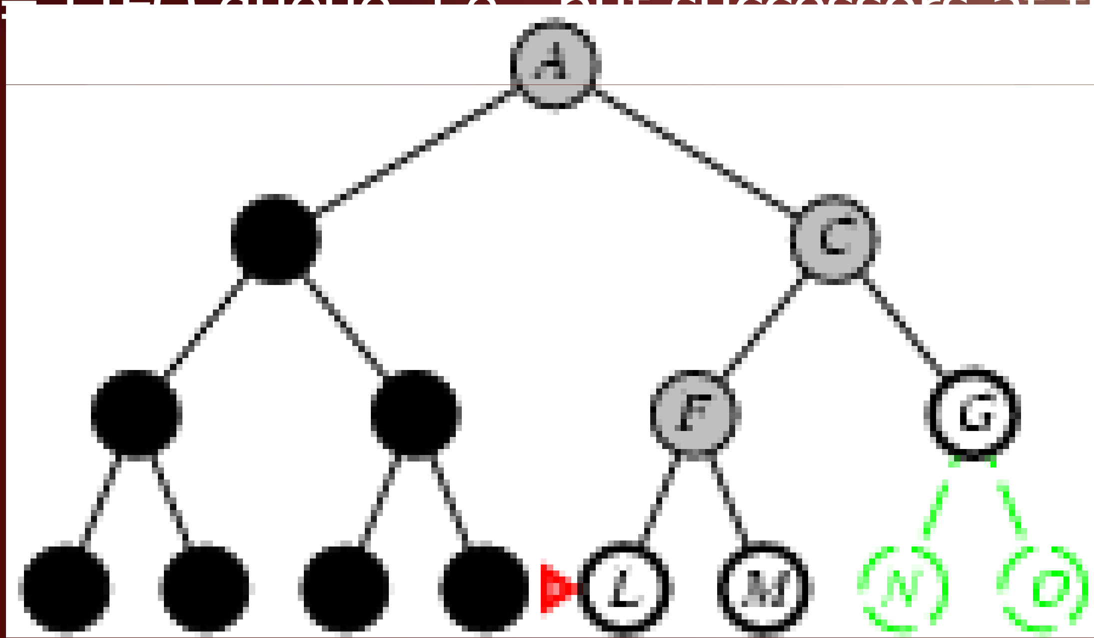
  -



Depth-first search

- Expand deepest unexpanded node
-
- Implementation:
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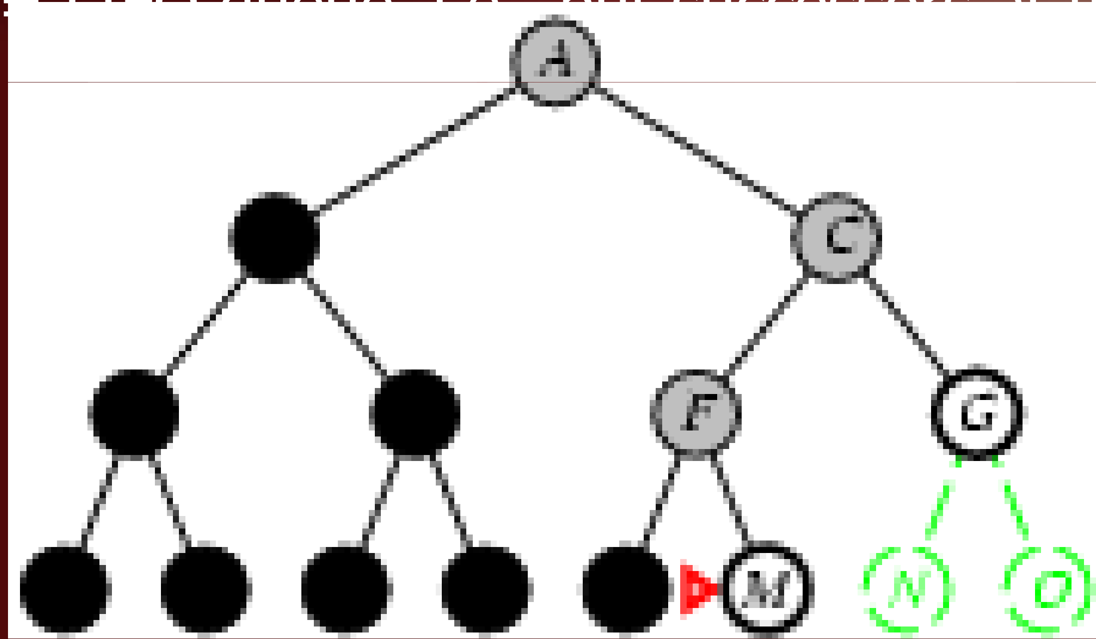
  -



