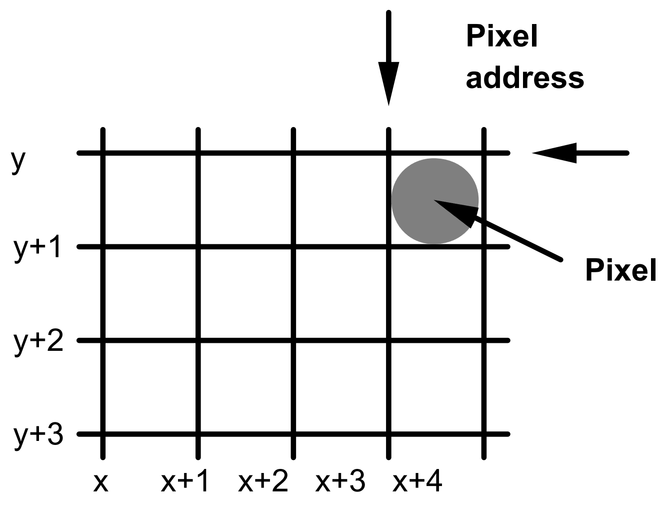
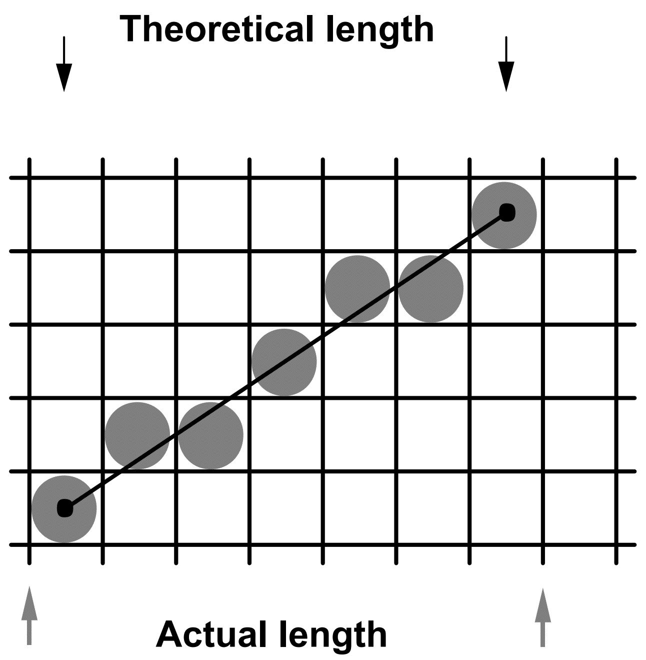
Raster Conversion Algorithms

DDA line algorithm

Bresenham line algorithm

Pixel Addressing in Raster Graphics

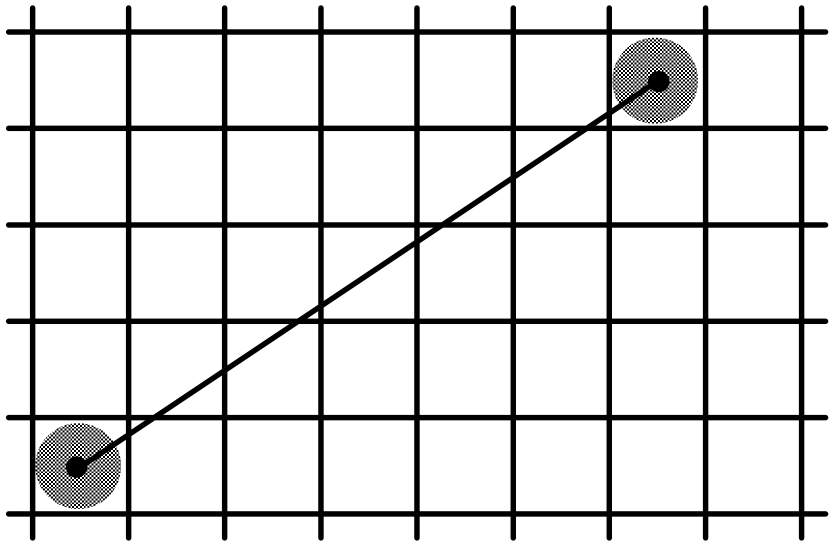




Raster conversion Algorithms: Requirements

- visual accuracy
- spatial accuracy
- speed

Line – Raster Representation



Basis for Line Drawing Algorithms

- The Cartesian slope-intercept for a straight line is
  - **$y = m.x + b$** --------------------→ 1
    - **m= Slope of the line**
    - **b= y intercept**
- The slope of the line is defined as

  $m=(y_{2-}y_1)/(x_{2-}x_1)$ ----------------→ 2
- The y intercept b is defined as

  $b=y1- m . x1$-----------------------→3
- Algorithms for displaying straight lines are based on the above 3 eqns.
- For any given x interval Δx along a line, we compute the y-interval Δy.
    - **$\Delta y=m . \Delta x$** ------------------------------→4
  - Similarly we can obtain Δx .
    - **$\Delta x= \Delta y/m$** ------------------------------------→5
  - The eqns 4 and 5 form the basis for determining the deflection voltage in analog devices.

