

## Causal Ordering of Messages

if  $\text{send}(m_1) \rightarrow \text{send}(m_2)$   
 then  $\text{receive}(m_1) \rightarrow \text{receive}(m_2)$ .

### Assumptions:

Group communication (broadcast)

No loss in communication channel

Customized vector clocks

→ Communication events only

→ On receive, no increment

### Algorithm:

Sender's rule :

Increment  $i^{\text{th}}$  value of send timestamp.

Receiver's rule : ( $i$ -sender,  $j$ -receiver)

$\rightarrow c_j[i] = tm[i] - 1$

$\rightarrow c_j[k] \geq tm[k], k \neq i$

( $>$  because send events can be concurrent)

### Case (i) (Basic Case)

$e_{21} : (0, 1, 0) \rightarrow \text{timestamp}$

$e_{11} : (i=2, j=1) \in [0, 0, 0]$

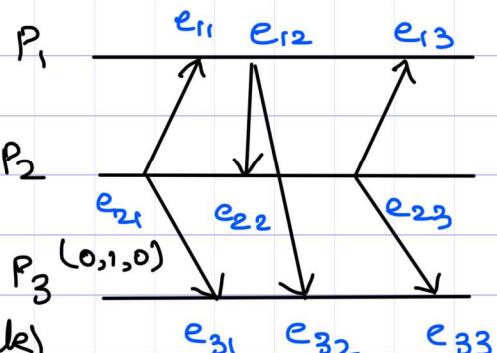
(current clock)

(i)  $c_1[2] == tm[2] - 1$  ? true

(ii)  $c_1[1, 3] \geq tm[1, 3]$  ? true.

∴ Accept message (∴ in causal order)

$P_1$  clock :  $(0, 1, 0)$



$e_{31} : (i=2, j=3) \quad C_3[0,0,0] \rightarrow tm[0,1,0]$   
 (i)  $C_3[2] = tm[2]-1$  ? true  
 (ii)  $C_3[1,3] \geq tm[1,3]$  ? true  
 ∴ Accept message  
 $P_3$  clock :  $(0,1,0)$

$e_{12}$  : Sender's rule.  $P_1$  clock :  $(1,1,0)$

$e_{22} : C_2[0,1,0] \rightarrow tm[1,1,0]$   
 ( $i=1, j=2$ )  
 (i)  $C_2[1] = tm[1]-1$  ? true  
 (ii)  $C_2[2,3] \geq tm[2,3]$  ? true  
 ∴ Accept message  
 $P_2$  clock :  $(1,1,0)$

$e_{32} : C_3[0,1,0] \rightarrow tm[1,1,0]$   
 ( $i=1, j=3$ )  
 (i)  $C_3[1] = tm[1]-1$  ? true  
 (ii)  $C_3[2,3] \geq tm[2,3]$  ? true  
 ∴ Accept message  
 $P_3$  clock :  $(1,1,0)$

$e_{23}$  : Sender's rule.  $P_2$  clock :  $(1,2,0)$

$e_{13} : C_1[1,1,0] \rightarrow tm[1,2,0]$   
 ( $i=2, j=1$ )  
 (i)  $C_1[2] = tm[2]-1$  ? true  
 (ii)  $C_1[1,3] \geq tm[1,3]$  ? true  
 ∴ Accept message  
 $P_1$  clock :  $(1,2,0)$

$e_{33} : C_3 [1, 1, 0] \text{ tm } [1, 2, 0]$   
 $(i = 2, j = 3)$

(i)  $C_3 [2] == \text{tm}[2] - 1 ? \text{true}$

(ii)  $C_3 [1, 3] \geq \text{tm}[1, 3] ? \text{true}$

$\therefore \text{Accept message}$

$P_3 \text{ clock} : (1, 2, 0)$

Case (ii)

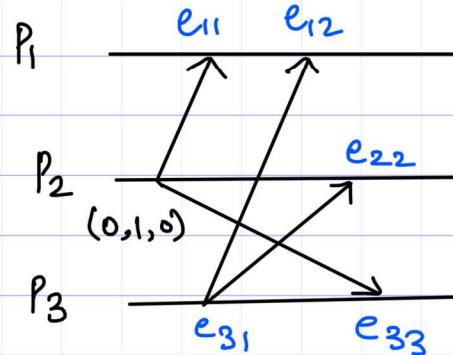
$e_{11} : C_1 (0, 0, 0) \text{ tm } (0, 1, 0)$   
 $(i = 2, j = 1)$

