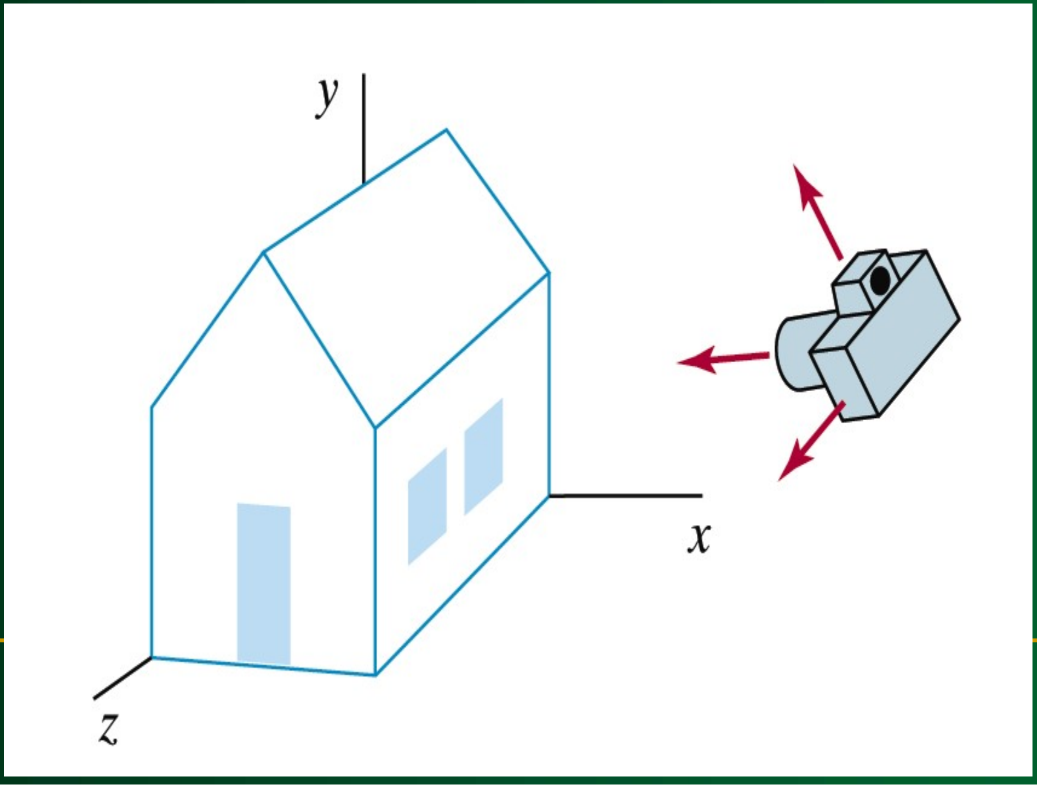
Three Dimensional Viewing

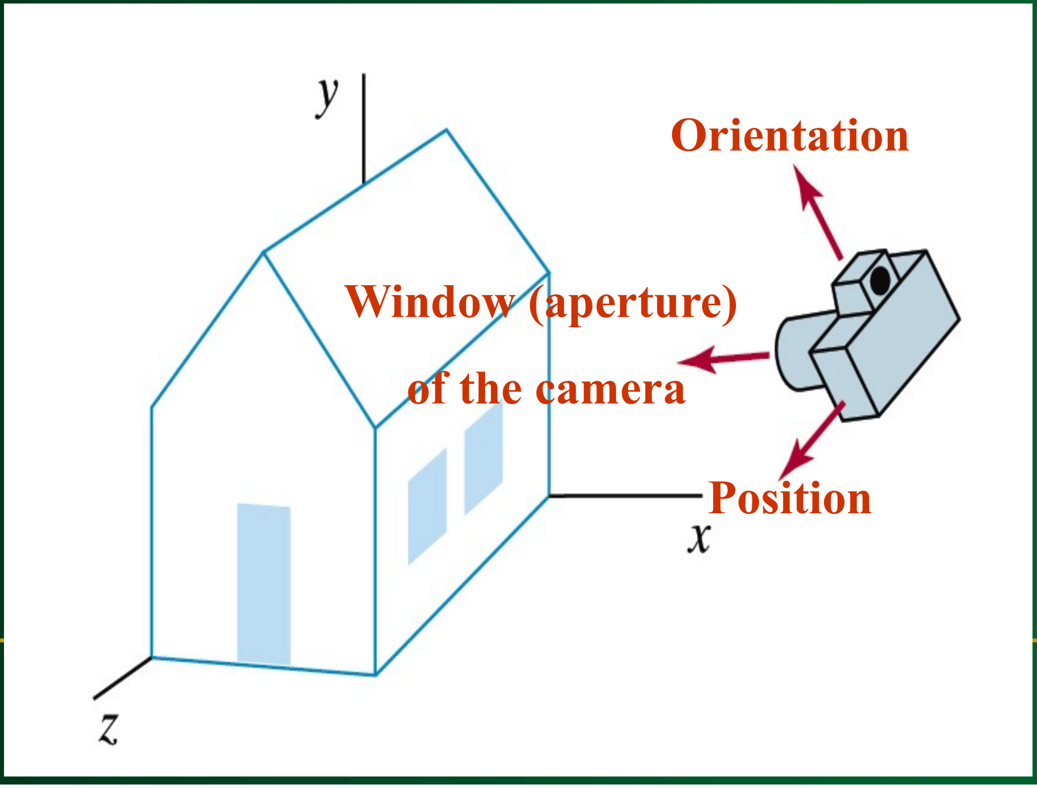
3D Viewing

- The steps for computer generation of a view of a three dimensional scene are somewhat analogous to the processes involved in taking a photograph.



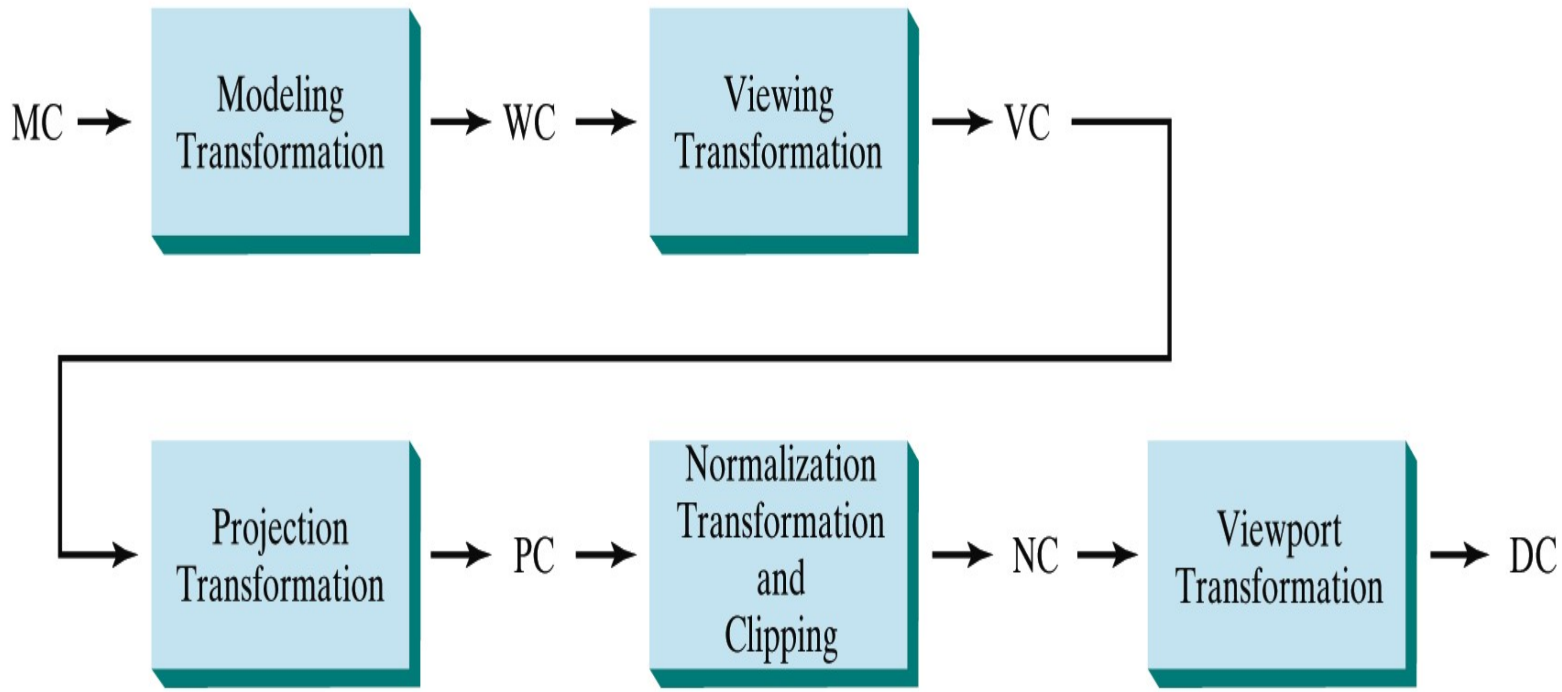
Camera Analogy

1. Viewing position
2. Camera orientation
3. Size of clipping window



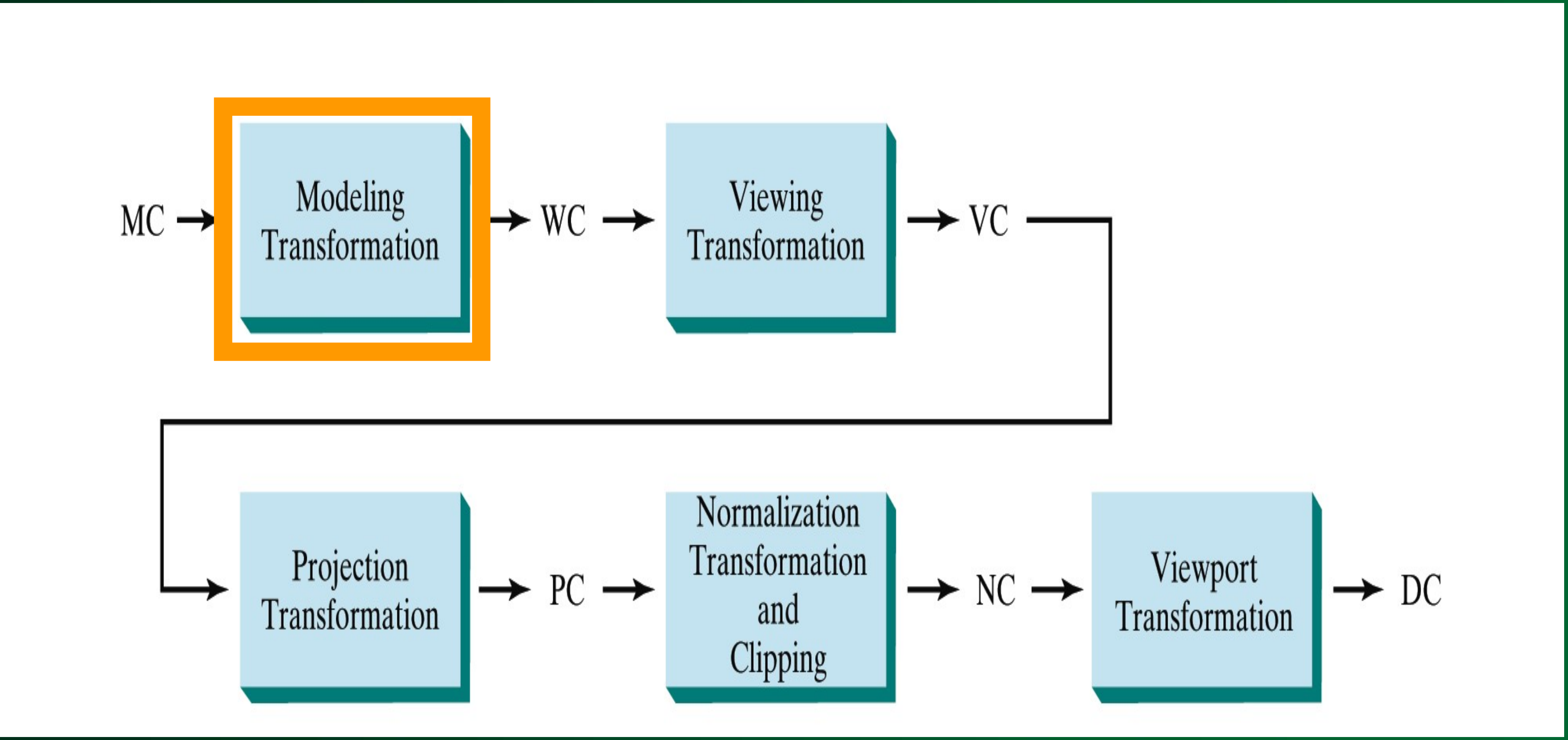
Viewing Pipeline

- The general processing steps for modeling and converting a world coordinate description of a scene to device coordinates:



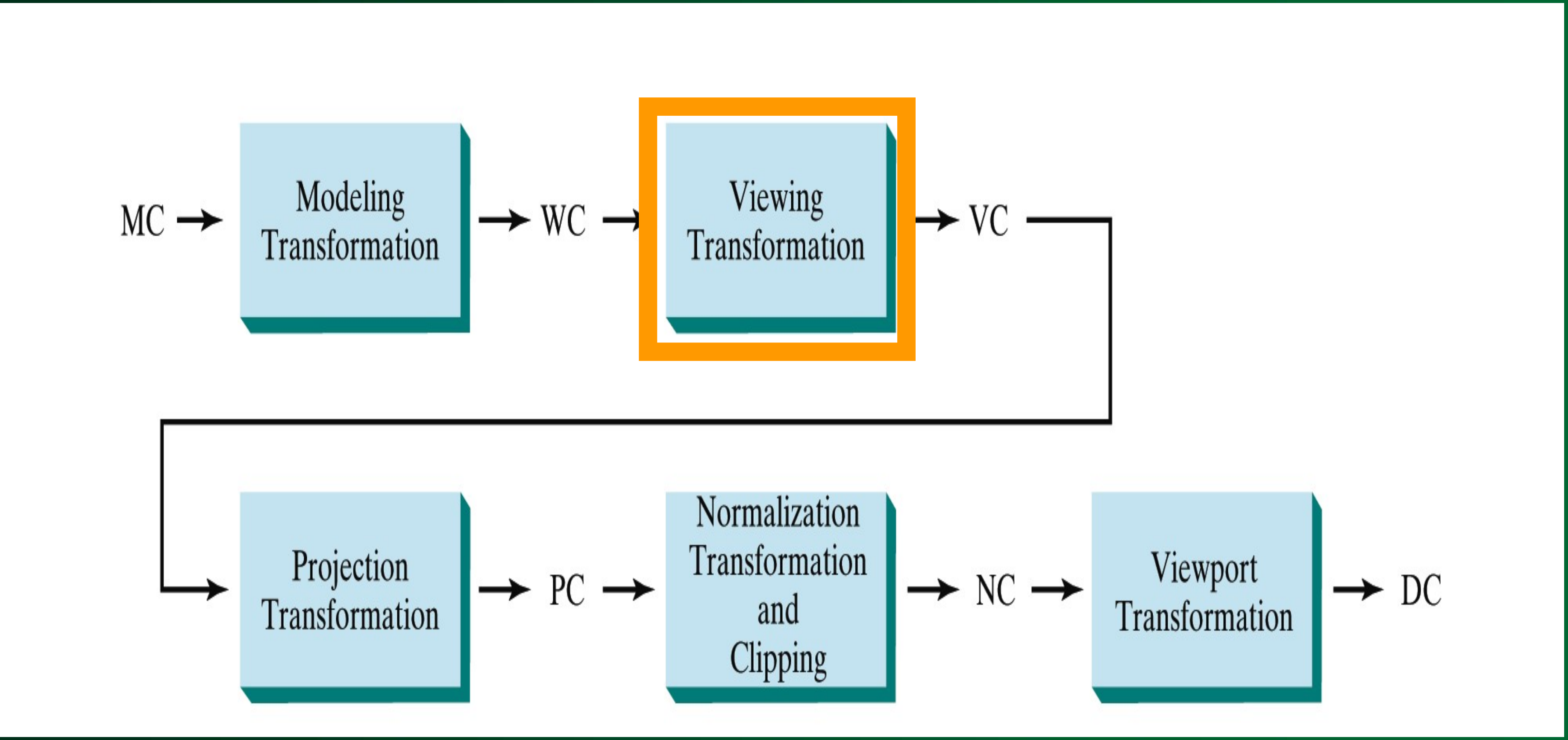
Viewing Pipeline

1. Construct the shape of individual objects in a scene within modeling coordinate, and place the objects into appropriate positions within the scene (world coordinate).