Properties of depth-first search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path
  -
    - → complete in finite spaces
- Time? $O(b^m)$: terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first
  -
- Space? $O(bm)$, i.e., linear space!
-
- Optimal? No

Depth-limited search

= depth-first search with depth limit *l*,
i.e., nodes at depth *l* have no successors

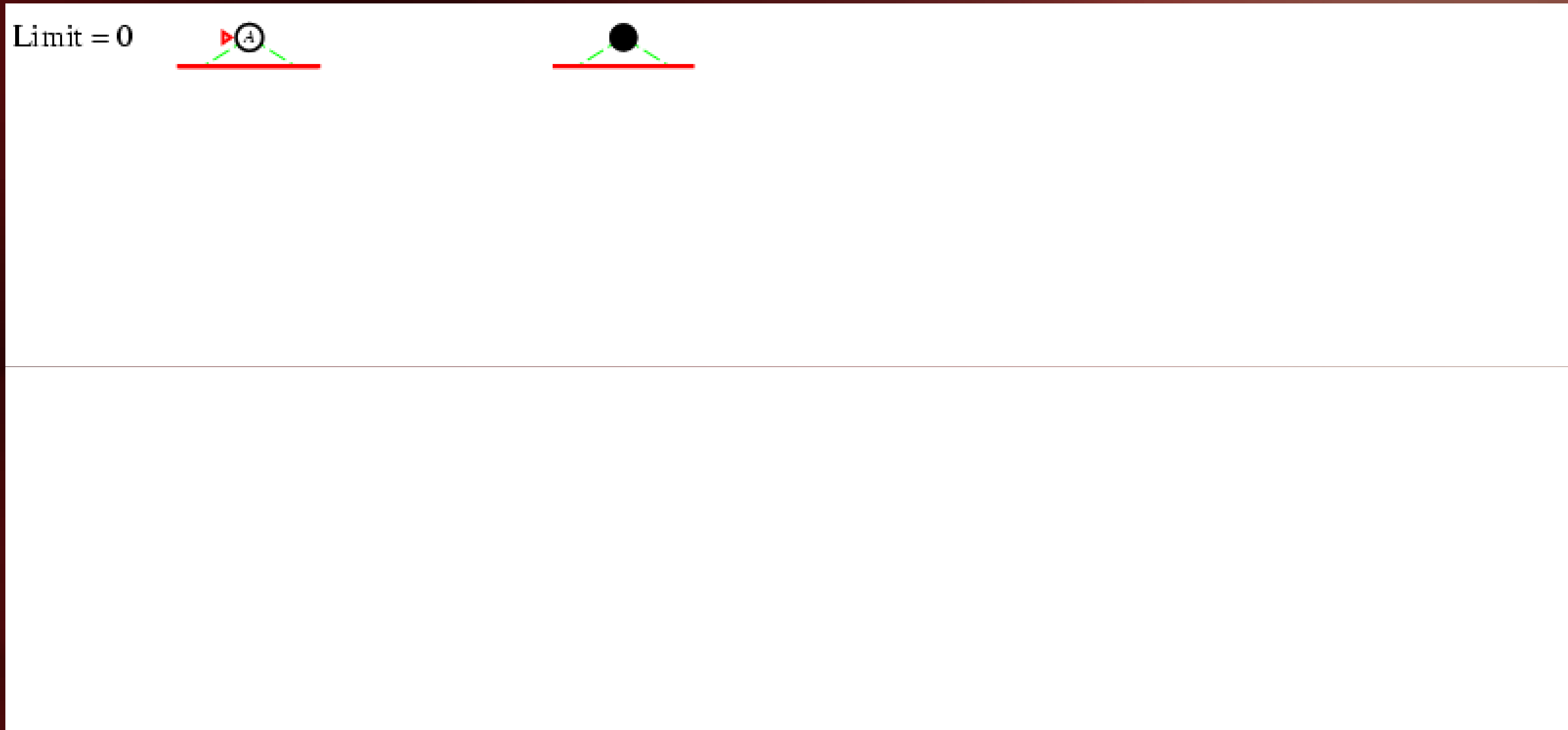
- Recursive implementation:

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

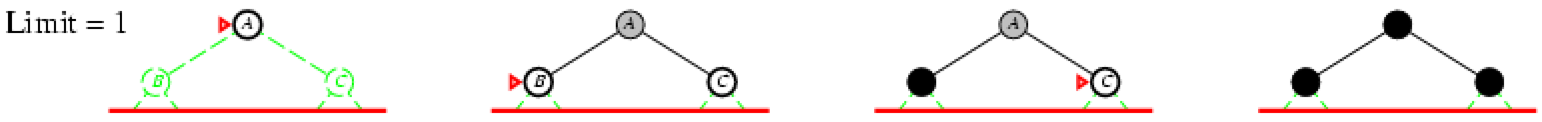
Iterative deepening search

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or failure
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
```

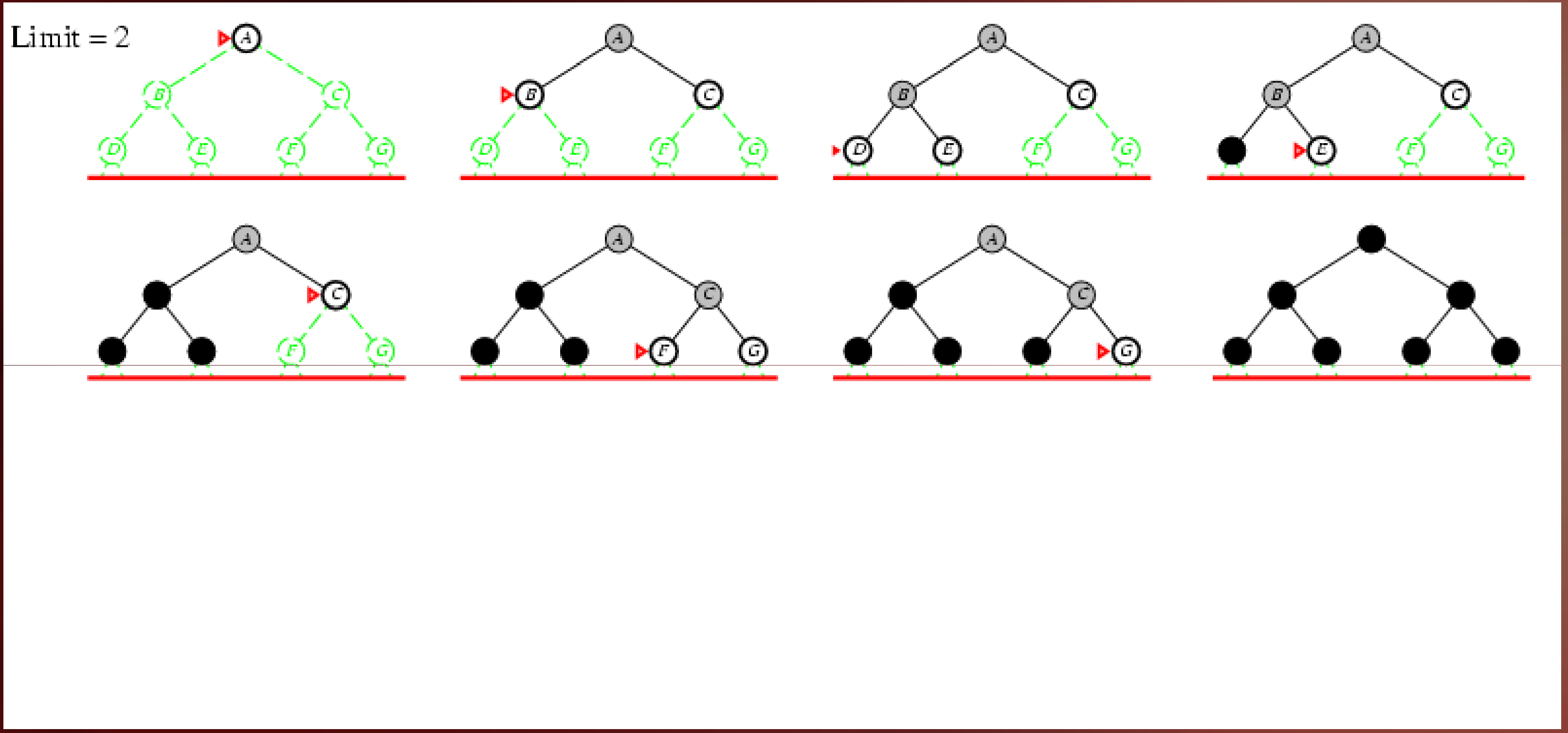
Iterative deepening search *l*=0



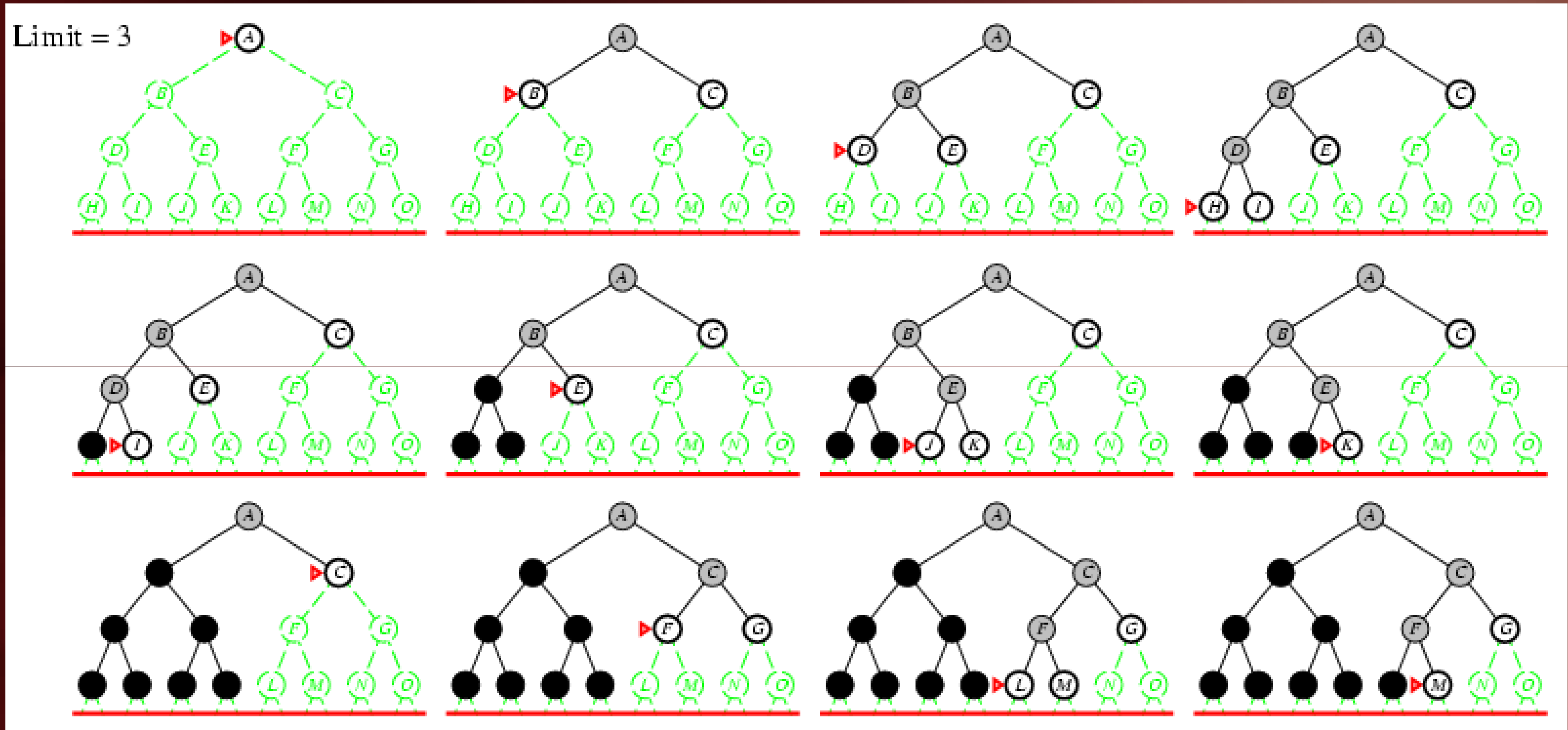
Iterative deepening search l=1



Iterative deepening search *l*=2



Iterative deepening search *l* =3



Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$N_{IDS} = (d+1)b^0 + d\ b\text{^}^1 + (d-1)b\text{^}^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$

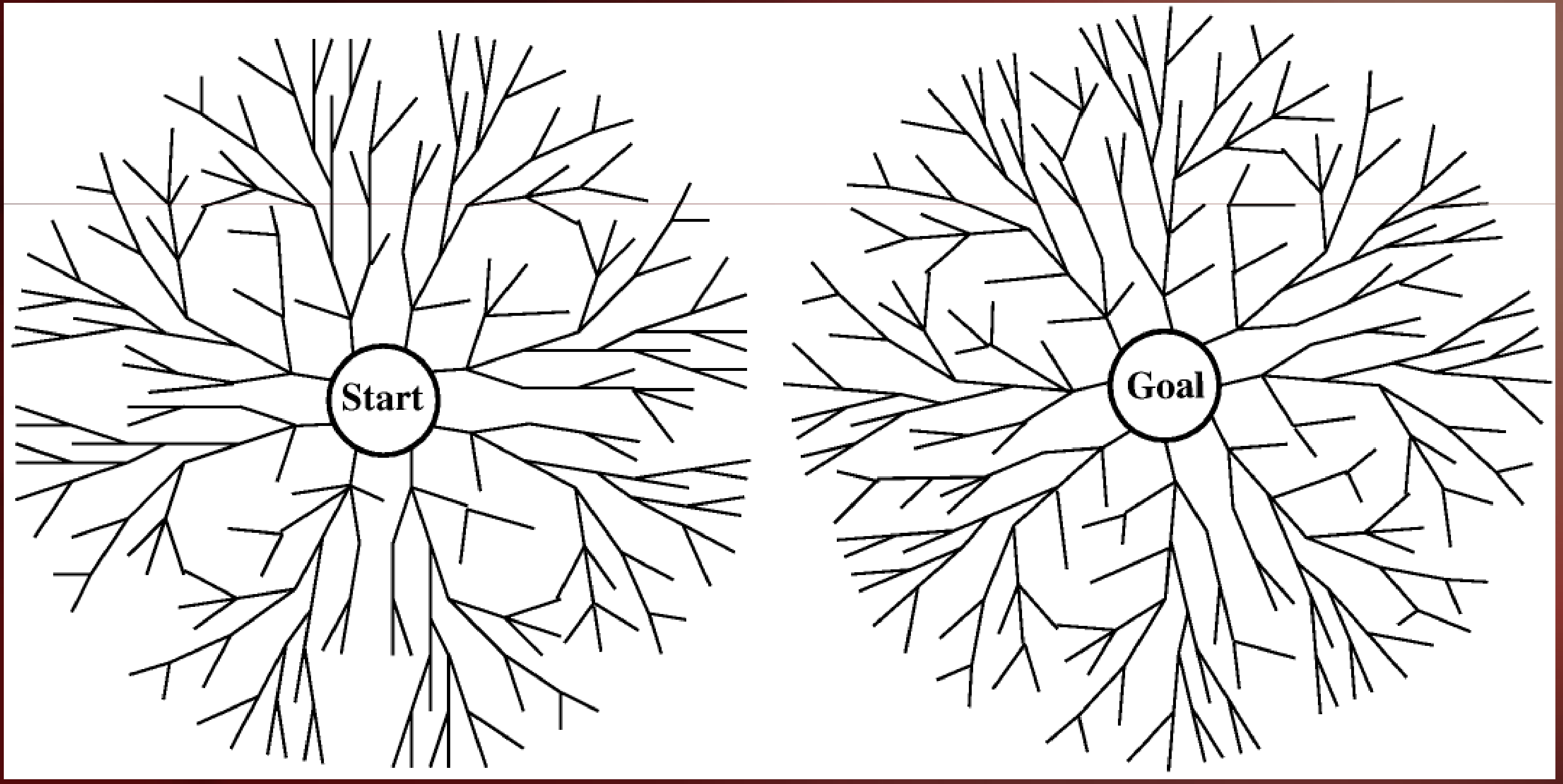
- For $b = 10$, $d = 5$,
-
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  -
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$
  -
- Overhead = (123,456 - 111,111)/111,111 = 11%

Properties of iterative deepening search

- Complete? Yes
-
- Time? $(d+1)b^0 + d\ b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
-
- Space? $O(bd)$
-
- Optimal? Yes, if step cost = 1

Bidirectional Search

- To go both ways, Top Down and Bottom Up.



Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal?	Yes	Yes	No	No	Yes	Yes
Complete?	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
  - Breadth First Search
  - Depth First Search
  - Uniform Cost Search (Best First Search)
  - Depth Limited Search
  - Iterative Deepening
  - Bidirectional Search