

Classical Encryption

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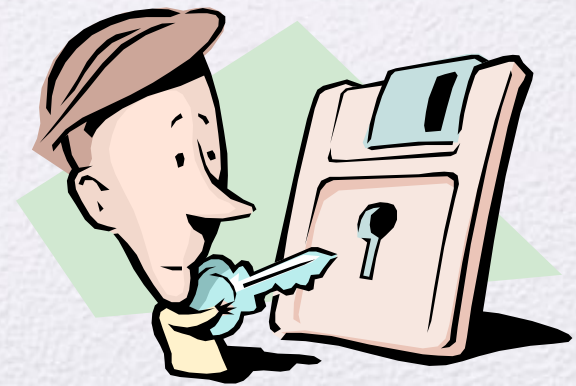


Objectives

- Symmetry key Cipher Model
- Substitution Techniques
- Transposition Techniques
- Steganography

Symmetric Encryption

- Also referred to as conventional encryption or single-key encryption
- Was the only type of encryption in use prior to the development of public-key encryption in the 1970s
- Remains by far the most widely used of the two types of encryption



Basic Terminology

- Plaintext
 - The original message
- Ciphertext
 - The coded message
- Enciphering or encryption
 - Process of converting from plaintext to ciphertext
- Deciphering or decryption
 - Restoring the plaintext from the ciphertext
- Cryptography
 - Study of encryption
- Cryptographic system or cipher
 - Schemes used for encryption
- Cryptanalysis
 - Techniques used for deciphering a message without any knowledge of the enciphering details
- Cryptology
 - Areas of cryptography and cryptanalysis together

Simplified Model of Symmetric Encryption

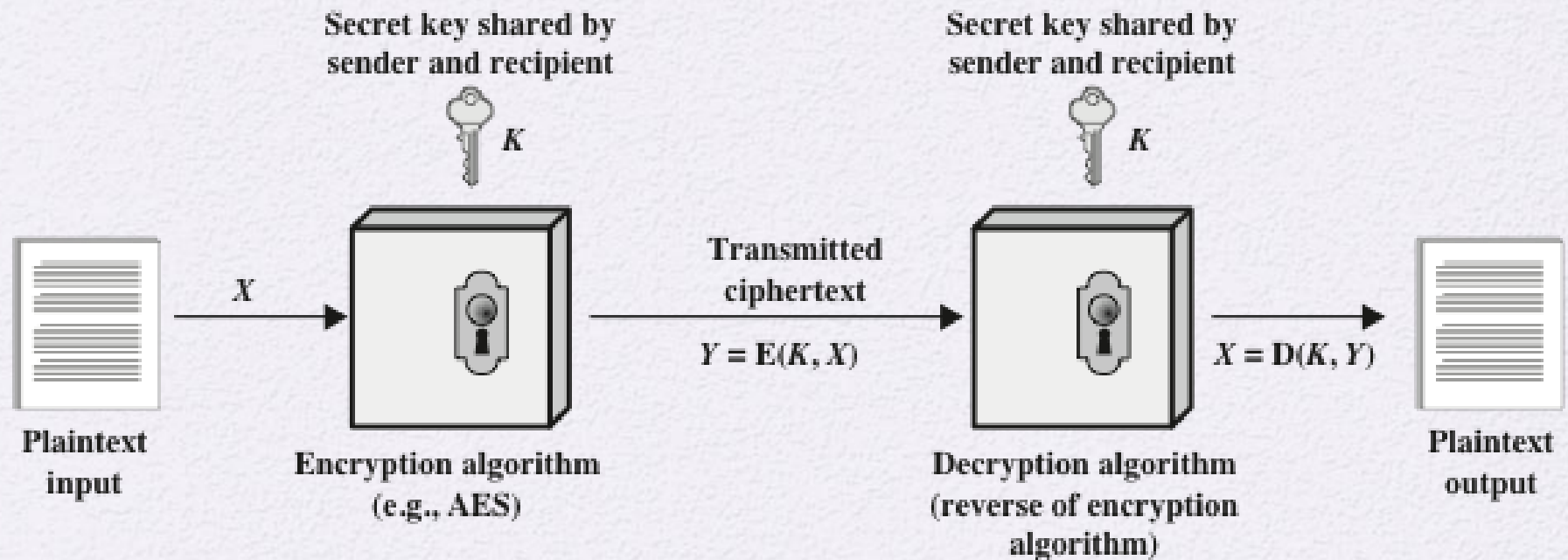


Figure 2.1 Simplified Model of Symmetric Encryption

Model of Symmetric Cryptosystem

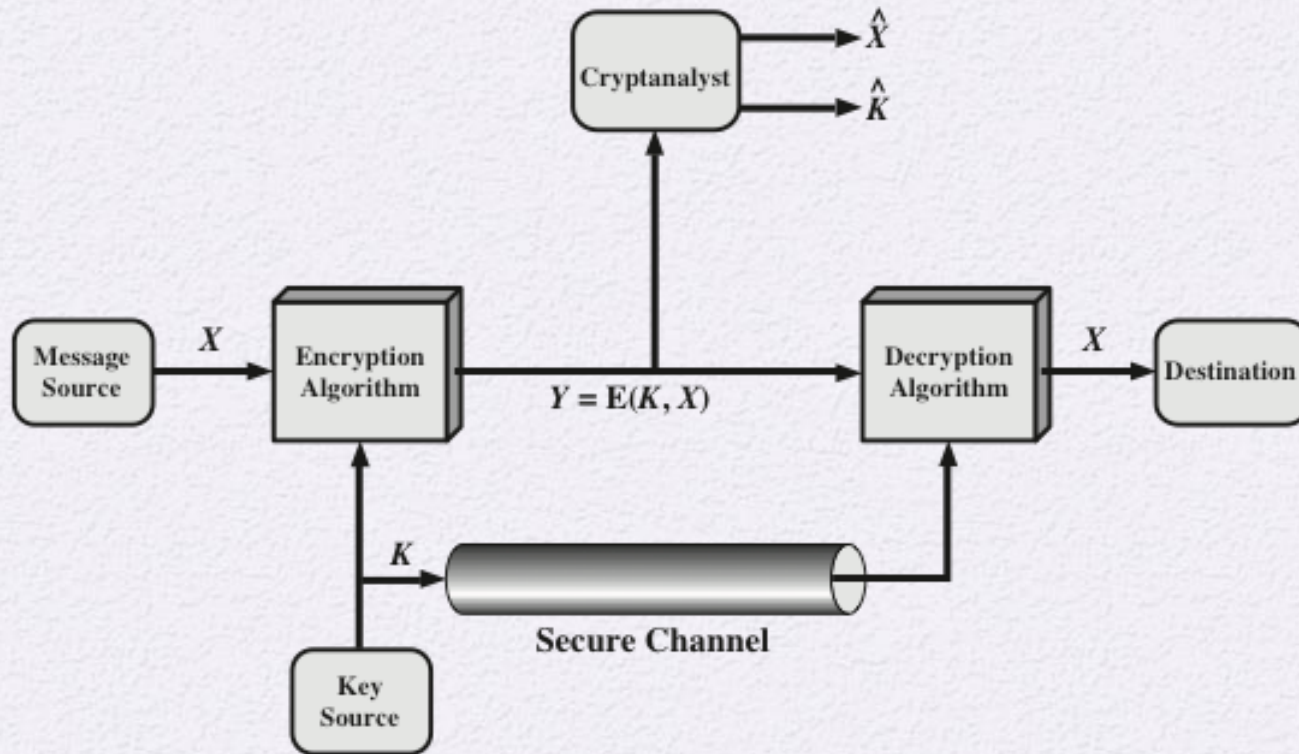
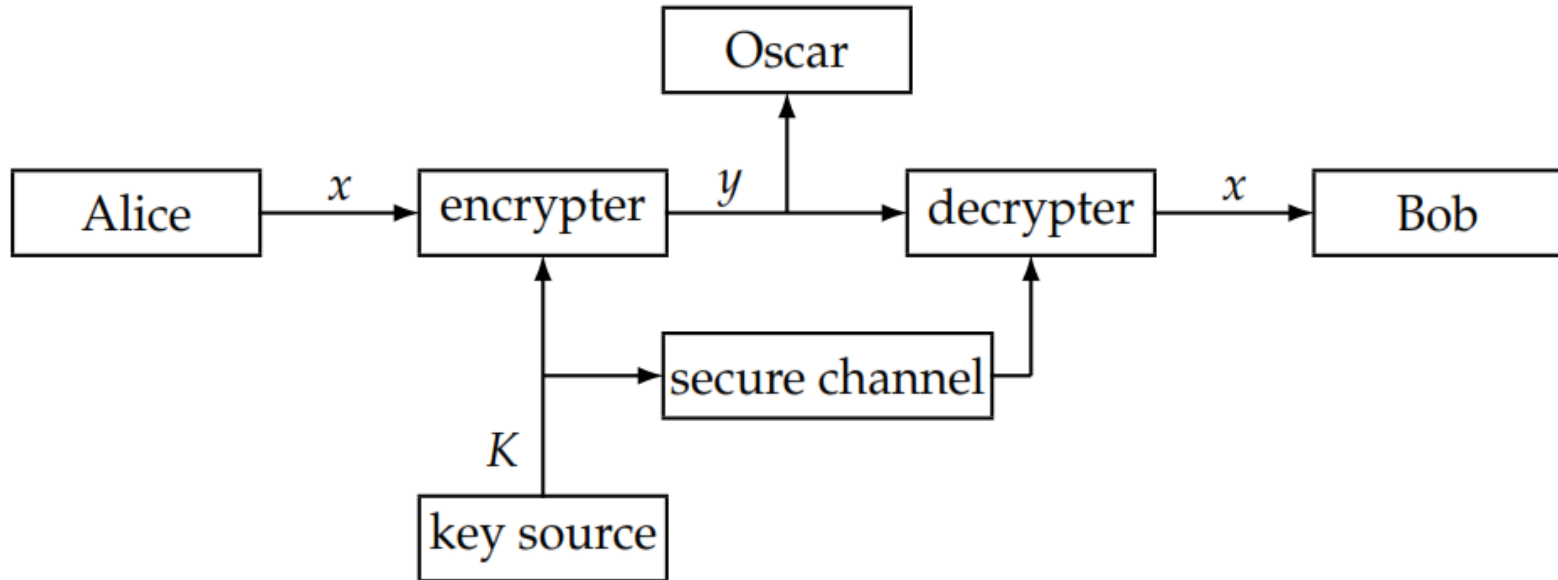


