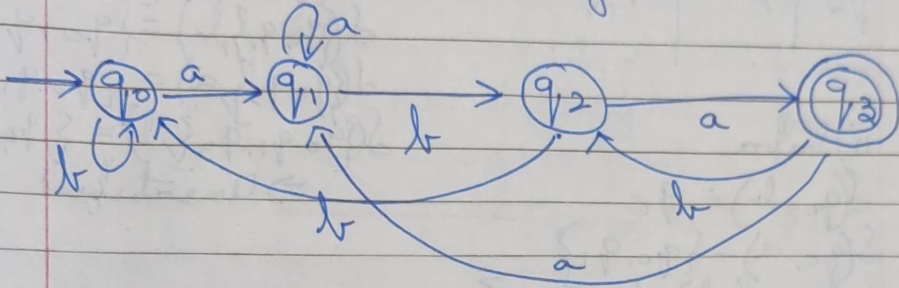


# UCS1503 - Theory of Computation Assignment - 1

1)  $L = \{ abba, aaba, baab, \dots \}$

DFA Transition diagram



Transition table

$\delta$	a	b
$q_0$	$q_1$	$q_0$
$q_1$	$q_1$	$q_2$
$q_2$	$q_3$	$q_0$
$q_3$	$q_1$	$q_2$

Checking for

i)  $bbabab$

$$\delta(q_0, b) = q_0$$

$$\delta(q_0, b) = q_0$$

$$\delta(q_0, a) = q_1$$

$$\delta(q_1, b) = q_2$$

$$\delta(q_2, a) = q_3$$

$$\delta(q_3, b) = q_2$$

$\Rightarrow$  Not satisfied

ii)  $baaba$

$$\delta(q_0, b) = q_0$$

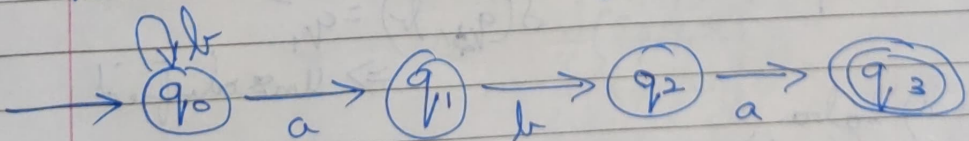
$$\delta(q_0, a) = q_1$$

$$\delta(q_1, b) = q_2$$

$$\delta(q_2, a) = q_3$$

$\Rightarrow$  Satisfied

NFA Transition diagram



# Transition Table

$\delta$	a	b
$q_0$	$\{q_0, q_1\}$	$\phi, q_0$
$q_1$	$\phi$	$q_2$
$q_2$	$q_3$	$\phi$
$q_3$	$\phi$	$\phi$

