2D Viewing

Part II

Clipping Algorithms

Types of Clipping

Point Clipping

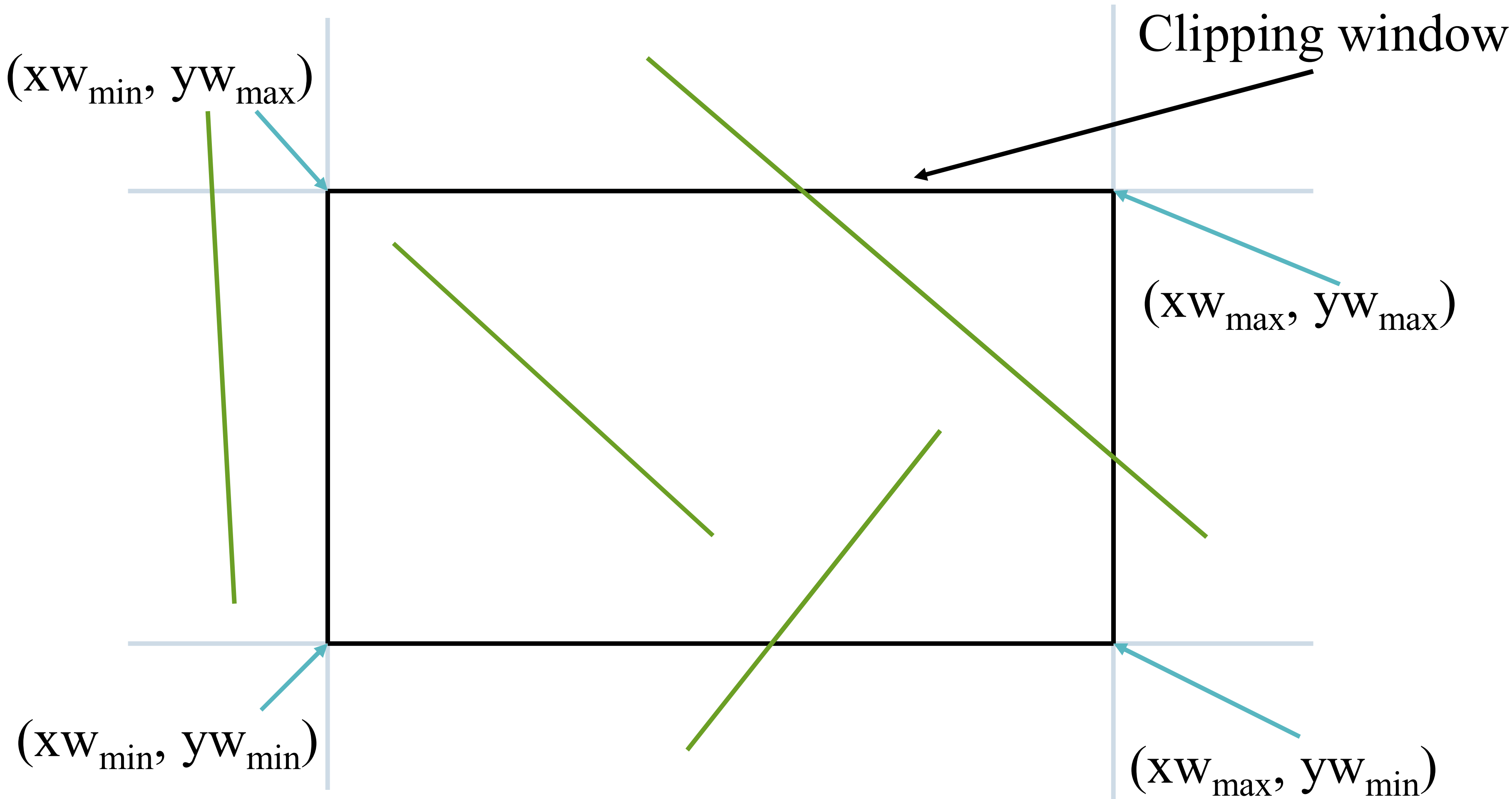
Line Clipping

Polygon Clipping

Curve Clipping

Text Clipping

Line Clipping



Line Clipping Algorithms

Simple Line Clipping

Cohen-Sutherland Algorithm

Liang-Barsky Algorithm

Simple Line Clipping

Using **point-clipping test** to determine completely INSIDE or OUTSIDE line.

$$xw_{\min} \le x \le xw_{\max}$$

$$yw_{\min} \le y \le yw_{\max}$$

Using **parametric equations** to determine partially INSIDE or OUTSIDE line.

$$x = x_0 + u(x_{end} - x_0)$$

$$y = y_0 + u(y_{end} - y_0) \quad 0 \le u \le 1$$

Cohen-Sutherland Line Clipping Algorithm

Oldest and most popular line-clipping algorithms.

