# 2D Representations and Transformations

# Overview

Coordinate Representations

Definition

2 Dimensional Geometric Transformations
- Translation
- Rotation
- Scaling

Homogeneous Coordinates

Reflection

Shearing

# Coordinate Reprsentations

A **Cartesian coordinate system** specifies each point uniquely in a plane by a pair of numerical **coordinates**, which are the signed distances from the point .

Graphics package are designed to use with Cartesian coordinate specifications.

Several different Cartesian reference frames are used to construct and display a scene.

The geometric part of the rendering process is that it consists of the application of a series of coordinate transformations that takes an object database through a series of coordinate systems.

# Coordinate Representations

*Local or Modelling Coordinate system* :For ease of modeling store the vertices of an object with respect to some point conveniently located in or near the object.

Ex:Construct the individual objects such as trees or furniture in a scene within separate coordinate reference frames .

Once an object has been modeled, the next stage is to place it in the scene that we wish to render

The global coordinate system of the scene is known as the *world coordinate system*

# Coordinate Representations

The world coordinate description of the scene is transferred to one or more output device reference frames for display called device  coordinates.

Modeling and world coordinate definitions allow us to any convenient dimensions.

Graphics system first converts the world coordinate positions to normalized device coordinates in the range of 0 to 1 before final conversion  to specific device coordinates.

$(x_{mc},y_{mc}) \rightarrow (x_{wc},y_{wc}) \rightarrow (x_{nc},y_{nc}) \rightarrow (x_{dc},y_{dc})$

# Geometric Transformation Definition

In many applications there is need for altering and manipulating displays.

Geometric Transformations: Operations that are applied to change the geometric description of an object by changing its position, orientation, or size.

The basic transformations are translation, rotation and scaling.

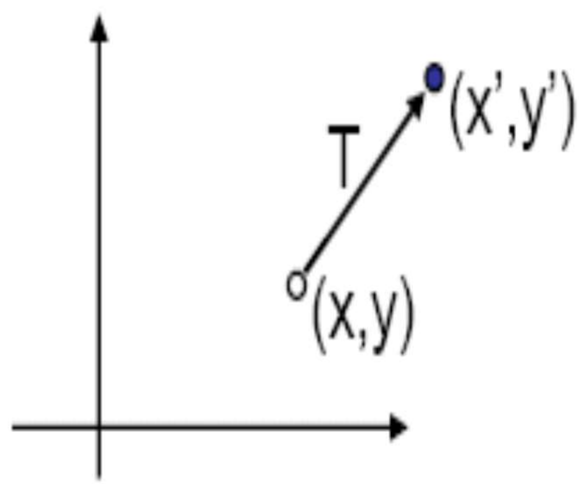
# TRANSLATION

Translation is applied to an object by repositioning it along a line path from one coordinate location to another.

Translate a two dimensional point by adding translation distances $t_x$ , $t_y$

- $x' = x + t_x$ , $y' = y + t_y$

The translation distance pair (tx, ty) is the translation vector or shift vector.



# TRANSLATION

Translation equations can be expressed as a single matrix equation

$$P = \begin{bmatrix} x \\ y \end{bmatrix} P' = \begin{bmatrix} x' \\ y' \end{bmatrix} T = \begin{bmatrix} tx \\ ty \end{bmatrix}$$

2D translation equation

$$P' = P + T$$

# TRANSLATION

Rigid body transformation → moves object without deformation

Every point is translated by the same amount

Straight line segment is translated by applying the transformations to each of the line endpoints and redrawing the line between the new endpoint positions.

A triangle with position (10,2), (20,2) and (15,5) is translated with the translation vector (-5.5,3.75). Determine the new positions of the triangle.

# 2D Rotation

A rotation transformation of an object is generated by specifying a rotation axis and a rotation angle.

All points of the object are then transformed to new positions by rotating the points through the specified angle about the rotation axis.

2D rotation is obtained by repositioning the object along a circular path in the *xy* plane.