Figure 2.2 Model of Symmetric Cryptosystem

Communication Channel



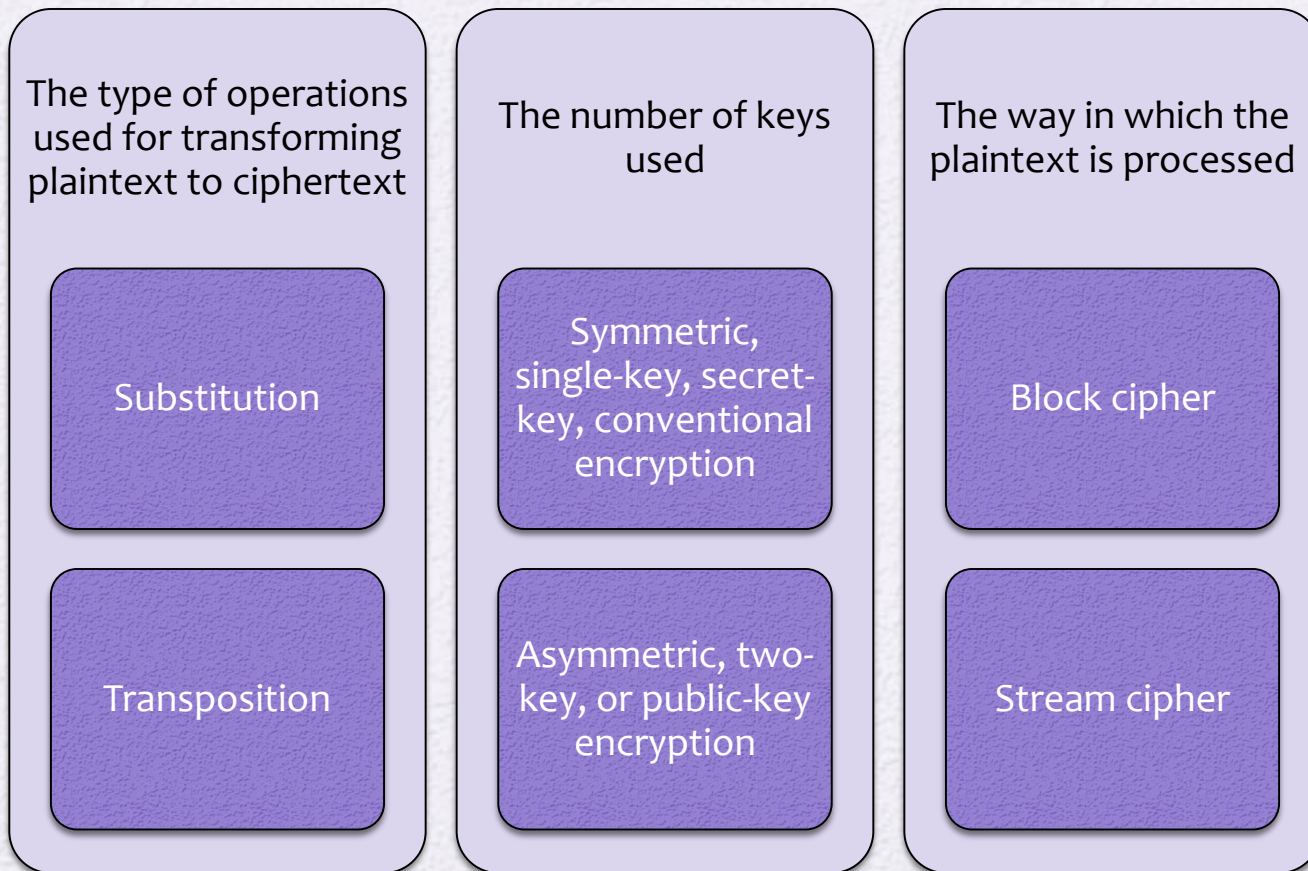
Simple Cryptosystem

Definition 2.1: A *cryptosystem* is a five-tuple $(\mathcal{P}, \mathcal{C}, \mathcal{K}, \mathcal{E}, \mathcal{D})$, where the following conditions are satisfied:

1. \mathcal{P} is a finite set of possible *plaintexts*;
2. \mathcal{C} is a finite set of possible *ciphertexts*;
3. \mathcal{K} , the *keyspace*, is a finite set of possible *keys*;
4. For each $K \in \mathcal{K}$, there is an *encryption rule* $e_K \in \mathcal{E}$ and a corresponding *decryption rule* $d_K \in \mathcal{D}$. Each $e_K : \mathcal{P} \rightarrow \mathcal{C}$ and $d_K : \mathcal{C} \rightarrow \mathcal{P}$ are functions such that $d_K(e_K(x)) = x$ for every plaintext element $x \in \mathcal{P}$.