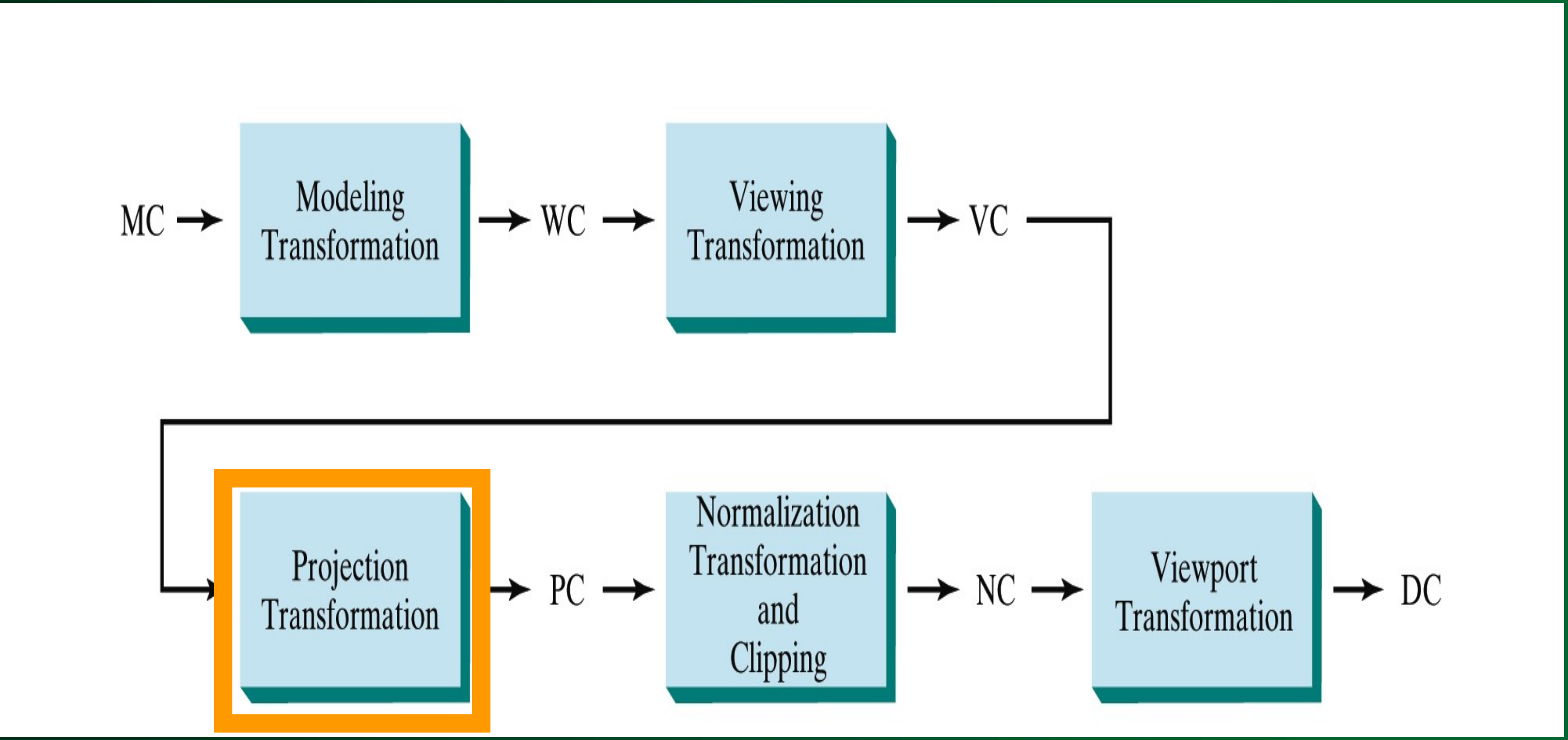
Viewing Pipeline

2. World coordinate positions are converted to viewing coordinates.



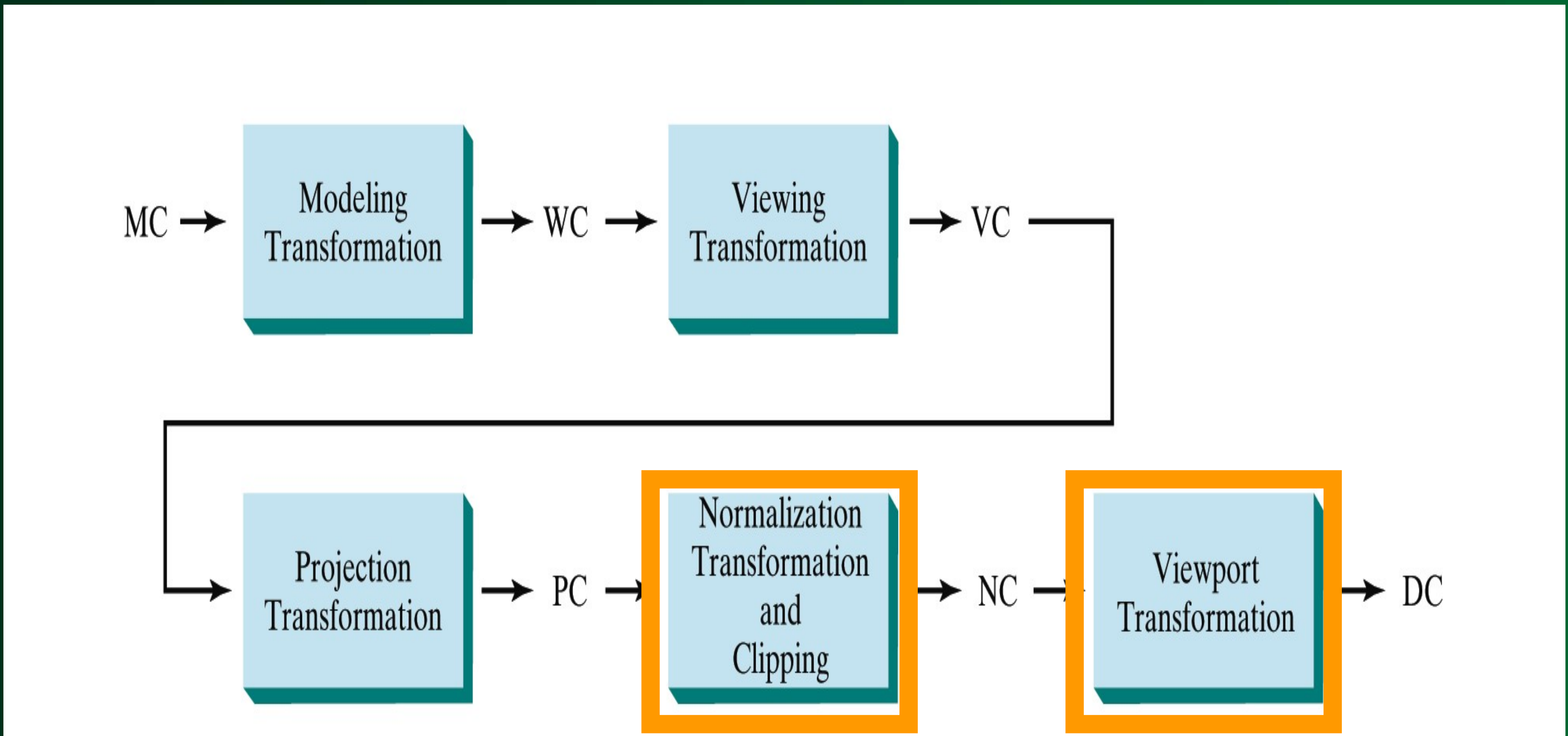
Viewing Pipeline

3. Convert the viewing coordinate description of the scene to coordinate positions on the projection plane.

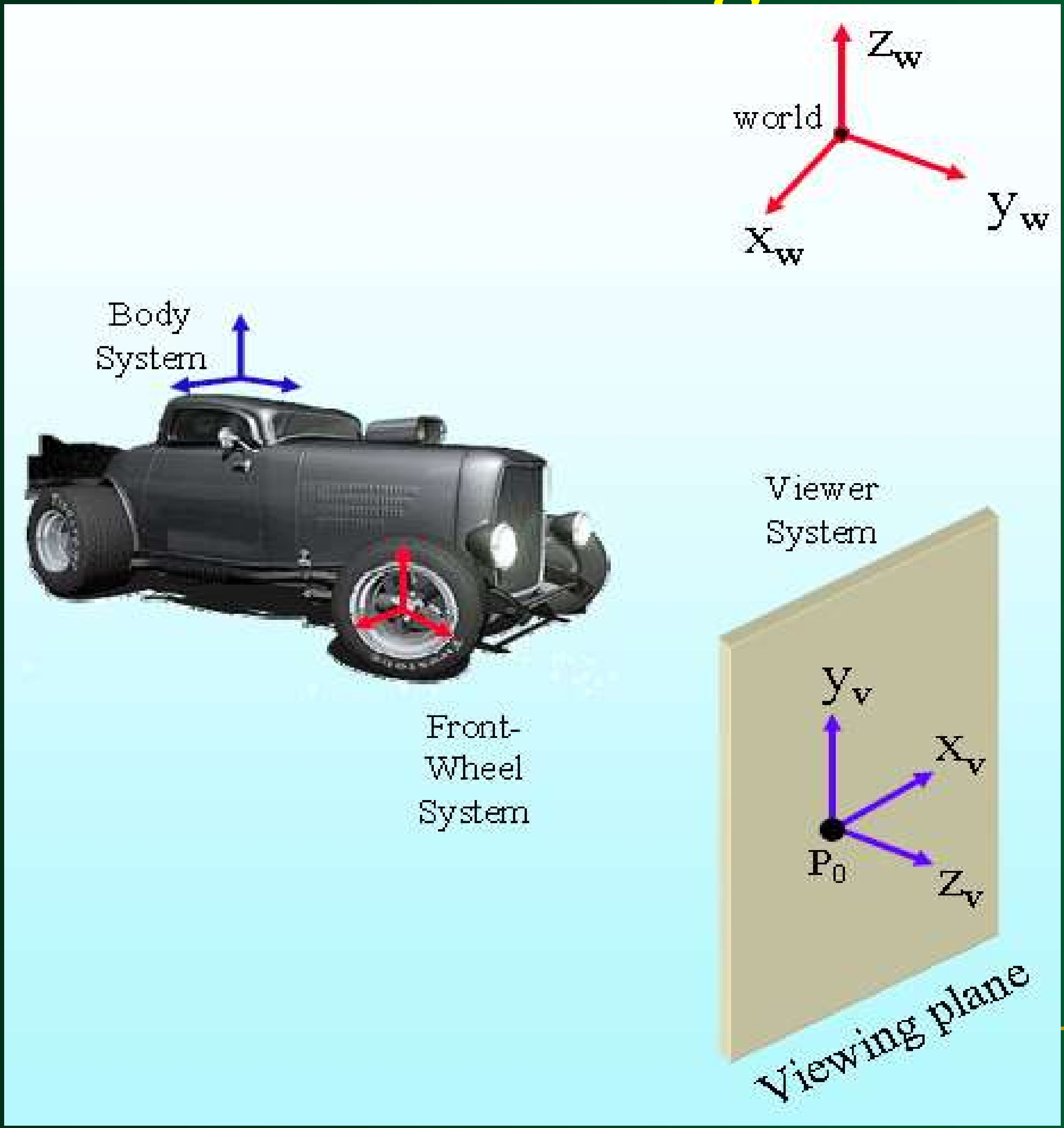


Viewing Pipeline

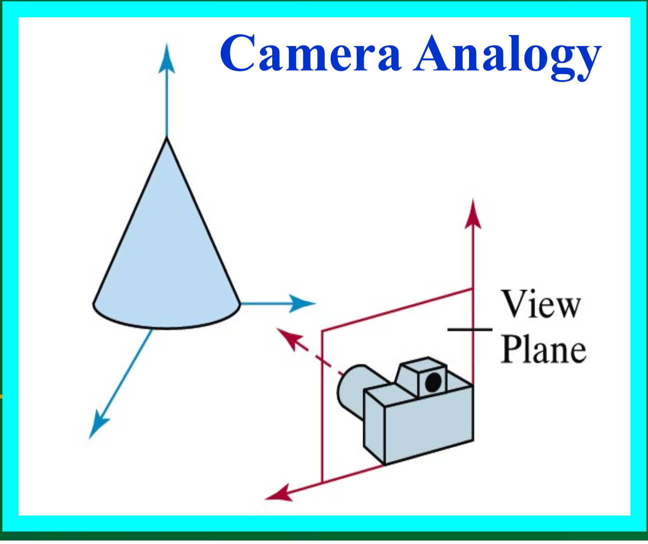
4. Positions on the projection plane, will then mapped to the Normalized coordinate and output device.



Viewing Coordinates

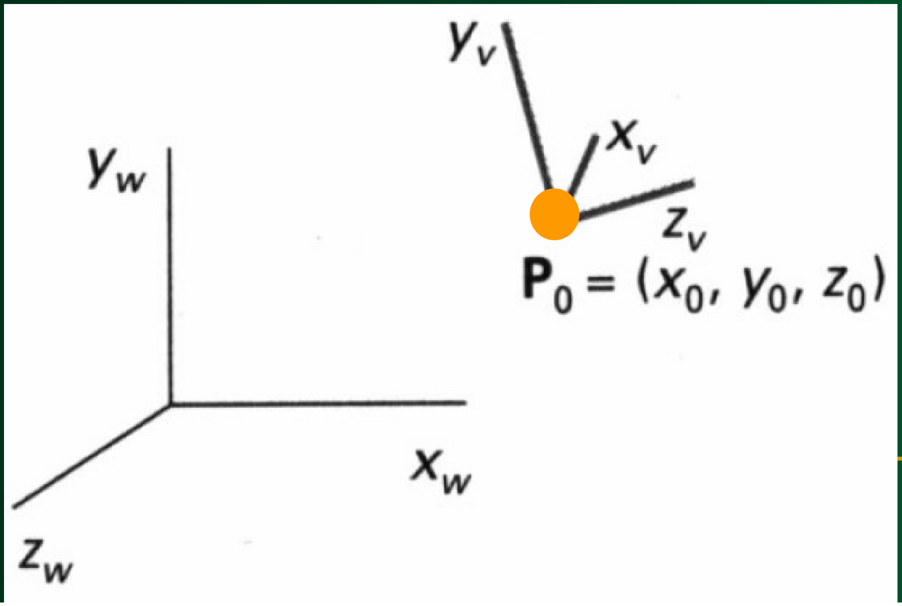


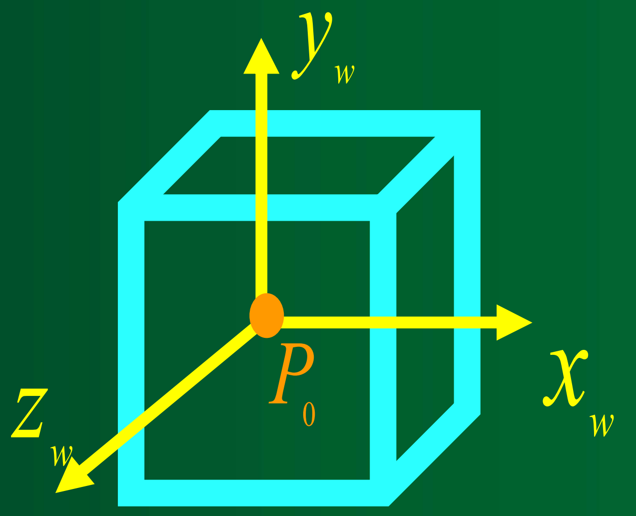
- Viewing coordinates system describes 3D objects with respect to a viewer.
- A Viewing (Projector) plane is set up perpendicular to z_v and aligned with (x_v,y_v).

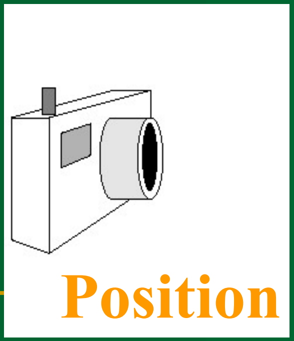


Specifying the Viewing Coordinate System (View Reference Point)

- We first pick a world coordinate position called **view reference point** (origin of our viewing coordinate system).
- P_0 is a point where a camera is located.
- The view reference point is often chosen to be close to or on the surface of some object, or at the center of a group of objects.

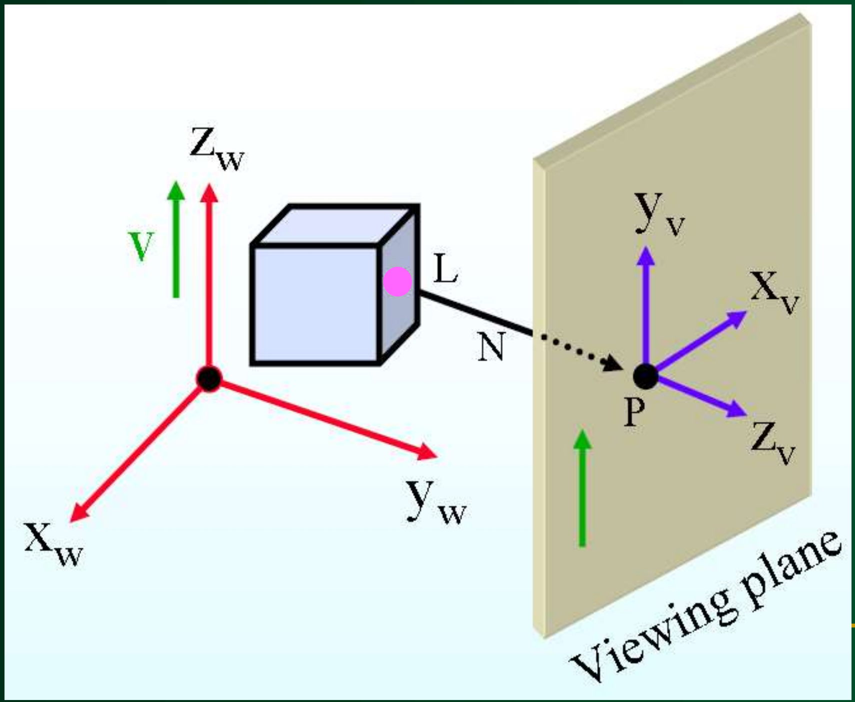


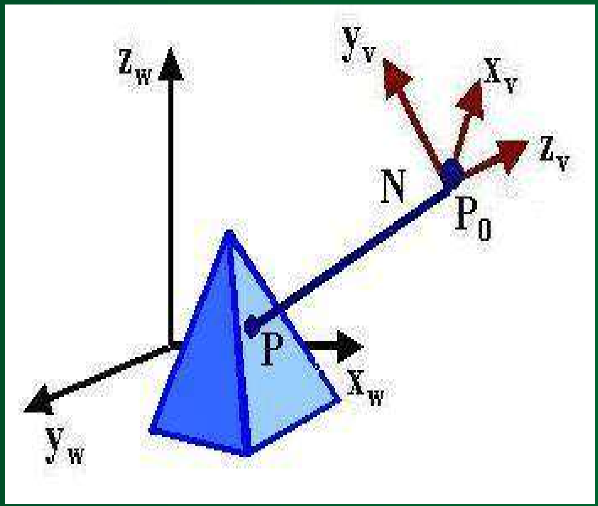


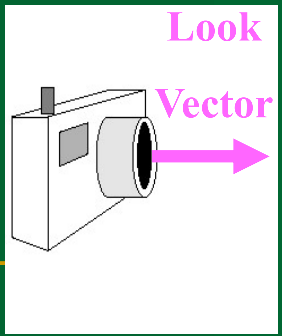


Specifying the Viewing Coordinate System (Z_v Axis)

- Next, we select the positive direction for the viewing z_v axis, by specifying the **view plane normal vector**, **N**.
- The direction of N, is from the **look at point** (L) to the view reference point.

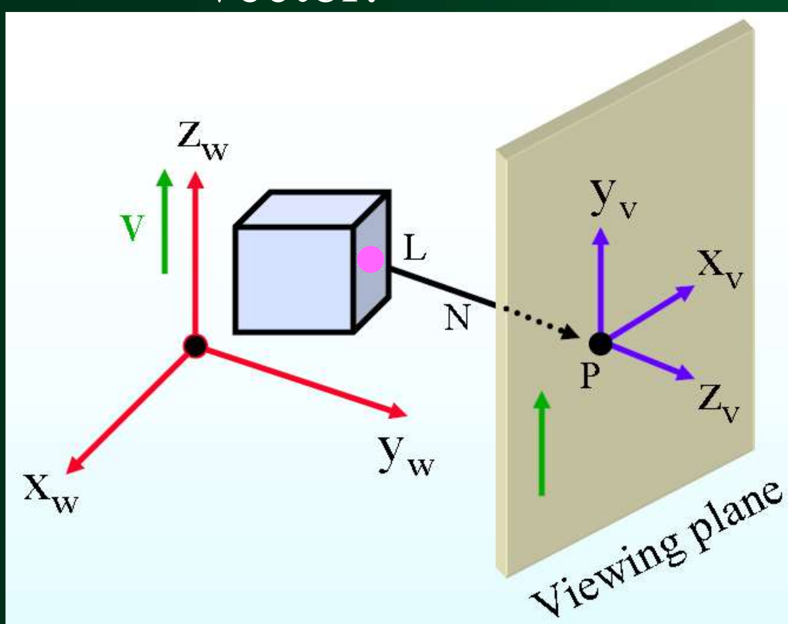


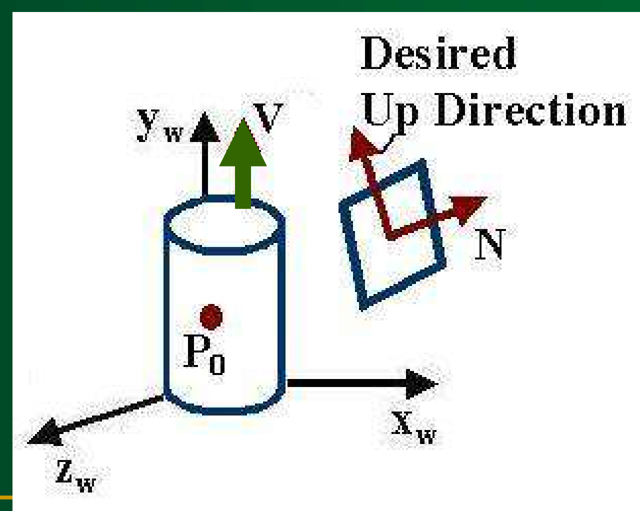


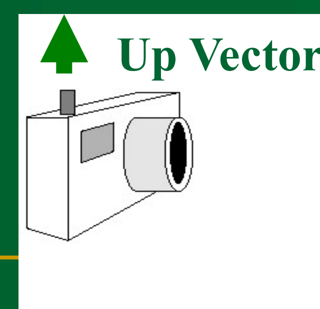


Specifying the Viewing Coordinate System (y_v Axis)

- Finally, we choose the ***up direction*** for the view by specifying a vector V, called the ***view up vector***.
- This vector is used to establish the positive direction for the y_v axis.
- V is projected into a plane that is perpendicular to the normal vector.