Cases to handle

- Case 1:**Slope |m| < 1**
  - Δx=small horizontal deflection voltage
  - Δy is calculated proportional to slope.(Δy =m. Δx)
- Case 2:**Slope |m| >1**
  - Δy=small vertical deflection voltage
  - Δx is calculated proportional to slope.(Δx =Δy/m)
- Case 3:**Slope m = 1**
  - Δx=Δy (Both the voltages are same)

Line drawing – DDA algorithm

- (DDA) is a scan-conversion line algorithm based on calculating either Δy or Δx.
  - $\Delta y = m * \Delta x$
  - $\Delta x = \Delta y / m$
- we have many cases based on sign of the slope, value of the slope, and the direction of drawing.
  - Slope sign: positive or negative.
  - Slope value: <= 1 or >1.
  - Direction: (left – right) or (right – left)

DDA – case 1

- **Positive slope and left to right:**

- If slope <= 1 then:

  $\mathbf{x_{k+1} = x_k + 1}$

  $\mathbf{y_{k+1} = y_k + m}$

- If slope > 1 then:

  $\mathbf{x_{k+1} = x_k + 1/m}$

  $\mathbf{y_{k+1} = y_k + 1}$

DDA – case 2

- **Positive slope and right to left:**

- If slope <= 1 then:

  $\mathbf{x_{k+1} = x_k - 1}$

  $\mathbf{y_{k+1} = y_k - m}$

- If slope > 1 then:

  $\mathbf{x_{k+1} = x_k - 1/m}$

  $\mathbf{y_{k+1} = y_k - 1}$

DDA – case 3

- **Negative slope and left to right:**

- If $|m| <= 1$ then:

  $\mathbf{x_{k+1} = x_k + 1}$

  $\mathbf{y_{k+1} = y_k - m}$

- If $|m| > 1$ then:

  $\mathbf{x_{k+1} = x_k + 1/m}$

  $\mathbf{y_{k+1} = y_k - 1}$

DDA – case 4

- **Negative slope and right to left:**

- If $|m| <= 1$ then:

  $\mathbf{x_{k+1} = x_k - 1}$

  $\mathbf{y_{k+1} = y_k + m}$

- If $|m| > 1$ then:

  $\mathbf{x_{k+1} = x_k - 1/m}$

  $\mathbf{y_{k+1} = y_k + 1}$

Digital Differential Algorithm

- Input line endpoints, (x_1,y_1) and (x_2, y_2)
- set pixel at position (x_1,y_1)
- calculate slope $m=(y_2-y_1)/(x_2-x_1)$
- **For +ve slope (left to right)**
  - **Case $|m|\leq 1$**: Sample at unit x intervals and compute each successive y.
  - Repeat the following steps until (x_2, y_2) is reached:

    $\mathbf{y_{k+1} = y_k + m}$ where$(m=\Delta y/ \Delta x)$

    $\mathbf{x_{k+1} = x_k + 1}$

    set pixel at position **$(\mathbf{x_{k+1}},\mathbf{Round(y_{k+1})})$**
  - **Case $|m|>1$**: Sample at unit y intervals and compute each successive x.
  - Repeat the following steps until (x_2, y_2) is reached:

    $\mathbf{x_{k+1} = x_k+ 1/m}$

    $\mathbf{y_{k+1} = y_k + 1}$

    set pixel at position **$(\mathbf{Round(x_{k+1}), y_{k+1}})$**

Digital Differential Algorithm

- **For +ve slope (right endpoint to left endpoint)**
  - **Case |m|≤1**: Sample at unit x intervals and compute each successive y.
  - Repeat the following steps until (x_2, y_2) is reached:

    $\mathbf{y_{k+1} = y_k - m}$ where$(m=\Delta y/ \Delta x)$

    $\mathbf{x_{k+1} = x_k - 1}$

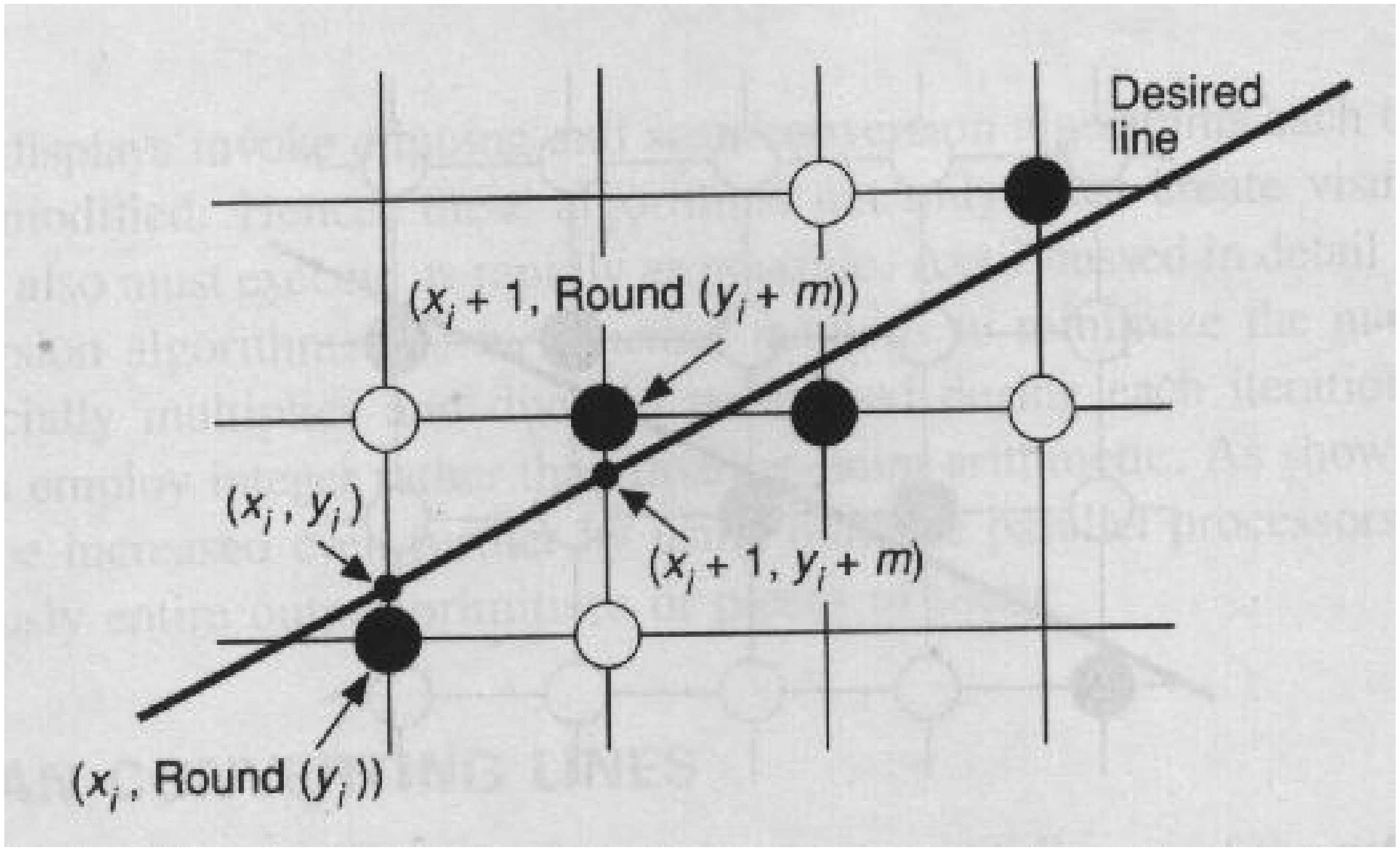
    set pixel at position $(x_{k-1},\text{Round}(y_{k-1}))$
  - **Case |m|>1**: Sample at unit y intervals and compute each successive x.
  - Repeat the following steps until (x_2, y_2) is reached:

    $\mathbf{x_{k+1} = x_{k\ -}1/m}$

    $\mathbf{y_{k+1} = y_k - 1}$

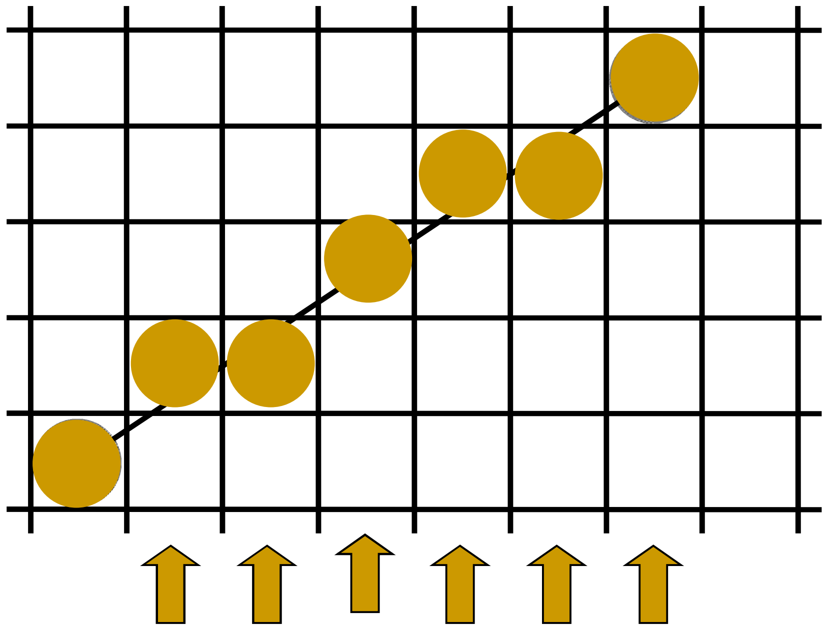
    set pixel at position $(\text{Round}(x_{k-1}), y_{k-1})$

Scan Conversion Process


Desired line
$(x_i + 1, \text{Round}(y_i + m))$
(x_i, y_i)
$(x_i + 1, y_i + m)$
$(x_i, \text{Round}(y_i))$

DDA (Digital Differential Algorithm)

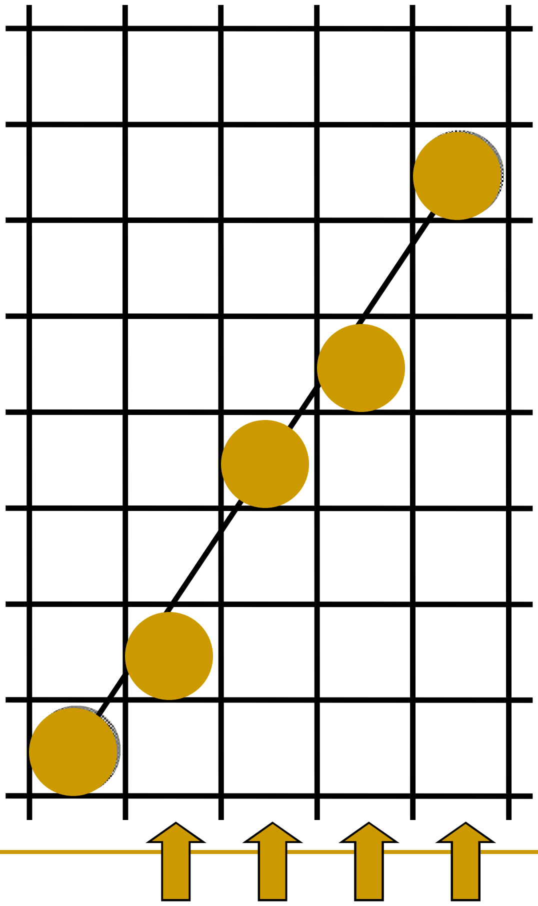
m < 1



- Δx=small horizontal deflection voltage (Δx=1)
- Δy proportional to slope m (Δy= Δx . m)

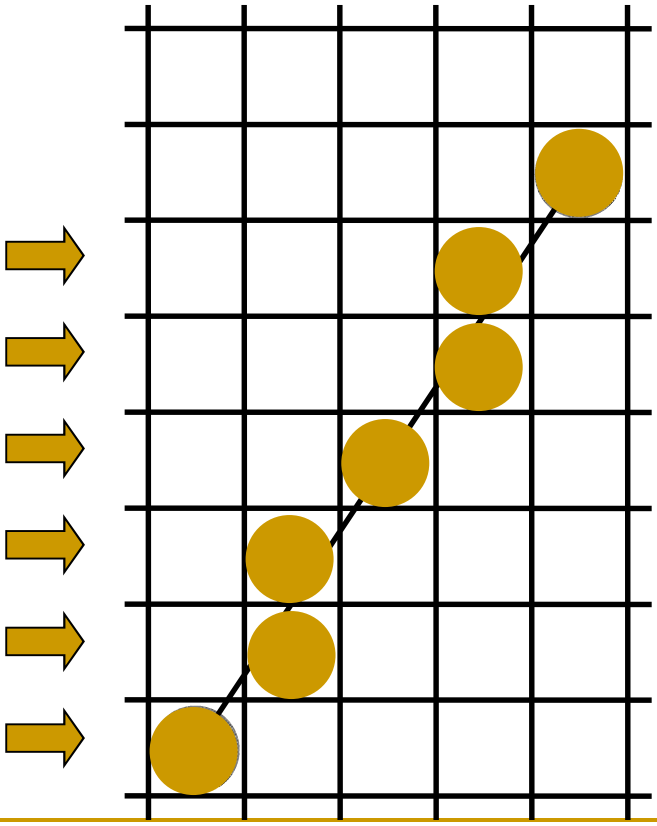
DDA (Digital Differential Algorithm)