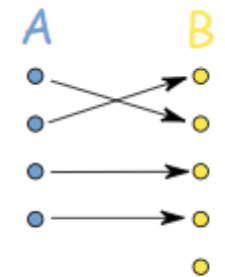
Cryptographic Systems

- Characterized along three independent dimensions:

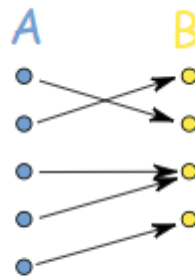


Injective

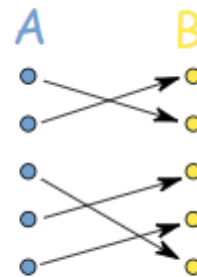
- Clearly, it must be the case that each encryption function e is an **injective** function (i.e., one-to-one); otherwise, decryption could not be accomplished.
- $y = e_{k_1}(x_1) = e_{k_2}(x_2)$ where $x_1 \neq x_2$



Injective
(not surjective)



Surjective
(not injective)



Bijjective
(injective, surjective)

Cryptanalysis and Brute-Force Attack

Cryptanalysis

- Attack relies on the nature of the algorithm plus some knowledge of the general characteristics of the plaintext
- Attack exploits the characteristics of the algorithm to attempt to deduce a specific plaintext or to deduce the key being used

Brute-force attack

- Attacker tries every possible key on a piece of ciphertext until an intelligible translation into plaintext is obtained
- On average, half of all possible keys must be tried to achieve success

Type of Attack	Known to Cryptanalyst
Ciphertext Only	<ul style="list-style-type: none"> • Encryption algorithm • Ciphertext
Known Plaintext	<ul style="list-style-type: none"> • Encryption algorithm • Ciphertext • One or more plaintext-ciphertext pairs formed with the secret key
Chosen Plaintext	<ul style="list-style-type: none"> • Encryption algorithm • Ciphertext • Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key
Chosen Ciphertext	<ul style="list-style-type: none"> • Encryption algorithm • Ciphertext • Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key
Chosen Text	<ul style="list-style-type: none"> • Encryption algorithm • Ciphertext • Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key • Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key

Table 2.1
Types of
Attacks
on
Encrypted
Messages

ciphertext-only attack

The opponent possesses a string of ciphertext, y .

known plaintext attack

The opponent possesses a string of plaintext, x , and the corresponding ciphertext, y .

chosen plaintext attack

The opponent has obtained temporary access to the encryption machinery. Hence he can choose a plaintext string, x , and construct the corresponding ciphertext string, y .

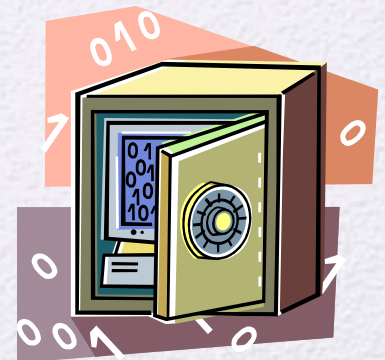
chosen ciphertext attack

The opponent has obtained temporary access to the decryption machinery. Hence he can choose a ciphertext string, y , and construct the corresponding plaintext string, x .



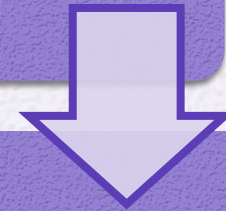
Encryption Scheme Security

- Unconditionally secure
 - No matter how much time an opponent has, it is impossible for him or her to decrypt the ciphertext simply because the required information is not there
- Computationally secure
 - The cost of breaking the cipher exceeds the value of the encrypted information
 - The time required to break the cipher exceeds the useful lifetime of the information

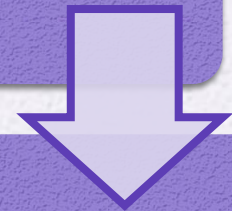


Brute-Force Attack

Involves trying every possible key until an intelligible translation of the ciphertext into plaintext is obtained



On average, half of all possible keys must be tried to achieve success



To supplement the brute-force approach, some degree of knowledge about the expected plaintext is needed, and some means of automatically distinguishing plaintext from garble is also needed

Substitution Technique

- Is one in which the letters of plaintext are replaced by other letters or by numbers or symbols
- If the plaintext is viewed as a sequence of bits, then substitution involves replacing plaintext bit patterns with ciphertext bit patterns



Shift Cipher

- Shift Cipher, which is based on modular arithmetic.

Definition 2.2: Suppose a and b are integers, and m is a positive integer. Then we write $a \equiv b \pmod{m}$ if m divides $b - a$. The phrase $a \equiv b \pmod{m}$ is called a *congruence*, and it is read as “ a is *congruent* to b modulo m .” The integer m is called the *modulus*.



Property

1. addition is *closed*, i.e., for any $a, b \in \mathbb{Z}_m$, $a + b \in \mathbb{Z}_m$
2. addition is *commutative*, i.e., for any $a, b \in \mathbb{Z}_m$, $a + b = b + a$
3. addition is *associative*, i.e., for any $a, b, c \in \mathbb{Z}_m$, $(a + b) + c = a + (b + c)$
4. 0 is an *additive identity*, i.e., for any $a \in \mathbb{Z}_m$, $a + 0 = 0 + a = a$
5. the *additive inverse* of any $a \in \mathbb{Z}_m$ is $m - a$, i.e., $a + (m - a) = (m - a) + a = 0$ for any $a \in \mathbb{Z}_m$

Group: Property 1, 3-5

Abelian Group: 1-5



Ring

6. multiplication is *closed*, i.e., for any $a, b \in \mathbb{Z}_m$, $ab \in \mathbb{Z}_m$
7. multiplication is *commutative*, i.e., for any $a, b \in \mathbb{Z}_m$, $ab = ba$
8. multiplication is *associative*, i.e., for any $a, b, c \in \mathbb{Z}_m$, $(ab)c = a(bc)$
9. 1 is a *multiplicative identity*, i.e., for any $a \in \mathbb{Z}_m$, $a \times 1 = 1 \times a = a$
10. the *distributive property* is satisfied, i.e., for any $a, b, c \in \mathbb{Z}_m$, $(a + b)c = (ac) + (bc)$ and $a(b + c) = (ab) + (ac)$.

Shift Cipher

Cryptosystem 2.1: *Shift Cipher*

Let $\mathcal{P} = \mathcal{C} = \mathcal{K} = \mathbb{Z}_{26}$. For $0 \leq K \leq 25$, define

$$e_K(x) = (x + K) \bmod 26$$

and

$$d_K(y) = (y - K) \bmod 26$$

$(x, y \in \mathbb{Z}_{26})$.





Caesar Cipher



- Simplest and earliest known use of a substitution cipher
- Used by Julius Caesar
- Involves replacing each letter of the alphabet with the letter standing three places further down the alphabet
- Alphabet is wrapped around so that the letter following Z is A

plain: meet me after the toga party

cipher: PHHW PH DIWHU WKH WRJD SDUWB

Caesar Cipher Algorithm

- Can define transformation as:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

- Mathematically give each letter a number

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

- Algorithm can be expressed as:

$$c = E(3, p) = (p + 3) \bmod (26)$$

- A shift may be of any amount, so that the general Caesar algorithm is:

$$C = E(k, p) = (p + k) \bmod 26$$

- Where k takes on a value in the range 1 to 25; the decryption algorithm is simply:

$$p = D(k, C) = (C - k) \bmod 26$$

Example with $k=11$

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>
0	1	2	3	4	5	6	7	8	9	10	11	12

<i>N</i>	<i>O</i>	<i>P</i>	<i>Q</i>	<i>R</i>	<i>S</i>	<i>T</i>	<i>U</i>	<i>V</i>	<i>W</i>	<i>X</i>	<i>Y</i>	<i>Z</i>
13	14	15	16	17	18	19	20	21	22	23	24	25

wewillmeetatmidnight

22 4 22 8 11 11 12 4 4 19 0 19 12 8 3 13 8 6 7 19

7 15 7 19 22 22 23 15 15 4 11 4 23 19 14 24 19 17 18 4

HPHTWWXPPELEXTOTYTRSE.