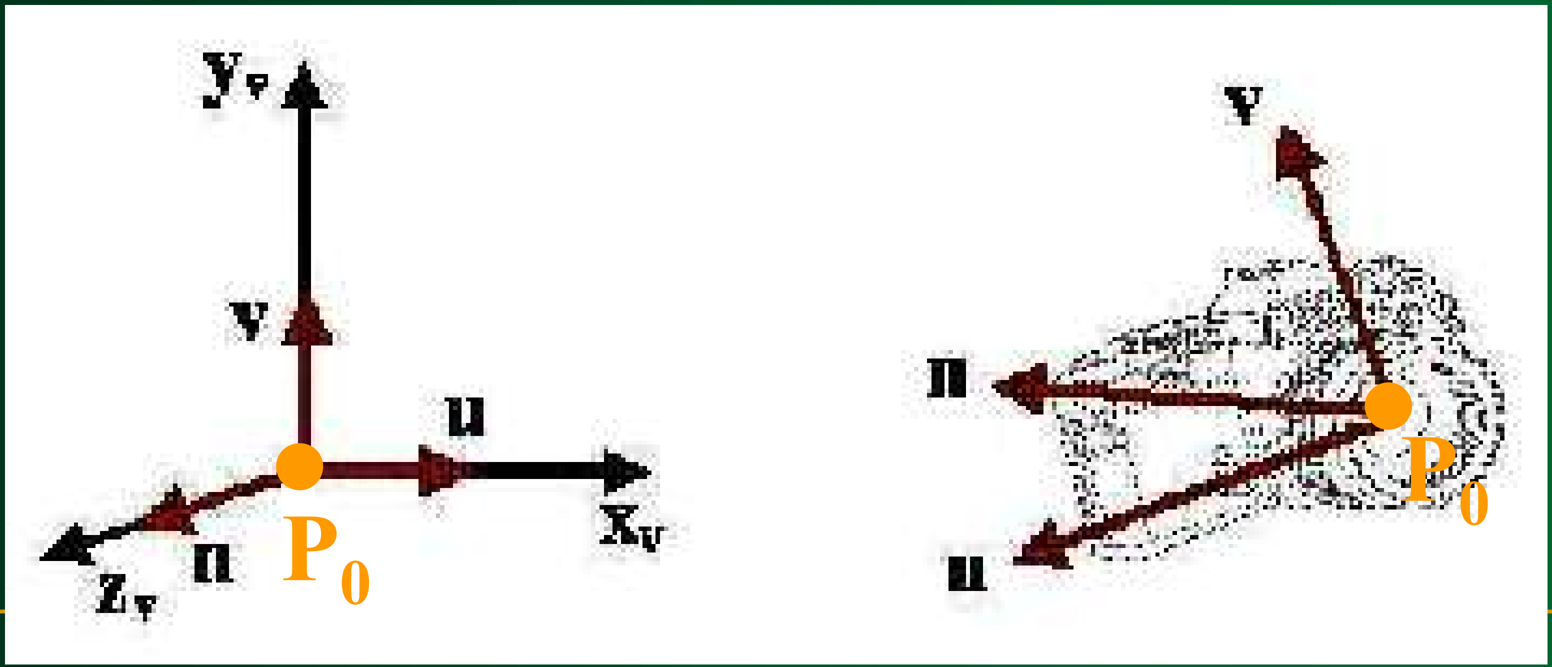






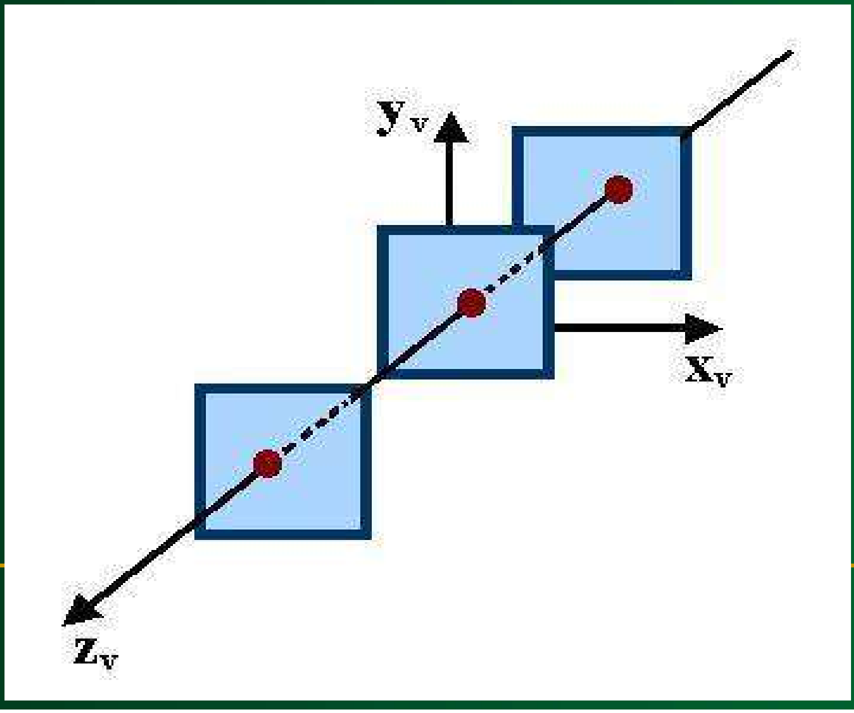
Specifying the Viewing Coordinate System (x_v Axis)

- Using vectors **N** and **V**, the graphics package computer can compute a third vector **U**, perpendicular to both **N** and **V**, to define the direction for the x_v axis.



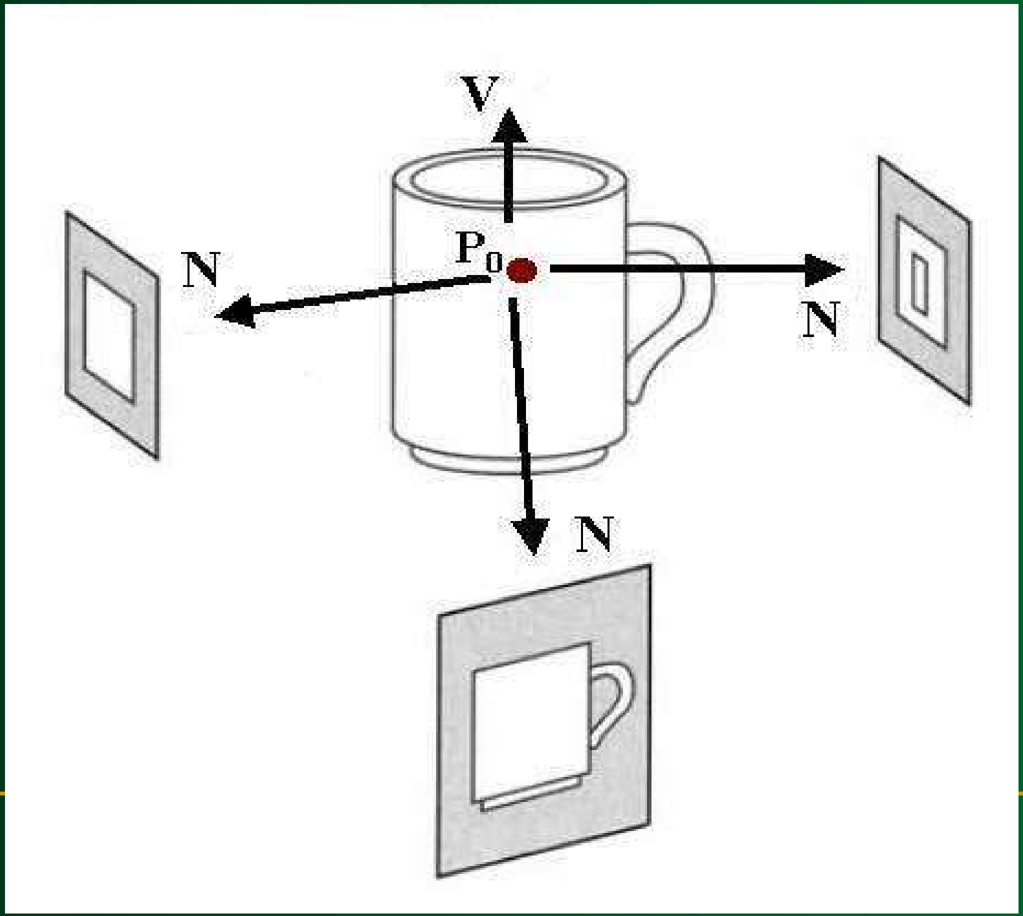
The View Plane

- Graphics package allow users to choose the position of the view plane along the z_v axis by specifying the **view plane distance** from the the viewing origin.
- The view plane is always parallel to the $x_v y_v$ plane.



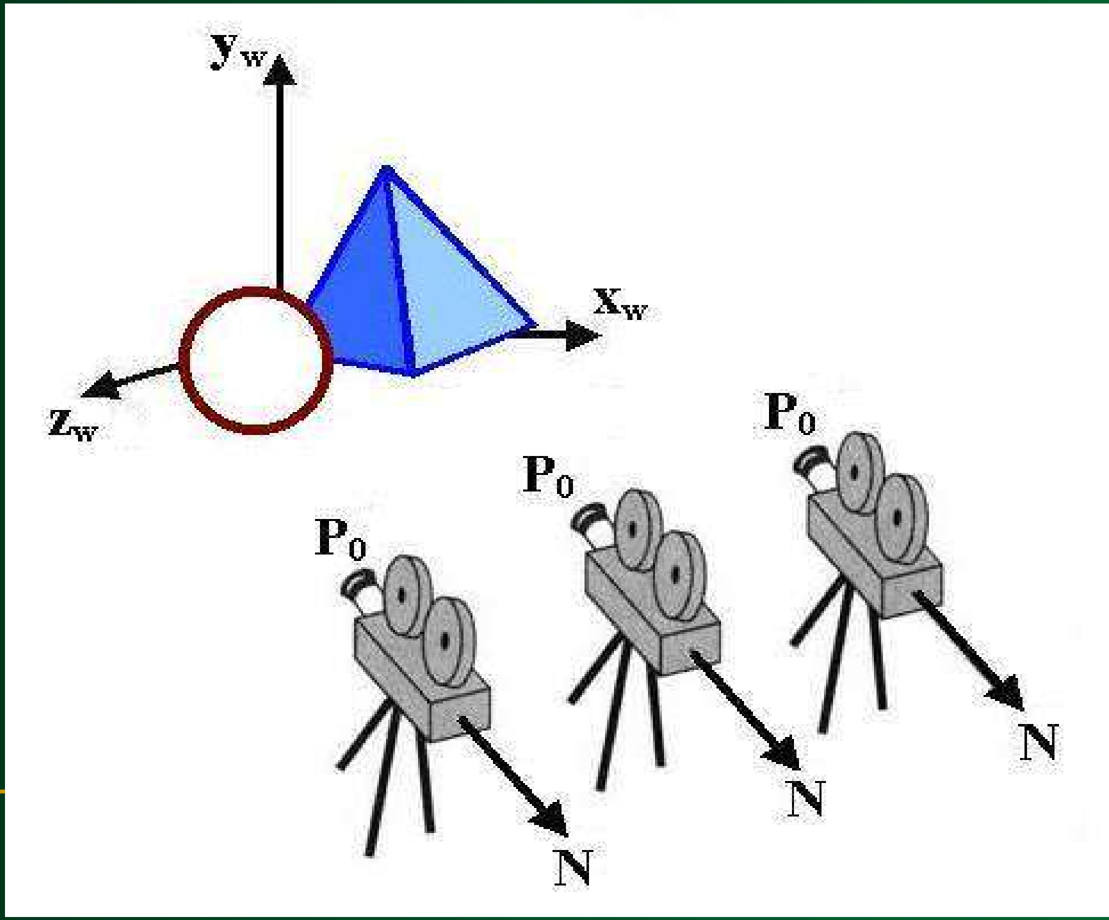
Obtain a Series of View

- To obtain a series of view of a scene, we can keep the view reference point fixed and **change** the direction of **N**.



Simulate Camera Motion

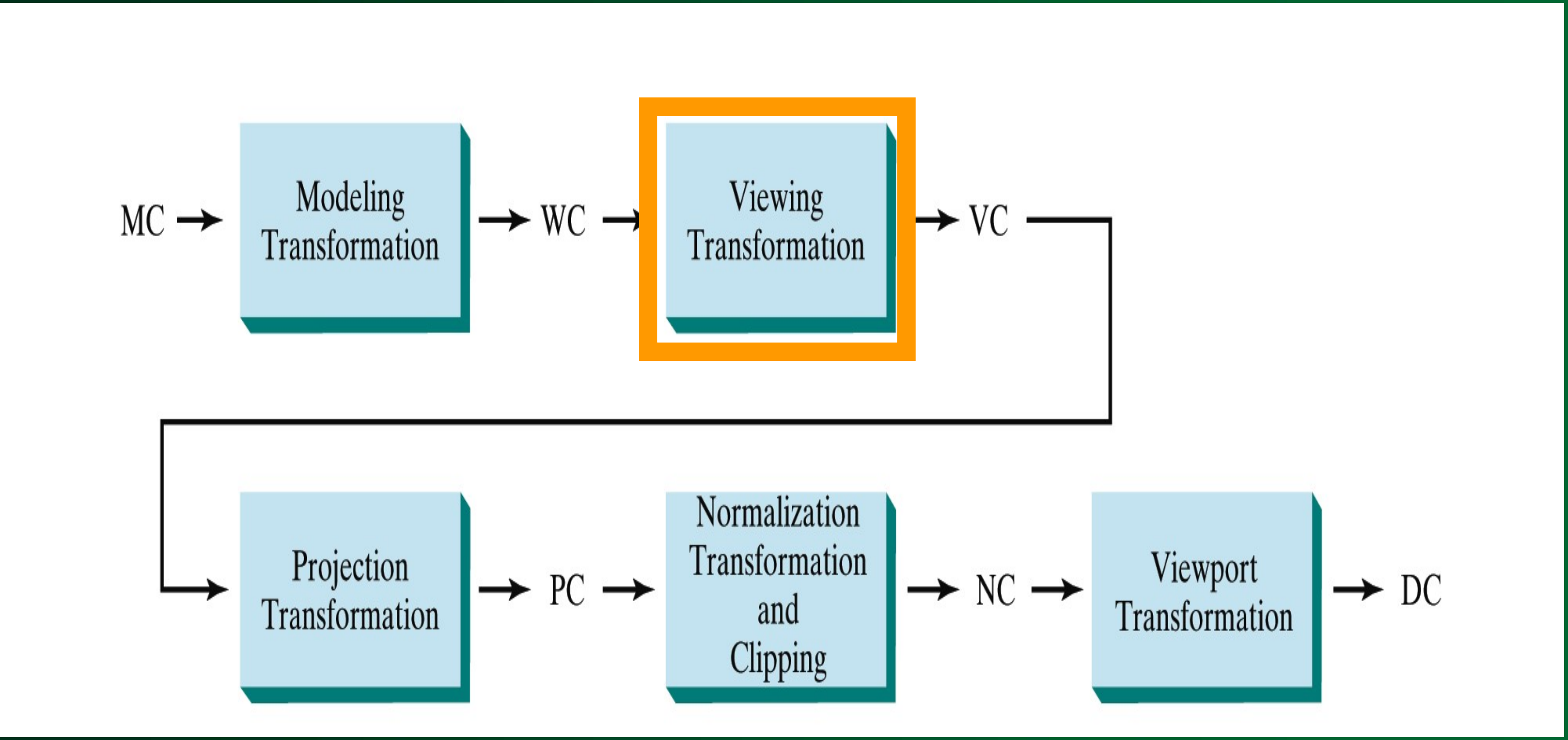
- To simulate camera motion through a scene, we can keep **N fixed** and **move** the view reference **point** around.



Transformation from World to Viewing Coordinates

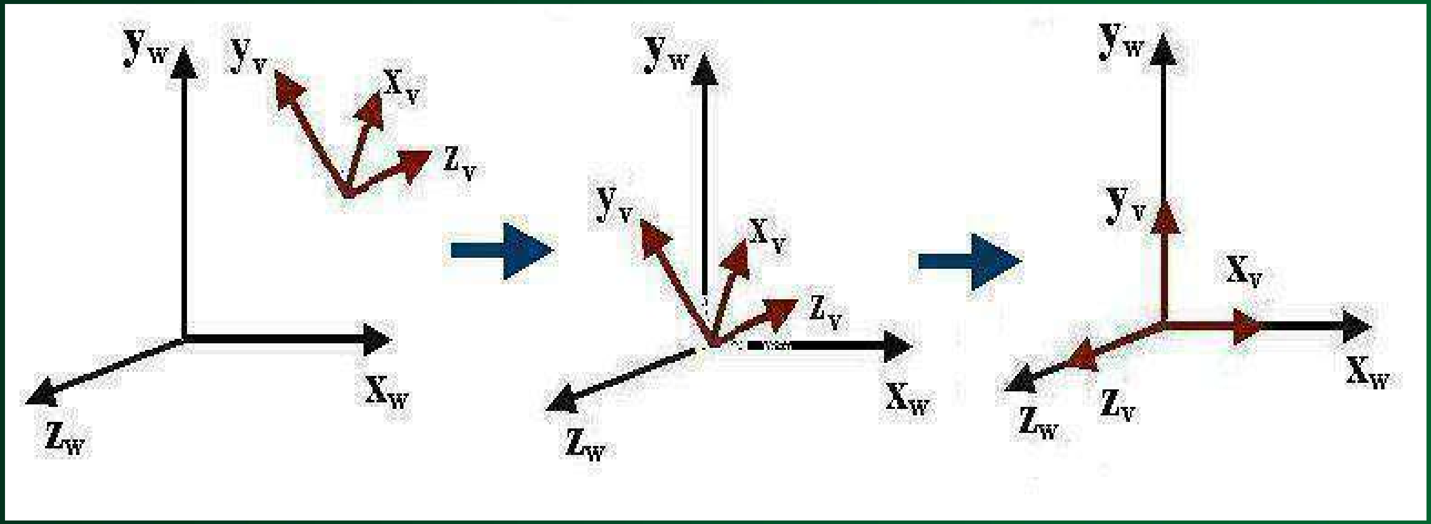
Viewing Pipeline

- Before object description can be projected to the view plane, they must be transferred to viewing coordinates.
- World coordinate positions are converted to viewing coordinates.



Transformation from World to Viewing Coordinates

- Transformation sequence from world to viewing coordinates:
- Translate the view reference point to the origin of the world-coordinate system.
- Apply rotations to align the x_v, y_v and z_v axes with the world x_w, y_w and z_w axes.

