CLOSURE PROPERTIES OF REGULAR SETS

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AP/CSE

LEARNING OBJECTIVE

- To construct finite automata for any given pattern and find its equivalent regular expressions
  - To understand the properties of FA and RE

DEFINITION OF REGULAR EXPRESSION

- Let Σ be an alphabet. The regular expressions over Σ and the sets that they denote are defined recursively as follows:

1. ϕ is a regular expression and denotes the empty set.
2. ε is a regular expression and denotes the set {ε}
3. For each a ∈ Σ, 'a' is a regular expression and denotes the set {a}.
4. If r and s are regular expressions denoting the languages R and S respectively then (r + s), (rs), (r)* are regular expressions that denotes the sets RUS, RS and R* respectively.

THEOREM 1

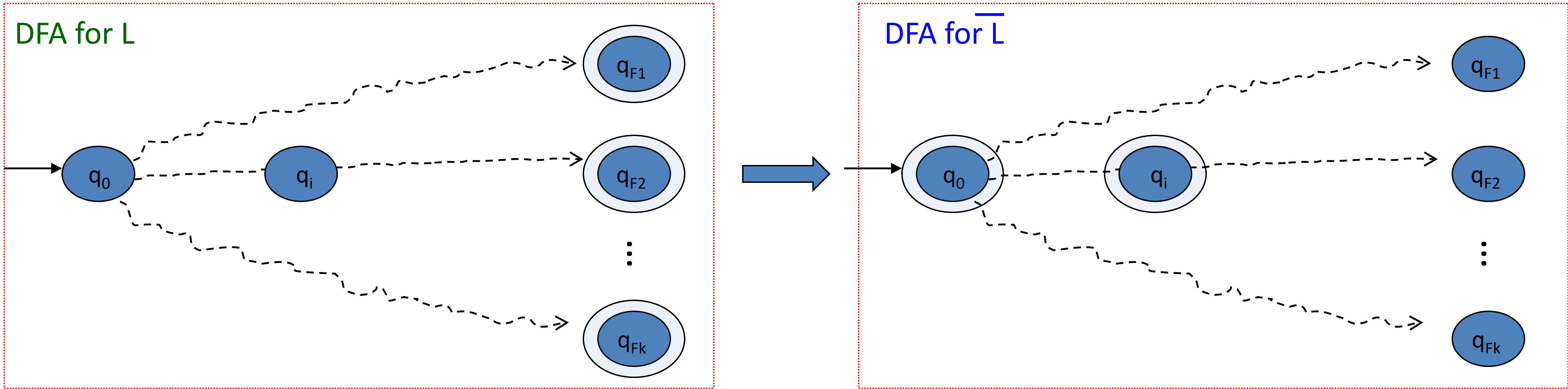
- Regular sets are closed under union, concatenation, and closure
- Proof
  - If L_1 and L_2 are regular, then there are regular expressions r_1 and r_2 denoting the languages L_1 and L_2 ,respectively.
  - By definition of RE "If r and s are regular expressions denoting the languages R and S respectively then (r + s), (rs), (r)* are regular expressions that denotes the sets RUS, RS and R* respectively"
  - Therefore $(r_1 + r_2)$, $(r_{1.} r_2)$ and $(r_1{}^*)$ are regular expressions denoting the languages $L_1 \cup L_{2,}$ $L_1 . L_2$ and $L_1{}^*$

THEOREM 2

- The class of regular sets is closed under complementation. ie If L is a regular set over $\sum$, then L=$\sum^*$-L is a regular set
- Proof
  - Suppose that L is a regular over an alphabet $\sum$.
  - There is a DFA M=(Q, $\sum$, δ, q_0, F) accepting L
  - Design a DFA M' = (Q, $\sum$, δ, q_0, F'), where F' = Q – F
  - Now, we have that L(M') = $\sum^*$ - L.
  - Hence, the complement of L is regular

THEOREM 2

Convert every final state into non-final, and every non-final state into a final state



Assumes q0 is a non-final state. If q0 is a final state, invert it.