Brute-Force Cryptanalysis of Caesar Cipher

(This chart can be found on
page 35 in the textbook)

KEY	PHHW	PH	DIWHU	WKH	WRJD	SDUWB
1	oggv	og	chvgt	vjg	vqic	rctva
2	nffu	nf	bgufs	uif	uphb	qbsuz
3	meet	me	after	the	toga	party
4	ldds	ld	zesdq	sgd	snfz	ozqsx
5	kccr	kc	ydrpc	rfc	rmey	nyprw
6	jbbq	jb	xcqbo	qeb	qldx	mxoqv
7	iaap	ia	wbpan	pda	pkcw	lwnpu
8	hzzo	hz	vaozm	ocz	objv	kvmot
9	gyyn	gy	uznyl	nby	niau	julns
10	fxxm	fx	tymxk	max	mhzt	itkmr
11	ewwl	ew	sxlwj	lzw	lgys	hsjlg
12	dvvk	dv	rwkvi	kyv	kfxr	grikp
13	cuuj	cu	qvjuh	jxu	jewq	fghjo
14	btti	bt	puitg	iwt	idvp	epgin
15	assh	as	othsf	hvs	hcuo	dofhm
16	zrrg	zr	nsGRE	gur	gbtn	cnegl
17	yqqf	yq	mrfqd	ftq	fasm	bmdfk
18	xppe	xp	lqepc	esp	ezrl	alcej
19	wood	wo	kpdob	dro	dyqk	zkbdi
20	vnnc	vn	jocna	cqn	cxpj	yjach
21	ummb	um	inbmz	bpm	bwoi	xizbg
22	tlla	tl	hmaly	aol	avnh	whyaf
23	skkz	sk	glzKX	znk	zumg	vgxze
24	rjyy	rj	fkyjw	ymj	ytlf	ufwyd
25	qiix	qi	ejxiv	xli	xske	tevxc

Figure 2.3 Brute-Force Cryptanalysis of Caesar Cipher

Sample of Compressed Text

```
~+Wu"- Ω-O)S4(=†, e-Ωträu.-î 0~Z-  
Û#2Ô#Åæð æ=q7,Ωn-@3NÔÛ æz'Y-f=î[±Ô_ èΩ,<NO~t«"xâ Åæfè03Å  
x)85k°Å  
_yî ^ΔÉ] ,= J/'iTê&1 'c<uΩ-  
ÅD(G WÄC~y_IðÄW PÔ1<îÛ†ç],=,~î^uNπ~="L~9OgfiO~&ES ~S øÔ5":  
~@!SQqèvo" ú\,S>h<-*6ø†%x'"|fió#~"myt~ZNP<,fi Åj Å0_~Zû-  
Ω~Ô~6ay{%,ΩBó ,I π+Ái~ú02ç8y'O-  
2ÅnB1 /ø~"ΠK~*Pαπ,úé^'jΣ~ø~ÔZî~Y~YΩmY> Ω+eð/'<Kf_~*+~"S0~  
B ZøK~Q8yú/.!òñîzeS/)>èQ ú
```

Figure 2.4 Sample of Compressed Text

Monoalphabetic Cipher

- Permutation
 - Of a finite set of elements S is an ordered sequence of all the elements of S , with each element appearing exactly once
- If the “cipher” line can be any permutation of the 26 alphabetic characters, then there are $26!$ or greater than 4×10^{26} possible keys
 - This is 10 orders of magnitude greater than the key space for DES
 - Approach is referred to as a *monoalphabetic substitution cipher* because a single cipher alphabet is used per message

Substitution Cipher

Cryptosystem 2.2: *Substitution Cipher*

Let $\mathcal{P} = \mathcal{C} = \mathbb{Z}_{26}$. \mathcal{K} consists of all possible permutations of the 26 symbols $0, 1, \dots, 25$. For each permutation $\pi \in \mathcal{K}$, define

$$e_{\pi}(x) = \pi(x),$$

and define

$$d_{\pi}(y) = \pi^{-1}(y),$$

where π^{-1} is the inverse permutation to π .



Plain text and Cipher text

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>
<i>X</i>	<i>N</i>	<i>Y</i>	<i>A</i>	<i>H</i>	<i>P</i>	<i>O</i>	<i>G</i>	<i>Z</i>	<i>Q</i>	<i>W</i>	<i>B</i>	<i>T</i>

<i>n</i>	<i>o</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>	<i>u</i>	<i>v</i>	<i>w</i>	<i>x</i>	<i>y</i>	<i>z</i>
<i>S</i>	<i>F</i>	<i>L</i>	<i>R</i>	<i>C</i>	<i>V</i>	<i>M</i>	<i>U</i>	<i>E</i>	<i>K</i>	<i>J</i>	<i>D</i>	<i>I</i>

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>
<i>d</i>	<i>l</i>	<i>r</i>	<i>y</i>	<i>v</i>	<i>o</i>	<i>h</i>	<i>e</i>	<i>z</i>	<i>x</i>	<i>w</i>	<i>p</i>	<i>t</i>

<i>N</i>	<i>O</i>	<i>P</i>	<i>Q</i>	<i>R</i>	<i>S</i>	<i>T</i>	<i>U</i>	<i>V</i>	<i>W</i>	<i>X</i>	<i>Y</i>	<i>Z</i>
<i>b</i>	<i>g</i>	<i>f</i>	<i>j</i>	<i>q</i>	<i>n</i>	<i>m</i>	<i>u</i>	<i>s</i>	<i>k</i>	<i>a</i>	<i>c</i>	<i>i</i>

The number of possible permutations is $26!$, which is more than 4.0×10^{26} a very large number.