Method speeds up the processing of line segments by performing initial tests

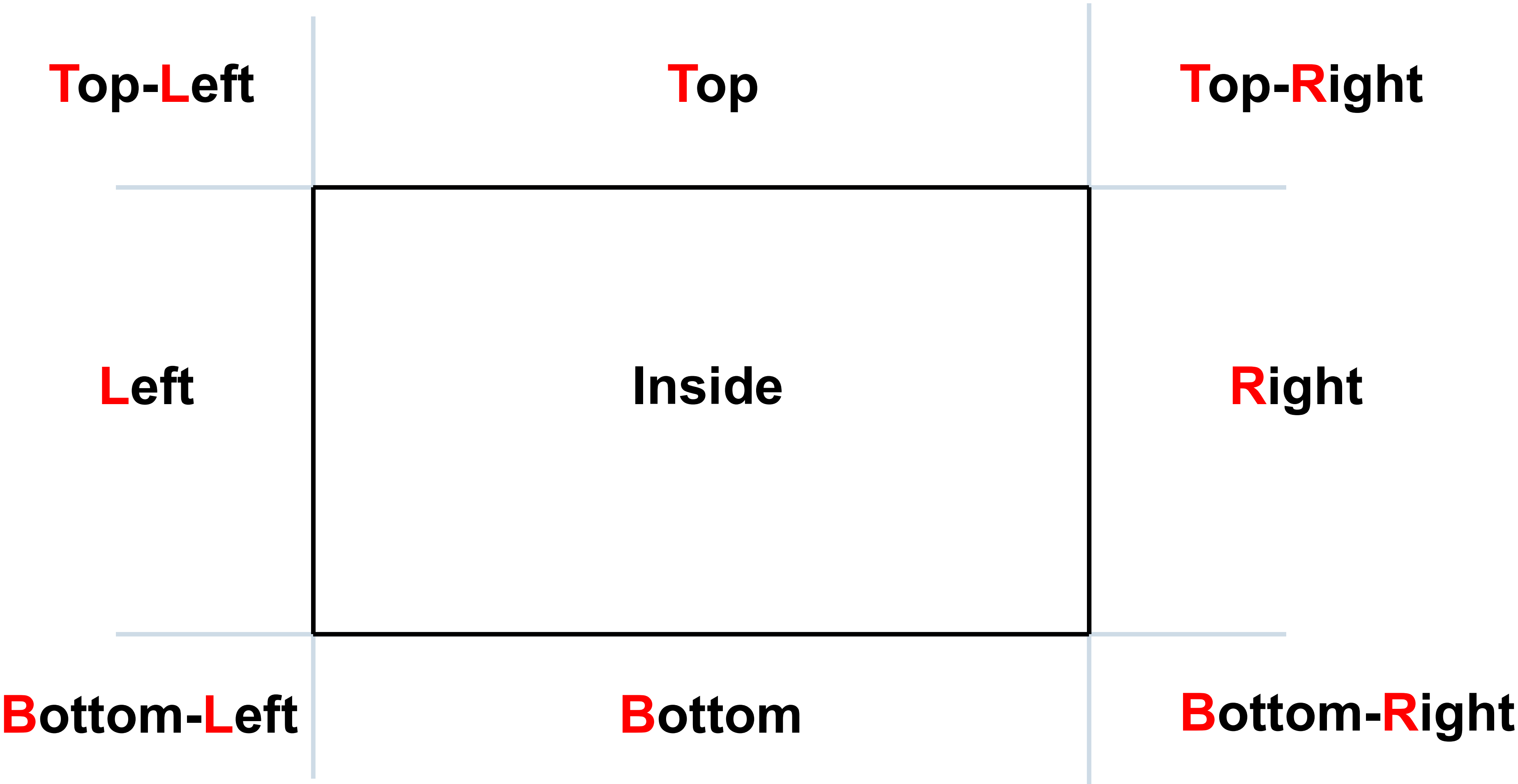
Reduces the no of intersections that must be calculated.

Each line endpoint in picture is assigned a four digit binary code called **region code** .

Region code identifies the location of the point relative to the boundaries of the clip window.

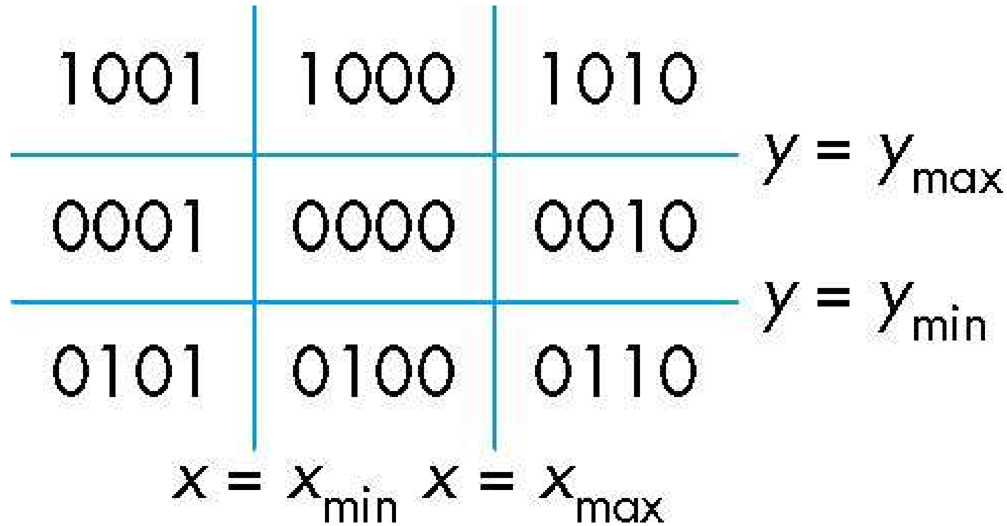
TBRL

Cohen-Sutherland Algorithm



Region Codes

	T	B	R	L
Bit	4	3	2	1



Each region is represented by a 4-bit **region code** TBRL:

$$T = \begin{cases} 1 \text{ if } y > y_{\max} \\ 0 \text{ otherwise} \end{cases} \quad B = \begin{cases} 1 \text{ if } y < y_{\min} \\ 0 \text{ otherwise} \end{cases} \quad R = \begin{cases} 1 \text{ if } x > x_{\max} \\ 0 \text{ otherwise} \end{cases} \quad L = \begin{cases} 1 \text{ if } x < x_{\min} \\ 0 \text{ otherwise} \end{cases}$$

Trivial accept cases

The trivial accept case corresponds to ensuring that no region code bits are set for both endpoints. This can be nicely accomplished with a bitwise OR operation.

RC (A)	0101
RC (B)	0100
bitwise OR	0101 → fails trivial accept

RC(C)	0000
RC (D)	0000 → trivial accept

Trivial reject cases

The trivial reject case can be nicely accomplished with a bitwise AND operation. We can trivially reject a line segment which has a result not equal to 0000

RC (A) 0101

RC (B) 0100

bitwise AND 0100 → trivial reject

RC (E) 1000

RC (F) 0010

bitwise AND 0000 →fails trivial reject

Cohen-Sutherland Algorithm (cont.)

Lines that cannot be identified as completely inside or outside by above tests can be test as follows

- Choose an endpoint of the line that is outside the window.
- Find the intersection point at the window boundary (base on region code).
- Replace endpoint with the intersection point and update the region code.
- Repeat steps until we find a clipped line either trivially accepted or trivially rejected.

