Social Network Analysis

GRAPH - Part2

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Social Network Analysis

Eccentricity of a Node

It is the largest geodesic distance between a node and any other node in a graph

Formally, the eccentricity of node n_i in a connected graph is equal to the maximum d(i,j), for all j, (or $\max_j$ d(i,j))

It shows how far a node is from the node most distant from it in the graph

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Diameter of a Graph

Largest geodesic distance between any pair of nodes in a graph

It is the largest eccentricity of any node

Diameter range from 1 to g-1

It quantifies how far apart the farthest two nodes in the graph are

Message takes shortest route over a path of length no greater than the diameter of the graph

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Diameter of a subgraph

Geodesic of a subgraph is the length of the shortest path between the nodes n_i and n_j within the subgraph

Any path, and any geodesic, including nodes and lines outside the subgraph, is not considered.

The diameter of a sub graph is the length of the largest geodesic within the subgraph.

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Connectivity of Graphs

Connectivity of a graph is a function of whether a graph remains connected when nodes and/or lines are deleted

Cutpoints

A node n_i is a cutpoint if the number of components in the graph g that contains n_i is fewer than the number of components in the subgraph g_s that results from deleting n_i from the graph.

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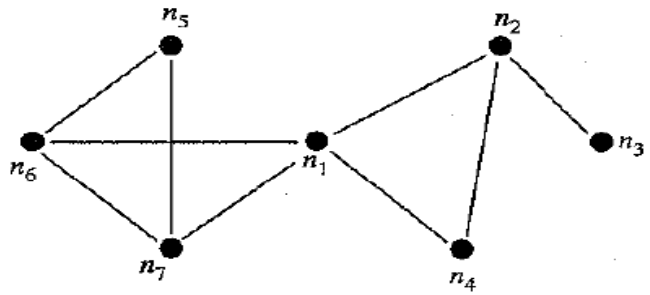
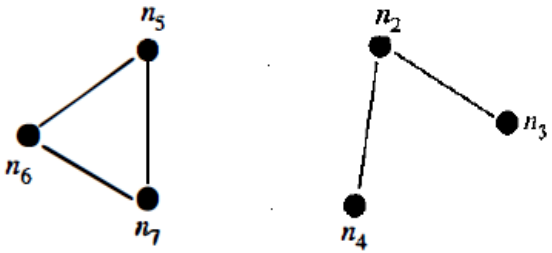
Example Cutpoint



Node n_1 is a node cut, or cutpoint



The graph without node n_1

Typical Scenario:

In a communications network, an actor who is a cutpoint is critical, if removed, in the remaining network that has two subsets of actors, between whom no communication can travel

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Cutset

Cutpoint can be extended from a single node to a set of nodes necessary to keep the graph connected

Cutset is the set of nodes is necessary to maintain the connectedness of a graph

If the set is of size k, then it is called a k-node cut

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Bridges

A bridge is a line that is critical to the connectedness of the graph.

A bridge is a line such that the graph containing the line has fewer components than the subgraph that is obtained after the line is removed

The removal of a bridge leaves more components than when the bridge is included

I-line cut is a set of I lines that, if deleted, disconnects the graph

A bridge is a 1-line cut

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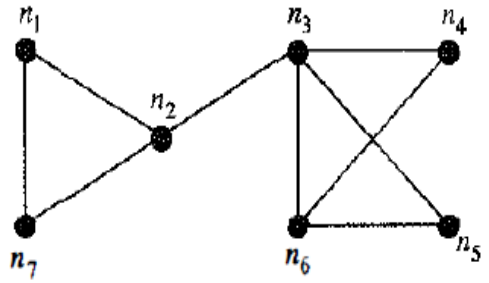
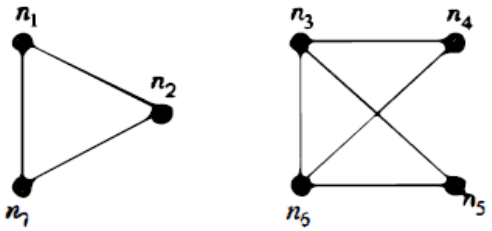
In social networks, a bridge is a critical tie, or a critical interaction between two actors