## Checking word

i) lbalal

$$\delta(q_0, l) = \phi \neq q_0$$

$\Rightarrow$  Unsatisfied

$$\delta(q_0, l) = q_0$$

$$\delta(q_0, a) = \{q_0, q_1\}$$

$$\delta(\{q_0, q_1\}, b) = \{q_0, q_2\}$$

$$\delta(\{q_0, q_2\}, a) = \{q_0, q_1, q_3\}$$

$$\delta(\{q_0, q_1, q_3\}, l) = \{q_0, q_2, \phi\}$$

$\Rightarrow$  Unsatisfied

ii) balba

$$\delta(q_0, b) = q_0$$

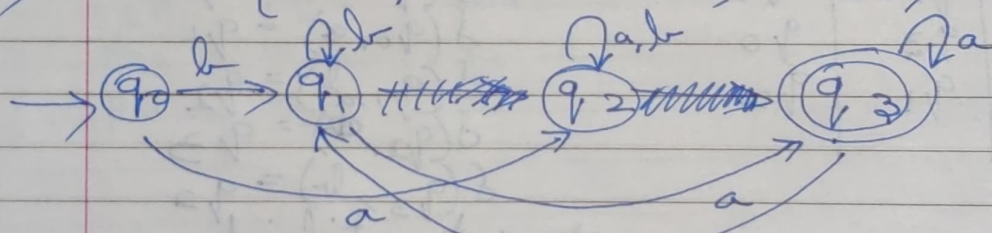
$$\delta(q_0, a) = \{q_0, q_1\}$$

$$\delta(\{q_0, q_1\}, b) = \{q_0, q_2\}$$

$$\delta(\{q_0, q_2\}, a) = \{q_0, q_1, q_3\}$$

$\Rightarrow$  Satisfied

2)  $L = \{ba, lba, lba, \dots\}$



i) lbalal

$$\delta(q_0, l) = q_1$$

$$\delta(q_1, b) = q_1$$

$$\delta(q_1, a) = q_3$$

$$\delta(q_3, b) = q_1$$

$$\delta(q_1, a) = q_3$$

$$\delta(q_3, l) = q_1$$

$\Rightarrow$  Unsatisfied

ii) balba

$$\delta(q_0, b) = q_1$$

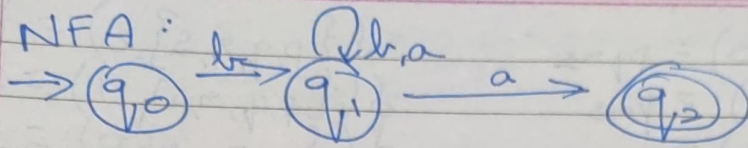
$$\delta(q_1, a) = q_3$$

$$\delta(q_3, b) = q_1$$

$$\delta(q_1, l) = q_3$$







$\delta$	a	b
$q_0$	$\emptyset$	$\{q_1\}$
$q_1$	$\{q_1, q_2\}$	$\{q_1\}$
$q_2$	$\emptyset$	$\emptyset$

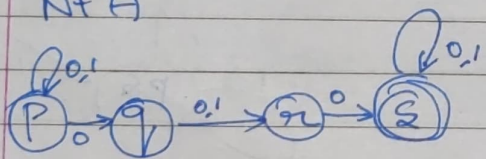
i) b b a b a b

$$\begin{aligned} \delta(q_0, b) &= q_1 \\ \delta(q_1, b) &= q_1 \\ \delta(q_1, a) &= q_1, q_2 \\ \delta(\{q_1, q_2\}, b) &= q_1 \\ \delta(\{q_1\}, a) &= q_1, q_2 \\ \delta(\{q_1, q_2\}, b) &= q_1 \end{aligned}$$

ii) b a b a

$$\begin{aligned} \delta(q_0, b) &= q_1 \\ \delta(q_1, a) &= \{q_1, q_2\} \\ \delta(\{q_1, q_2\}, b) &= q_1 \\ \delta(\{q_1\}, a) &= \{q_1, q_2\} \Rightarrow \text{Satisfied} \end{aligned}$$

3) NFA



$$\begin{aligned} M &= \{Q, \Sigma, p, F, \delta\} \\ Q &= \{p, q, r, s\} \\ \Sigma &= \{0, 1\} \\ F &= \{s\} \end{aligned}$$

DFA:

$$M' = \{Q', \Sigma, p', F', \delta'\}$$

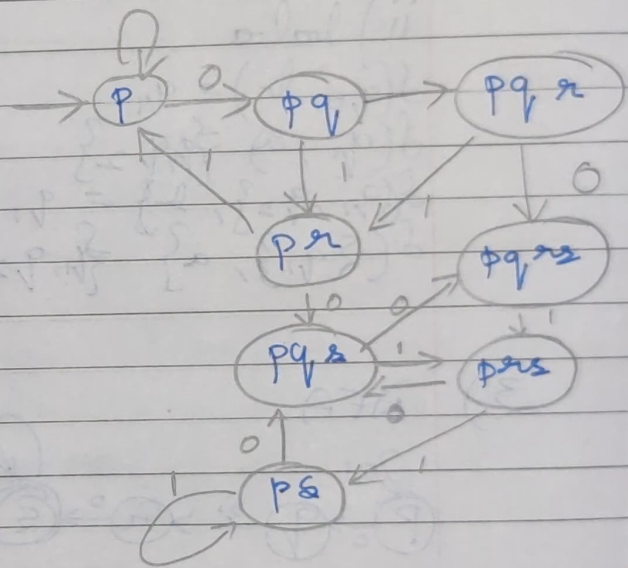
$$\begin{aligned} \delta(p, 0) &= pq \\ \delta(p, 1) &= p \\ \delta(pq, 0) &= pq, r \\ \delta(pq, 1) &= pr \\ \delta(pq, r, 0) &= pq, r, s \\ \delta(pq, r, 1) &= pr \\ \delta(pr, 0) &= pq, s \\ \delta(pr, 1) &= p \end{aligned}$$