(i)  $C_1 [2] == \text{tm}[2] - 1 ?$   
 $\text{true}$

(ii)  $C_1 [1, 3] \geq \text{tm}[1, 3] ? \text{true}$

$\therefore \text{Accept message}$

$P_1 \text{ clock} : (0, 1, 0)$



$e_{31} : \text{Sender's rule . } P_3 \text{ clock} : (0, 0, 1)$

$e_{33} : C_3 (0, 0, 1) \text{ tm } (0, 1, 0)$   
 $(i = 2, j = 3)$

(i)  $C_3 [2] == \text{tm}[2] - 1 ? \text{true}$

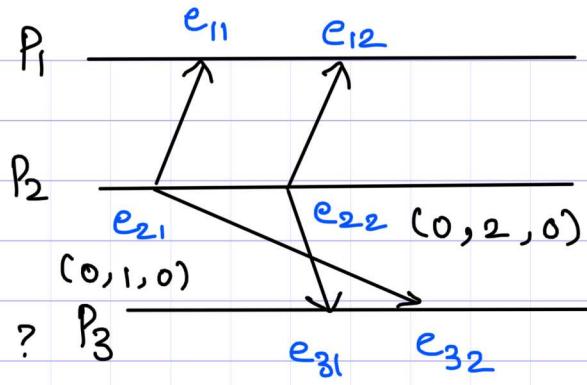
(ii)  $C_3 [1, 3] \geq \text{tm}[1, 3] ? \text{true}$

$\therefore \text{Accept message.}$

$P_3 \text{ clock} : (0, 1, 1)$

Similarly,  $e_{12}$  &  $e_{22}$  are accepted.

Case (iii) (not in causal order)



$e_{11} : C_1(0,0,0) \text{ tm}(0,1,0)$   
( $i = 2, j = 1$ )

(i)  $C_1[2] == \text{tm}[2] - 1$ ? true

(ii)  $C_1[1,3] \geq \text{tm}[1,3]$ ? true

∴ Accept message

$P_1$  clock:  $(0,1,0)$

→ 2nd msg delivered first.

$e_{31} : C_3(0,0,0) \text{ tm}(0,2,0)$   
( $i = 2, j = 3$ )

(i)  $C_3[2] == \text{tm}[2] - 1$ ? false

There is a gap!  $P_3$  missed a message from  $P_2$ .

∴ Do not accept the message, buffer it.

$P_3$  buffer:  $(0,2,0)_{e_{31}}$

$e_{32} : C_3(0,0,0) \text{ tm}(0,1,0)$   
( $i = 2, j = 3$ )

(i) & (ii) are true.

∴ Accept the message.

$P_3$  clock:  $(0,1,0)$

Retrieve message from Buffer & try delivery.

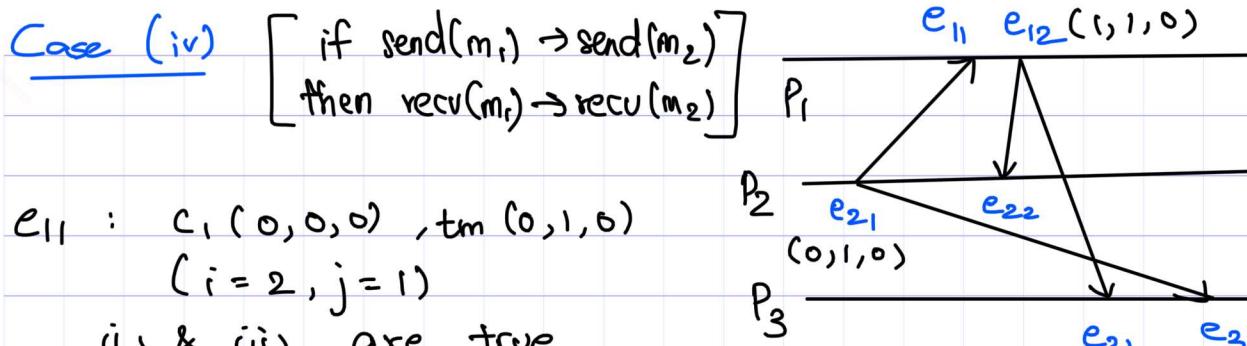
$e_{31}' : C_3(0,1,0) \text{ tm}(0,2,0)$

(i) & (ii) are true now.

∴ Accept message.

$P_3$  clock:  $(0,2,0)$ .

$e_{12}$  will be accepted.



$e_{22}$  : will be accepted similarly.  
 $C_2$  clock :  $(1,1,0)$

$e_{31}$  :  $C_3(0,0,0)$   $tm(1,1,0) \downarrow (i = 1, j = 3)$

$(i_1) C_3[1] == tm[1] - 1 ?$  true.