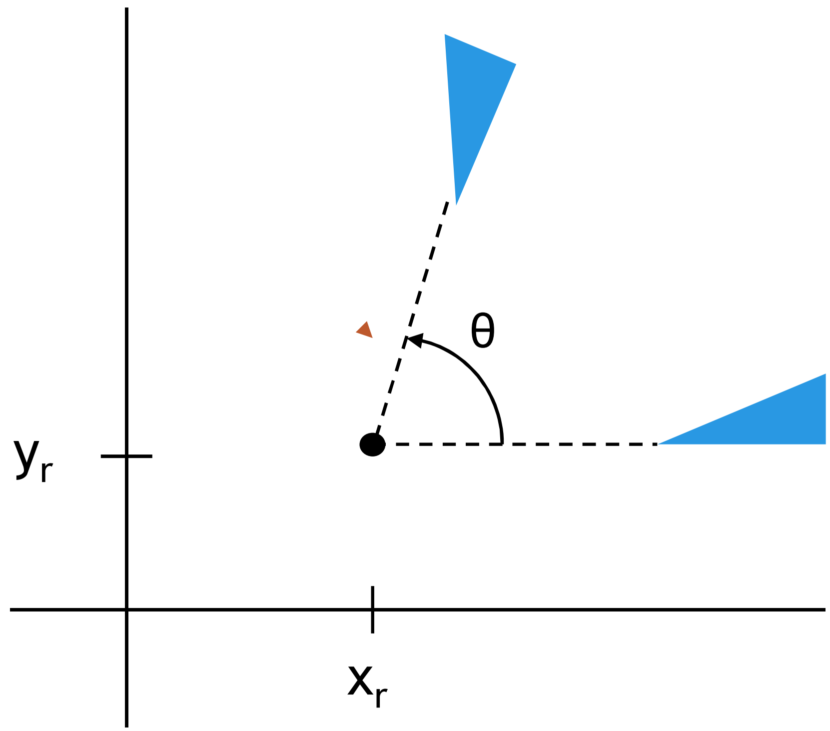
# 2D ROTATION

Parameter for 2D rotation:

- Rotation angle, $\theta$
- Rotation point (pivot point), $(xr, yr)$

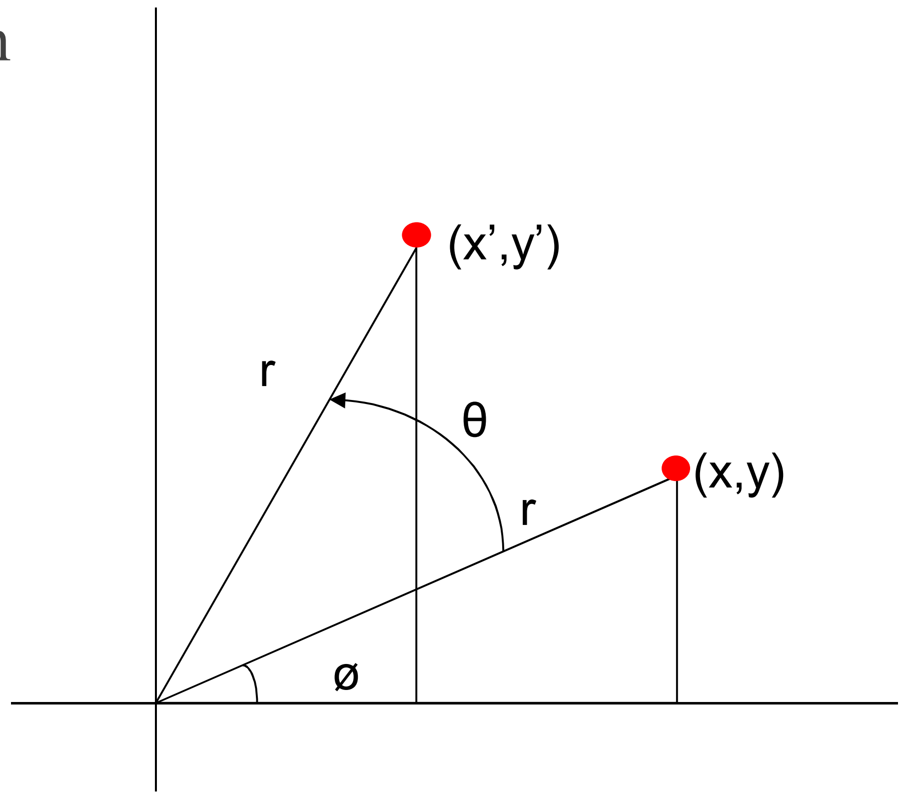
Positive $\theta$ >> counterclockwise rotation about the pivot point