Repeat step for other lines.

How to check for intersection?

if bit 4 = 1 → there is intersection on TOP boundary.

if bit 3 = 1 → .. .. BOTTOM ..

if bit 2 = 1 → .. .. RIGHT ..

if bit 1 = 1 → .. .. LEFT ..

How to find intersection point?

- using slope intercept of the line equation

m=(y2-y1)/(x2 –x1)

intersection with LEFT or RIGHT boundary.

$x = xw_{min}$ (LEFT) $x = xw_{max}$ (RIGHT)

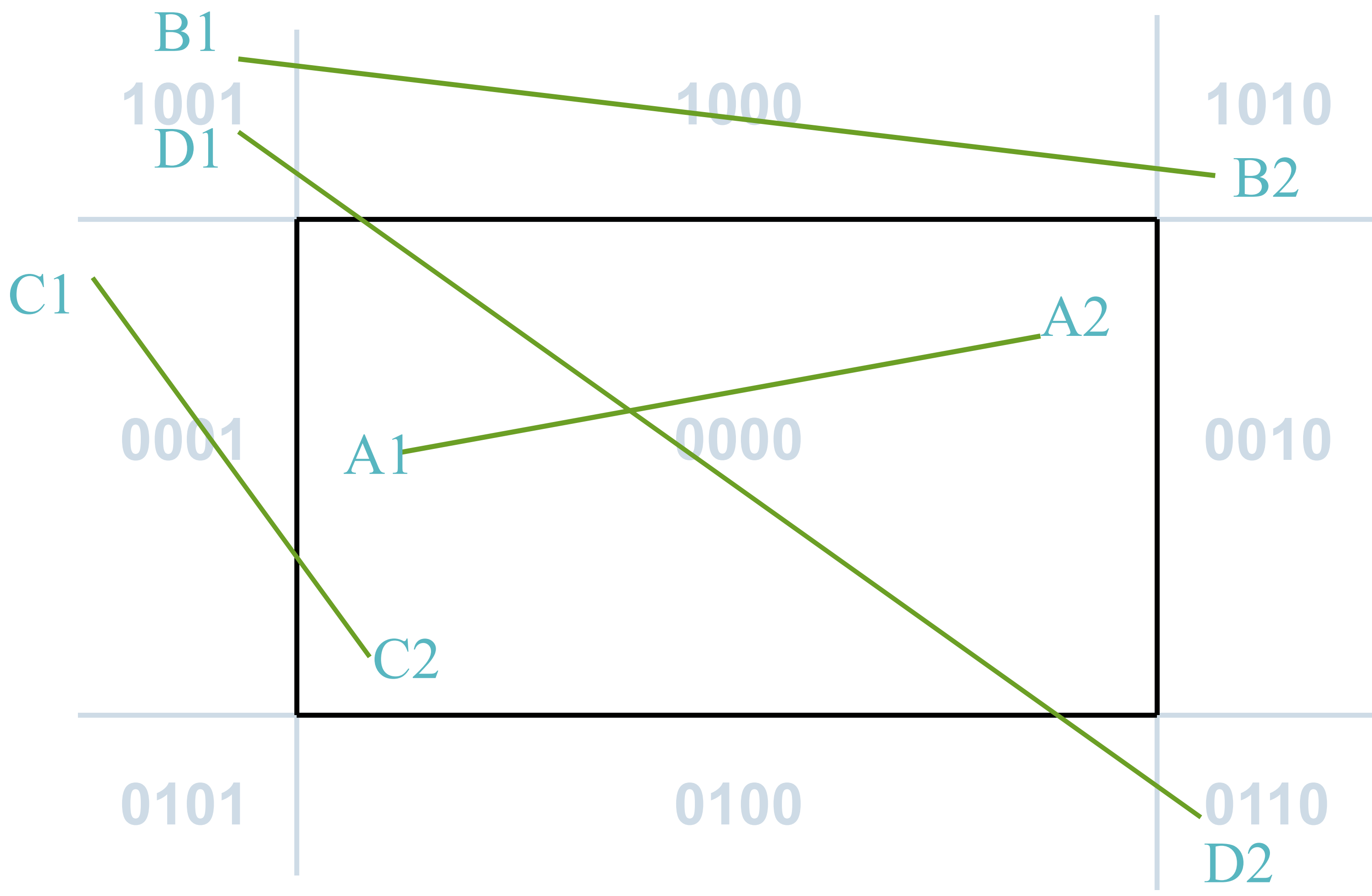
$$\mathbf{y = y1 + m(x - x1)}$$

intersection with BOTTOM or TOP boundary.