THEOREM 3

- Regular sets are closed under intersection
- Proof
  - By DeMorgan's law:

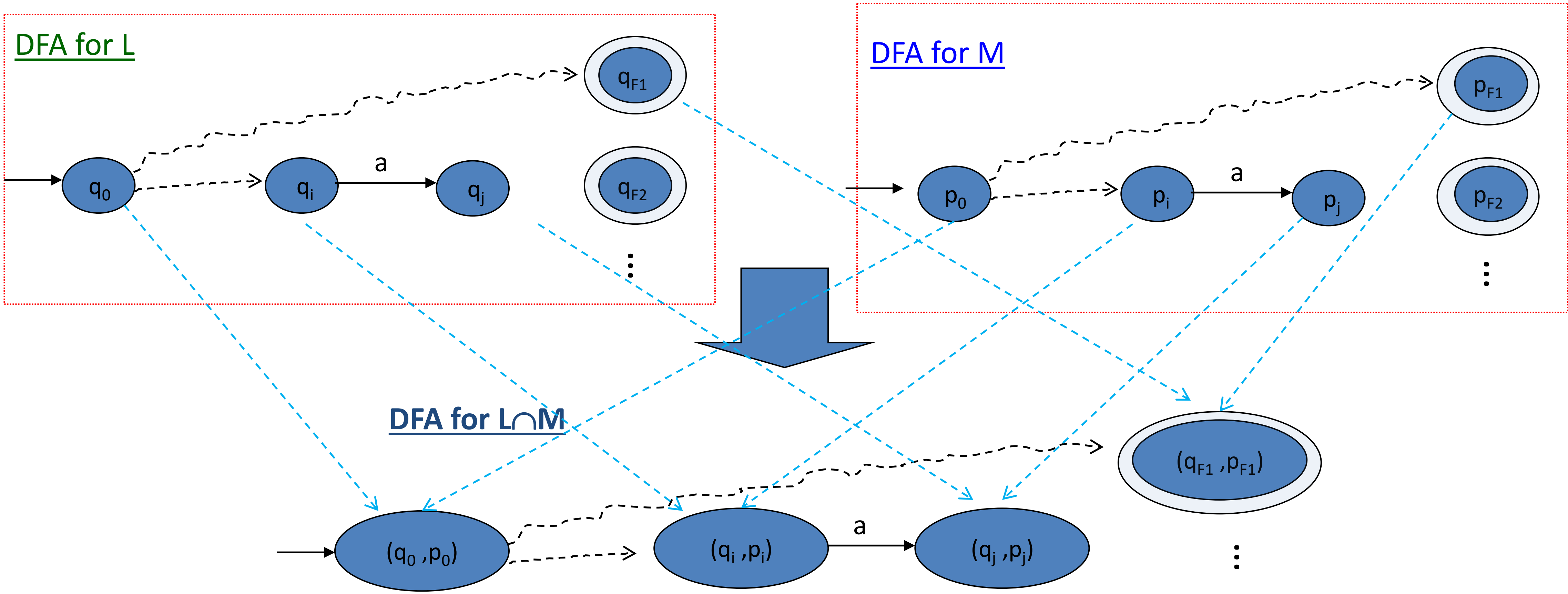
$$L \cap M = \overline{(\overline{L} \cup \overline{M})}$$

  - Since Regular Sets are closed under union and complementation, they are also closed under intersection

THEOREM 3

- A_L = DFA for L = $\{Q_L, \Sigma, q_L, F_L, \delta_L\}$
- A_M = DFA for M = $\{Q_M, \Sigma, q_M, F_M, \delta_M\}$
- Build $A_{L \cap M} = \{Q_L \times Q_M, \Sigma, (q_L, q_M), F_L \times F_M, \delta\}$ such that:
  - $\delta((p,q),a) = (\delta_L(p,a), \delta_M(q,a))$, where p in Q_L, and q in Q_M
- This construction ensures that a string w will be accepted if and only if w reaches an accepting state in both input DFAs.