Example:



Line $(n_2\ n_3)$ is a bridge



Graph without line $(n_2\ n_3)$

The line (n2, n3) is a bridge. If the line (n2, n3) is removed from the graph, there is no path between nodes n1 and n5 and the graph becomes disconnected

If the line (n2, n3) were nonexistent, nodes n1, n2, and n7 would not be reachable from nodes n3, n4, n5 and n6

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Node and Line-Connectivity

- One way to measure the cohesiveness of a graph is by its connectivity

A graph is cohesive if:

- There are relatively frequent lines
- Many nodes with relatively large degrees
- Relatively short or numerous paths between pairs of nodes

- Cohesive graphs have many short geodesics, and small diameters, relative to their sizes

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If a graph is not cohesive then it is "vulnerable" to the removal of a few nodes or lines

A vulnerable graph is more likely to become disconnected if a few nodes or lines are removed

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Point-connectivity or node-connectivity of a graph

It is the minimum number of nodes that must be removed to make the graph disconnected, or to leave a trivial graph

K(W), is the minimum number K for which the graph has a K-node cut.

If the graph is disconnected, then k=0, since no node must be removed

If the graph contains a cutpoint, then K = 1 since the removal of the single node leaves the graph disconnected

If a graph contains a pair of nodes whose removal together would disconnect the graph, then K = 2

Higher values of K indicate higher levels of connectivity of the graph

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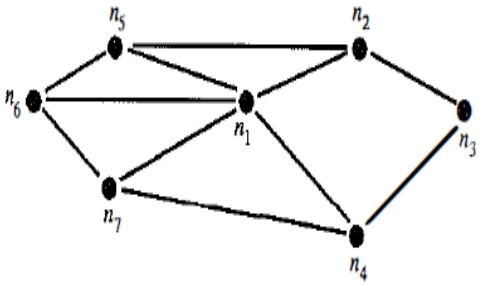
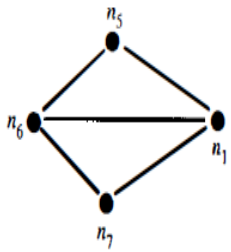


n_2 and n_4 comprise a 2-node cut



The graph without n_2 and n_4

The 2-node cut consists of n2 and n4, because without them n3 would not be connected to the remainder of the graph

The value K is the minimum number of nodes that must be removed to make the graph disconnected

For any value k less than K the graph is said to be k-node connected.

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A complete graph has no cutpoint as all nodes are adjacent to all others

To disconnect a complete graph, one would need to remove g - 1 nodes resulting in a trivial graph (g = 1)

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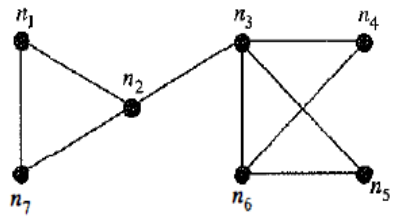
The line-connectivity or edge-connectivity of a graph, λ (G), is the minimum number λ for which the graph has a λ -line cut

The value, λ, is the minimum number of lines that must be removed to disconnect the graph or leave a trivial graph



Line $(n_2\ n_3)$ is a bridge

l1=(n2,n3) is a bridge, λ(G)=1, the minimum number of lines whose removal disconnects the graph is 1

The graph is said to be l-line connected, since I is the minimum number of lines that must be removed to make the graph disconnected.

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Summary

Diameter is the largest geodesic distance between any pair of nodes

Connectivity of a graph is a function of whether a graph remains connected when nodes and/or lines are deleted

Cutpoints are critical nodes when removed graph gets disconnected

Bridge is a critical line when removed graph splits into components or subgraph

Node connectivity - minimum number of nodes that must be removed to make the graph disconnected

Line Connectivity - minimum number of lines that must be removed to make the graph disconnected