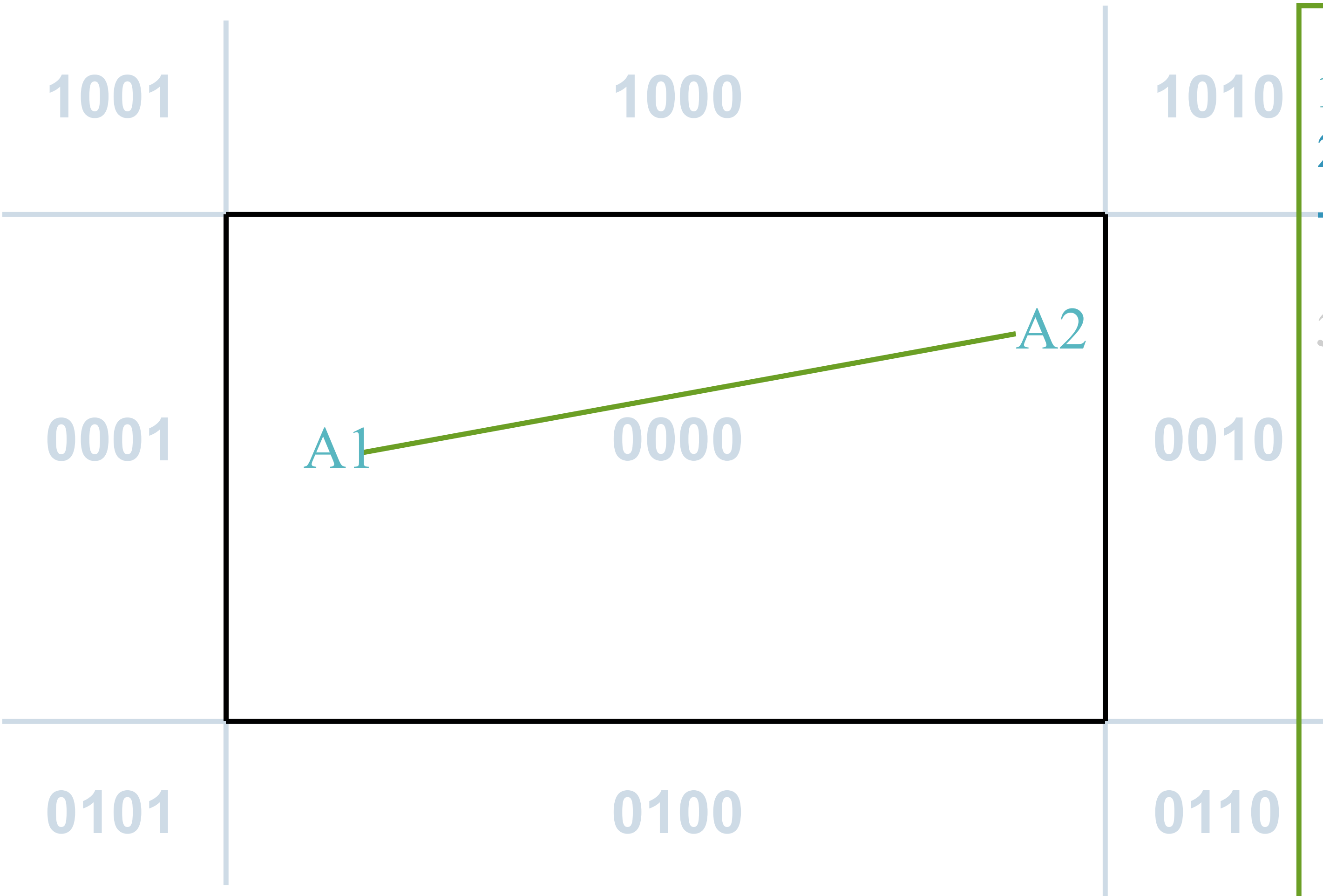
$y = yw_{min}$ (BOTTOM) $y = yw_{max}$ (TOP)

$$\mathbf{x = x1 + (y - y1)/m}$$

Trivial accept & reject



Example



algorithm

1. A1=0000,A2=0000

2. (both 0000) – Yes -> accept & draw

3.