Negative $\theta$ >> clockwise rotation about the pivot point



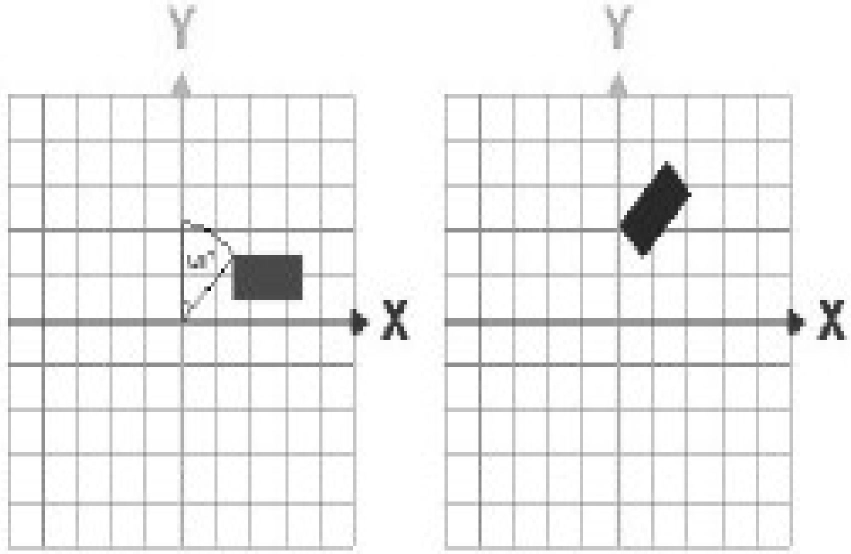
# 2D ROTATION

- Rotation of a point from position $(x, y)$ to position $(x', y')$ through an angle $\theta$ relative to the coordinate origin. The original angular displacement of the point from the $x$ axis is $\Phi$



# 2D ROTATION

Rotation by 45° counter-clockwise about origin.



# 2D Rotation

Using standard trigonometric identities, transformed coordinates can be expressed in terms of angles $\theta$ and $\Phi$ as

$$x' = r\cos(\Phi + \theta)$$
$$= r\cos\Phi\cos\theta - r\sin\Phi\sin\theta$$

$$y' = r\sin(\Phi + \theta)$$
$$= r\cos\Phi\sin\theta - r\sin\Phi\cos\theta$$

The original coordinates of the point in polar coordinates are

$$x = r\cos ø, \qquad y = r\sin ø$$

# 2D ROTATION

Substituting expression (5) into (4), we obtain the transformation equations for rotating a point at position ($x$, $y$) through an angle $\theta$ about the origin:

- $x' = x \cos\theta - y \sin\theta$
- $y' = y \sin\theta + y \cos\theta$

Rotation equation in matrix form, P’ = R . P where the rotation matrix is

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

# 2D Rotation

- Rotation of a point about an arbitrary pivot point.

