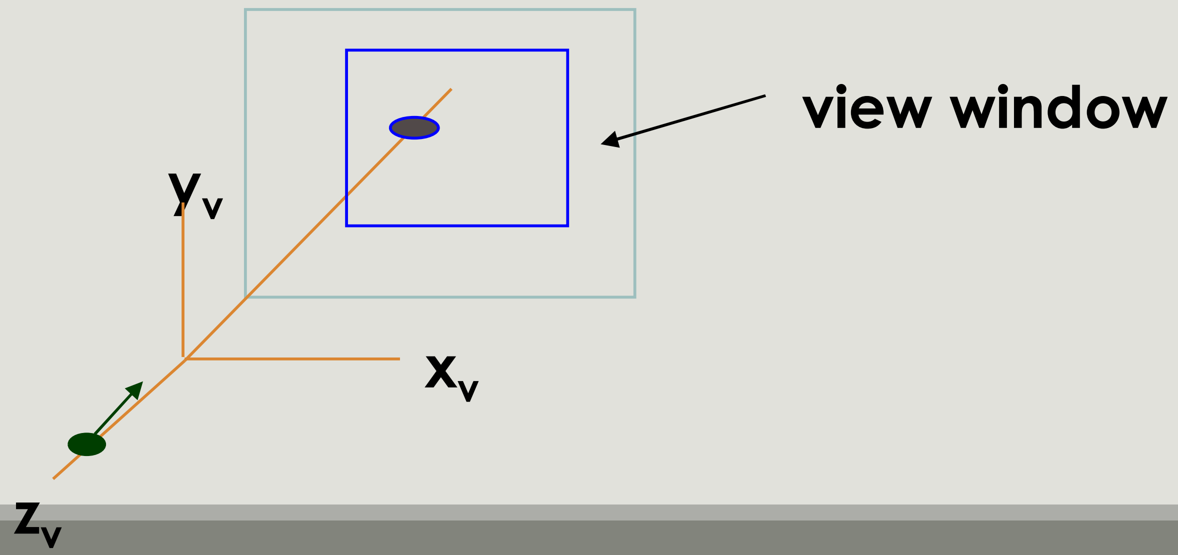
View Volumes and General Projection Transformations

View Volumes - View Window

- Type of lens in a camera is one factor which determines how much of the view is captured
  - wide angle lens captures more than regular lens
- Analogy in computer graphics is the view window, a rectangle in the view plane

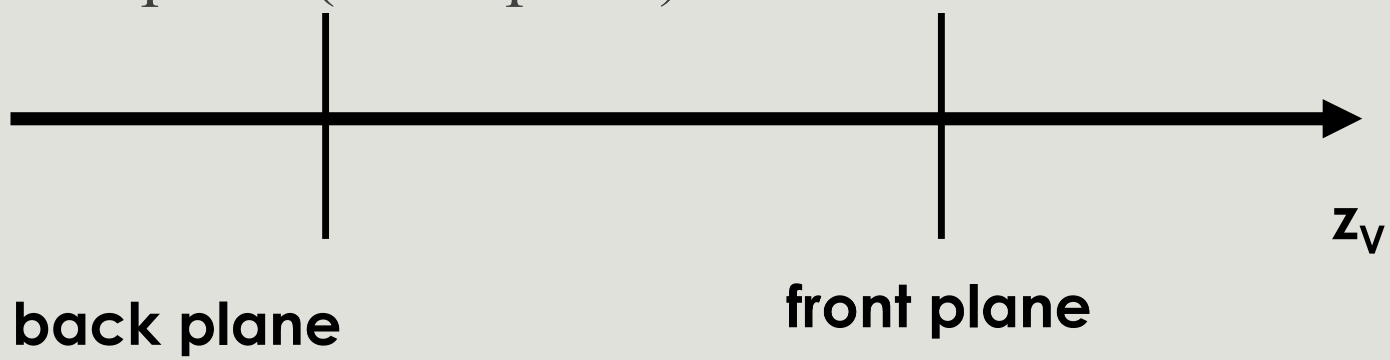


View Volumes

- Edges of the view window are parallel to the $x_v y_v$ axes and window boundary positions are specified in the viewing coordinates.
- View volume can be set up using the window boundaries.
- Objects within the view volume will appear on an output device all others are clipped from the display.
- The size of the view volume depends on the size of the window while the shape depends on the type of projection to be used.
- For parallel projection, the view volume forms an infinite parallelepiped and for perspective view volume is a pyramid.