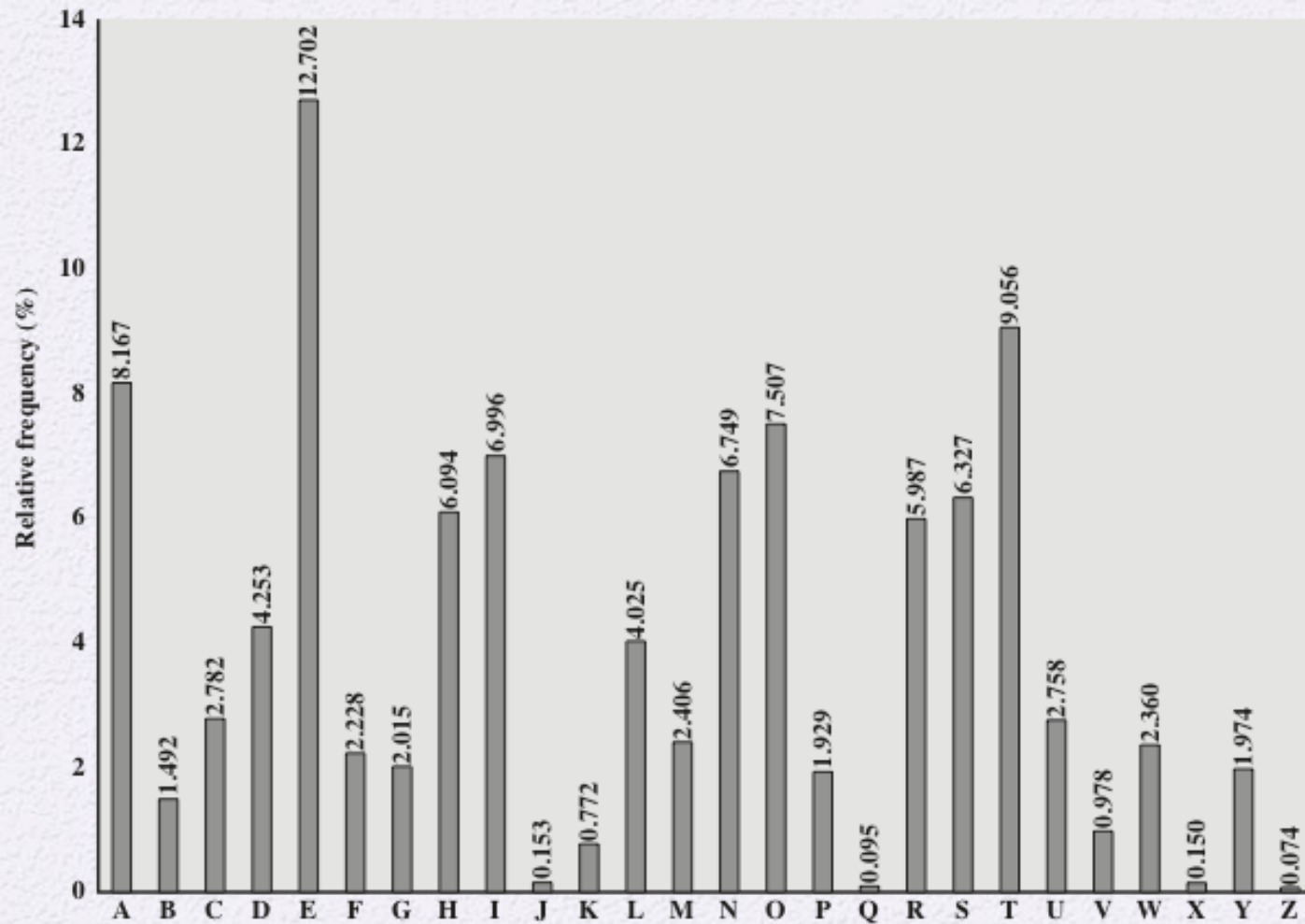


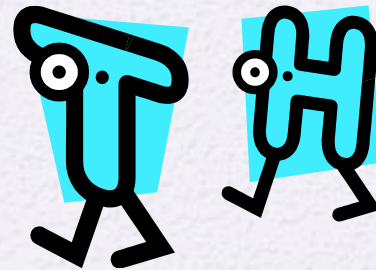
Figure 2.5 Relative Frequency of Letters in English Text

Monoalphabetic Ciphers

- Easy to break because they reflect the frequency data of the original alphabet
- Countermeasure is to provide multiple substitutes (homophones) for a single letter

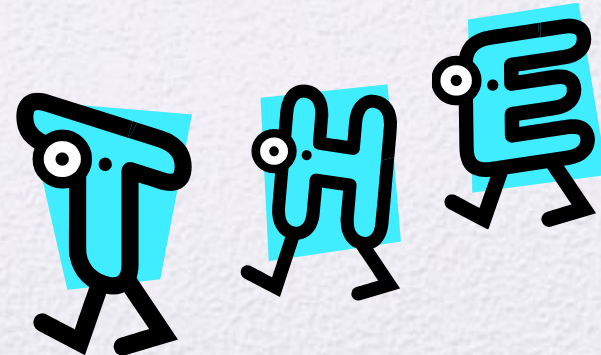
- Digram

- Two-letter combination
 - Most common is *th*



- Trigram

- Three-letter combination
 - Most frequent is *the*



30 Digrams, 12 Trigrams

*TH, HE, IN, ER, AN, RE, ED, ON, ES, ST,
EN, AT, TO, NT, HA, ND, OU, EA, NG, AS,
OR, TI, IS, ET, IT, AR, TE, SE, HI, OF.*

*THE, ING, AND, HER, ERE, ENT,
THA, NTH, WAS, ETH, FOR, DTH*



Affine Cipher

- The Shift Cipher is a special case of the Substitution Cipher.
- Affine Cipher, we restrict the encryption functions to functions of the form
- $e(x) = (ax + b) \bmod 26$ $a, b \in \mathbb{Z}_{26}$
- when $a = 1$, we have a Shift Cipher. This congruence has unique solution iff $\gcd(a, 26) = 1$
- $a, 26$ are relatively prime.

$$ax + b \equiv y \pmod{26}$$

$$ax \equiv y - b \pmod{26}.$$



Contd...

- $A=1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23,$ and $25.$ are relatively prime to $26.$
- multiplicative inverses of the elements relatively prime to 26

$$1^{-1} = 1,$$

$$3^{-1} = 9,$$

$$5^{-1} = 21,$$

$$7^{-1} = 15,$$

$$11^{-1} = 19,$$

$$17^{-1} = 23,$$

$$25^{-1} = 25.$$



Example

Cryptosystem 2.3: *Affine Cipher*

Let $\mathcal{P} = \mathcal{C} = \mathbb{Z}_{26}$ and let

$$\mathcal{K} = \{(a, b) \in \mathbb{Z}_{26} \times \mathbb{Z}_{26} : \gcd(a, 26) = 1\}.$$

For $K = (a, b) \in \mathcal{K}$, define

$$e_K(x) = (ax + b) \bmod 26$$

and

$$d_K(y) = a^{-1}(y - b) \bmod 26$$

$(x, y \in \mathbb{Z}_{26})$.

Example

Example 2.3 Suppose that $K = (7, 3)$. As noted above, $7^{-1} \bmod 26 = 15$. The encryption function is

$$e_K(x) = 7x + 3,$$

and the corresponding decryption function is

$$d_K(y) = 15(y - 3) = 15y - 19,$$

where all operations are performed in \mathbb{Z}_{26} . It is a good check to verify that $d_K(e_K(x)) = x$ for all $x \in \mathbb{Z}_{26}$. Computing in \mathbb{Z}_{26} , we get

$$\begin{aligned} d_K(e_K(x)) &= d_K(7x + 3) \\ &= 15(7x + 3) - 19 \\ &= x + 45 - 19 \\ &= x. \end{aligned}$$



Plain text = hot

To illustrate, let's encrypt the plaintext *hot*. We first convert the letters *h*, *o*, *t* to residues modulo 26. These are respectively 7, 14, and 19. Now, we encrypt:

$$\begin{aligned}(7 \times 7 + 3) \bmod 26 &= 52 \bmod 26 = 0 \\(7 \times 14 + 3) \bmod 26 &= 101 \bmod 26 = 23 \\(7 \times 19 + 3) \bmod 26 &= 136 \bmod 26 = 6.\end{aligned}$$

So the three ciphertext characters are 0, 23, and 6, which corresponds to the alphabetic string *AXG*. We leave the decryption as an exercise for the reader. \square



Playfair Cipher

- Best-known multiple-letter encryption cipher
- Treats digrams in the plaintext as single units and translates these units into ciphertext digrams
- Based on the use of a 5 x 5 matrix of letters constructed using a keyword
- Invented by British scientist Sir Charles Wheatstone in 1854
- Used as the standard field system by the British Army in World War I and the U.S. Army and other Allied forces during World War II