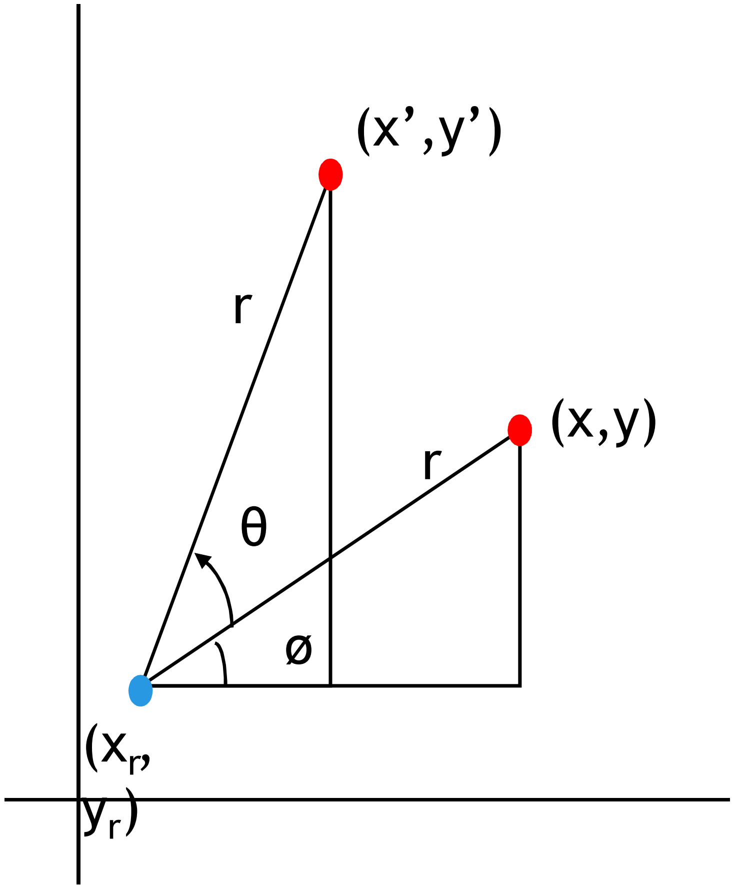
For rotation of a point about any specified rotation position $(x_r, y_r)$:

$$x' = x_r + (x - x_r)\cos\theta - (y - y_r)\sin\theta$$

$$y' = y_r + (x - x_r)\sin\theta + (y - y_r)\cos\theta$$

Rotations are also rigid body transformations that move object without deformation

Every point in the object is rotated through the same angle



# 2D SCALING

To change the size of an object.

A simple operation is by multiplying object positions $(x, y)$ by scaling factors $s_x$ and $s_y$ to produce the transformed coordinates (x’,y’) :

$$x' = x \cdot s_x \;,\quad y' = y \cdot s_y$$

can also be written in matrix form

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

or $$P' = S \cdot P$$

# 2D Scaling

Any positive value can be assigned to scaling factors $s_x$ and $s_y$

Values less than 1 reduce the size and greater than 1 enlarge it.

Specifying a value of 1 for both $s_x$ and $s_y$ leaves the size unchanged.

Uniform scaling: maintain relative object proportions (size) when $s_x$ and $s_y$ is assigned same value.

# 2D Scaling

Differential scaling: applying unequal values for $s_x$ and $s_y$.

- Often use in design applications, where pictures are constructed from few basic shape that can be adjusted by scaling and positioning transformations .

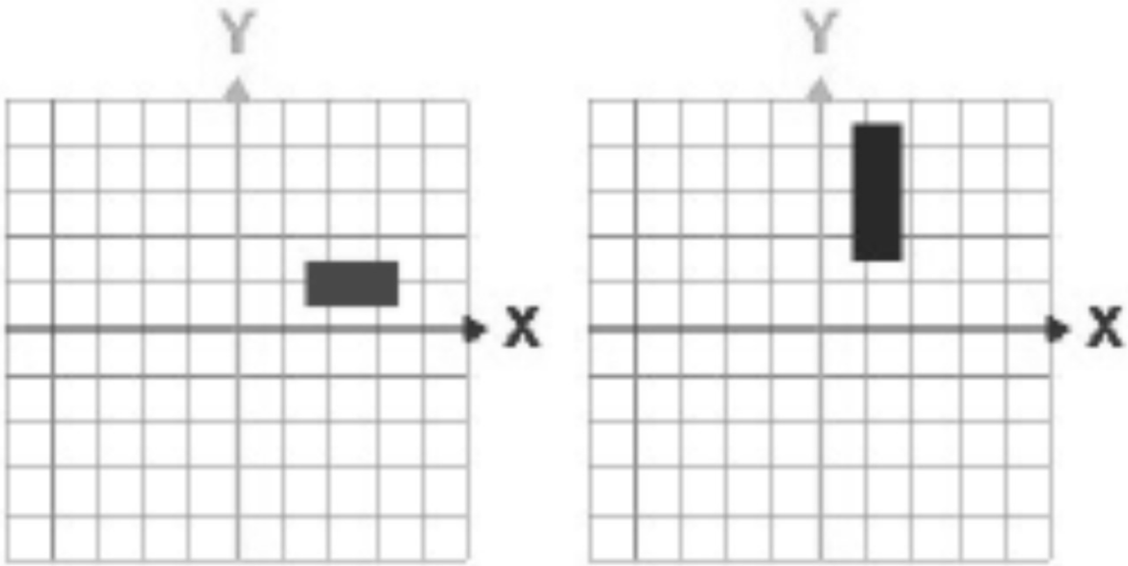
Objects transformed are BOTH scaled and repositioned.