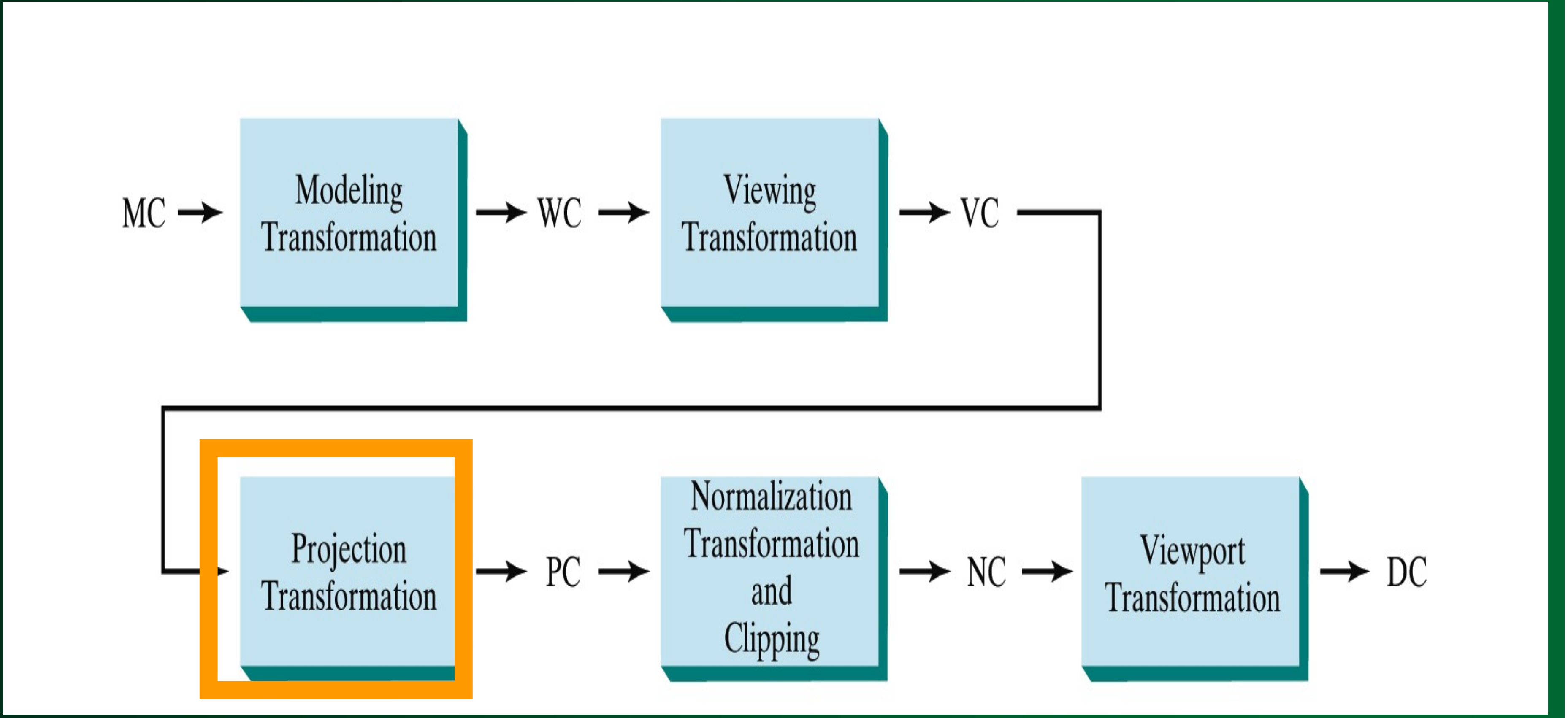


$$\mathbf{M}_{WC,VC} = \mathbf{R}_z \cdot R_y \cdot R_x . \mathbf{T}$$

Projection

Viewing Pipeline

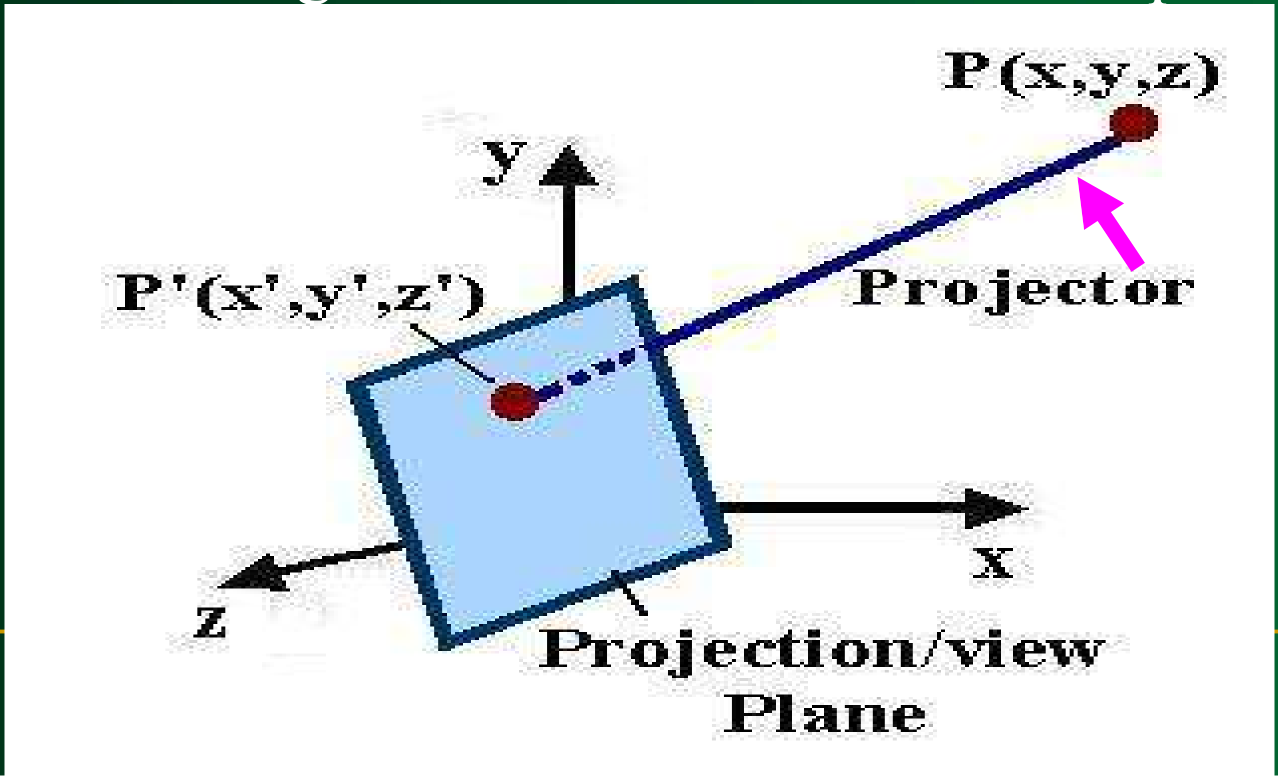
- Convert the viewing coordinate description of the scene to coordinate positions on the projection plane.
- Viewing 3D objects on a 2D display requires a mapping from 3D to 2D.



Projection

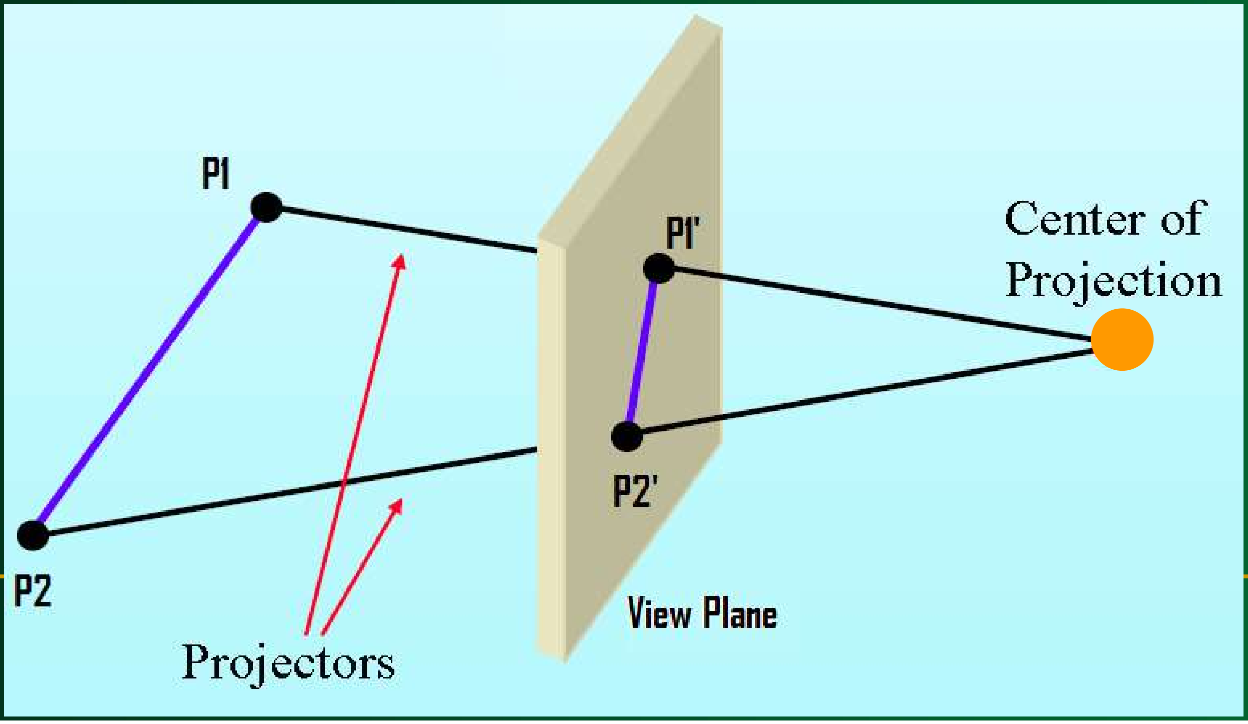
- **Projection** can be defined as a mapping of point P(x,y,z) onto its image $P'(x',y',z')$ in the projection plane.

- The mapping is determined by a ***projector*** that passes through P and intersects the view plane (P').

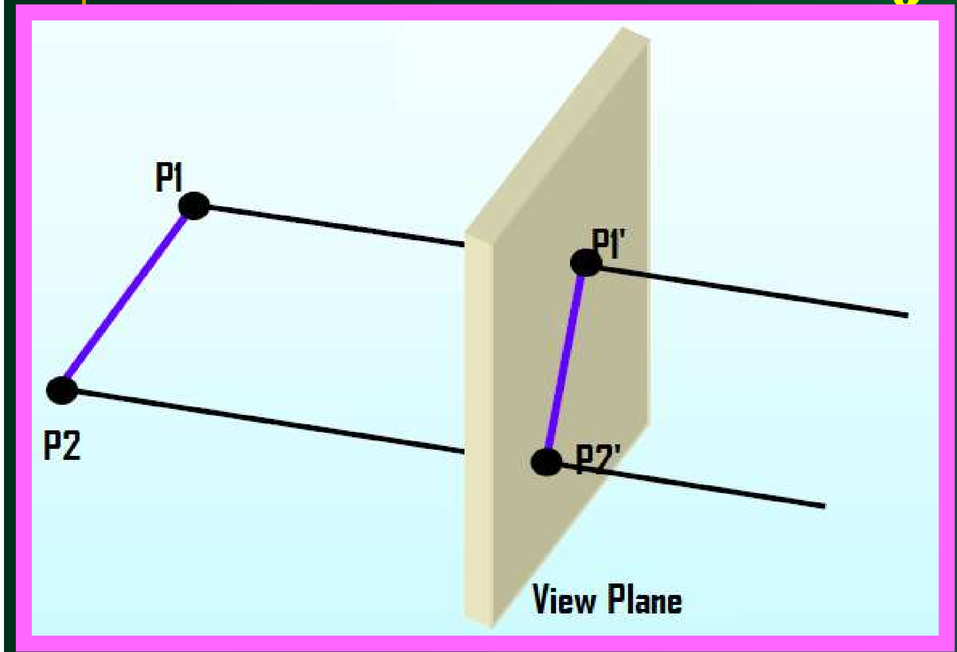


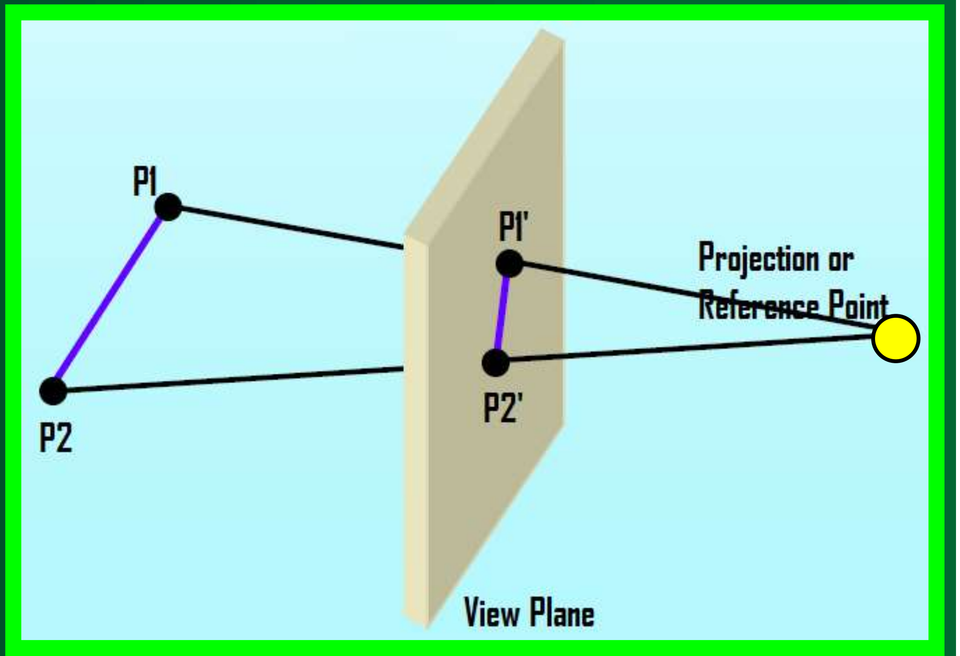
Projection

- Projectors are lines from **center (reference) of projection** through each point in the object.
- The projected view of the object is determined by calculating the intersection of projection lines with the view plane.



Projection



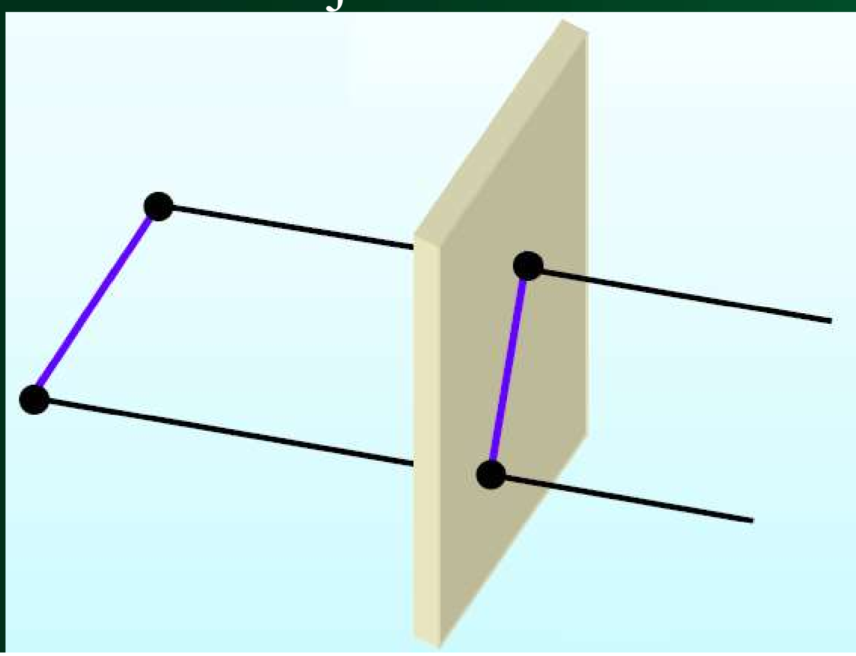


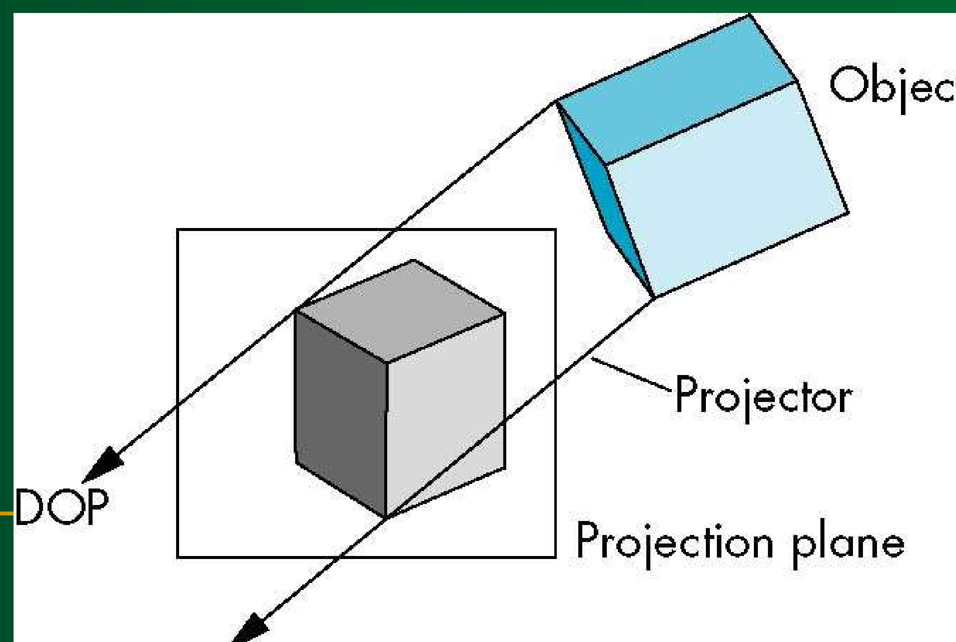
Parallel Projection : Coordinate positions are transformed to the view plane along **parallel lines**.

Perspective Projection: Object positions are transformed to the view plane along lines that converge to the **projection reference (center) point**.

Parallel Projection

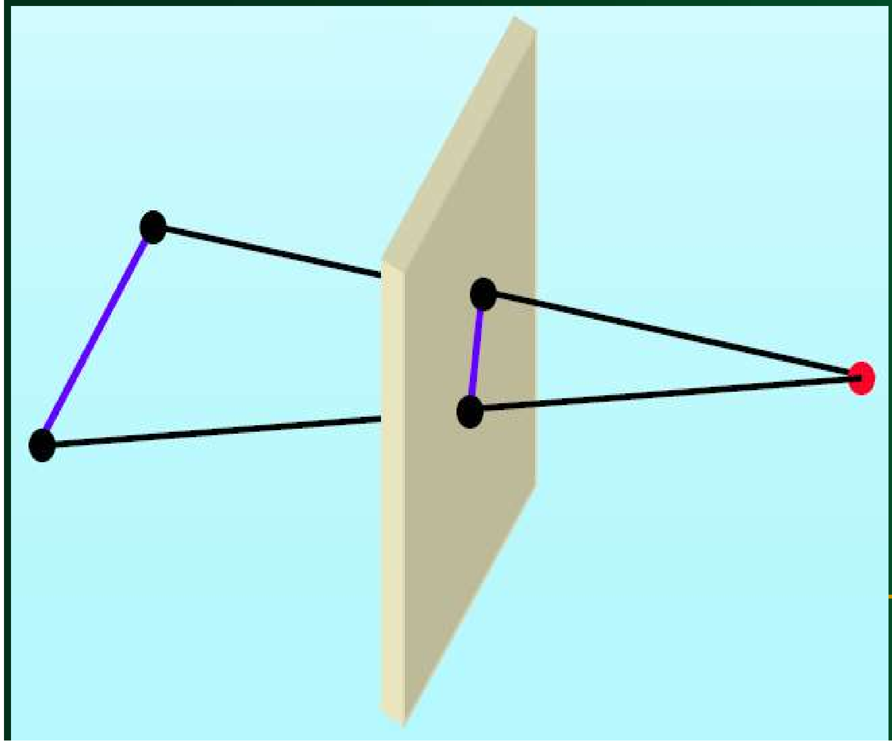
- Coordinate position are transformed to the view plane along parallel lines.
- Center of projection at infinity results with a parallel projection.
- A parallel projection preserves relative proportion of objects, but does not give us a realistic representation of the appearance of object.