Playfair Key Matrix

- Fill in letters of keyword (minus duplicates) from left to right and from top to bottom, then fill in the remainder of the matrix with the remaining letters in alphabetic order
- Using the keyword MONARCHY:

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

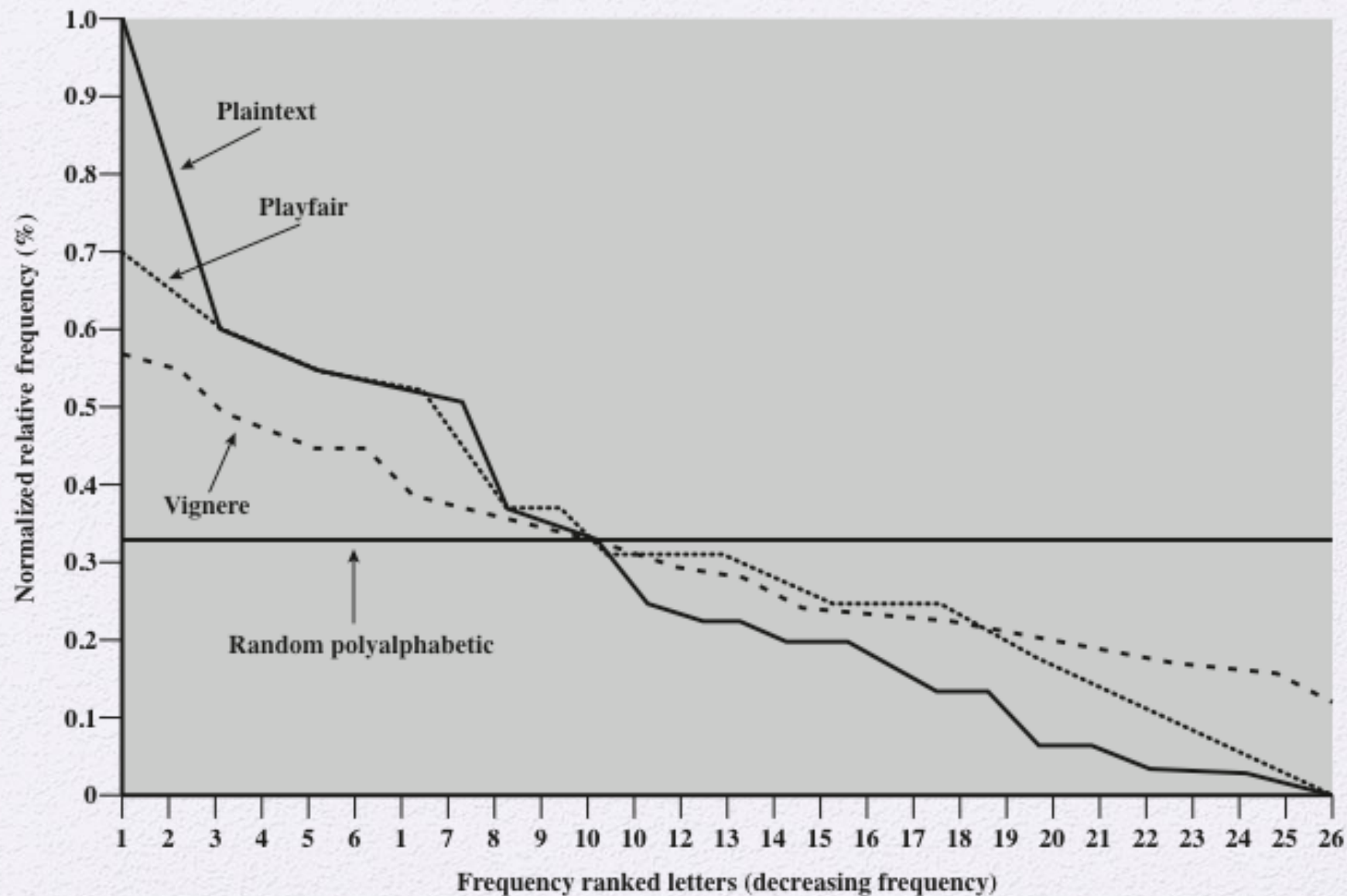


Figure 2.6 Relative Frequency of Occurrence of Letters

Hill Cipher

- Developed by the mathematician Lester Hill in 1929
- Strength is that it completely hides single-letter frequencies
 - The use of a larger matrix hides more frequency information
 - A 3×3 Hill cipher hides not only single-letter but also two-letter frequency information
- Strong against a ciphertext-only attack but easily broken with a known plaintext attack

Example

Example 2.5 Suppose the key is

$$C = PK \bmod 26$$

$$K = \begin{pmatrix} 11 & 8 \\ 3 & 7 \end{pmatrix}.$$

From the computations above, we have that

$$K^{-1} = \begin{pmatrix} 7 & 18 \\ 23 & 11 \end{pmatrix}.$$

Suppose we want to encrypt the plaintext *july*. We have two elements of plaintext to encrypt: $(9, 20)$ (corresponding to *ju*) and $(11, 24)$ (corresponding to *ly*). We compute as follows:

$$(9, 20) \begin{pmatrix} 11 & 8 \\ 3 & 7 \end{pmatrix} = (99 + 60, 72 + 140) = (3, 4)$$

and

$$(11, 24) \begin{pmatrix} 11 & 8 \\ 3 & 7 \end{pmatrix} = (121 + 72, 88 + 168) = (11, 22).$$

Hence, the encryption of *july* is *DELW*. To decrypt, Bob would compute:



Decryption

Hence, the encryption of *july* is *DELW*. To decrypt, Bob would compute:

$$(3,4) \begin{pmatrix} 7 & 18 \\ 23 & 11 \end{pmatrix} = (9,20)$$

and

$$(11,22) \begin{pmatrix} 7 & 18 \\ 23 & 11 \end{pmatrix} = (11,24).$$

Hence, the correct plaintext is obtained.

Polyalphabetic Ciphers

- Polyalphabetic substitution cipher
 - Improves on the simple monoalphabetic technique by using different monoalphabetic substitutions as one proceeds through the plaintext message

All these techniques have the following features in common:

- A set of related monoalphabetic substitution rules is used
- A key determines which particular rule is chosen for a given transformation

Vigenère Cipher

- Best known and one of the simplest polyalphabetic substitution ciphers
- In this scheme the set of related monoalphabetic substitution rules consists of the 26 Caesar ciphers with shifts of 0 through 25
- Each cipher is denoted by a key letter which is the ciphertext letter that substitutes for the plaintext letter a

Example of Vigenère Cipher

- To encrypt a message, a key is needed that is as long as the message
- Usually, the key is a repeating keyword
- For example, if the keyword is *deceptive*, the message “we are discovered save yourself” is encrypted as:

key: deceptivedeceptivedeceptive

plaintext: wearediscoveredsaveyourself

ciphertext: ZICVTWQNGRZGVTWAVZHCQYGLMGJ

Example

Example 2.4 Suppose $m = 6$ and the keyword is *CIPHER*. This corresponds to the numerical equivalent $K = (2, 8, 15, 7, 4, 17)$. Suppose the plaintext is the string

thiscryptosystemisnotsecure.