For m > 1 , If we sample along x



DDA (Digital Differential Algorithm)

For m > 1, we sample along y



- Δy=small horizontal deflection voltage (Δy=1)
- Δx proportional to slope m (Δx= Δy/ m)

DDA Line algorithm - Procedure

- Procedure lineDDA(xa,xb,ya,yb:integer);
- Var
  - dx,dy,steps,k : integer;
  - xIncrement, yIncrement, x , y: real;
  - Begin
    - dx:=xb-xa;
    - dy:=yb-ya;
    - if abs(dx) >abs(dy) then steps:=abs(dx)
    - else steps:=abs(dy)
    - xIncrement :=dx/steps;
    - yIncrement :=dy/steps;
    - x:=xa;
    - y:=ya;

DDA Line algorithm – Procedure contd.

```
setPixel(round(x),round(y),1);
for k:=1 to steps do
Begin
    x:=x+xIncrement;
    y:=y+yIncrement;
    setPixel(round(x), round(y),1);
End
End {lineDDA}
```

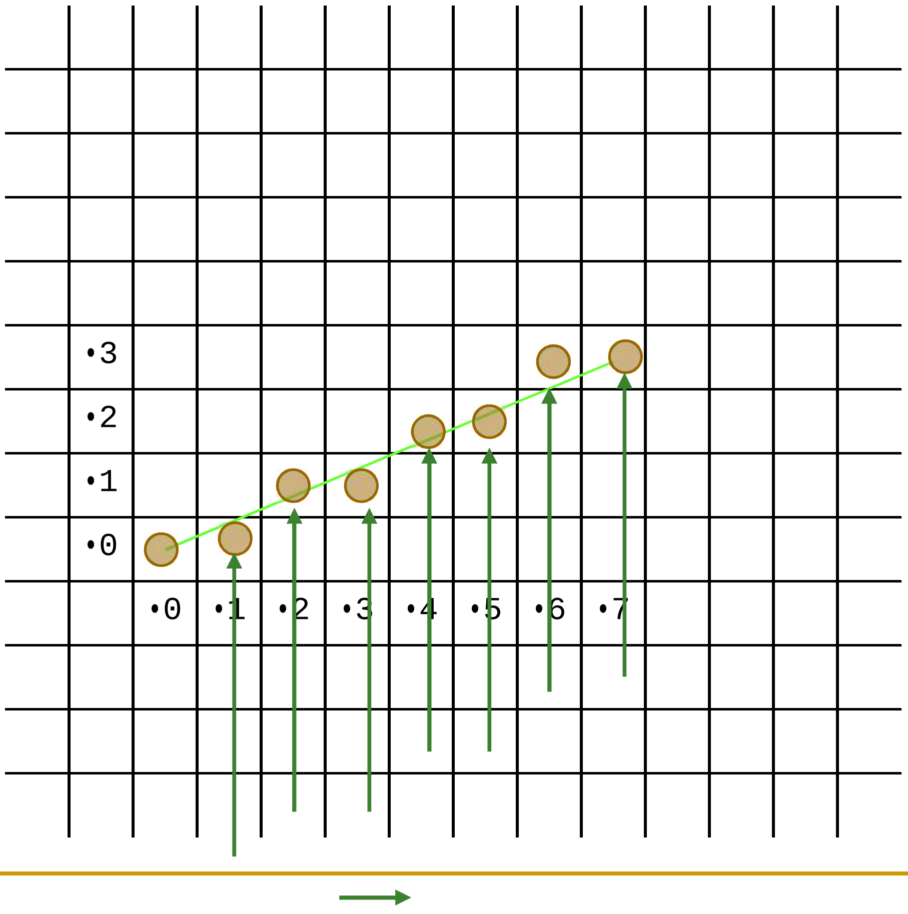
Example

- **Consider endpoints:** P1 (0,0) P2 (7,3)
- m = (3 – 0)/(7 - 0)

  = 0.429 (m<1)(+ve slope)
- dx = 1
- $x_0 = 0,\ y_0 = 0$
- $x_1 = x_0 + 1 = 1$
- $y_1 = y_0 + 0.429$

  $= 0.429 \approx 0$
- $x_1 = 1,\ y_1 = 0.429$
- $x_2 = x_1 + 1 = 2$
- $y_2 = y_1 + 0.429$

  $= 0.859 \approx 1$



Example contd.