3.1

3.2

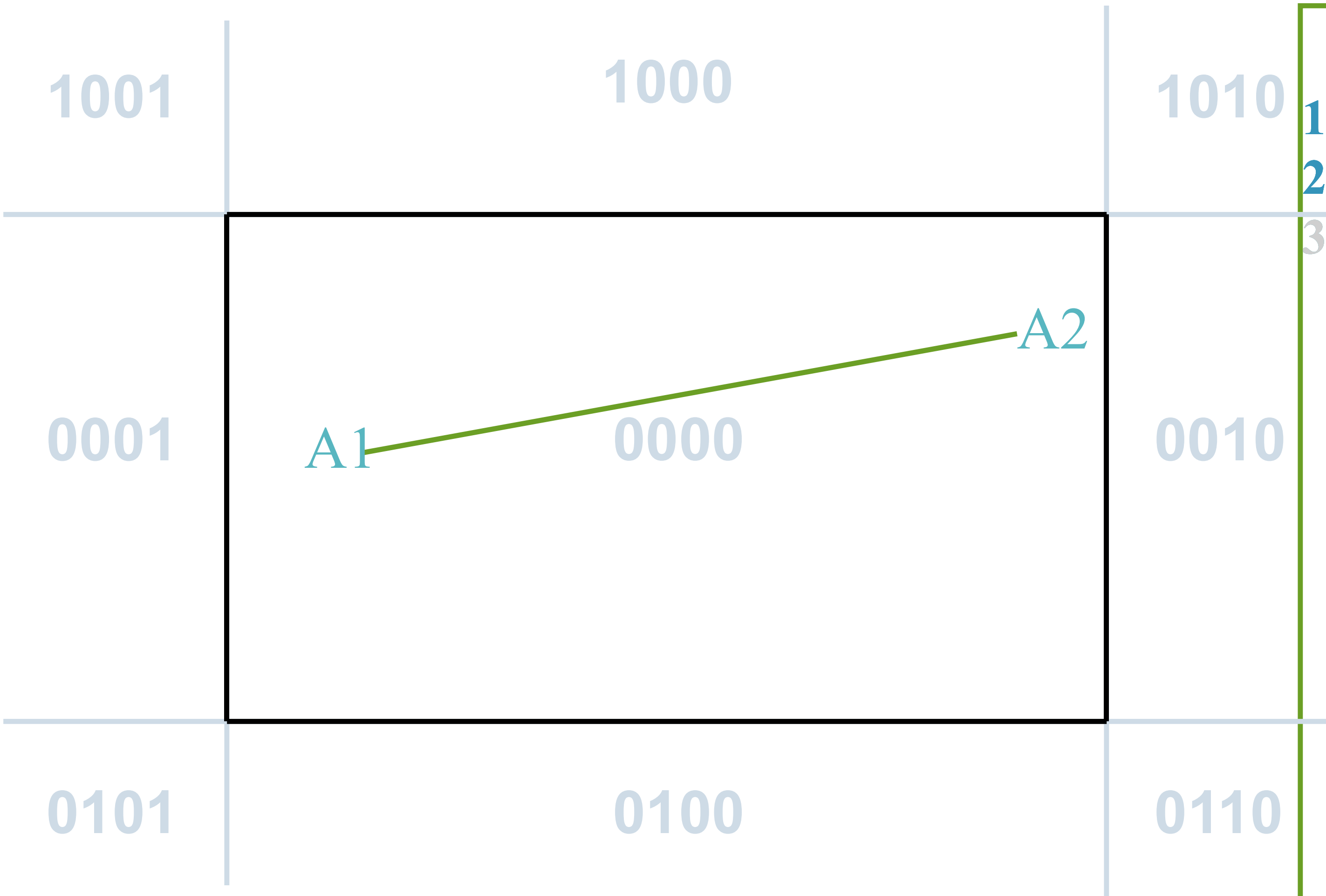
3.2.1

3.2.2

3.2.3

3.2.4

Example



algorithm

1. B1=1001,B2=1010

2. (both 0000) – No

3. AND Operation

B1 → 1001

B2 → 1010

Result 1000

3.1 (not 0000) – Yes

→ reject

3.2

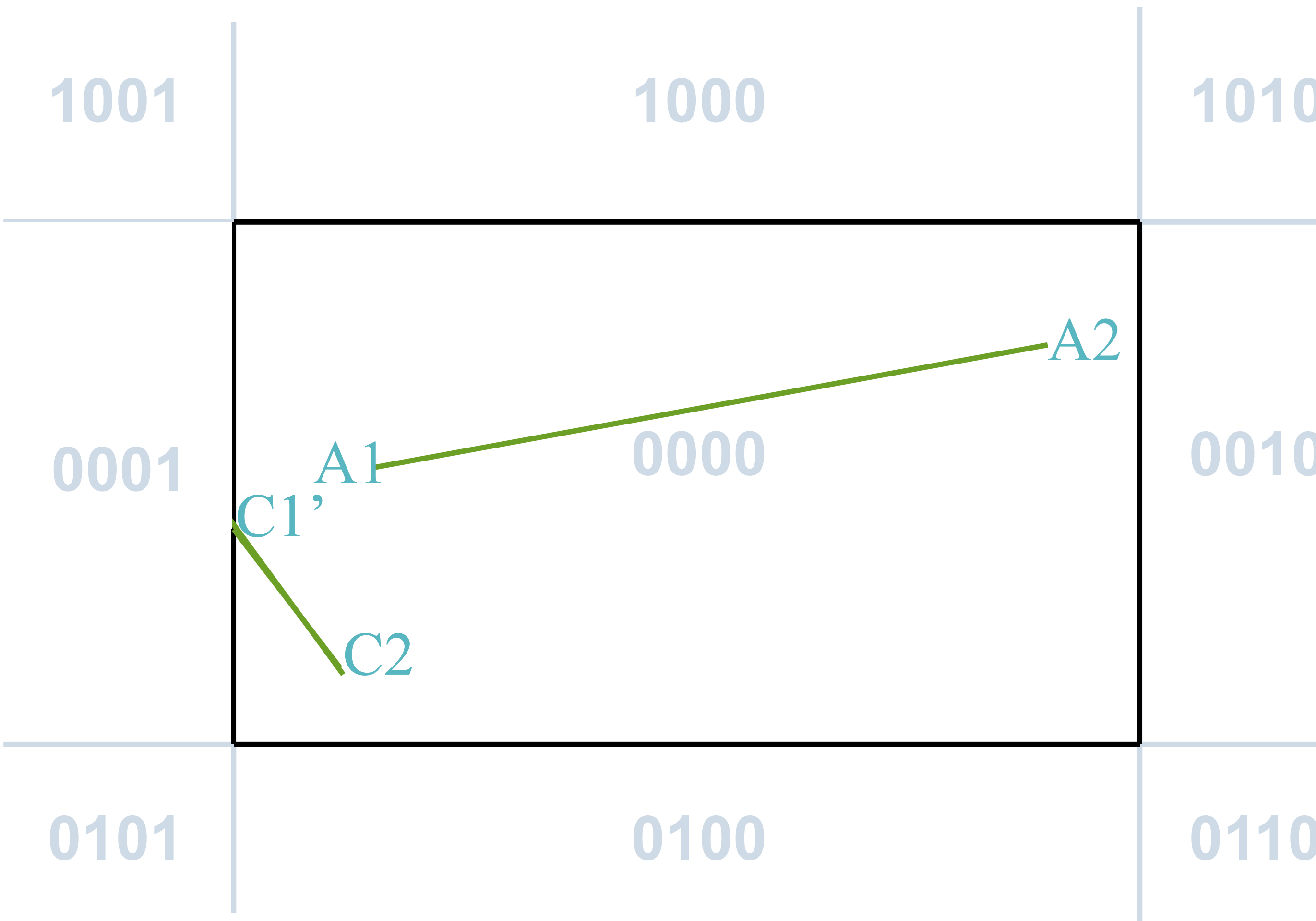
3.2.1

3.2.2

3.2.3

3.2.4

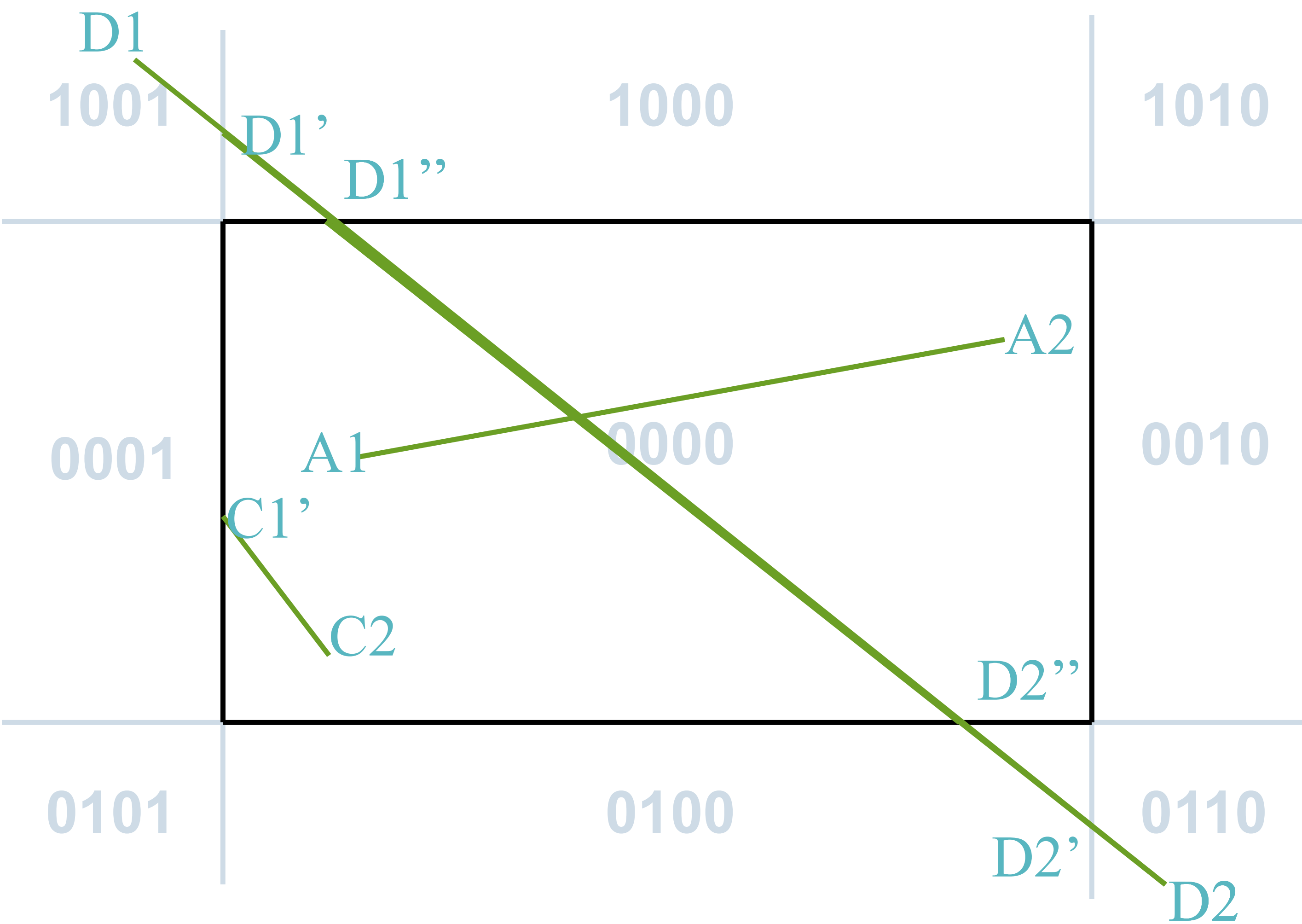
Example



algorithm

1. **C1’=0000,C2=0000**
2. **(both 0000) – Yes -> accept & draw**
3.
   3.1
   3.2
      3.2.1
      3.2.2
      3.2.3
      3.2.4

Example



algorithm

Advantages & Disadvantages of Cohen-Sutherland Algorithm

Easy to implement

Early accept/reject tests

Slow for many clipped lines

Exercise

$(xw_{min}, xw_{max}) = (10, 150)$

$(yw_{min}, yw_{max}) = (10, 100)$

$P_0 = (0, 120)$

$P_1 = (130, 5)$

Liang-Barsky Line Clipping

This algorithm uses the parametric equations for a line

Solves four inequalities to find the range of the parameter for which the line is in the viewport (window).

Consider the parametric definition of a line:

- $x = x_1 + u\Delta x$
- $y = y_1 + u\Delta y$
- $\Delta x = (x_2 - x_1)$, $\Delta y = (y_2 - y_1)$, $0 \leq u \leq 1$

Liang-Barsky Line Clipping

Mathematically, this can be written using point clipping conditions in the parametric form

- $x_{min} \leq x_1 + u\Delta x \leq x_{max}$
- $y_{min} \leq y_1 + u\Delta y \leq y_{max}$

Rearranging, we get

- $-u\Delta x \leq (x_1 - x_{min})$
- $u\Delta x \leq (x_{max} - x_1)$