Scaling factor:

- |<1| - move objects closer to origin
- |>1| - move objects farther from the origin

# 2D SCALING

- Scaling vector : (0.5, 3.0) about origin.



# 2D SCALING

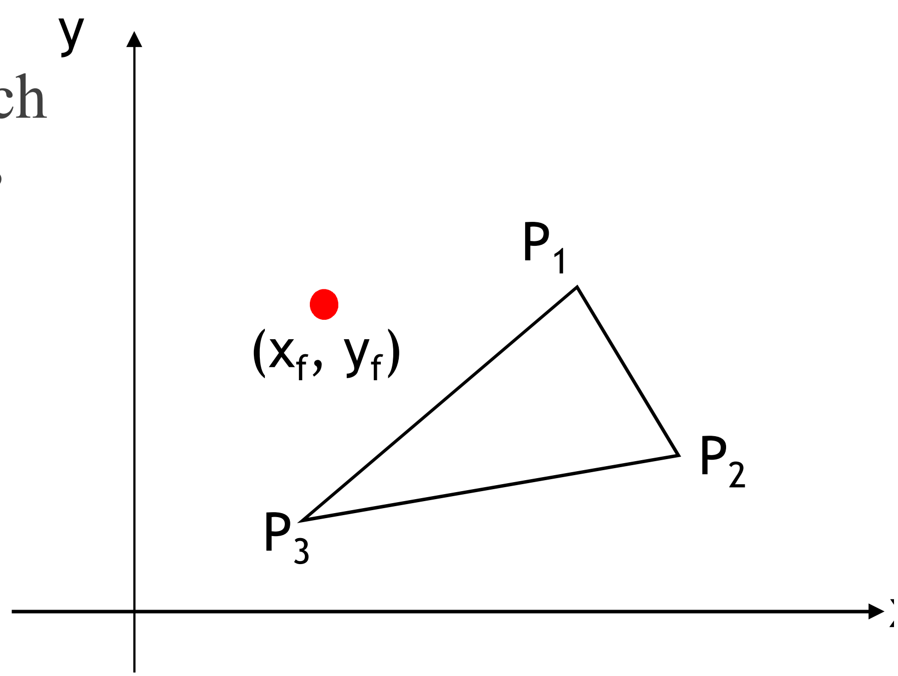
The location of the scaled object can be controlled by choosing a position, fixed point, that is to remain unchanged after the transformation.

The coordinate for fixed point ($x_f$, $y_f$) are often chosen at some object position, but any other position can be selected.

Objects are now resized by scaling the distances between object points and the fixed point.

# 2D SCALING

- Scaling relative to a chosen fixed point (xf,yf). The distance from each polygon vertex to the fixed point is scaled by transformation equation (13).




$$x' = x \,.\, s_x + x_f\,(1 - s_x)$$

$$y' = y \,.\, s_y + y_f\,(1 - s_y)$$

# Summary

Local and World Coordinates

Cartesian and Polar coordinate system

Geometric transformations

- Rigid body transformations