$\boxed{\begin{array}{l} \text{Conveys to } P_3 \text{ that } P_1 \text{ recv'd.} \\ \text{a msg. from } P_2 \text{ which} \\ P_3 \text{ hasn't yet recv'd.} \end{array}}$

$(ii_1) C_3[2,3] \geq tm[2,3] ?$  false.  
 Buffer  $e_{31}$  in  $P_3$ . Do not accept.

$e_{32}$  :  $C_3(0,0,0)$   $tm(0,1,0)$   
 $(i = 2, j = 3)$

$(i_1 \& ii_1)$  are true.  
 $\therefore$  Accept message.  
 $P_3$  clock :  $(0,1,0)$

Attempt redelivery.

$e'_{31}$  :  $C_3(0,1,0)$   $tm(1,1,0)$   
 $(i = 1, j = 3)$

$(i_1 \& ii_1)$  are true.  
 $\therefore$  Accept message.  
 $P_3$  clock :  $(1,1,0)$

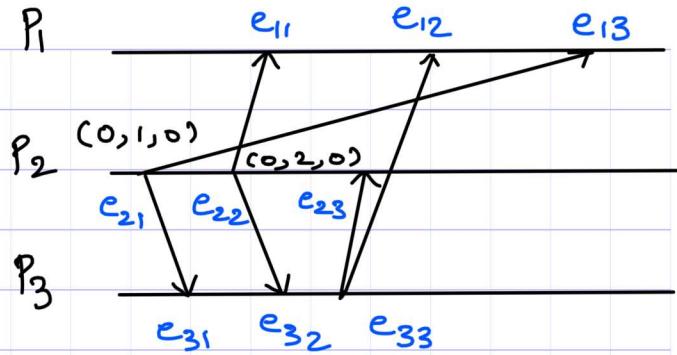
Case (v)

$e_{31} : C_3(0,0,0)$   
 $tm(0,1,0)$

( $i = 2, j = 3$ )  
(i) & (ii) are true.

∴ Accept message.

$C_3 \text{ clock} : (0,1,0)$



$e_{32} : C_3(0,1,0) \quad tm(0,2,0)$   
( $i = 2, j = 3$ )

(i) & (ii) are true.

∴ Accept message.

$C_3 \text{ clock} : (0,2,0)$

$e_{33} : \text{Sender's rule. } C_3 \text{ clock} : (0,2,1)$

$e_{23} : \text{will be accepted.}$

$C_2 \text{ clock} : (0,2,1)$

$e_{11} : C_1(0,0,0) \quad tm(0,2,0) \quad (i=3, j=1)$

(i) false.

Buffer the message.

$P_1 \text{ buffer} : (0,2,0)_{e_{11}}$

$e_{12} : C_1(0,0,0) \quad tm(0,2,1) \quad (i=3, j=1)$

(i) true

(ii) false

Buffer the message

$P_1 \text{ buffer} : e_{11}, (0,2,1)_{e_{12}}$

$e_{13} : C_1(0, 0, 0) \text{ tm}(0, 1, 0) \quad (i=2, j=1)$

Rule (i) & (ii) are true.

∴ Accept message.

$C_1 \text{ clock} : (0, 1, 0).$

Attempt redelivery.

$e_{11}' : C_1(0, 1, 0) \text{ tm}(0, 2, 0)$

(i) & (ii) are true.

∴ Accept message.

$C_1 \text{ dock} : (0, 2, 0).$

$e_{12}' : C_1(0, 2, 0) \text{ tm}(0, 2, 1)$

(i) & (ii) are true.

∴ Accept message.

$C_1 \text{ clock} : (0, 2, 1)$

Case (vi)

$e_{31} : C_3(0, 0, 0)$   
 $\text{tm}(0, 3, 0)$   
 $(i=2, j=3)$

(i) false

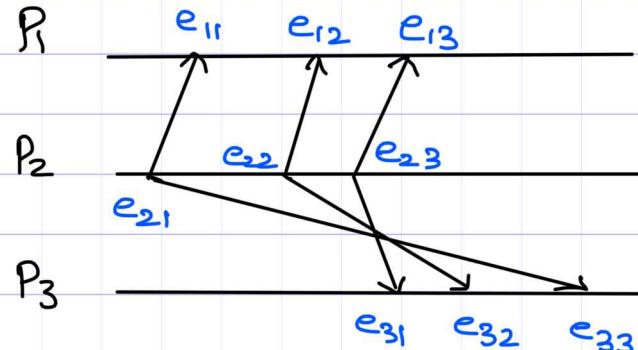
Buffer  $e_{31}$  in  $P_3$ .