- $-u\Delta y \leq (y_1 - y_{min})$
- $u\Delta y \leq (y_{max} - y_1)$

Liang-Barsky Line Clipping

In general: u * pk ≤ qk

Where the parameters p and q are defined as

$$\overline{P}_1 = -\Delta x, \quad \overline{q}_1 = x_1 - x_{\min}$$

$$\overline{P}_2 = \Delta x, \quad \overline{q}_2 = x_{\max} - x_1$$

$$\overline{P}_3 = -\Delta y, \quad \overline{q}_3 = y_1 - y_{\min}$$

$$\overline{P}_4 = \Delta y, \quad \overline{q}_4 = y_{\max} - y_1$$

Liang-Barsky Line Clipping

Cases: pk = 0

- Line is parallel to boundaries

If for the same k, $qk < 0$,

- The line is completely outside , so reject it

Else,

The line is inside the parallel clipping boundary accept the line

For a non zero value of **pk,**

- Calculate the value of u that corresponds to the point where the infinitely extended line intersects with the extension of boundary k as

$$u = qk / pk$$

Calculate the parameters u1 and u2 the part of the line within clip rectangle

Liang-Barsky Line Clipping

Initialize u1=0,u2=1

Case **pk < 0**

Line proceeds from outside to inside of the infinite extension

- rk = qk / pk
- u1 = max(rk, u1)

Case **p_k > 0**

Line proceeds from inside to outside boundary

rk = qk / pk

$u_2 = \min(r_k, u_2)$

If $u_1 > u_2$, the line is completely outside and can rejected

Else

The endpoints of the clipped line is calculated from the two values of parameters u

If u1 < u2 then draw a line from:

$(x0 + \Delta x \cdot u1, y0 + \Delta y \cdot u1)$ to

$(x0 + \Delta x \cdot u2, y0 + \Delta y \cdot u2)$

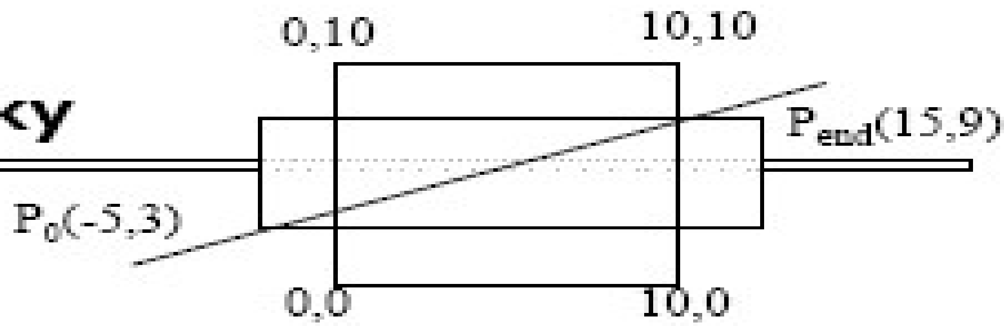
Liang-Barsky Line Clipping

In most cases, Liang-Barsky is slightly more efficient

- Intersection calculations are reduced
- Avoids multiple shortenings of line segments
- Window intersection of the line is computed only once

Liang-Barsky Line Clipping

Example Liang-Barsky



$$u_{left} = \frac{q_1}{p_1} = \frac{x_0 - xw_{min}}{-\Delta x} = \frac{-5-0}{-(15-(-5))} = \frac{1}{4}$$

Entering ⇒ $u_{min} = 1/4$

$$u_{right} = \frac{q_2}{p_2} = \frac{xw_{max} - x_0}{\Delta x} = \frac{10-(-5)}{15-(-5)} = \frac{3}{4}$$

Exiting ⇒ $u_{max} = 3/4$

$$u_{bottom} = \frac{q_3}{p_3} = \frac{y_0 - yw_{min}}{-\Delta y} = \frac{3-0}{-(9-3)} = -\frac{1}{2}$$

$u < 0$ then ignore

$$u_{top} = \frac{q_4}{p_4} = \frac{yw_{max} - y_0}{\Delta y} = \frac{10-3}{9-3} = \frac{7}{6}$$

$u > 1$ then ignore

Liang-Barsky Line Clipping

Liang-Barsky Line-Clipping

- We have $u_{min} = 1/4$ and $u_{max} = 3/4$

$$P_{end} - P_0 = (\underbrace{15+5}_{\Delta x},\underbrace{9-3}_{\Delta y}) = (20,6)$$

- If $u_{min} < u_{max}$, there is a line segment
  - compute endpoints by substituting u values
- Draw a line from

  $(-5+(20)\cdot(1/4),\ 3+(6)\cdot(1/4))$

  to

  $(-5+(20)\cdot(3/4),\ 3+(6)\cdot(3/4))$

Nicholl-Lee-Nicholl Line Clipping

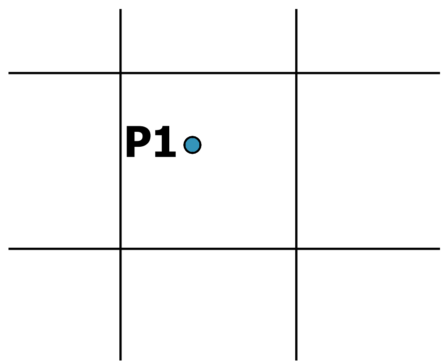
Create more regions around the clip window.

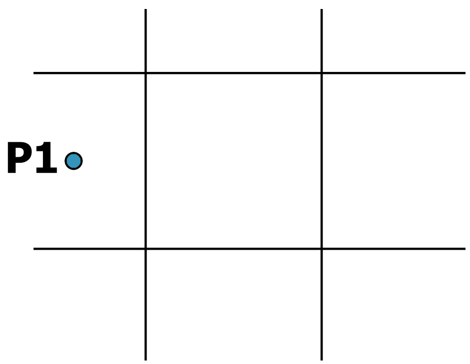
Avoids multiple clipping of an line segment

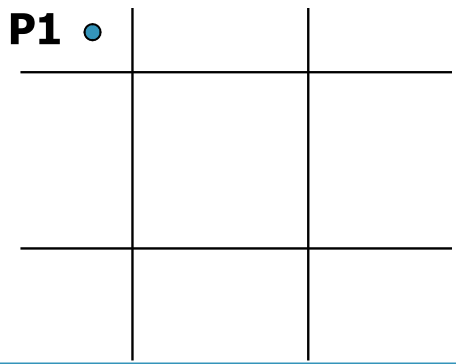
Performs fewer comparisons and divisions of the three.

Disadv: applied only to 2 D Clipping.