$$\begin{aligned} \delta(p, 0) &= \{p, q\} \\ \delta(p, 1) &= \{p\} \\ \delta(\{p, q\}, 0) &= \{p, q, r\} \\ \delta(\{p, q\}, 1) &= \{p, r\} \\ \delta(\{p, q, r\}, 0) &= \{p, r, s\} \\ \delta(\{p, q, r\}, 1) &= \{p, r\} \\ \delta(\{p, r\}, 0) &= \{p, q, s\} \\ \delta(\{p, r\}, 1) &= \{p\} \end{aligned}$$

$$\begin{aligned}
 \delta(pqrs, 0) &= pqrs \\
 \delta(pqrs, 1) &= prs \\
 \delta(pqs, 0) &= pqrs \\
 \delta(pqs, 1) &= prs \\
 \delta(prs, 0) &= pqs \\
 \delta(prs, 1) &= ps \\
 \delta(ps, 0) &= pqs \\
 \delta(ps, 1) &= ps
 \end{aligned}$$

$$\begin{aligned}
 \delta(\{p, q, r, s\}, 0) &= \{p, q, r, s\} \\
 \delta(\{p, q, r, s\}, 1) &= \{p, r, s\} \\
 \delta(\{p, q, r\}, 0) &= \{p, q, r, s\} \\
 \delta(\{p, q, r\}, 1) &= \{p, r, s\} \\
 \delta(\{p, r, s\}, 0) &= \{p, q, s\} \\
 \delta(\{p, r, s\}, 1) &= \{p, s\} \\
 \delta(\{p, s\}, 0) &= \{p, q, s\} \\
 \delta(\{p, s\}, 1) &= \{p, s\}
 \end{aligned}$$

$\delta$	0	1
p	pq	p
pq	pqr	pr
pqr	pqrs	pr
pr	pqs	p
pqrs	pqrs	prs
pqs	pqrs	prs
prs	pqs	ps
ps	pqs	ps



$$\begin{aligned}
 Q' &= \{p, pq, pqr, pr, pqs, pqrs, prs, ps\} \\
 \Sigma &= \{0, 1\} \\
 F &= \{pqrs, pqs, prs, ps\}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } M &= \{Q, \Sigma, p, F, \delta\} & Q &= \{p, q, r, s\} \\
 F &= \{q, s\} & \Sigma &= \{0, 1\}
 \end{aligned}$$

DFA :

