Graph and Matrices

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Introduction

- The information in a graph expressed in a variety of ways in matrix form

E.g. Sociomatrix, incidence matrix

- Sociomatrix – adjacency matrix represents whether two nodes are adjacent or not
- The sociomatrix for a graph (for a nondirectional relation) is symmetric
- Example of a sociomatrix : friendship relation between 4 friends

Incidence Matrix

	n_1	n_2	n_3	n_4
n_1	-	0	0	0
n_2	0	-	1	0
n_3	0	1	-	0
n_4	0	0	0	-

- Incidence matrix, I - records which lines are incident with which nodes
- Incidence matrix has nodes indexing the rows, and lines indexing the columns
- Incidence matrix is binary, as it records whether a line incident with a node or it is not
- It is not necessarily square matrix
- For digraph the entries are choice-based (if i node in row chooses j node in column the entry in the cell is 1 otherwise 0)
- So the entry for i and j may be different from j and i

Matrix Operations

- Permutations
- A permutation of a set of objects is any reordering of the objects (possible reordering)
- It is used in the study of cohesive subgroups
- Important in constructing block-models and in evaluating the goodness-of-fit of blockmodels

	n_1	n_2	n_3	n_4	n_5
n_1	-	0	1	0	1
n_2	0	-	0	1	0
n_3	1	0	-	0	1
n_4	0	1	0	-	0
n_5	1	0	1	0	-

X permuted

	n_5	n_1	n_3	n_2	n_4
n_5	-	1	1	0	0
n_1	1	-	1	0	0
n_3	1	1	-	0	0
n_2	0	0	0	-	1
n_4	0	0	0	1	-

- Reordering rows and cols of the matrix, helps to discover patterns, identify subsets

Transpose

- Transpose of a sociomatrix is analogous to reversing the direction of the ties between the actors
- For non-directional relations, transpose is identical to original matrix
- Matrix multiplication is a very important operation in social network analysis
- It can be used to study walks and reachability in a graph

Powers of the Matrix

- Power of the matrix and Boolean matrix multiplication is also used in social network analysis
- Studying powers of the matrix X used to find walk of specific length
- For example, elements of X^3 counts the number of walks of length 3 between each pair of nodes
- Also used to find walks of longer lengths
- It is also used to find the reachability matrix $\mathbf{X}^{[R]} = \{x_{ij}^{[R]}\}$, says that each pair of nodes whether they are reachable, or not
- The entries tell us total number of directed walks from row node n^i, to column node n^j

Geodesic and distances

- The first power *p* for which the (i, j) element is non-zero gives the length of the shortest path and is equal to d(i,j)

$$d(i, j) = min_p x_{ij}^{[p]} > 0.$$

- If the graph is connected or if the digraph is at least strongly connected, the diameter of the graph is then the largest entry in the distance matrix; otherwise, the diameter is infinite or undefined

Example

X

	n_1	n_2	n_3	n_4	n_5	n_6
n_1	-	1	0	0	1	0
n_2	0	-	1	0	0	1
n_3	0	1	-	0	0	0
n_4	0	0	0	-	1	0
n_5	0	0	0	0	-	1
n_6	0	1	0	0	0	-

$\mathbf{X}^2$

	n_1	n_2	n_3	n_4	n_5	n_6
n_1	0	0	1	0	0	2
n_2	0	2	0	0	0	0
n_3	0	0	1	0	0	1
n_4	0	0	0	0	0	1
n_5	0	1	0	0	0	0
n_6	0	0	1	0	0	1

$\mathbf{X}^3$

	n_1	n_2	n_3	n_4	n_5	n_6
n_1	0	3	0	0	0	0
n_2	0	0	2	0	0	2
n_3	0	2	0	0	0	0
n_4	0	1	0	0	0	0
n_5	0	0	1	0	0	1
n_6	0	2	0	0	0	0

$\mathbf{X}^4$

	n_1	n_2	n_3	n_4	n_5	n_6
n_1	0	0	3	0	0	3
n_2	0	4	0	0	0	0
n_3	0	0	2	0	0	2
n_4	0	0	1	0	0	1
n_5	0	2	0	0	0	0
n_6	0	0	2	0	0	2

$\mathbf{X}^5$

	n_1	n_2	n_3	n_4	n_5	n_6
n_1	0	6	0	0	0	0
n_2	0	0	4	0	0	4
n_3	0	4	0	0	0	0
n_4	0	2	0	0	0	0
n_5	0	0	2	0	0	2
n_6	0	4	0	0	0	0

- For Example : From N1 to N6, the shortest path is 1, though there exist 2 and 3.
- The diameter (largest entry in the distance matrix) is 6

Computing Nodal Degrees

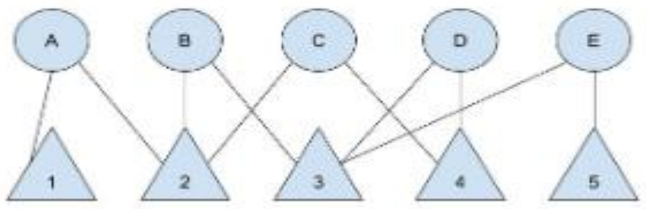
- For a nondirectional relation, the nodal degrees are equal to either the row sums or the column sums.

Indegree & Outdegree of directed graph:

- Indegree - Summing over columns (that is, lines) gives the number of lines incident with the node

- Outdegree - Summing over rows (that is, lines) gives the number of lines originate from the node

	1	2	3	4	5
A	1	1	0	0	0
B	0	1	1	0	0
C	0	1	0	1	0
D	0	0	1	1	0
E	0	0	1	0	0



Computing Density

- The density of a graph, digraph, or valued (di)graph can be calculated as the sum of all entries in the matrix, divided by the possible number of entries

$$\Delta = \frac{\sum_{i=1}^{g} \sum_{j=1}^{g} x_{ij}}{g(g-1)}$$