NLN

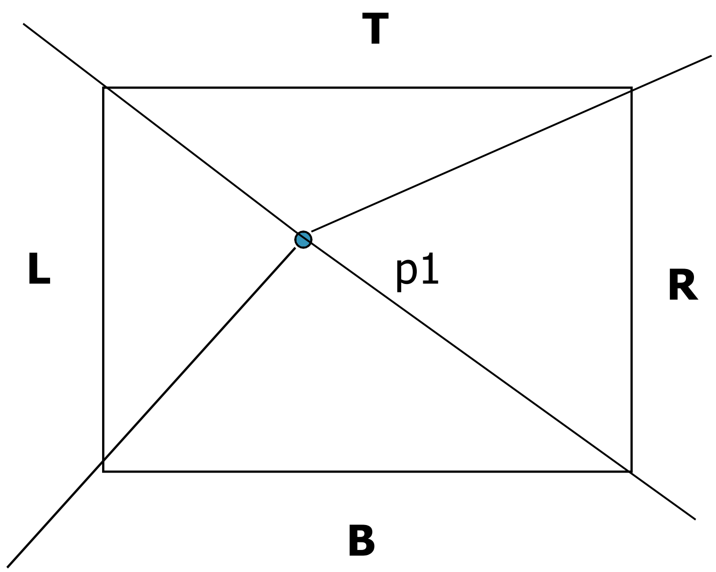


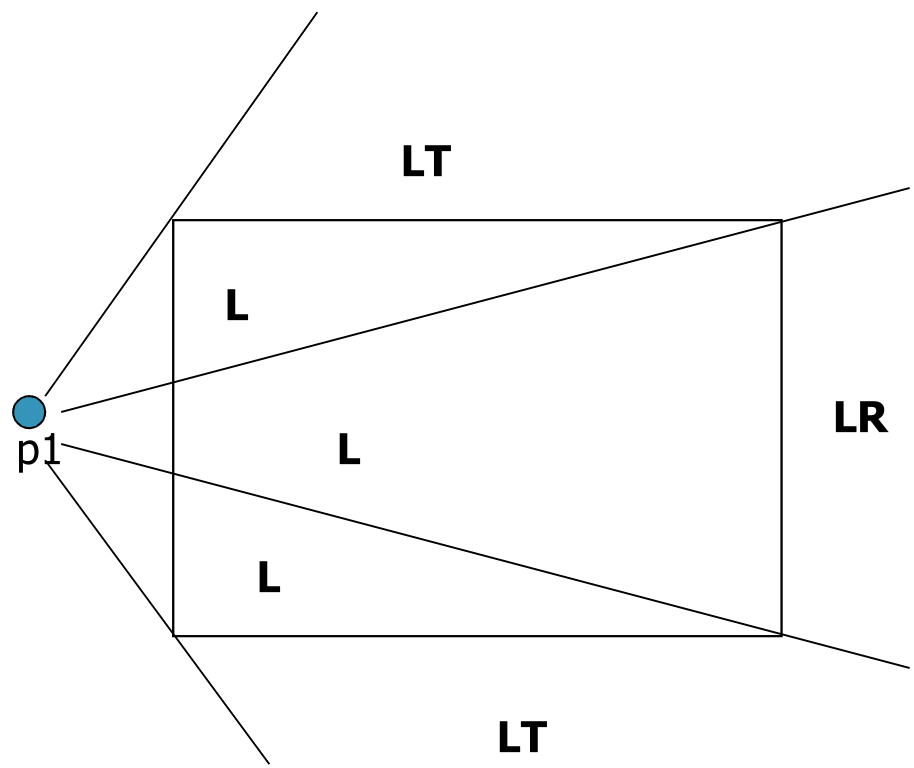




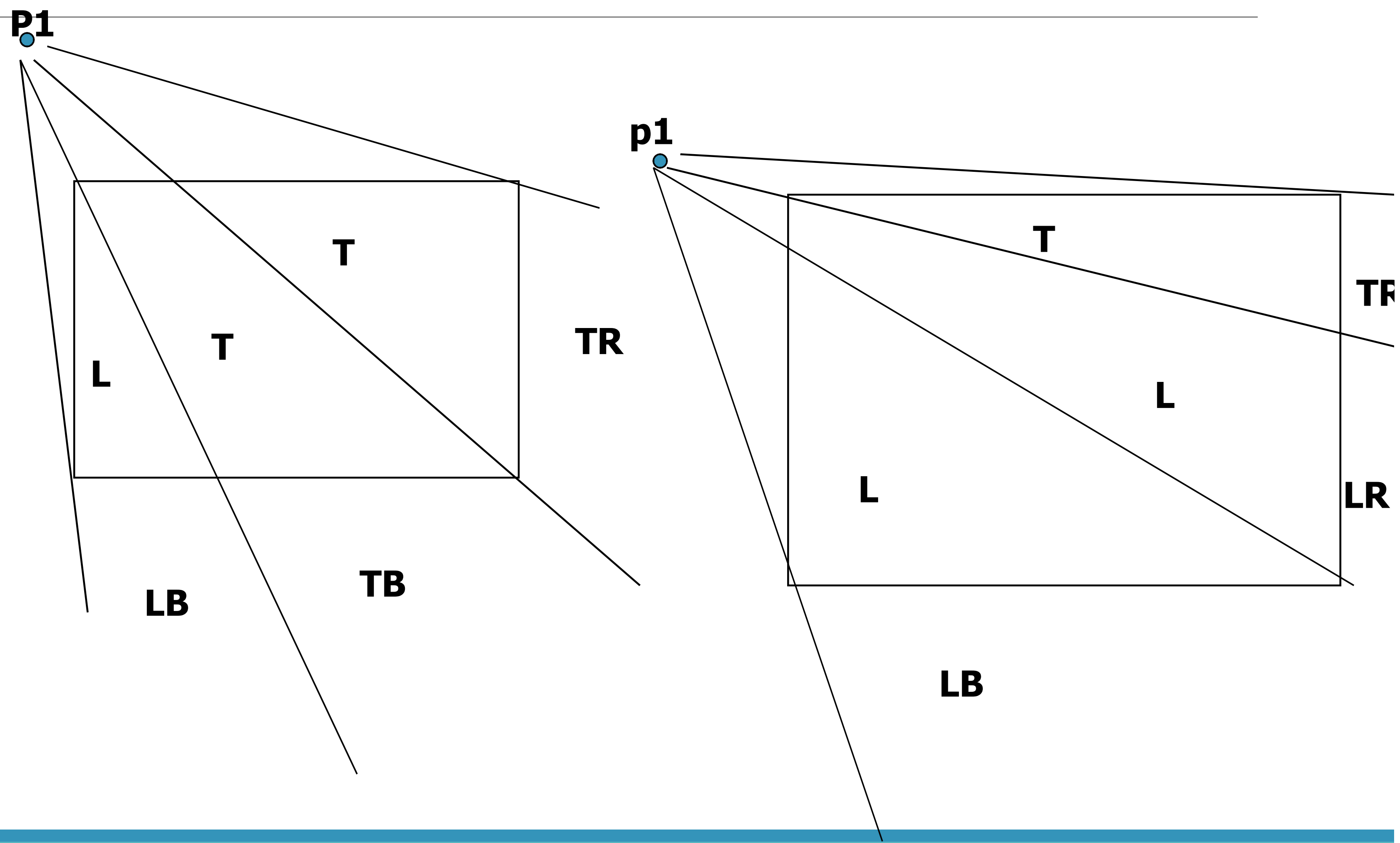
NLN

Find p2 relative to p1.





NLN



NLN

To determine where p2 is located,

- Slope p1ptr < slope p1p2 < slope p1ptl

Find x and y intersection points.

Thank You