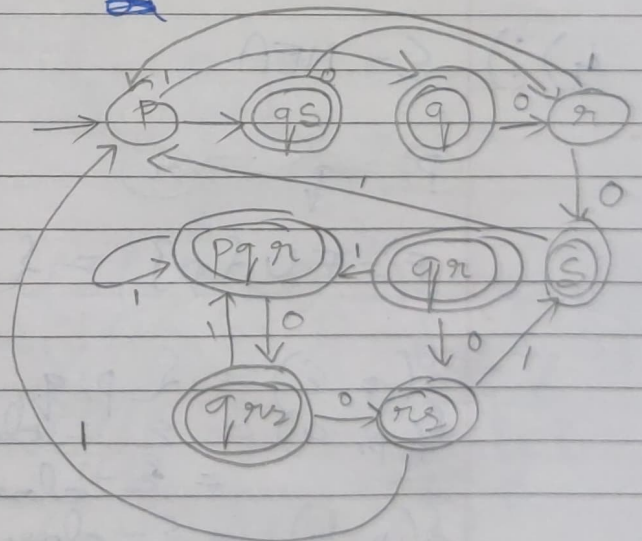
$$M' = \{Q', \Sigma, p, F', \delta'\}$$



$$\begin{aligned}
 \delta(p, 0) &= qs \\
 \delta(p, 1) &= q \\
 \delta(qs, 0) &= r \\
 \delta(qs, 1) &= pqn \\
 \delta(q, 0) &= r \\
 \delta(q, 1) &= qn \\
 \delta(r, 0) &= s \\
 \delta(r, 1) &= p \\
 \delta(pqn, 0) &= qns \\
 \delta(pqn, 1) &= pqn \\
 \delta(qn, 0) &= ns \\
 \delta(qn, 1) &= pqn \\
 \delta(s, 0) &= \emptyset \\
 \delta(s, 1) &= p \\
 \delta(qns, 0) &= ns \\
 \delta(qns, 1) &= pqn \\
 \delta(pns, 0) &= s \\
 \delta(pns, 1) &= p
 \end{aligned}$$

$$\begin{aligned}
 \delta(p, 0) &= \{qs\} \\
 \delta(p, 1) &= \{q\} \\
 \delta(qs, 0) &= \{r\} \\
 \delta(qs, 1) &= \{p, q, n\} \\
 \delta(q, 0) &= \{r\} \\
 \delta(q, 1) &= \{qn\} \\
 \delta(r, 0) &= \{s\} \\
 \delta(r, 1) &= \{p\} \\
 \delta(pqn, 0) &= \{q, n, s\} \\
 \delta(pqn, 1) &= \{p, q, n\} \\
 \delta(qn, 0) &= \{n, s\} \\
 \delta(qn, 1) &= \{p, q, n\} \\
 \delta(s, 0) &= \emptyset \\
 \delta(s, 1) &= \{p\} \\
 \delta(qns, 0) &= \{ns\} \\
 \delta(qns, 1) &= \{p, q, n\} \\
 \delta(pns, 0) &= \{s\} \\
 \delta(pns, 1) &= \{p\}
 \end{aligned}$$

$\delta'$	0	1
p	qs	q
qs	r	pqn
q	r	qn
r	s	p
pqn	qns	pqn
qn	ns	pqn
s	-	p
qns	ns	pqn
ns	s	p

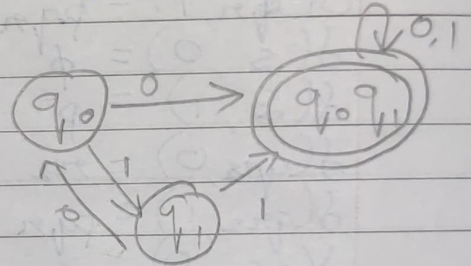


iii)  $M(Q, \Sigma, q_0, F, \delta)$   
 $Q = \{q_0, q_1\}$   
 $\Sigma = \{0, 1\}$   
 $F = \{q_0\}$

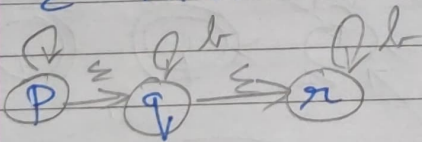
$$\begin{aligned}\delta(q_0, 0) &= q_0 q_1 \\ \delta(q_0, 1) &= q_1 \\ \delta(q_0 q_1, 0) &= q_0 q_1 \\ \delta(q_0 q_1, 1) &= q_0 q_1 \\ \delta(q_1, 0) &= q_0 \\ \delta(q_1, 1) &= q_0 q_1\end{aligned}$$

$$\begin{aligned}\delta(q_0, 0) &= \{q_0, q_1\} \\ \delta(q_0, 1) &= q_1 \\ \delta(\{q_0, q_1\}, 0) &= \{q_0, q_1\} \\ \delta(\{q_0, q_1\}, 1) &= \{q_0, q_1\} \\ \delta(q_1, 0) &= q_0 \\ \delta(q_1, 1) &= \{q_0, q_1\}\end{aligned}$$

$\delta'$	0	1
$q_0$	$q_0 q_1$	$q_1$
$q_0 q_1$	$q_0 q_1$	$q_0 q_1$
$q_1$	$q_0$	$q_0 q_1$