THEOREM 3



DFA for L
q_{F1}
q_0
q_i
a
q_j
q_{F2}
DFA for M
p_{F1}
p_0
p_i
a
p_j
p_{F2}
DFA for L∩M
(q_{F1}, p_{F1})
(q_0, p_0)
(q_i, p_i)
a
(q_j, p_j)

THEOREM 4

- Regular sets are closed under set difference
- Proof

Closed under intersection

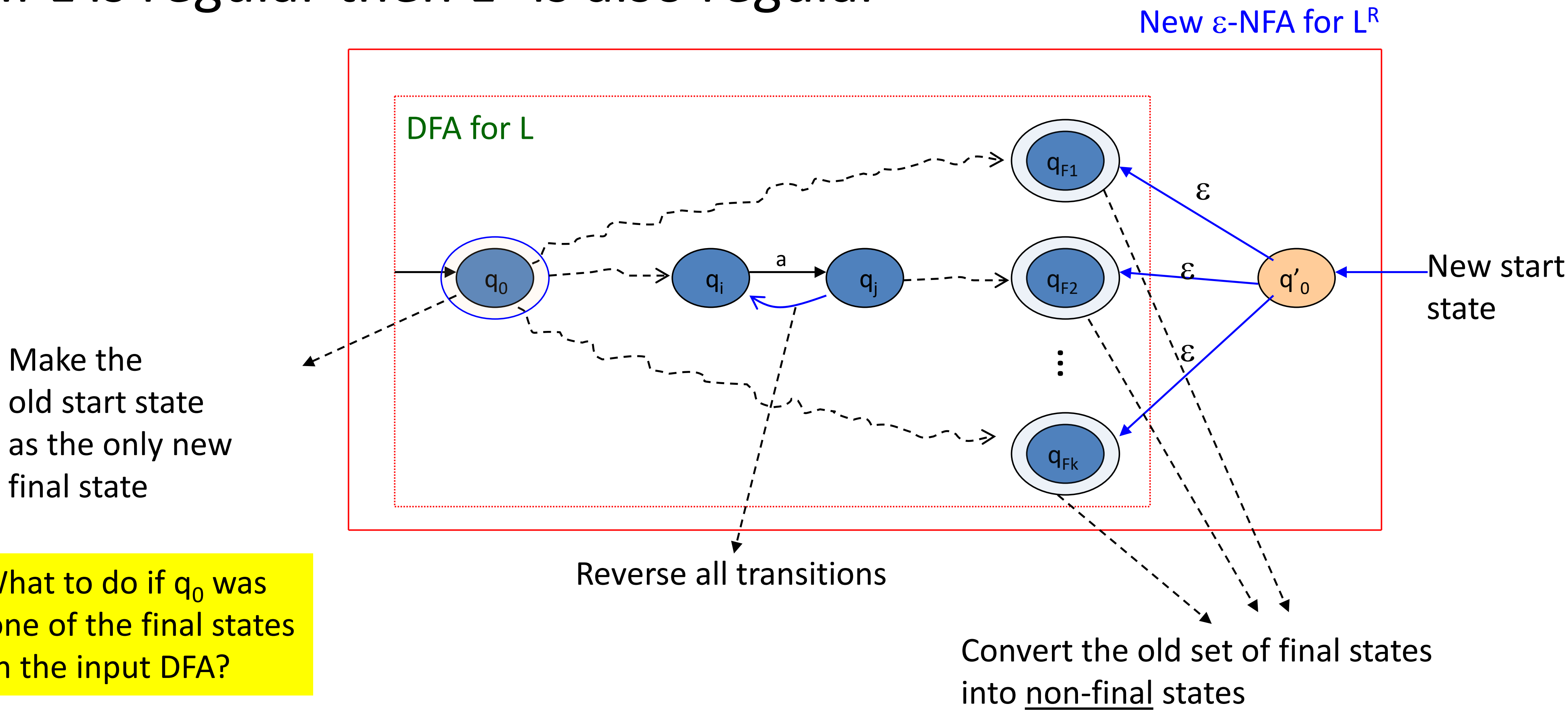
Closed under complementation

- $L - M = L \cap \overline{M}$

❖ Therefore, L - M is also regular

THEOREM 5

- If L is regular then L^R is also regular

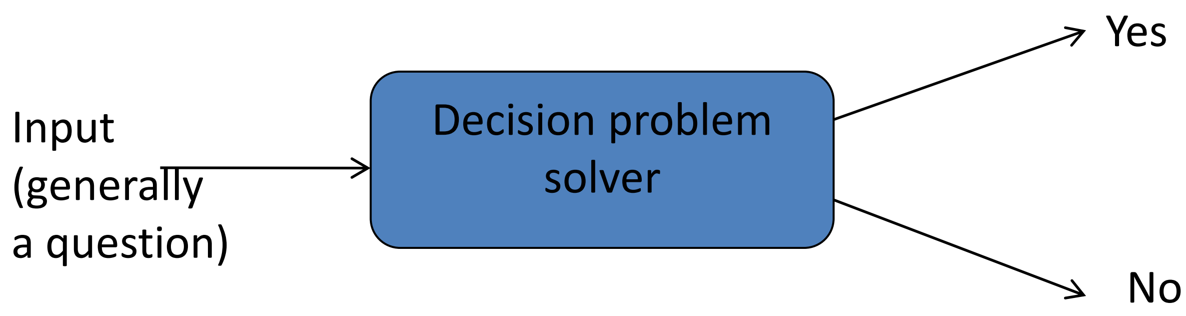


What to do if q_0 was one of the final states in the input DFA?

DECISION PROPERTIES OF REGULAR SETS

INTRODUCTION

- <u>Any “decision problem” looks like this</u>