View Volume - Front and Back Planes

- We will also typically want to limit the view in the z_V direction
- We define two planes, each parallel to the view plane, to achieve this
  - front plane (or near plane)
  - back plane (or far plane)



View Volume

- Front and back clipping planes allow us to eliminate parts of the scene from the viewing operations based on the depth.
- Both the planes must be on the same side of the projection reference point
- Back plane must be farther from the projection point than the front plane.
- Including the front and back planes a view volume is bounded by six planes.
- Orthographic parallel projection-->rectangular parallelepiped
- Oblique parallel projection--> oblique parallelepiped
- Perspective projection→ truncate the infinite pyramidal view volume to form a frustum

Parallel Projection View Volume

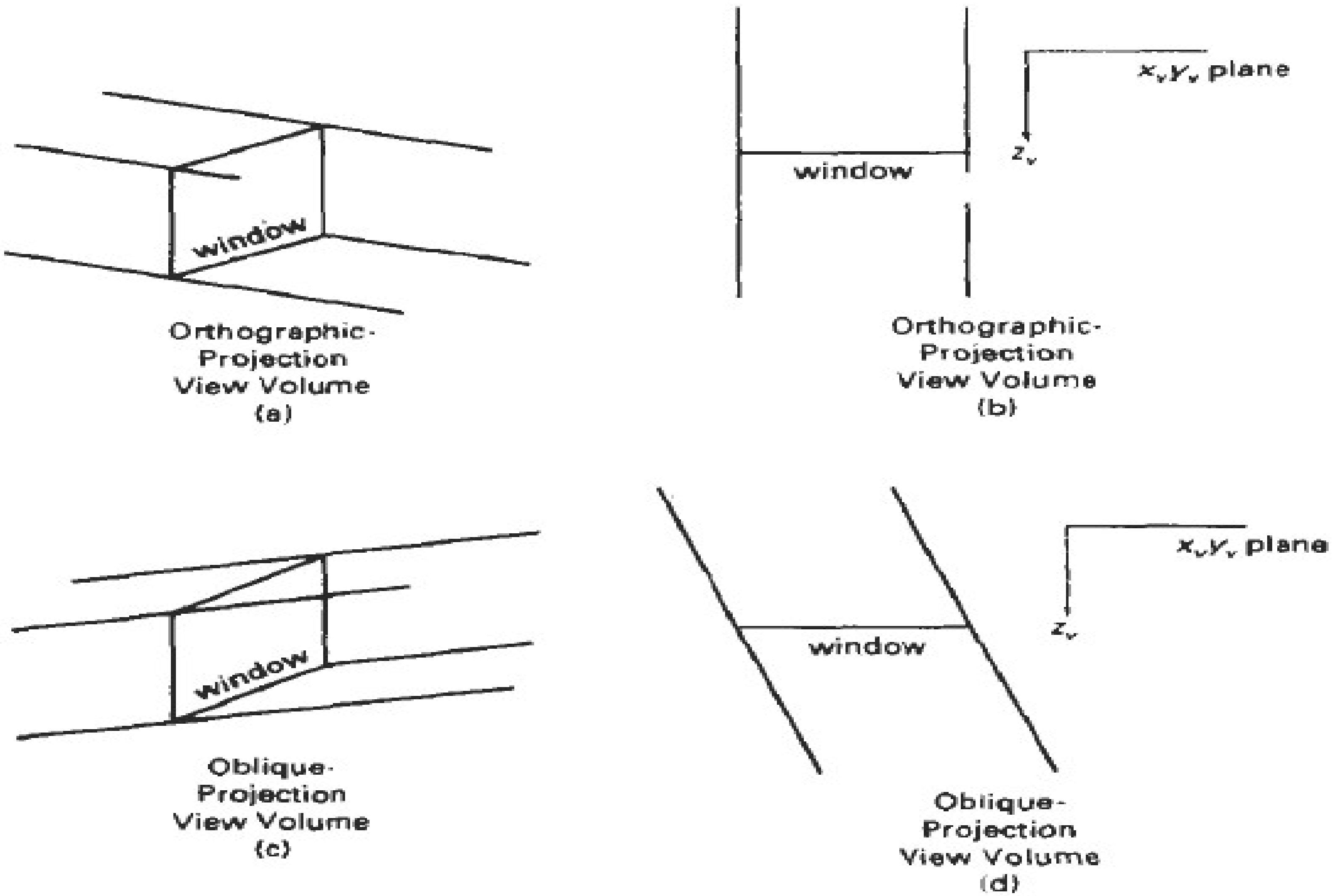


Figure 12-28
View volume for a parallel projection. In (a) and (b), the side and top views of the view volume for an orthographic projection are shown; and in (c) and (d), the side and top views of an oblique view volume are shown.