- **$x_2 = 2, y_2 = 0.858$**
- $x_3 = x_2 + 1 = 3$
- $y_3 = y_2 + 0.429$
  $= 1.287 \approx 1$
- **$x_3 = 3, y_3 = 1.287$**
- $x_4 = x_3 + 1 = 4$
- $y_4 = y_3 + 0.429$
  $= 1.716 \approx 2$
- **$x_4 = 4, y_4 = 2$**
- $x_5 = x_4 + 1 = 5$
- $y_5 = y_4 + 0.429$
  $= 2.145 \approx 2$

Example contd.

- **$x_5 = 5, y_5 = 2$**
- $x_6 = x_5 + 1 = 6$
- $y_6 = y_5 + 0.429$

  $= 2.574 \approx 3$
- **$x_6 = 6, y_6 = 3$**
- $x_7 = x_6 + 1 = 7$
- $y_7 = y_6 + 0.429$

  $= 3.003 \approx 3$

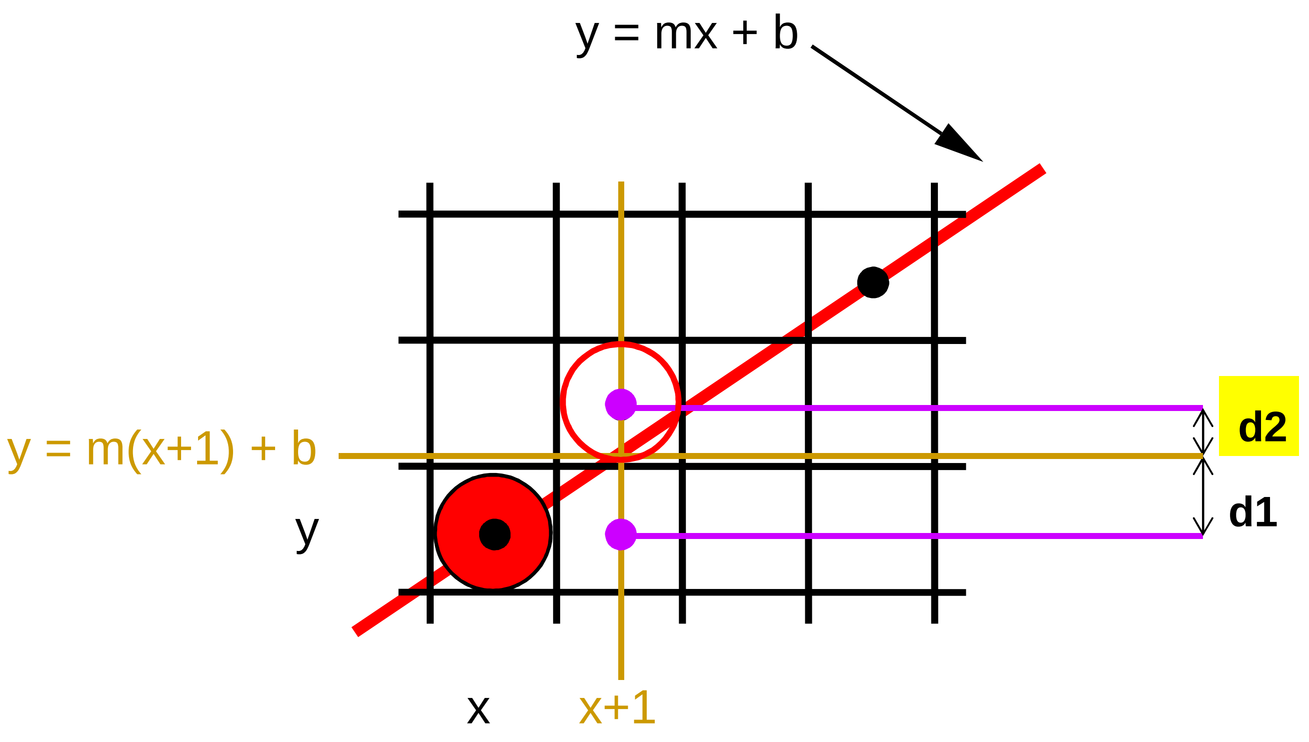
  $x_7=7,y_7=3$

Disadvantages of DDA algorithm

- DDA works with floating point arithmetic
- Rounding to integers necessary

Bresenham's Line Algorithm

- **An accurate and efficient raster line generating-algorithms developed by Bresenham.**
- **Scan converts lines using only incremental integer calculations.**