MEMBERSHIP QUESTION

- <u>Decision Problem:</u> Given L, is w in L?
- <u>Possible answers:</u> Yes or No
- <u>Approach:</u>
  1. Build a DFA for L
  2. Input w to the DFA
  3. If the DFA ends in an accepting state, then yes; otherwise no.

EMPTINESS TEST

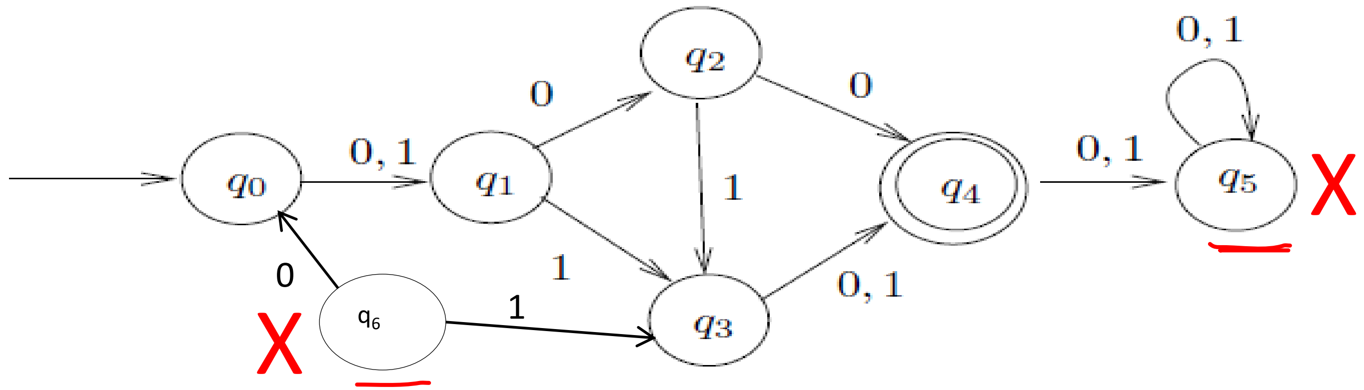
- Decision Problem: Is L=Ø ?
- Approach:
  1. Build a DFA for L
  2. From the start state, run a *reachability* test, which returns:
     1. success: if there is at least one final state that is reachable from the start state
     2. failure: otherwise
  3. L=Ø if and only if the reachability test fails