Perspective Projection View Volume

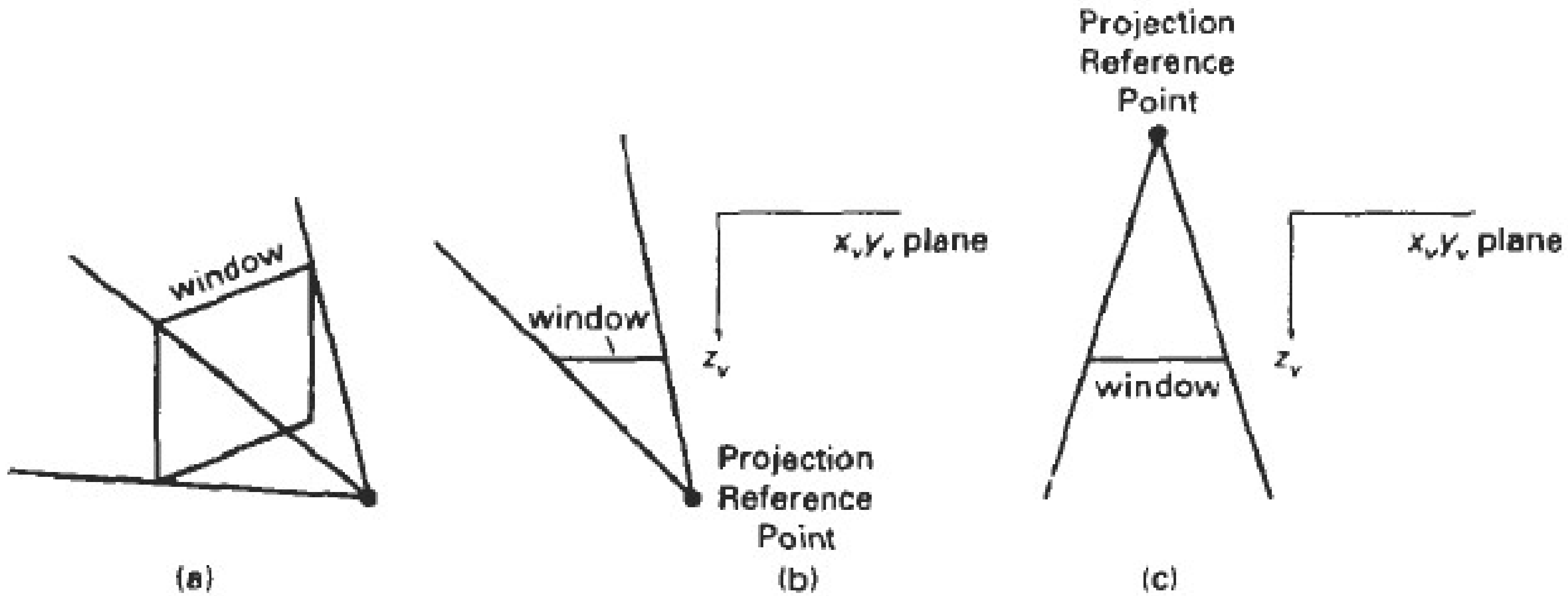
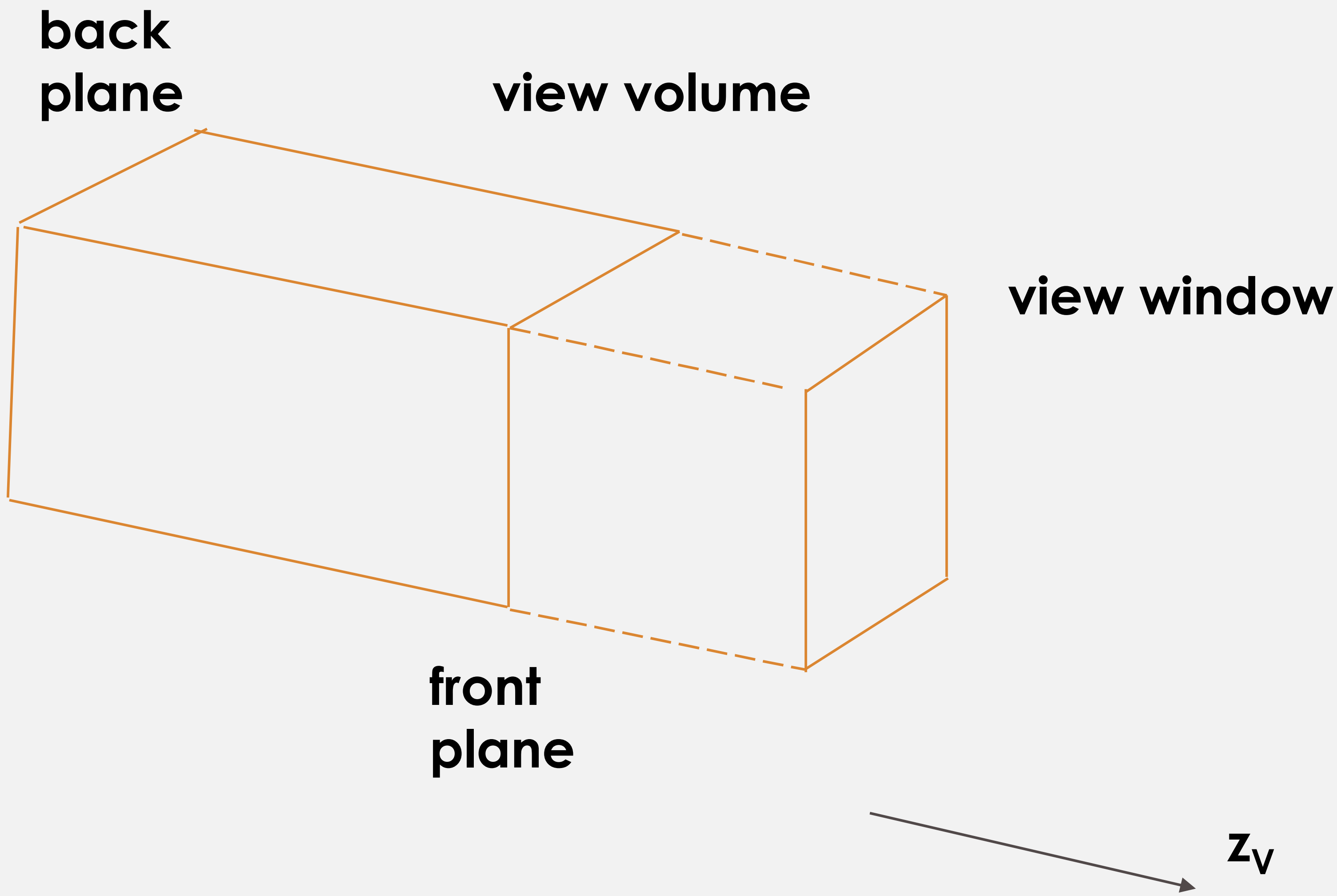