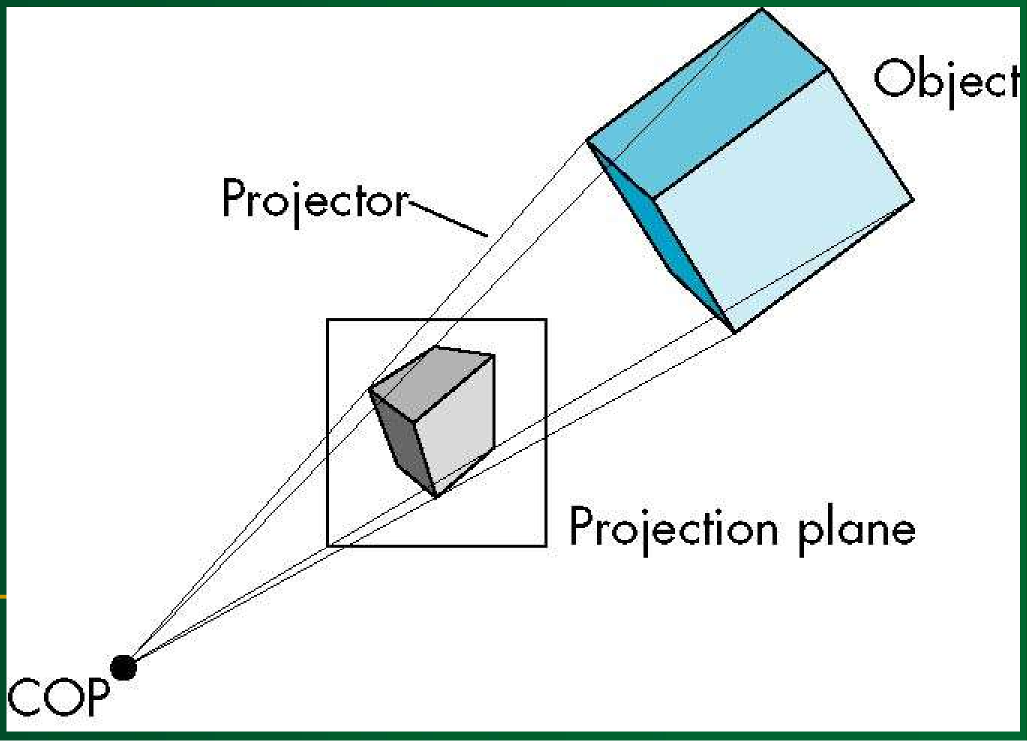




Perspective Projection

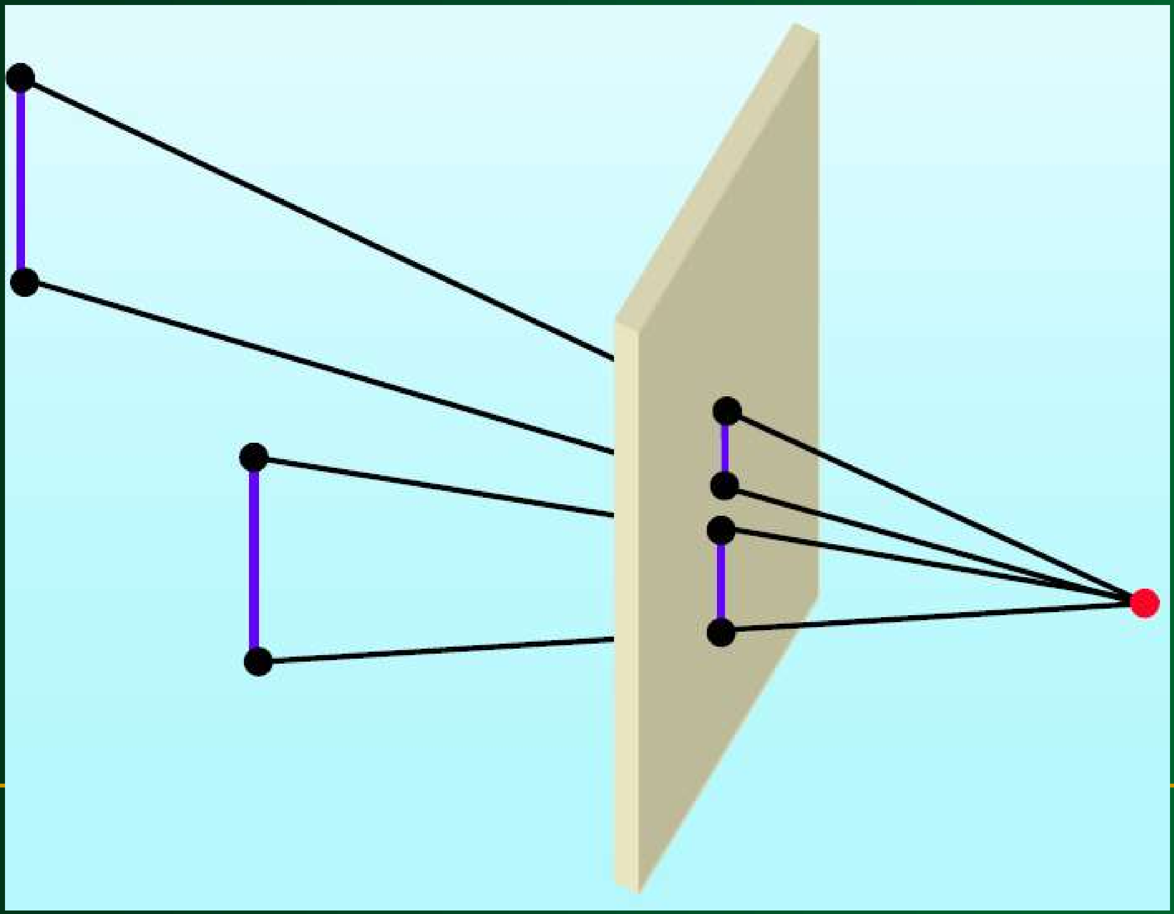
- Object positions are transformed to the view plane along lines that converge to the **projection reference (center) point.**
- Produces realistic views but does not preserve relative proportion of objects.



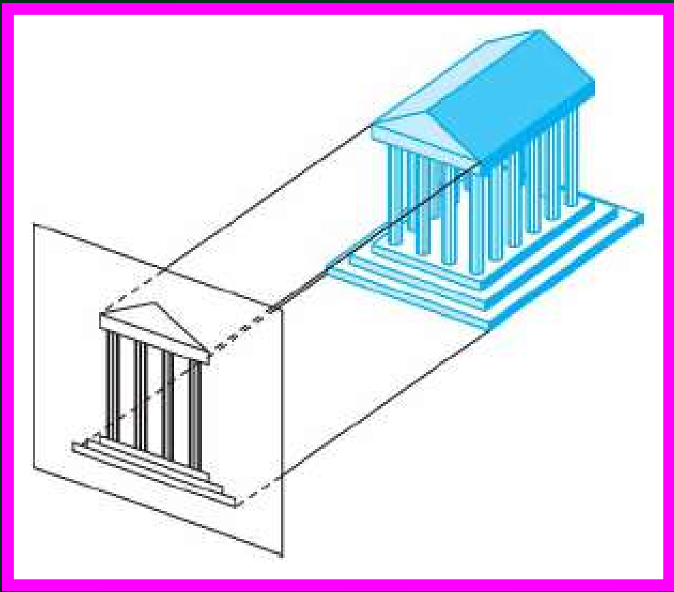


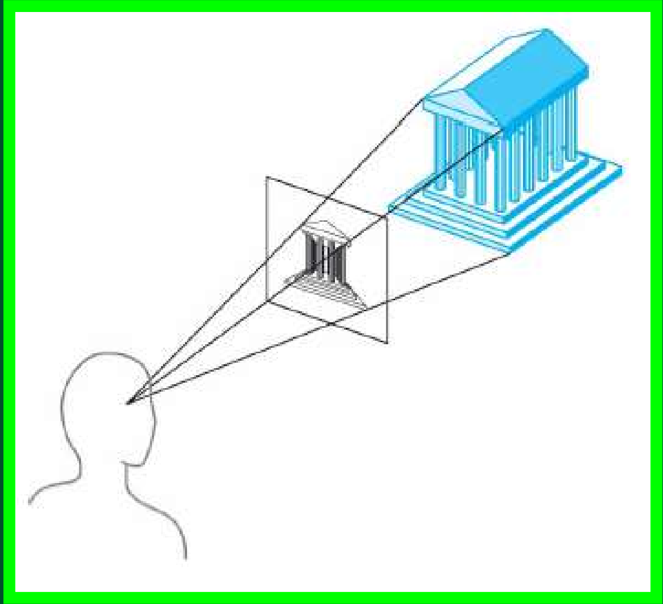
Perspective Projection

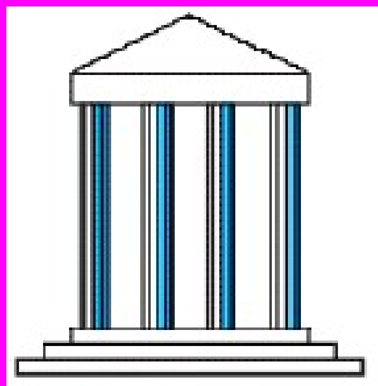
- Projections of distant objects are smaller than the projections of objects of the same size that are closer to the projection plane.



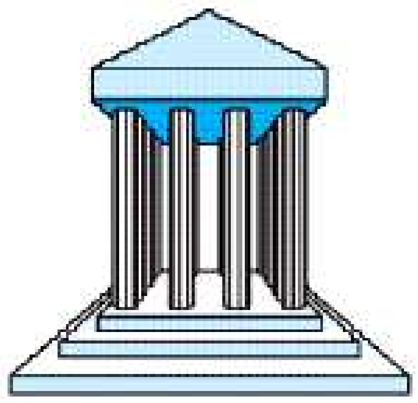
Parallel and Perspective Projection







parallel
(orthographic)

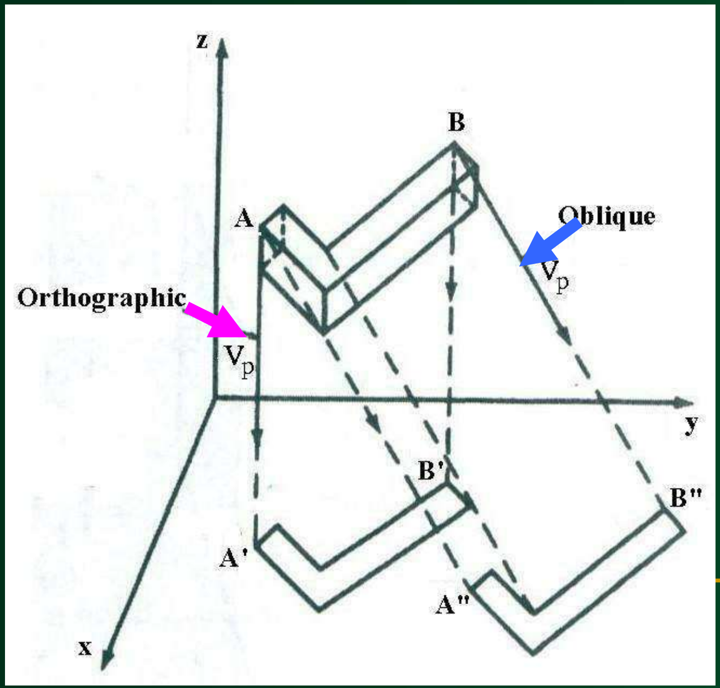


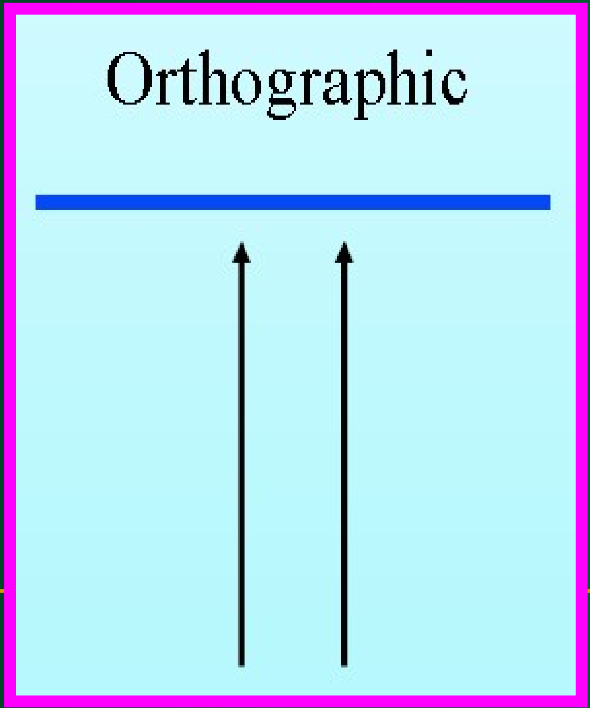
perspective

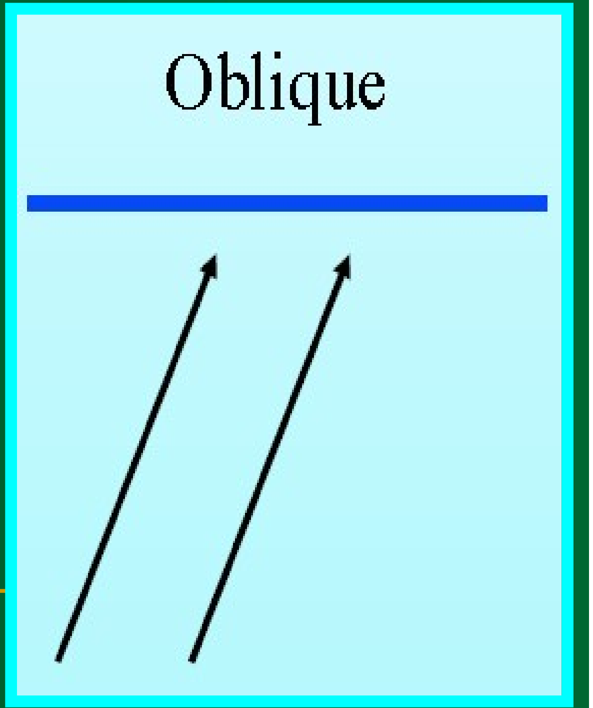
Parallel Projection

Parallel Projection

- **Projection vector**: Defines the direction for the projection lines (projectors).
- ***Orthographic Projection***: Projectors (projection vectors) are **perpendicular** to the projection plane.
- ***Oblique Projection***: Projectors (projection vectors) are ***not*** perpendicular to the projection plane.