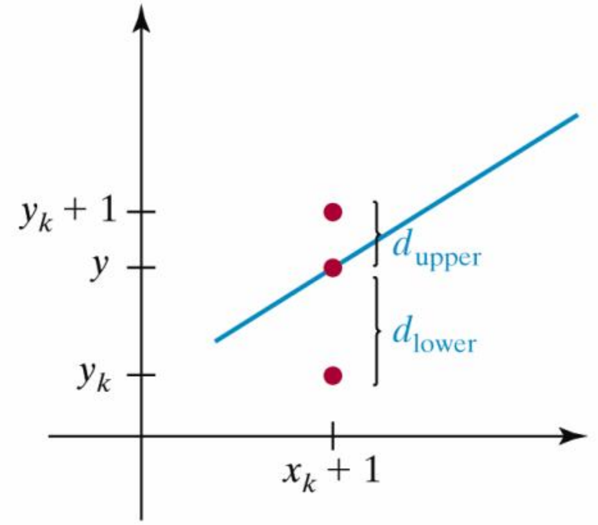


Figure 3-11

Vertical distances between pixel positions and the line y coordinate at sampling position $x_k + 1$.

Bresenham's line algorithm (slope ≤ 1)

- Input line endpoints, (x_0,y_0) and (x_n, y_n)
- calculate **$ax = x_n - x_0$** and **$ay = y_n - y_0$**
- Assuming the pixel (x_k , y_k) is to displayed
  - Determine the positions whether at **(x_k+1 , y_k) and (x_k+1 , y_k+1)**
- From x_k+1 we label vertical pixel separations from line as d1 and d2
- The y coordinate at pixel column position x_k+1 is calculated as
    - **$y=m(x_k+1)+b$.**
  - **Then**
    - **$d1=y-y_k=m(x_k+1)+b-y_k$**

and

  - **$d2=y_k+1-y=y_k+1- m(x_k+1)-b$.**

Bresenham's line algorithm (slope ≤ 1)

- d1-d2=2 **$m(x_k+1)-2y_k+2b-1$**
- Decision parameter p_k for the kth step in line algorithm obtained by rearranging the above equations.
- Substitute m=Δy/ Δx, where Δy, Δx are horizontal and vertical seperations.
  - $P_{k=}$ Δx(d1-d2)== **$2\,\Delta y.\,x_k - 2\,\Delta x\,.\,y_k + c$-------------------$\rightarrow$1**
  - C is constant has the value 2 Δy+ Δx(2b-1)
  - When P_k is negative plot the lower pixel else plot the upper pixel.
- Coordinate changes along the line occur in unit steps in x and y directions.
- The values of successive decision parameter can be evaluated from
  - $P_{k+1=}$**$2\,\Delta y.\,x_{k+1} - 2\,\Delta x\,.\,y_{k+1} + c$------------------------------$\rightarrow$2**
- Subtracting 1 & 2 we get
  - $P_{k+1}=P_k+$ **$2\,\Delta y - 2\,\Delta x\,(y_{k+1-}\,y_{k)}$ (since $x_{k+1=}x_k+1$)**
  - The term $(y_{k+1-}\,y_{k)}$ is either 1 or 0 depending on the sign of parameter p_k
  - The recursive calculation of decision parameters is performed at each integer x position.
- The parameter p_0 is evaluated from eq1 $p_0=2$ **Δy- Δx**

Bresenham's Line Algorithm

- Input line endpoints, (x_0,y_0) and (x_n, y_n)
- Load (x_0,y_0) into the frame buffer that is first point
- Calculate the constants Δx, Δy, $2\Delta y$ and $2\Delta y - 2\Delta x$
- calculate parameter $p_0 = 2\,\Delta y - \Delta x$
- Set pixel at position (x_0,y_0)
- repeat the following steps until (x_n, y_n) is reached:

```
    if $p_k < 0$
        set the next pixel at position $(x_k +1, y_k )$
        calculate new $p_{k+1} = p_k + 2\,\Delta y$
    if $p_k \geq 0$
        set the next pixel at position $(x_k +1, y_k + 1 )$
        calculate new $pk_{+1} = p_k + 2(\Delta y - \Delta x)$
```

- Repeat last step Δx times.