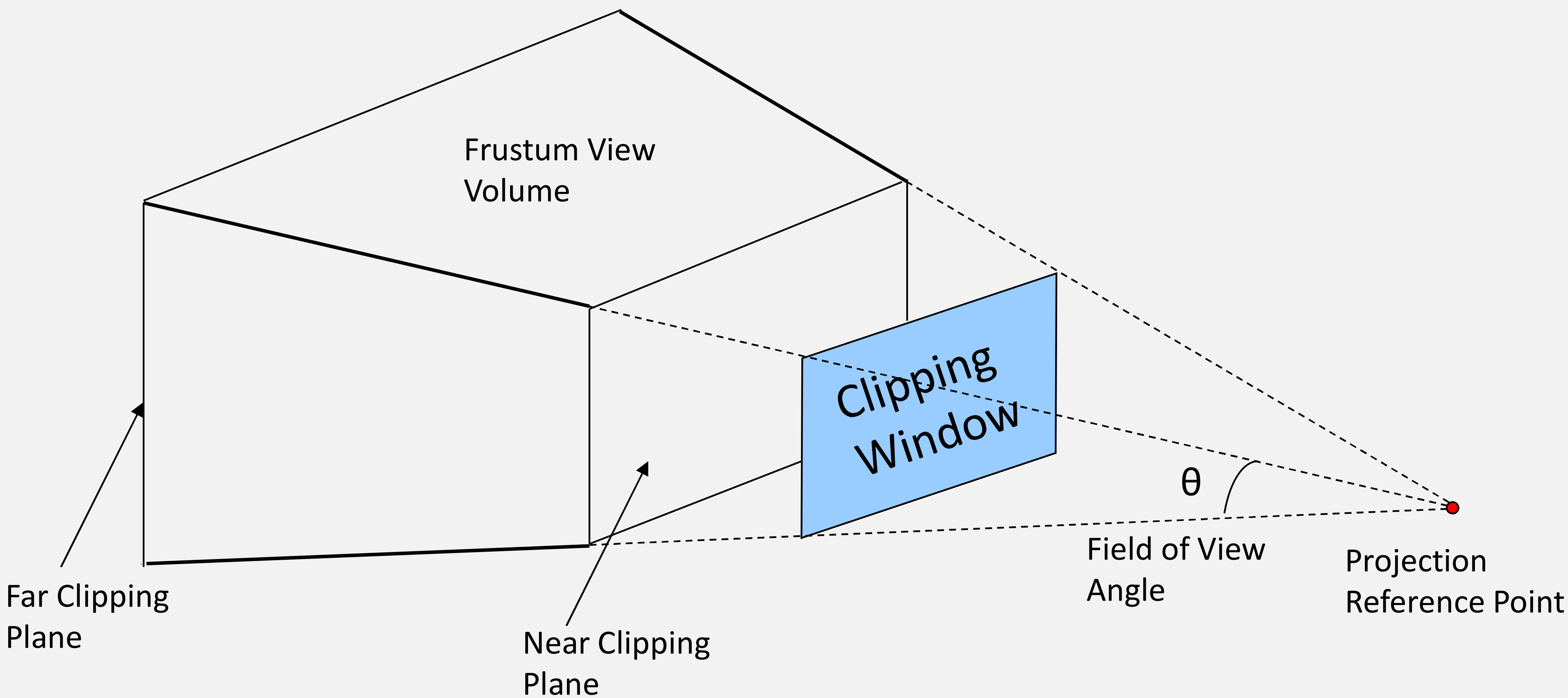
Figure 12-29
Examples of a perspective-projection view volume for various positions of the projection reference point.