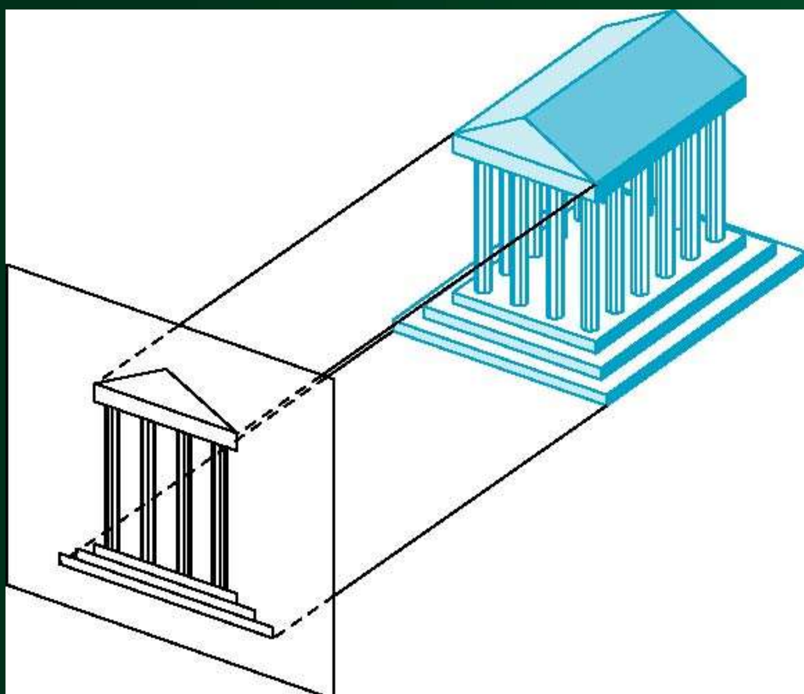


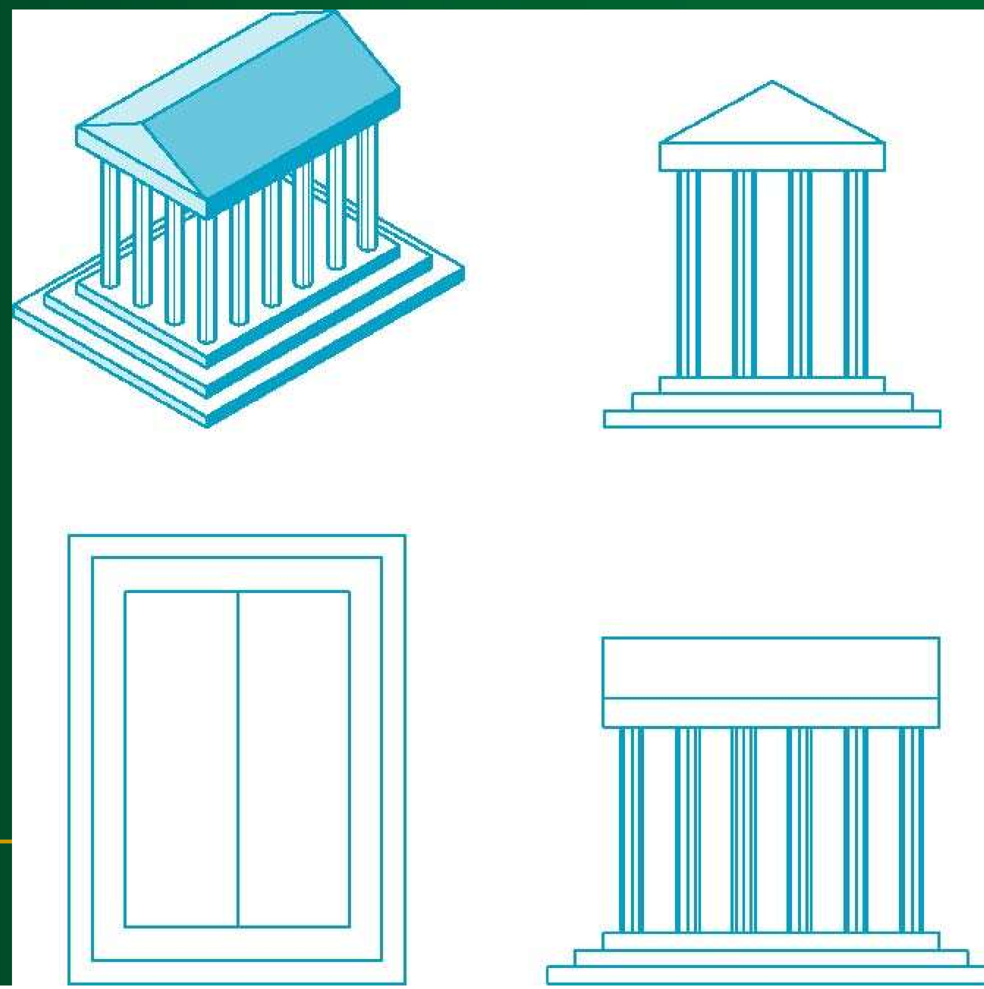
Orthographic

Parallel Projection

Orthographic Parallel Projection

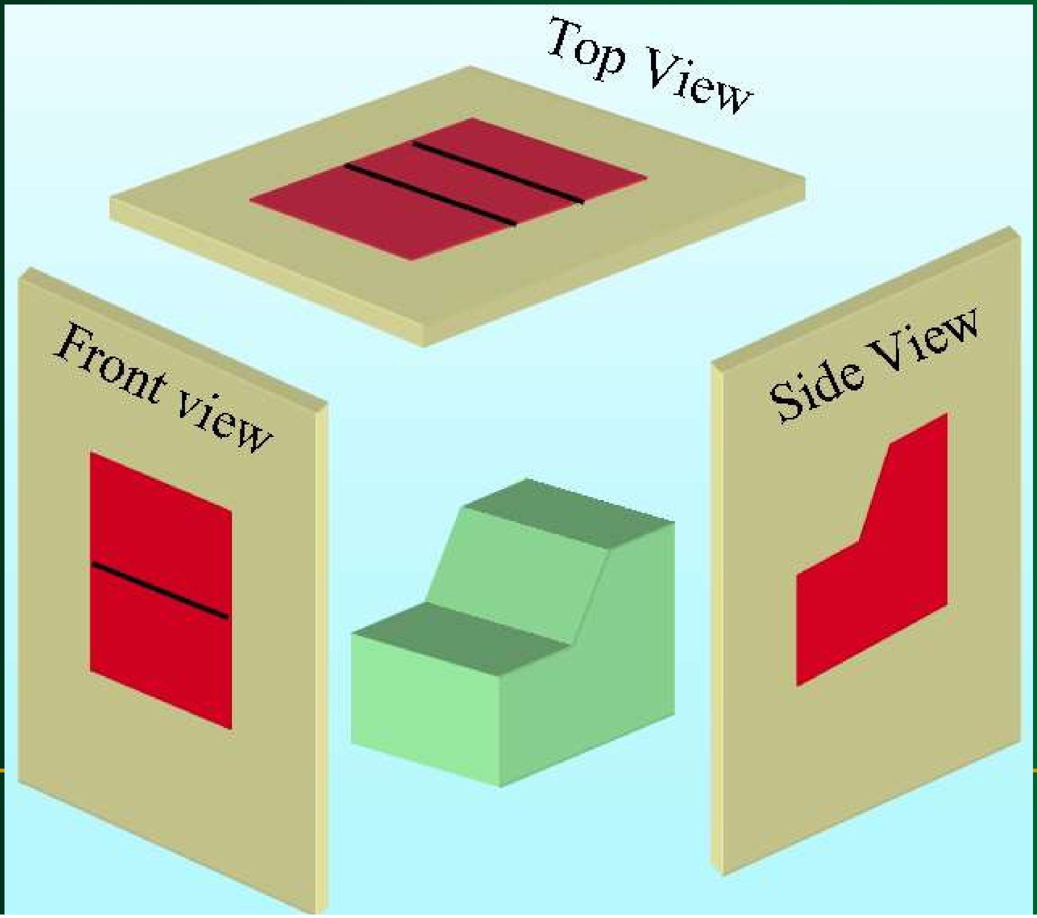
- Orthographic projection used to produce the **front**, **side**, and **top** views of an object.
- Engineering and architectural drawings employ orthographic projections.



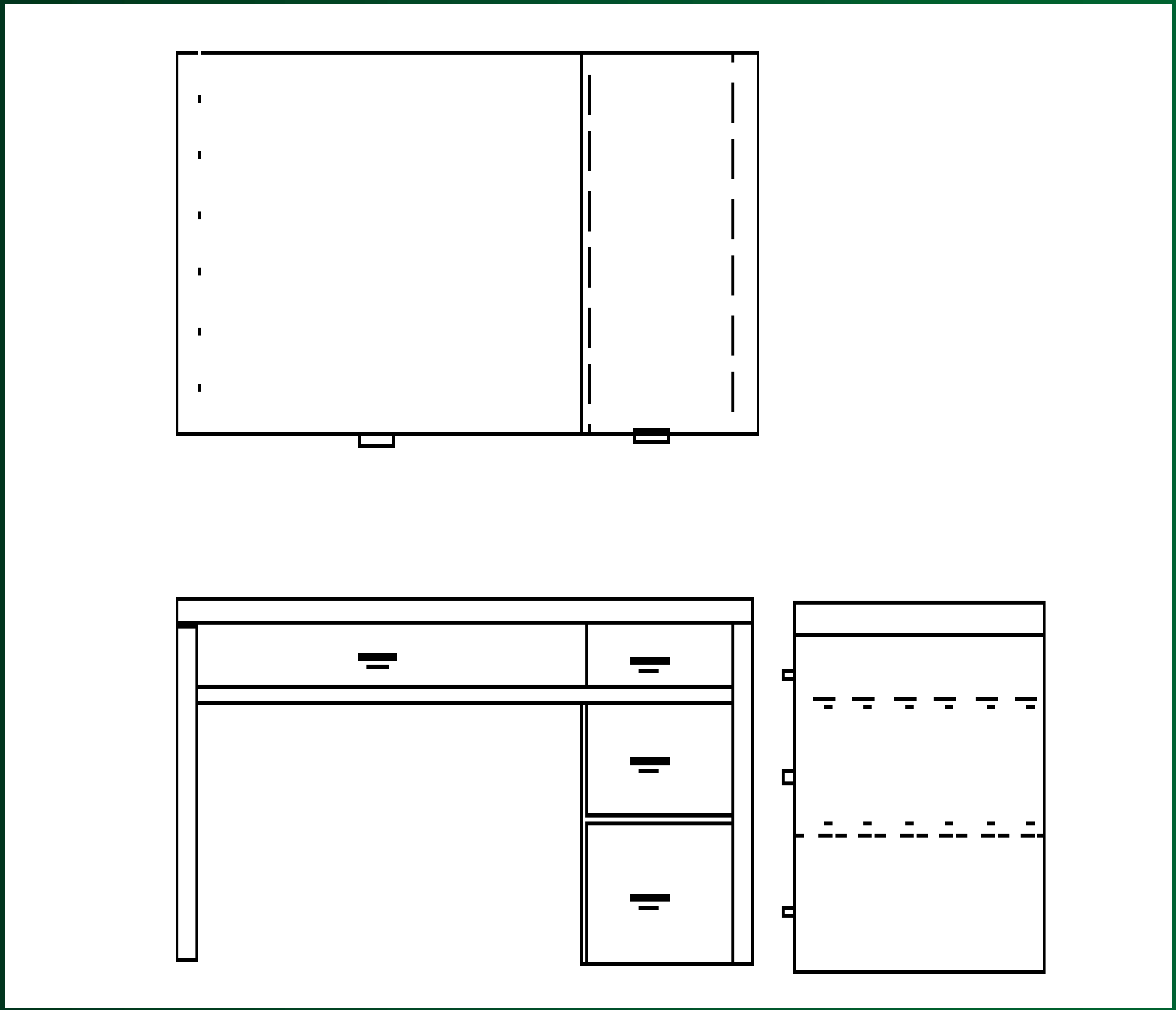


Orthographic Parallel Projection

- ***Front***, ***side***, and ***rear*** orthographic projections of an object are called ***elevations***.
- ***Top*** orthographic projection is called a ***plan*** view.



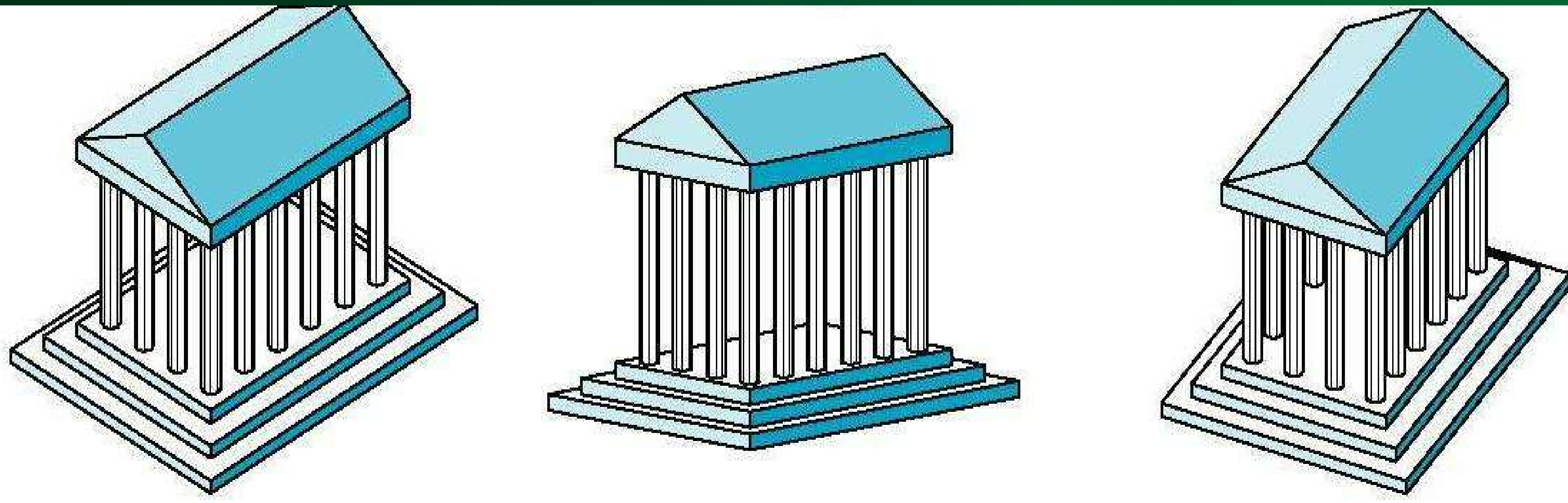
Orthographic Parallel Projection



Multi View Orthographic

Orthographic Parallel Projection

- ***Axonometric orthographic*** projections display more than one face of an object.

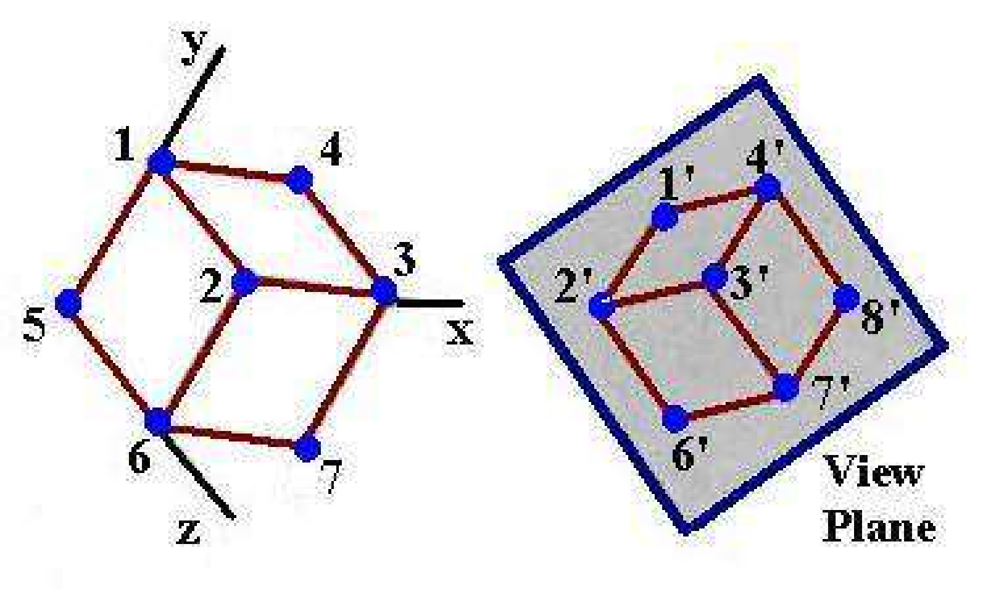


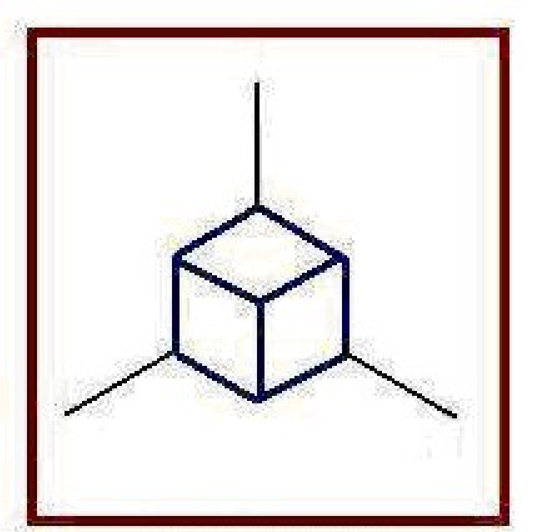
Isometric Dimetric Trimetric

Orthographic Parallel Projection

- ***Isometric Projection***: Projection plane intersects each coordinate axis in which the object is defined (principal axes) at the same distant from the origin.

- Projection vector makes equal angles with all of the **three principal axes**.



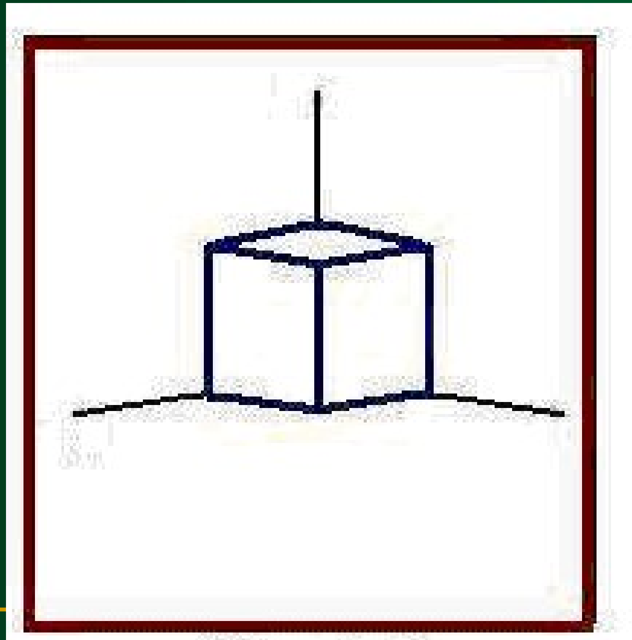


Isometric

Isometric projection is obtained by **aligning** the **projection vector** with the **cube diagonal**.

Orthographic Parallel Projection

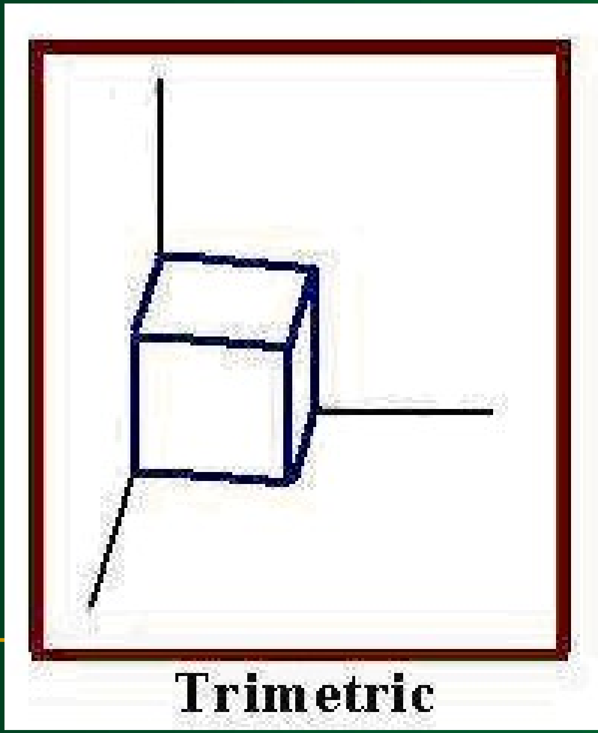
- ***Dimetric Projection***: Projection vector makes **equal angles** with exactly **two** of the principal axes.



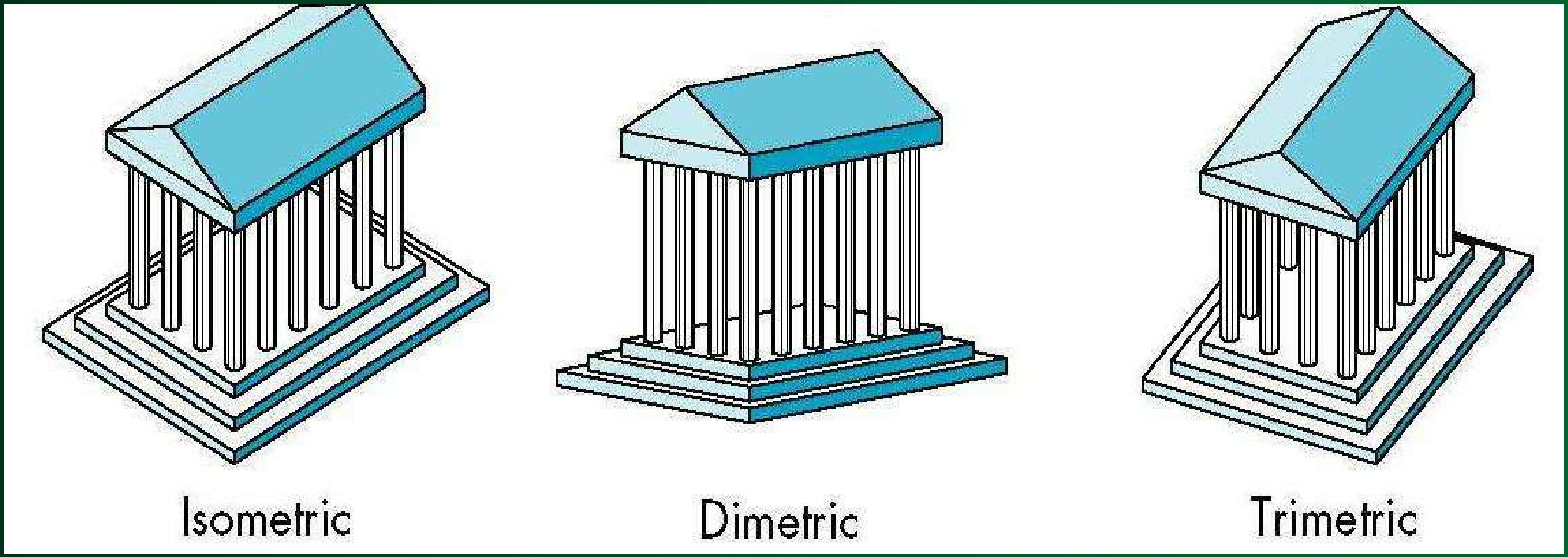
Dimetric

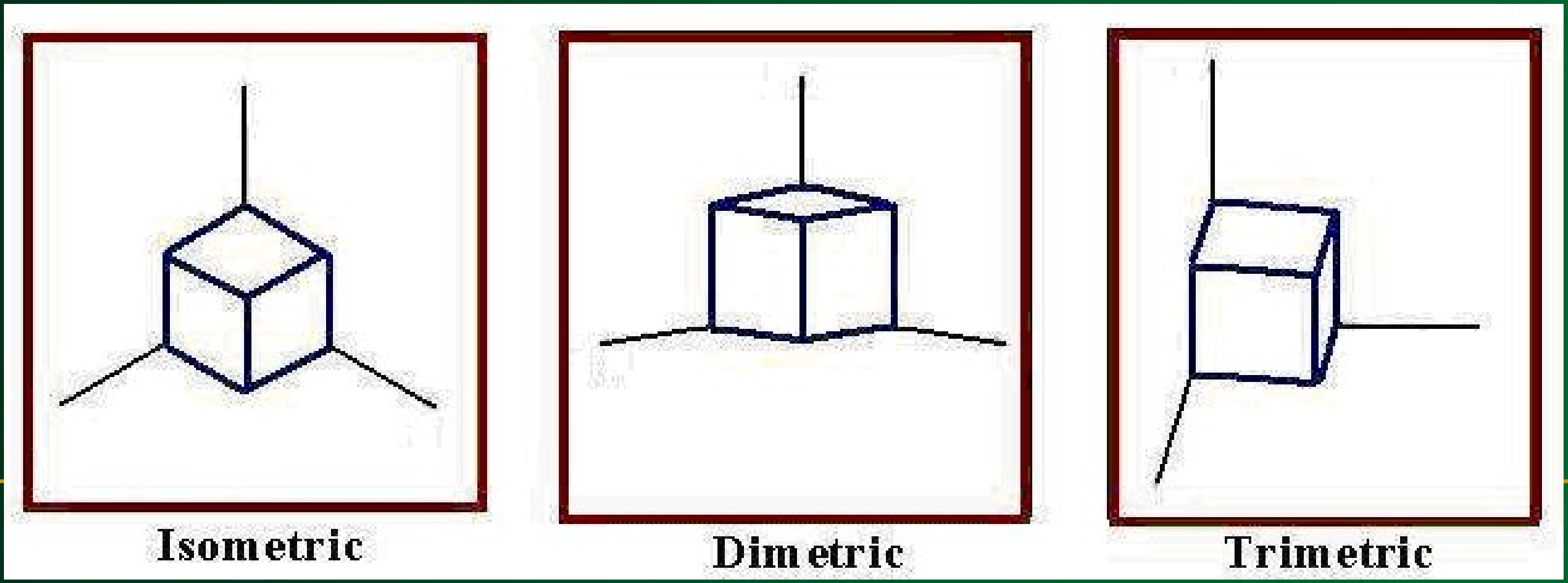
Orthographic Parallel Projection

- ***Trimetric Projection***: Projection vector makes **unequal angles** with the three principal axes.