Advantages of Bresenham's Line Algorithm

- Bresenham’s algorithm uses integer arithmetic
- Constants need to be computed only once
- Bresenham’s algorithm generally faster than DDA

Bresenham’s line drawing Procedure

```
procedure lineBres (xa, ya, xb, yb : integer)
var
    dx, dy, x, y, xEnd, p: integer;
begin
    dx :=abs(xa-xb);
    dy :=abs(ya-yb);
    p:=2 * dy – dx;
    if xa > xb then
        begin
            x:= xb;
            y:= yb;
            xEnd := xa;
    else
    begin
        x:=xa;
        y:=ya;
```

Contd…

```
xEnd := xb;
end;
setPixel (x,y,1);
while x < xEnd do
begin
    x:= x+1;
    if p < 0 then p:= p+2 * dy
    else
    begin
        y:=y+1;
    p:=p+2 * (dy-dx)
    end;
    setPixel (x,y,1);
    end
    end; {lineBres}
```

Bresenham's Line Algorithm (Example)

- Note: Bresenham's algorithm is used when slope is <= 1.

- using Bresenham's Line-Drawing Algorithm, Digitize the line with endpoints (20,10) and (30,18).

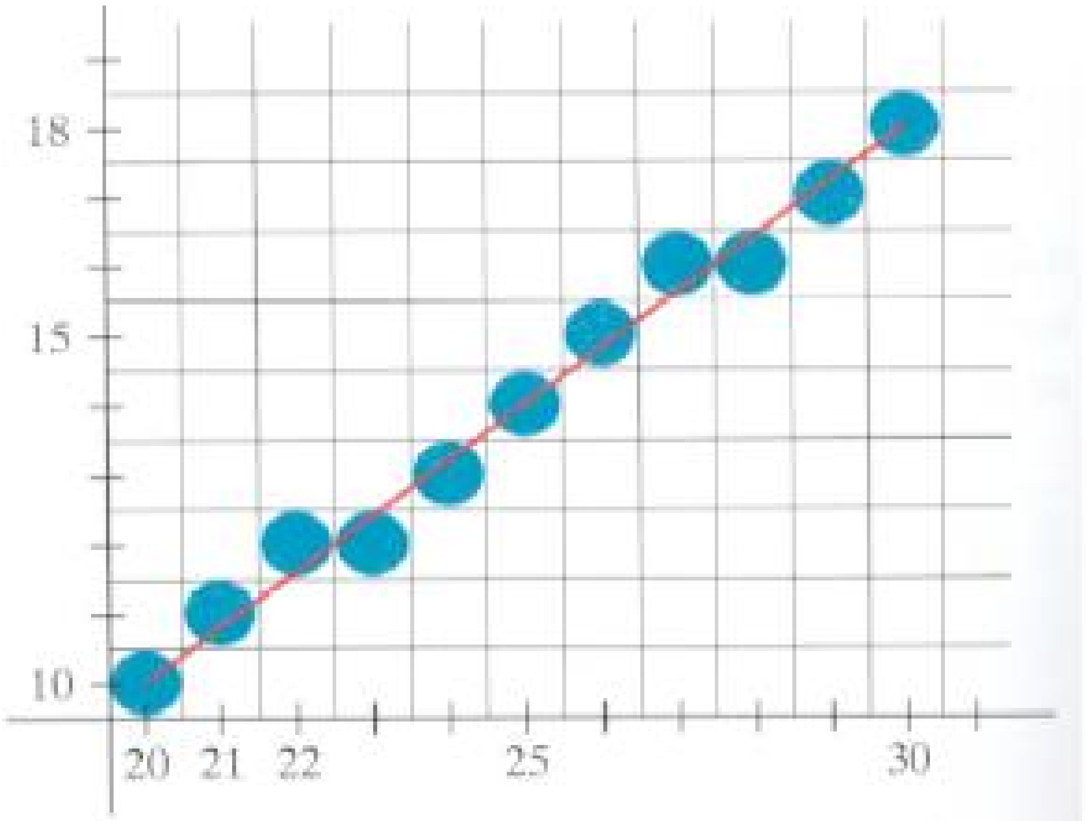
- $\Delta y = 18 – 10 = 8$
- $\Delta x = 30 – 20 = 10$
- $m = \Delta y / \Delta x = 0.8$

- plot the first point (x0, y0) = (20, 10)
- $p0 = 2 * \Delta y – \Delta x = 2 * 8 – 10 = 6$, so the next point is (21, 11)

Example (cont.)

K	P_k	(x_{k+1}, y_{k+1})	K	P_k	(x_{k+1}, y_{k+1})
0	6	(21,11)	5	6	(26,15)
1	2	(22,12)	6	2	(27,16)
2	-2	(23,12)	7	-2	(28,16)
3	14	(24,13)	8	14	(29,17)
4	10	(25,14)	9	10	(30,18)

Example (cont.)



- Thank you