FINITENESS

- <u>Decision Problem:</u> Is L finite or infinite?
- <u>Approach:</u>
  1. Build DFA for L
  2. Remove all states unreachable from the start state
  3. Remove all states that cannot lead to any accepting state.
  4. After removal, check for cycles in the resulting FA
  5. L is finite if there are no cycles; otherwise it is infinite

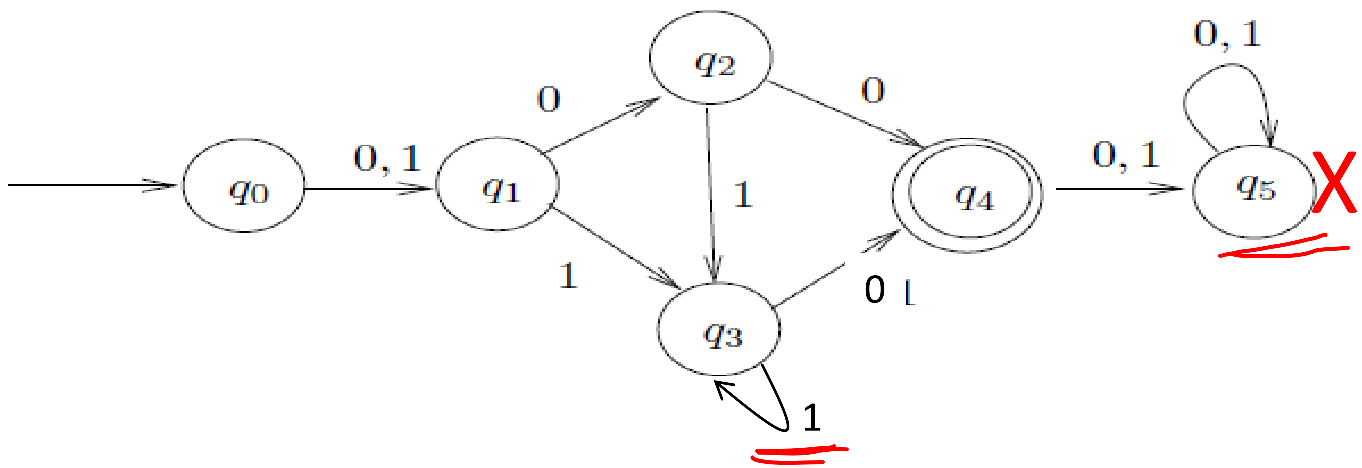
FINITENESS TEST - EXAMPLES

Ex 1) Is the language of this DFA finite or infinite?



FINITE

Ex 2) Is the language of this DFA finite or infinite?



INFINITE

SUMMARY

- Closure Properties of Regular Sets
- Decision Properties of Regular Sets

TEST YOUR KNOWLEDGE

- Regular sets are closed under union,concatenation and kleene closure.
  a) True
  b) False
  c) Depends on regular set
  d) Can’t say
- If L1 and L2 are regular sets then intersection of these two will be
  a) Regular
  b) Non Regular
  c) Recursive
  d) Non Recursive

TEST YOUR KNOWLEDGE

- Reverse of a DFA can be formed by
  a) using PDA
  b) making final state as non-final
  c) making final as starting state and starting state as final state
  d) None of the mentioned
- Complement of (a + b)* will be
  a) phi
  b) null
  c) a
  d) b

LEARNING OUTCOME

On successful completion of this topic, the student will be able to:

- Understand equivalence of FA and RE(K3)

REFERENCE

- Hopcroft J.E., Motwani R. and Ullman J.D, “Introduction to Automata Theory, Languages and Computations”, Second Edition, Pearson Education, 2008