Orthographic Parallel Projection



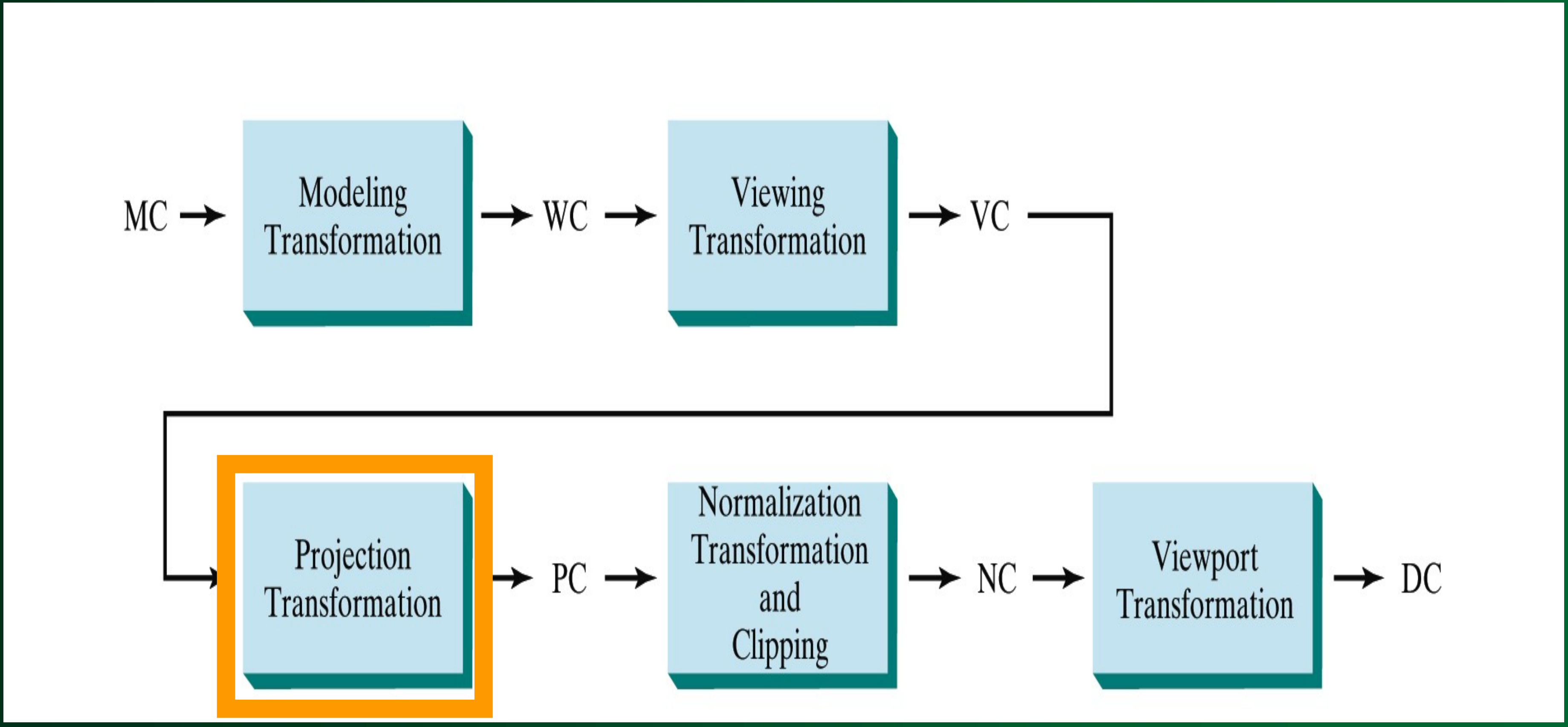


Orthographic Parallel Projection

Transformation

Orthographic Parallel Projection Transformation

- Convert the **viewing coordinate** description of the scene to coordinate positions on the **Orthographic parallel projection plane**.



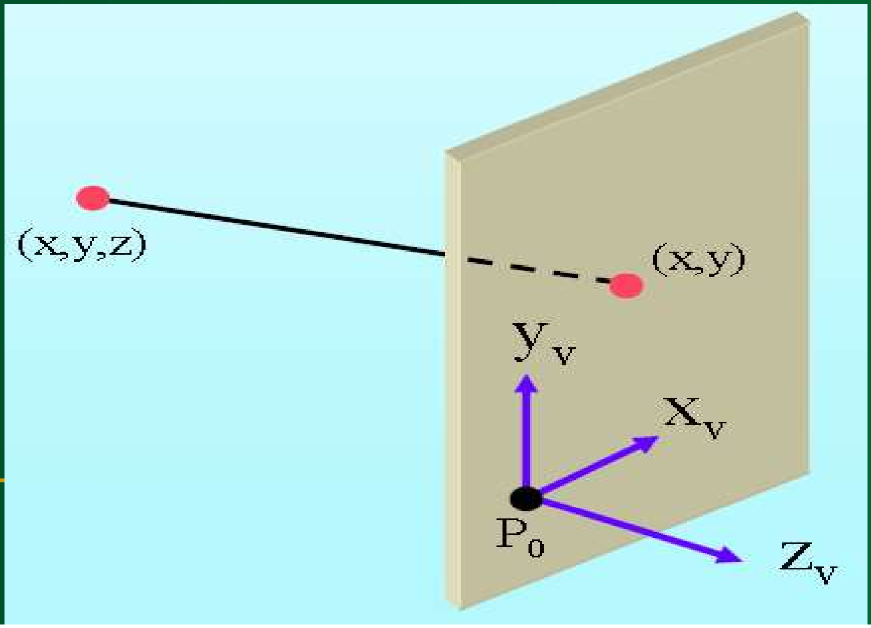
Orthographic Parallel Projection Transformation

- Since the view plane is placed at position z_{vp} along the z_v axis. Then any point (x,y,z) in viewing coordinates is transformed to projection coordinates as:

$$\boxed{x_p = x, \quad y_p = y}$$

The original z-coordinate value is preserved for the depth information.

$$\mathbf{M}_{Orthographic\ \ Parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

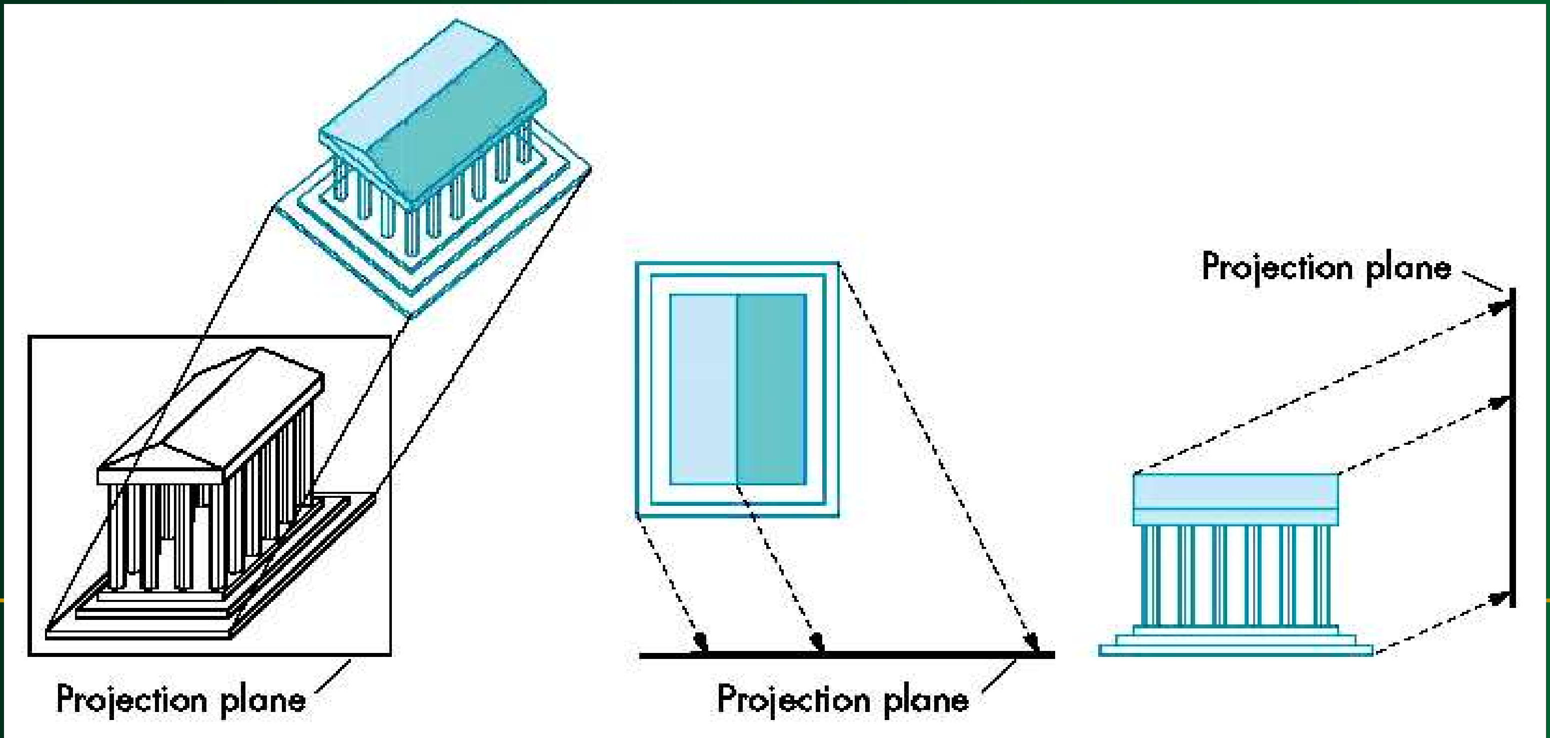


Oblique

Parallel Projection

Oblique Parallel Projection

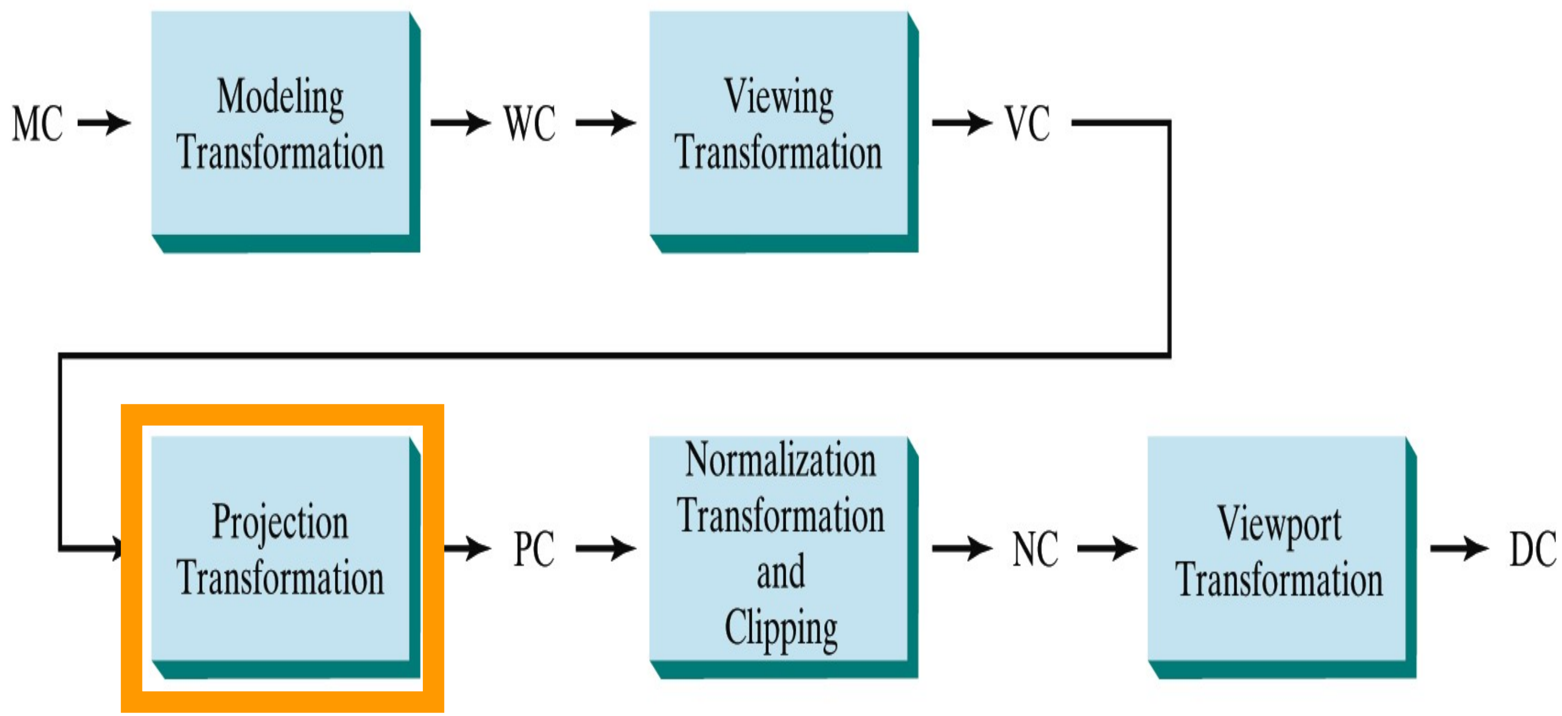
- Projections are **not** perpendicular to the viewing plane.
- Angles and lengths are preserved for faces parallel to the plane of projection.
- Preserves 3D nature of an object.



Oblique Parallel Projection Transformation

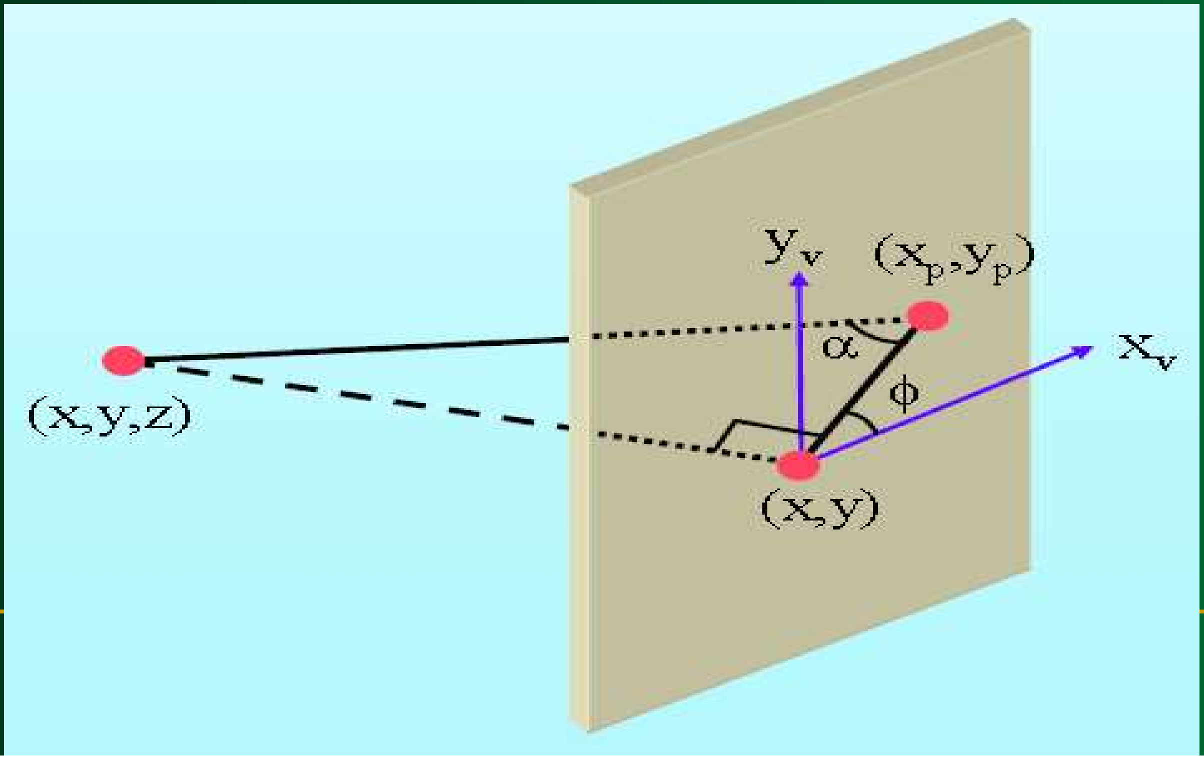
Oblique Parallel Projection Transformation

- Convert the **viewing coordinate** description of the scene to coordinate positions on the **Oblique parallel projection plane**.



Oblique Parallel Projection

- Point (x,y,z) is projected to position (x_p,y_p) on the view plane.
- Projector (oblique) from (x,y,z) to (x_p,y_p) makes an angle α with the line (of length L) on the projection plane that joins (x_p,y_p) and (x,y).
- Line L is at an angle ϕ with the horizontal direction in the projection plane.