View Volume - Parallel Projection

View Volume bounded by six planes



Perspective Projection View Volume



Effects of Moving the PRP w.r.t the view plane

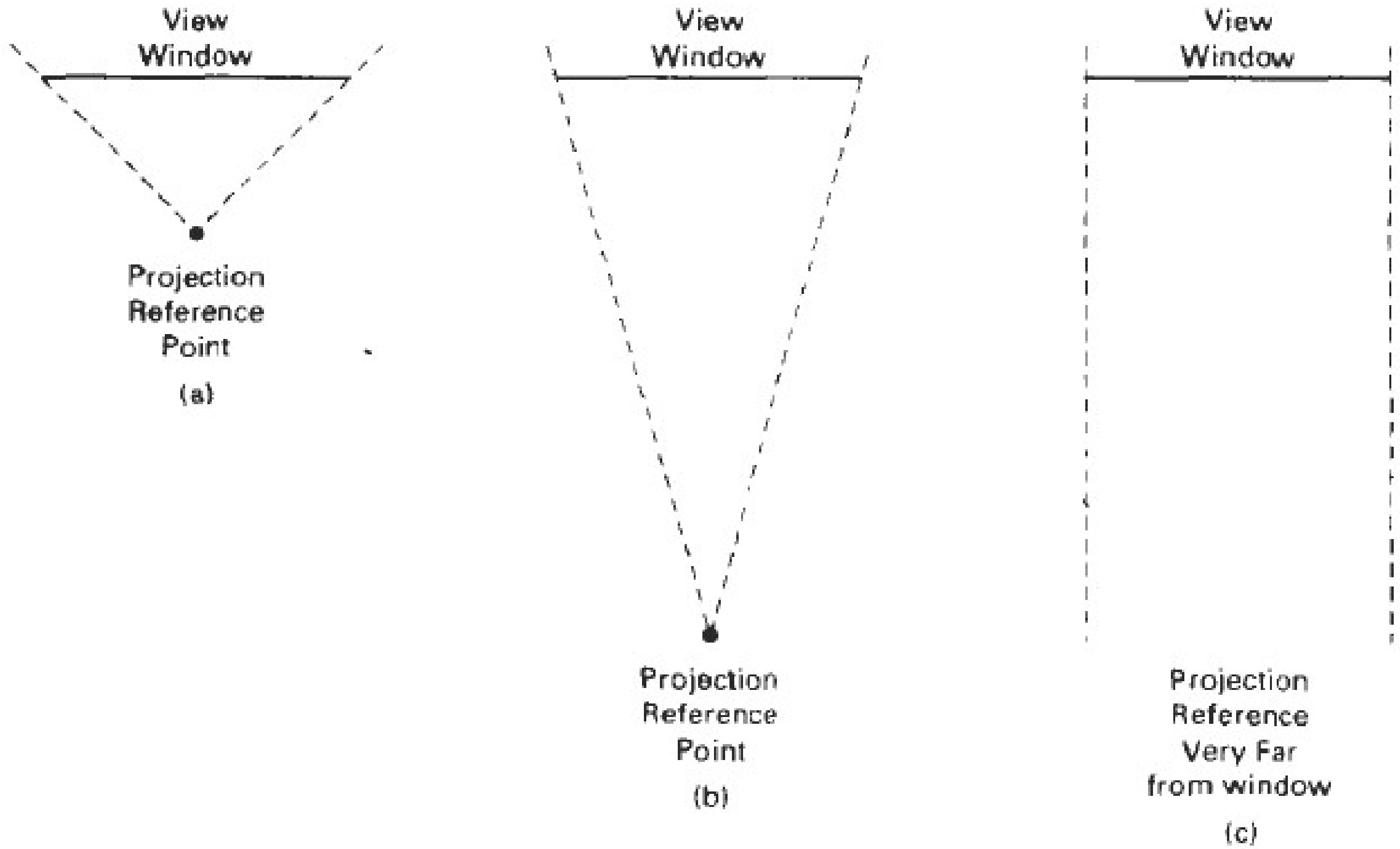


Figure 12-33
Changing perspective effects by moving the projection reference point away from the view plane