We convert the plaintext elements to residues modulo 26, write them in groups of six, and then “add” the keyword modulo 26, as follows:

$$\begin{array}{cccccccccccc}
 19 & 7 & 8 & 18 & 2 & 17 & 24 & 15 & 19 & 14 & 18 & 24 \\
 2 & 8 & 15 & 7 & 4 & 17 & 2 & 8 & 15 & 7 & 4 & 17 \\
 \hline
 21 & 15 & 23 & 25 & 6 & 8 & 0 & 23 & 8 & 21 & 22 & 15
 \end{array}$$

$$\begin{array}{cccccccccccc}
 18 & 19 & 4 & 12 & 8 & 18 & 13 & 14 & 19 & 18 & 4 & 2 \\
 2 & 8 & 15 & 7 & 4 & 17 & 2 & 8 & 15 & 7 & 4 & 17 \\
 \hline
 20 & 1 & 19 & 19 & 12 & 9 & 15 & 22 & 8 & 25 & 8 & 19
 \end{array}$$

$$\begin{array}{ccc}
 20 & 17 & 4 \\
 2 & 8 & 15 \\
 \hline
 22 & 25 & 19
 \end{array}$$



Complexity

- Possible key length = 26^m
- $m = 5 \cdot 26^5 = 1.1 \times 10^7$

Vigenère Autokey System

- A keyword is concatenated with the plaintext itself to provide a running key
- Example:
key: deceptivewearediscoveredsav
plaintext: wearediscoveredsaveyourself
ciphertext: ZICVTWQNGKZEIIGASXSTSLVWLA
- Even this scheme is vulnerable to cryptanalysis
 - Because the key and the plaintext share the same frequency distribution of letters, a statistical technique can be applied

Example

Example 2.9 Suppose the key is $K = 8$, and the plaintext is
rendezvous.

We first convert the plaintext to a sequence of integers:

17 4 13 3 4 25 21 14 20 18

The keystream is as follows:

8 17 4 13 3 4 25 21 14 20

Now we add corresponding elements, reducing modulo 26:

25 21 17 16 7 3 20 9 8 12

In alphabetic form, the ciphertext is:

ZVRQH DUJIM.



Decryption

25 21 17 16 7 3 20 9 8 12

Then we compute

$$x_1 = d_8(25) = (25 - 8) \bmod 26 = 17.$$

Next,

$$x_2 = d_{17}(21) = (21 - 17) \bmod 26 = 4,$$



Vernam Cipher

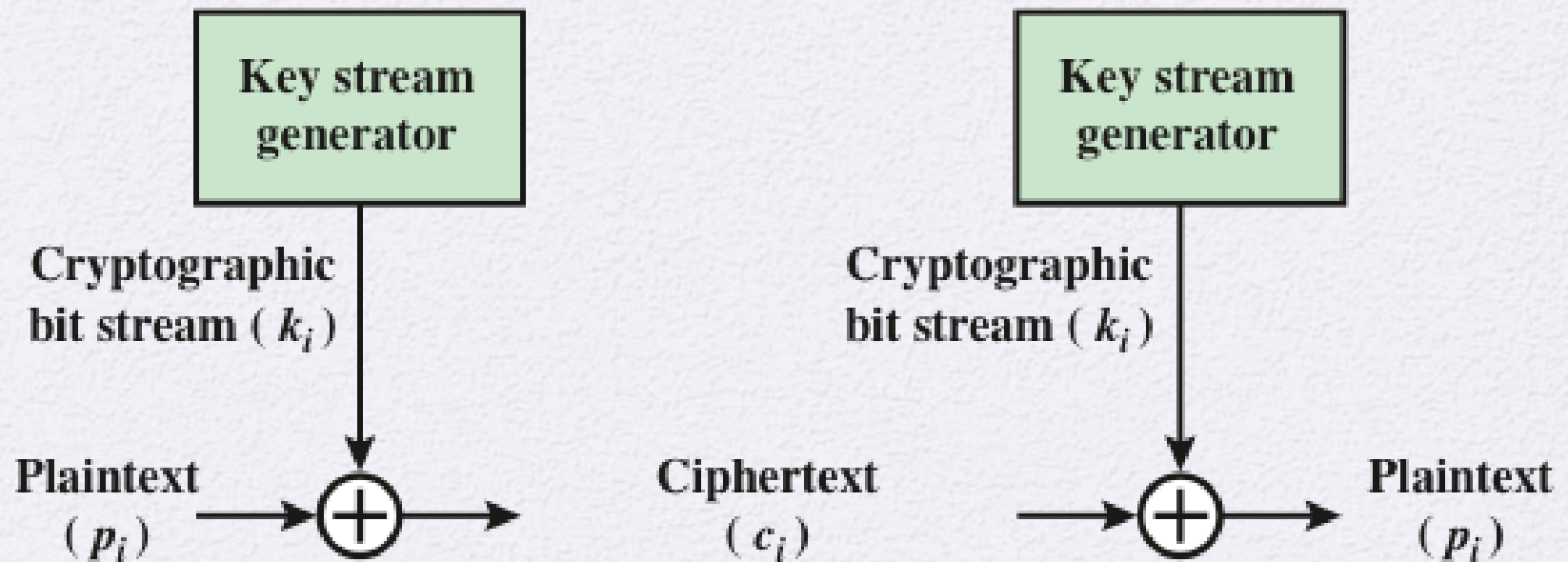


Figure 2.7 Vernam Cipher

One-Time Pad

- Improvement to Vernam cipher proposed by an Army Signal Corp officer, Joseph Mauborgne
- Use a random key that is as long as the message so that the key need not be repeated
- Key is used to encrypt and decrypt a single message and then is discarded
- Each new message requires a new key of the same length as the new message
- Scheme is unbreakable
 - Produces random output that bears no statistical relationship to the plaintext
 - Because the ciphertext contains no information whatsoever about the plaintext, there is simply no way to break the code



Difficulties

- The one-time pad offers complete security but, in practice, has two fundamental difficulties:
 - There is the practical problem of making large quantities of random keys
 - Any heavily used system might require millions of random characters on a regular basis
 - Mammoth key distribution problem
 - For every message to be sent, a key of equal length is needed by both sender and receiver
- Because of these difficulties, the one-time pad is of limited utility
 - Useful primarily for low-bandwidth channels requiring very high security
- The one-time pad is the only cryptosystem that exhibits *perfect secrecy* (see Appendix F)

Transposition / Permutation Cipher

- A permutation of a finite set X is a bijective function (one to one and onto function)

Rail Fence Cipher

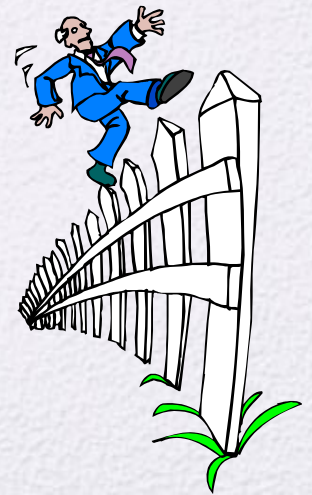
- Simplest transposition cipher
- Plaintext is written down as a sequence of diagonals and then read off as a sequence of rows
- To encipher the message “meet me after the toga party” with a rail fence of depth 2, we would write:

m e m a t r h t g p r y

e t e f e t e o a a t

Encrypted message is:

MEMATRHTGPRYETEFETEOAAT



Row Transposition Cipher

- Is a more complex transposition
- Write the message in a rectangle, row by row, and read the message off, column by column, but permute the order of the columns
 - The order of the columns then becomes the key to the algorithm

Key: 4 3 1 2 5 6 7

Plaintext: a t t a c k p
 o s t p o n e
 d u n t i l t
 w o a m x y z

Ciphertext: T T N A A P T M T S U O A O D W C O I X K N L Y P E T Z

Example

Example 2.7 Suppose $m = 6$ and the key is the following permutation π :

x	1	2	3	4	5	6
$\pi(x)$	3	5	1	6	4	2

shesellsseashellsbytheseashore.