4) i)  $\Sigma$ -NFA



$$\begin{aligned}\Sigma\text{-closure}(p) &= \{p, q, r\} \\ \Sigma\text{-closure}(q) &= \{q, r\} \\ \Sigma\text{-closure}(r) &= \{r\}\end{aligned}$$

$$F' = F \cup \{p\} = \{p, r\}$$

$$\begin{aligned}\delta(p, \epsilon) &= \{p, q, r\} \\ \delta(p, a) &= \Sigma\text{-closure}(\delta(\delta(p, \epsilon)), a) \\ &= \Sigma\text{-closure}(p) = \{p, q, r\} \\ \delta(p, b) &= \Sigma\text{-closure}(\delta(\delta(p, \epsilon), b)) \\ &= \Sigma\text{-closure}(q, r) = \{q, r\} \\ \delta(q, a) &= \Sigma\text{-closure}(\delta(\delta(q, \epsilon)), a) \\ &= \Sigma\text{-closure}(q, r, b) = \emptyset\end{aligned}$$



$$\delta(q, b) = \varepsilon\text{-closure}(\delta(\delta(q, \varepsilon), b))$$

$$= \{q, r\}$$

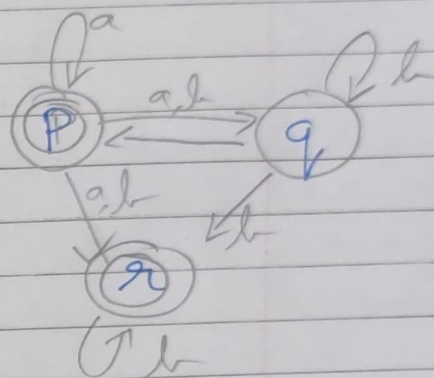
$$\delta(q, a) = \varepsilon\text{-closure}(\delta(\delta(q, \varepsilon), a))$$

$$= \phi$$

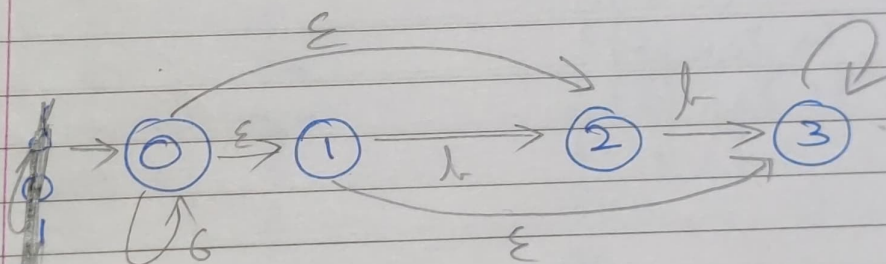
$$\delta(r, b) = \varepsilon\text{-closure}(\delta(\delta(r, \varepsilon), b))$$

$$= \{r\}$$

$\delta'$	a	b
* p	{p, q, r}	{q, r}
q	$\phi$	{q, r}
r	$\phi$	{r}



ii)



$\varepsilon$  closure of

$$0 = 0, 1, 2, 3$$

$$1 = 1, 3$$

$$2 = \phi$$

$$3 = 3$$

$$\delta(0, a) = \varepsilon\text{-closure}(\delta(\delta(0, a, \varepsilon), a)) = \{0, 1, 2, 3\}$$

$$\delta(0, b) = \varepsilon\text{-closure}(\delta(\delta(0, \varepsilon), b)) = \{2, 3\}$$

$$\delta(1, a) = \varepsilon\text{-closure}(\delta(\delta(1, \varepsilon), a)) = \phi$$

$$\delta(1, b) = \varepsilon\text{-closure}(\delta(\delta(1, \varepsilon), b)) = \{2, 3\}$$

$$\delta(2, a) = \varepsilon\text{-closure}(\delta(\delta(2, \varepsilon), a)) = \phi$$

$$\delta(2, b) = \varepsilon\text{-closure}(\delta(\delta(2, \varepsilon), b)) = \{3\}$$

$$\delta(3, a) = \varepsilon\text{-closure}(\delta(\delta(3, \varepsilon), a)) = \phi$$

$$\delta(3, b) = \varepsilon\text{-closure}(\delta(\delta(3, \varepsilon), b)) = \{3\}$$

$$\delta(0, c) = \{3\} \quad \delta(1, c) = \{3\} \quad \delta(2, c) = \phi$$

$$\delta(3, c) = \{3\}$$