Projected Object size w.r.t the View Plane Position

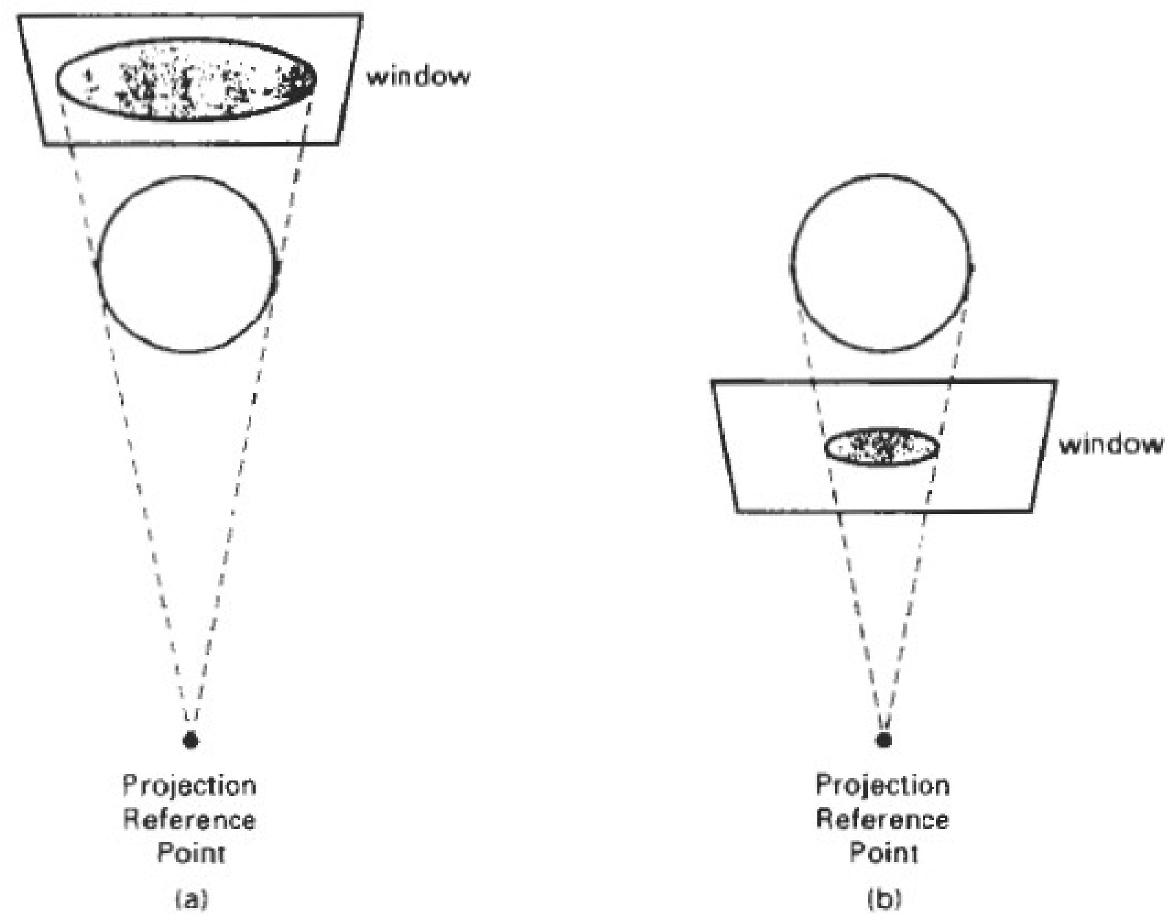


Figure 12-34
Projected object size depends on whether the view plane is positioned in front of the object or behind it, relative to the position of the projection reference point.

General Parallel Projection Transformation

General Parallel Projection Transformation

- The direction of the parallel projection is specified with a projection vector from the projection reference point to the center of the view window.

- Oblique parallel projection transformation is done by shearing to a regular parallelepiped.

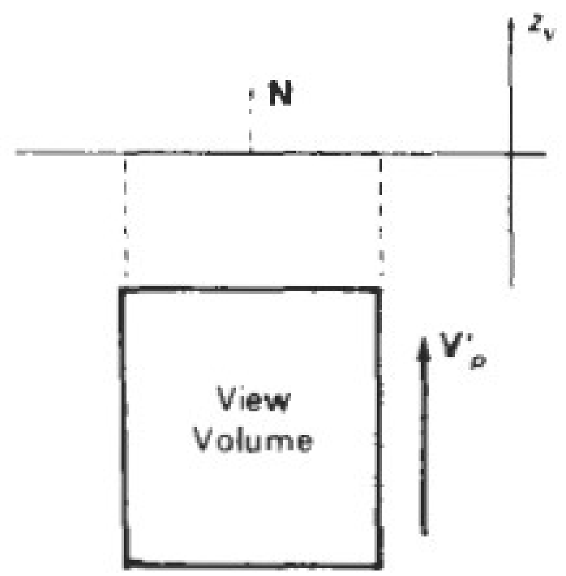


Figure 12-37
Regular parallelepiped view volume obtained by shearing the view volume in Fig. 12-36.