We first partition the plaintext into groups of six letters:

shesel | lsseas | hellsb | ythese | ashore

Now each group of six letters is rearranged according to the permutation π , yielding the following:

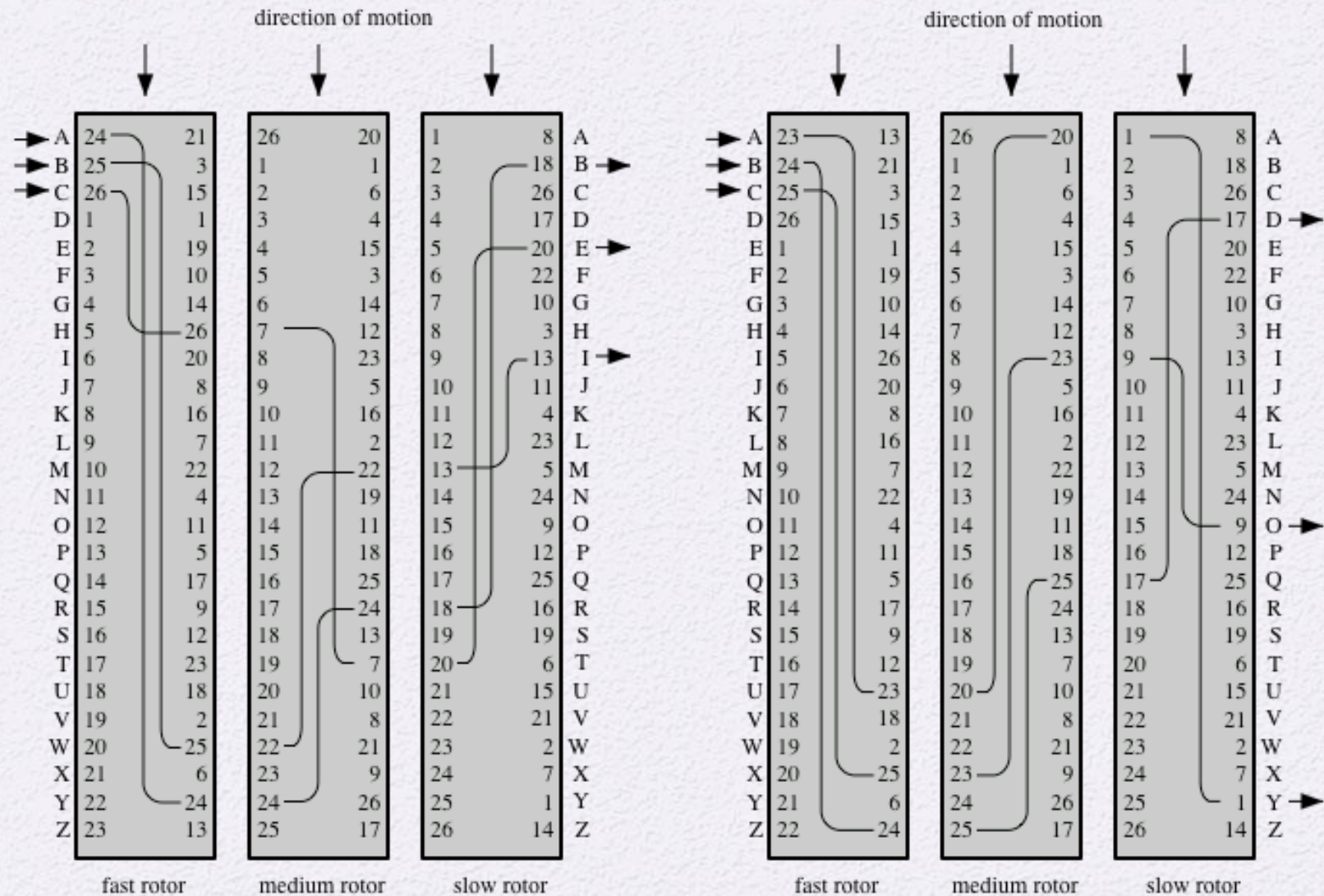
EESLSH | SALSES | LSHBLE | HSYEET | HRAEOS

So, the ciphertext is:

EESLSHSALSESLSHBLEHSYEETHRAEOS.



Rotor Machines



(a) Initial setting

(b) Setting after one keystroke

Figure 2.8 Three-Rotor Machine With Wiring Represented by Numbered Contacts

Steganography

3rd March

Dear George,

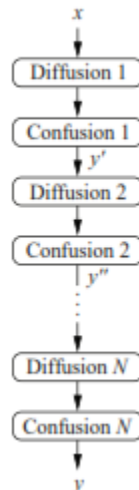
Greetings to all at Oxford. Many thanks for your letter and for the Summer examination package. All Entry Forms and Fees Forms should be ready for final despatch to the Syndicate by Friday 20th or at the very latest, I'm told, by the 21st. Admin has improved here, though there's room for improvement still; just give us all two or three more years and we'll really show you! Please don't let those wretched 16+ proposals destroy your basic O and A pattern. Certainly this sort of change, if implemented immediately, would bring chaos.

Sincerely yours,

Figure 2.9 A Puzzle for Inspector Morse
(from *The Silent World of Nicholas Quinn*, by Colin Dexter)

Product Cipher

1. **Confusion** is an encryption operation where the relationship between key and ciphertext is obscured. Today, a common element for achieving confusion is substitution, which is found in both DES and AES.
2. **Diffusion** is an encryption operation where the influence of one plaintext symbol is spread over many ciphertext symbols with the goal of hiding statistical properties of the plaintext. A simple diffusion element is the bit permutation, which is used frequently within DES. AES uses the more advanced Mixcolumn operation.



Example

Example 3.1. Let's assume a small block cipher with a block length of 8 bits. Encryption of two plaintexts x_1 and x_2 , which differ only by one bit, should roughly result in something as shown in Fig. 3.2.

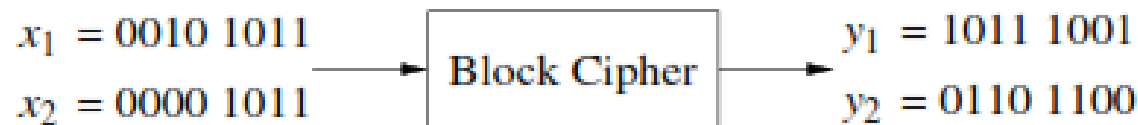


Fig. 3.2 Principle of diffusion of a block cipher

Note that modern block ciphers have block lengths of 64 or 128 bit but they show exactly the same behavior if one input bit is flipped.

Other Steganography Techniques



- Character marking
 - Selected letters of printed or typewritten text are over-written in pencil
 - The marks are ordinarily not visible unless the paper is held at an angle to bright light
- Invisible ink
 - A number of substances can be used for writing but leave no visible trace until heat or some chemical is applied to the paper
- Pin punctures
 - Small pin punctures on selected letters are ordinarily not visible unless the paper is held up in front of a light
- Typewriter correction ribbon
 - Used between lines typed with a black ribbon, the results of typing with the correction tape are visible only under a strong light

Summary

- Symmetric Cipher Model
 - Cryptography
 - Cryptanalysis and Brute-Force Attack
- Transposition techniques
- Rotor machines



- Substitution techniques
 - Caesar cipher
 - Monoalphabetic ciphers
 - Playfair cipher
 - Hill cipher
 - Polyalphabetic ciphers
 - One-time pad
- Steganography