$P_3 \text{ Buffer} : (0, 3, 0) e_{31}$

$e_{32} : C_3(0, 0, 0) \text{ tm}(0, 2, 0) \quad (i=2, j=3)$

(i) false.

$P_3 \text{ Buffer} : e_{31}, (0, 2, 0) e_{32}$



$e_{33} : C_3(0, 0, 0) \text{ tm } (0, 1, 0) \quad (i=2, j=3)$   
(i) & (ii) are true.

∴ Accept message.

$C_3 \text{ clock} : (0, 1, 0)$ .

Attempt redelivery.

$e_{31}' : C_3(0, 1, 0) \text{ tm } (0, 3, 0) \quad (i=2, j=3)$   
(i) false

Buffer  $e_{31}'$  again!

$P_3 \text{ Buffer} : e_{32}, (0, 3, 0)_{e_{31}}$ ,

$e_{32}' : C_3(0, 1, 0) \text{ tm } (0, 2, 0) \quad (i=2, j=3)$   
(i) & (ii) are true.

Accept message.

$C_3 \text{ clock} : (0, 2, 0)$

$e_{33}'' : C_3(0, 2, 0) \text{ tm } (0, 3, 0) \quad (i=2, j=3)$   
(i) & (ii) are true.

Accept message.

$C_3 \text{ clock} : (0, 3, 0)$ .

Case (vii)

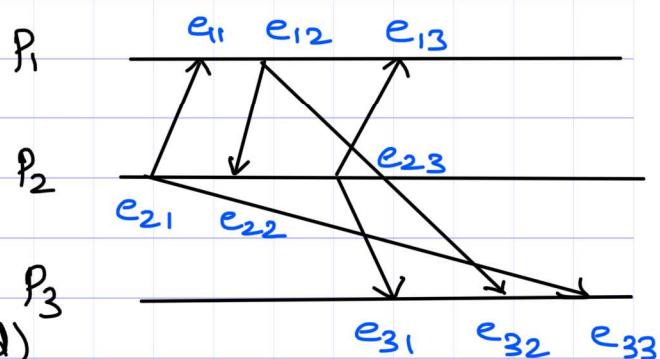
(Re-entrance,  
but not from  
the same  
sender)

$e_{12} : C_2 : (0, 1, 0) \text{ (send)}$

$e_{11} : \text{Accept. } C_1 : (0, 1, 0)$

$e_{12} : C_1 : (1, 1, 0) \text{ (send)}$

$e_{22} : \text{Accept. } C_2 : (1, 1, 0)$



$e_{23} : C_2 : (1, 2, 0)$  (send)

$e_{13} : \text{Accept. } C_1 : (1, 2, 0)$

$e_{31} : C_3 (0, 0, 0) \text{ tm}(1, 2, 0) (i=2, j=3)$   
(i) false.

$P_3 \text{ Buffer} : (1, 2, 0) e_{31}$

$e_{32} : C_3 (0, 0, 0) \text{ tm}(1, 1, 0) (i=1, j=3)$

(i) true

(ii) false

$P_3 \text{ Buffer} : e_{31}, (1, 1, 0) e_{32}$

$e_{33} : C_3 (0, 0, 0) \text{ tm}(0, 1, 0) (i=2, j=3)$

(i) & (ii) are true.

$\therefore \text{Accept. } P_3 \text{ clock} : (0, 1, 0)$

Attempt redelivery.

$e_{31}' : C_3 (0, 1, 0) \text{ tm}(1, 2, 0) (i=2, j=3)$

(i)  $C_3[2] == \text{tm}[2]-1$  ? true.

(ii)  $C_3[1, 3] \geq \text{tm}[1, 3]$  ? false.

$P_3 \text{ Buffer} : e_{32}, (1, 2, 0) e_{31}'$

$e_{32}' : C_3 (0, 1, 0) \text{ tm}(1, 1, 0) (i=1, j=3)$

(i)  $C_3[1] == \text{tm}[1]-1$  ? true.

(ii)  $C_3[2, 3] \geq \text{tm}[2, 3]$  ? true.

$\therefore \text{Accept message.}$

$C_3 \text{ clock} : (1, 1, 0).$

$e_{33}'' : C_3 (1, 1, 0) \text{ tm}(1, 2, 0) (i=2, j=3)$

(i) & (ii) are true.

$\therefore \text{Accept the message.}$

$C_3 \text{ clock} : (1, 2, 0).$

If 2 send events causally affect each other, then the corresponding 2 receive events must also causally affect each other.