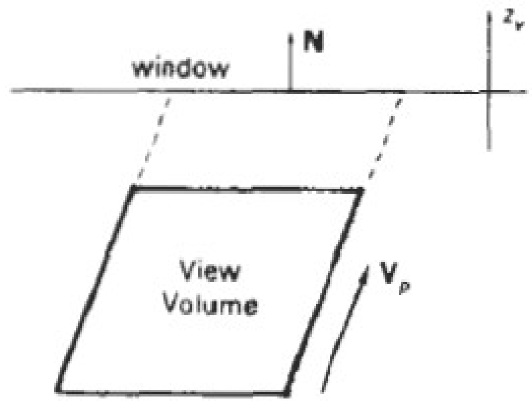


Figure 12-36
Oblique projection vector and associated view volume.

General Parallel Projection Transformation

The elements of the projection vector in viewing coordinates are

$$Vp = (p_x, p_y, p_z)$$

Shear the projection vector V_p with normal vector N

$$Vp' = M_{parallel} \, . \, Vp$$

$$M_{parallel} = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

General Parallel Projection Transformation

Explicit transformation equations in terms of shear parameters a and b are

$$0 = p_x + a p_z$$
$$0 = p_y + b p_z$$

$$a = -\frac{p_x}{p_z}, \qquad b = -\frac{p_y}{p_z}$$

General parallel projection matrix in terms of elements of projection vector is

$$\mathbf{M}_{\text{parallel}} = \begin{bmatrix} 1 & 0 & -p_x/p_z & 0 \\ 0 & 1 & -p_y/p_z & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

General Perspective Projection Transformation

General Perspective Projection Transformation

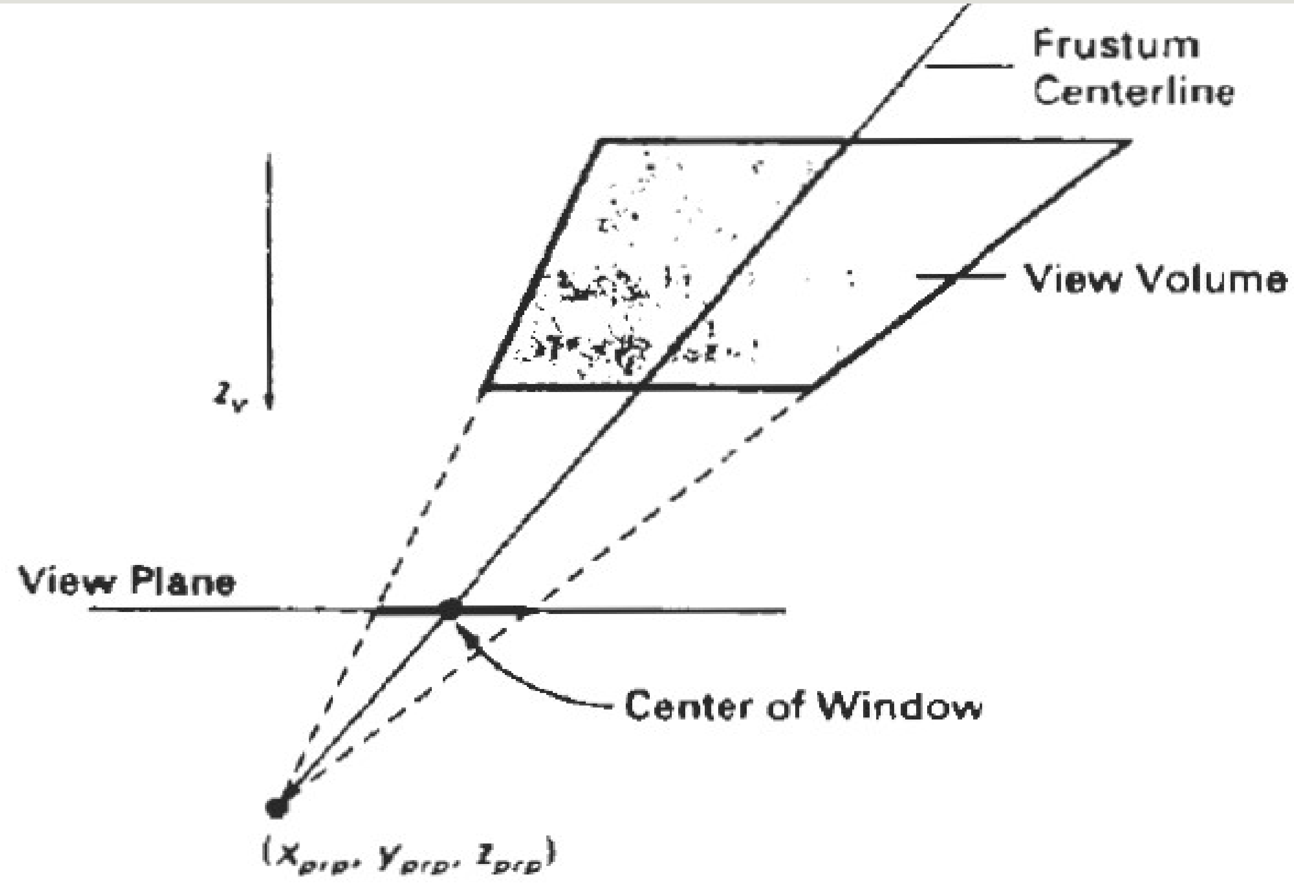


Figure 12-39
General shape for the perspective view volume with a projection reference point that is not on the z_v axis

General Perspective Projection Transformation

Done with the following 2 operations

1. Shear the view volume so that the centerline of the frustum is perpendicular to the view plane

2. Scale the view volume with a scaling factor that depends on 1/z

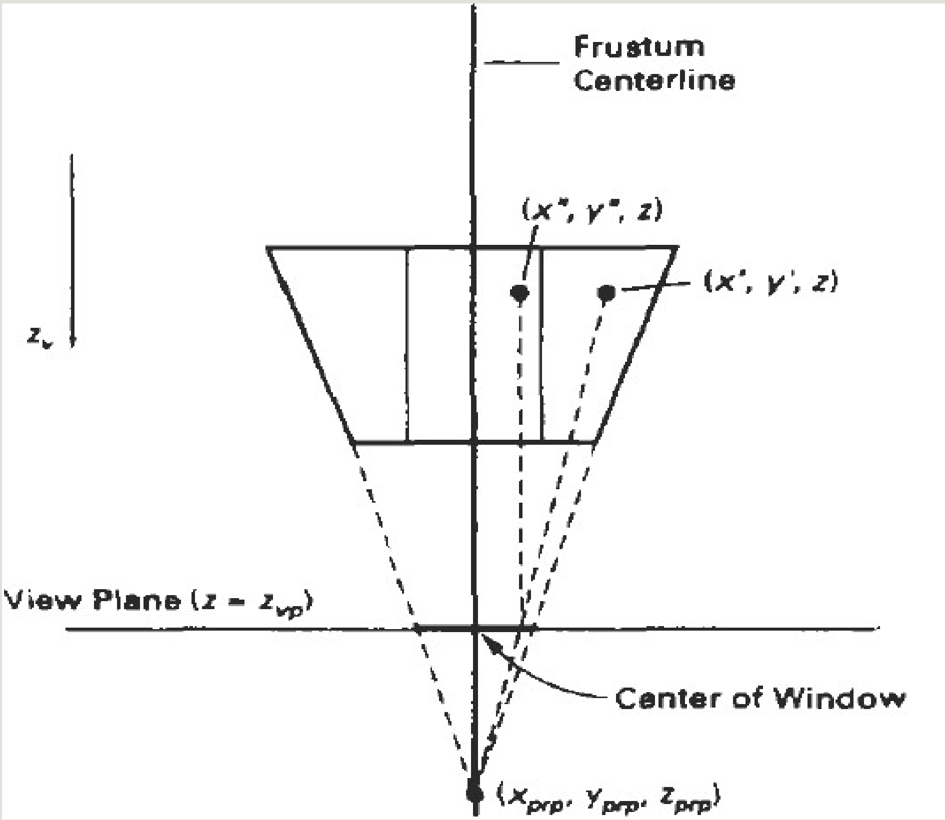


Figure 12-40
Shearing a general perspective view volume to center it on the projection window.

General Perspective Projection Transformation

The transformation matrix is

$$\mathbf{M}_{\text{shear}} = \begin{bmatrix} 1 & 0 & a & -a z_{prp} \\ 0 & 1 & b & -b z_{prp} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Points within the view volume are transformed by this operation as

$$x' = x + a(z - z_{prp})$$
$$y' = y + b(z - z_{prp})$$
$$z' = z$$

When projection reference point is on z axis

$$x_{prp} = y_{prp} = 0$$

General Perspective Projection Transformation

After shearing we apply scaling transformation to produce a regular parallelepiped.

$$x'' = x'\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + x_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$

$$y'' = y'\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + y_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$

General Perspective Projection Transformation

and the homogeneous matrix representation is

$$\mathbf{M}_{scale} = \begin{bmatrix} 1 & 0 & \dfrac{-x_{prp}}{z_{prp}-z_{vp}} & \dfrac{x_{prp}z_{vp}}{z_{prp}-z_{vp}} \\ 0 & 1 & \dfrac{-y_{prp}}{z_{prp}-z_{vp}} & \dfrac{y_{prp}z_{vp}}{z_{prp}-z_{vp}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \dfrac{-1}{z_{prp}-z_{vp}} & \dfrac{z_{prp}}{z_{prp}-z_{vp}} \end{bmatrix}$$

The general perspective projection transformation can be expressed as

$$M_{perspective} = M_{scale} \cdot M_{shear}$$