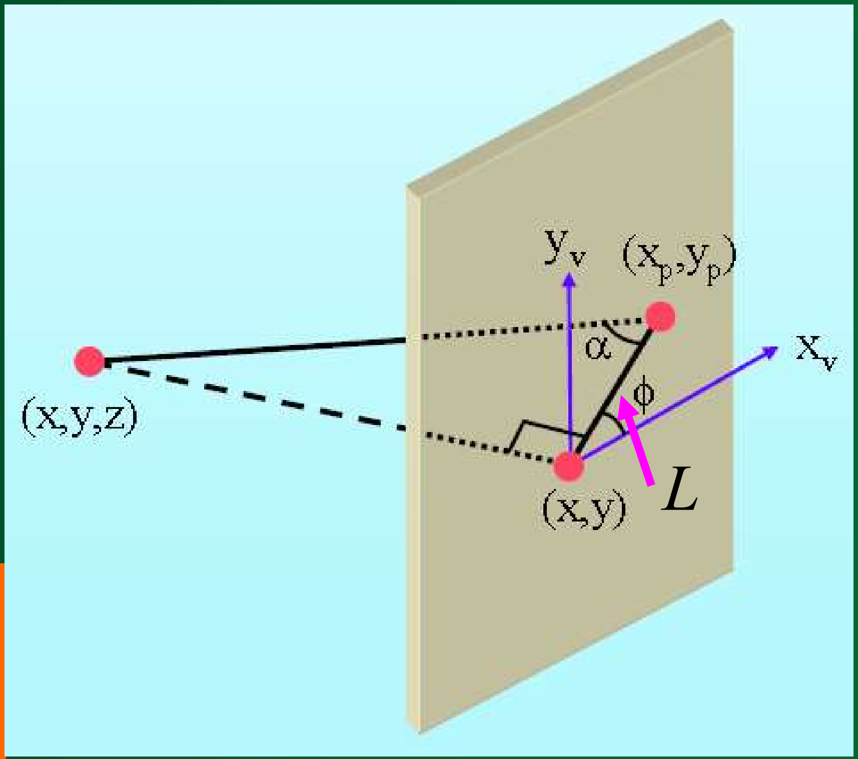
Oblique Parallel Projection

$$x_p = x + L\cos\varphi$$

$$y_p = y + L\sin\varphi$$

$$\tan\alpha = \frac{z}{L} \qquad L = \frac{z}{\tan\alpha} = zL_1$$

$$\boxed{\begin{aligned} x_p &= x + z(L_1\cos\varphi) \\ y_p &= y + z(L_1\sin\varphi) \end{aligned}}$$



$$\mathbf{M}_{Parallel} = \begin{bmatrix} 1 & 0 & L_1\cos\varphi & 0 \\ 0 & 1 & L_1\sin\varphi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

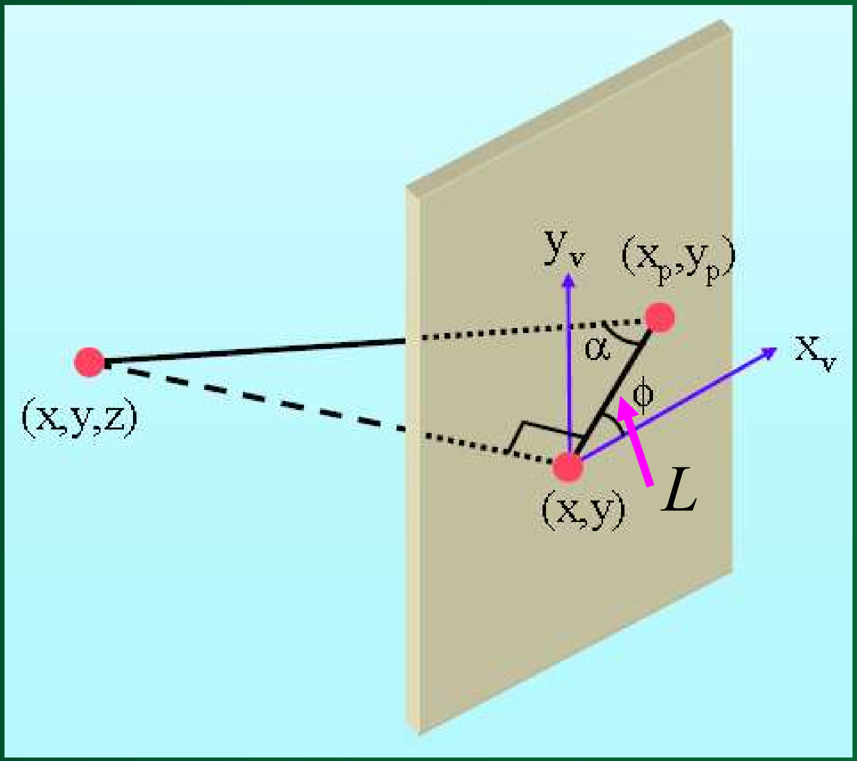
Oblique Parallel Projection

Orthographic Projection:

$$L_1 = 0$$

$$\alpha = 90^\circ$$

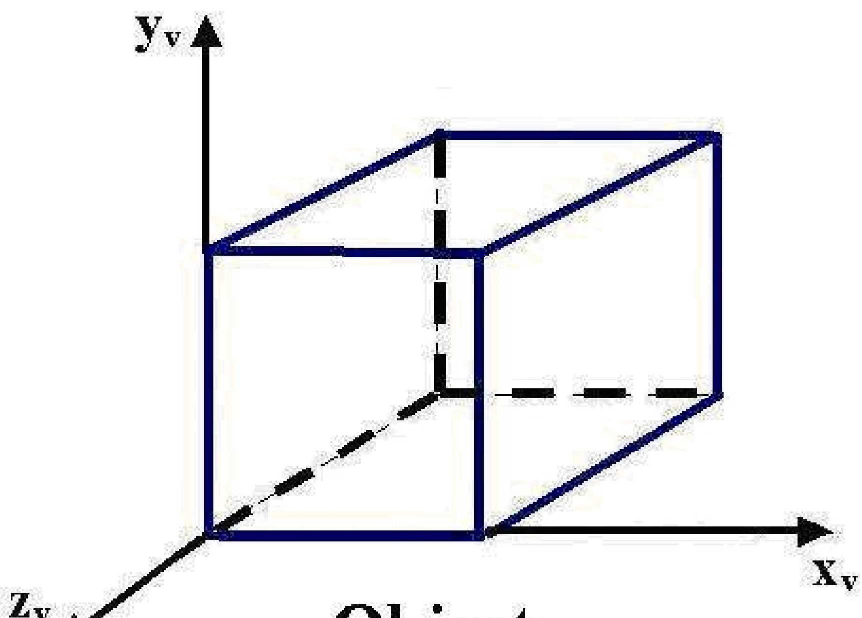
$$\boxed{x_p = x, \quad y_p = y}$$



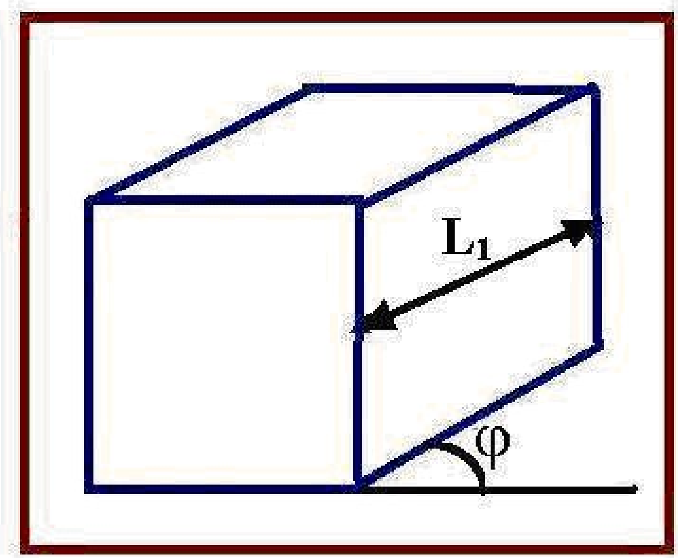
$$\mathbf{M}_{Orthographic\ \ Parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Oblique Parallel Projection

- **Angles**, **distances**, and **parallel lines** in the plane are projected accurately.



Object



Projection on the Viewing Plane

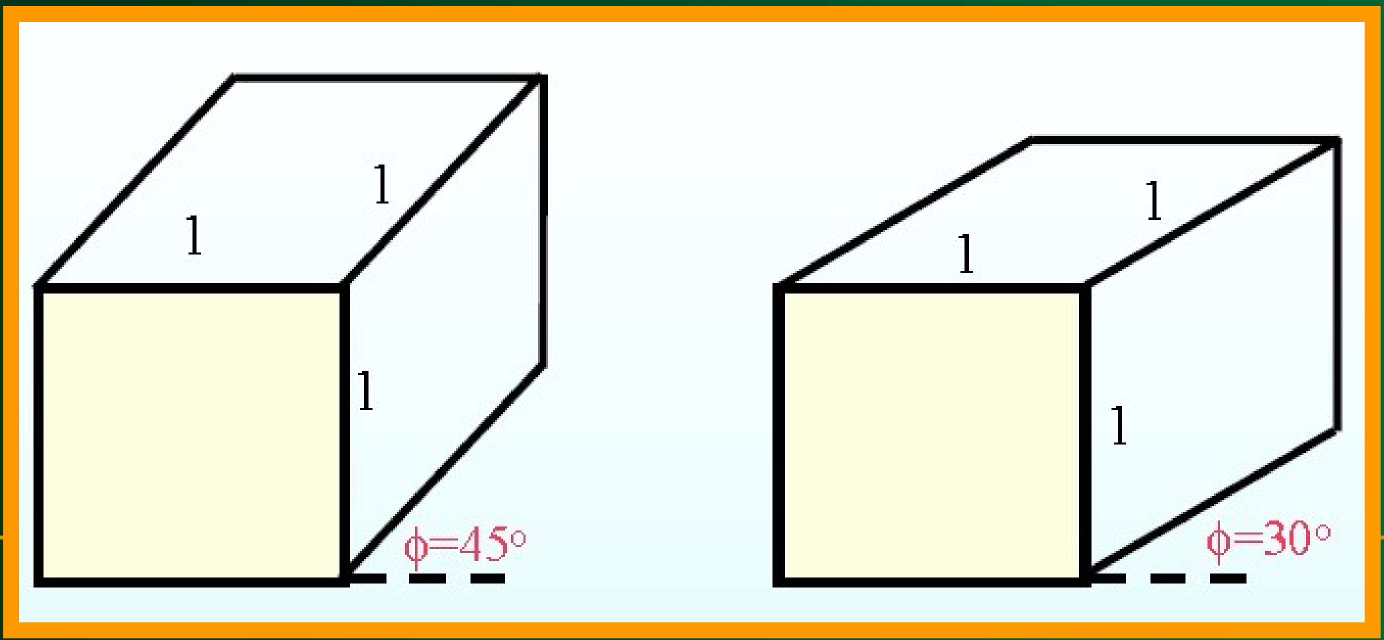
Cavalier Projection

Cavalier Projection:

$$\phi = 30^\circ \quad and \quad 45^\circ$$

$$\boxed{\begin{array}{l} \tan\alpha = 1 \\ \alpha = 45^\circ \end{array}}$$

- Preserves lengths of lines perpendicular to the viewing plane.
- 3D nature can be captured but shape seems distorted.
- Can display a combination of front, and side, and top views.



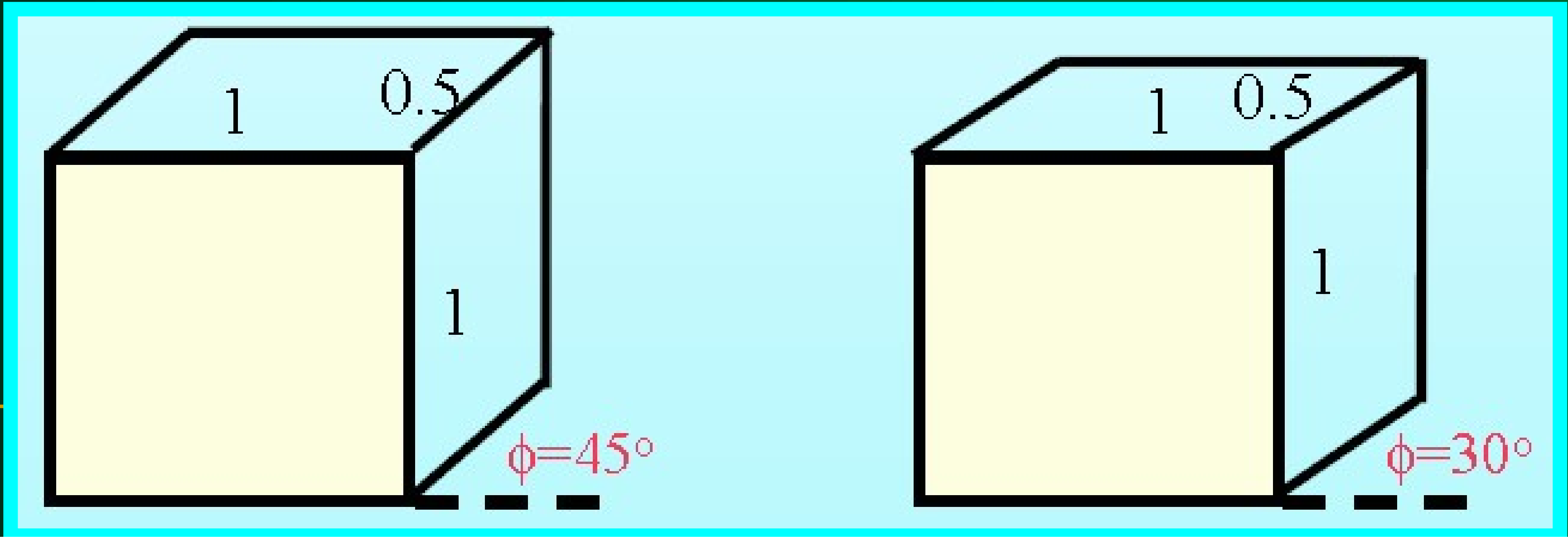
Cabinet Projection

Cabinet Projection:

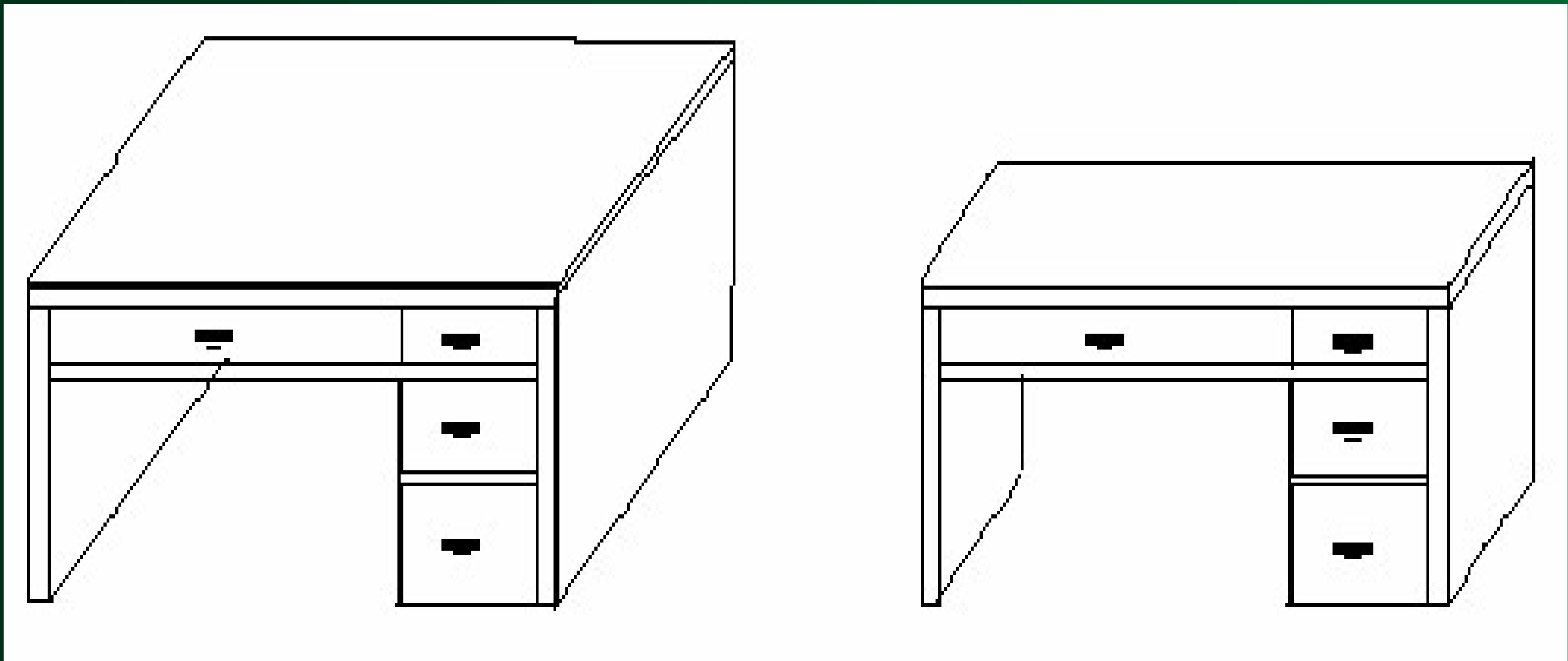
$\phi = 30^{\circ} \quad and \quad 45^{\circ}$

$$\tan\alpha = 2$$
$$\alpha \approx 63.4^{\circ}$$

- Lines perpendicular to the viewing plane are projected at ***½*** of their **length**.
- A more realistic view than the cavalier projection.
- Can display a combination of front, and side, and top views.



Cavalier & Cabinet Projection

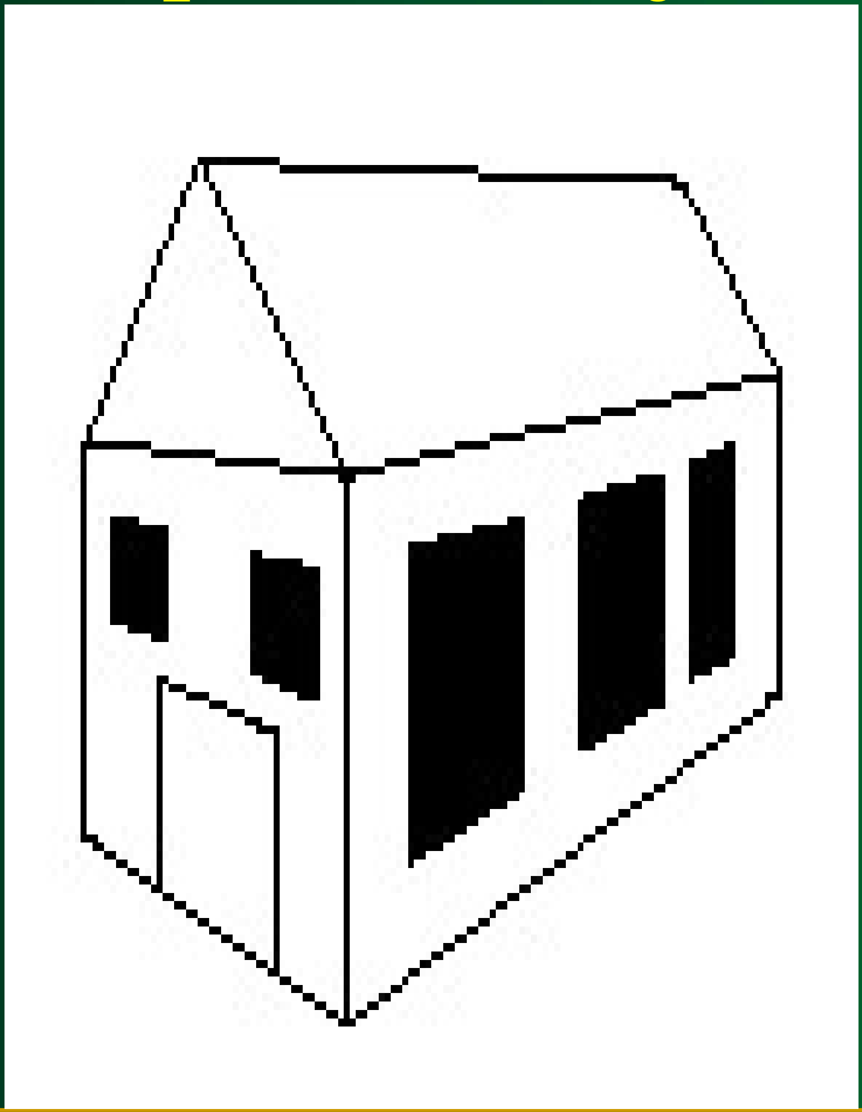


Cavalier

Cabinet

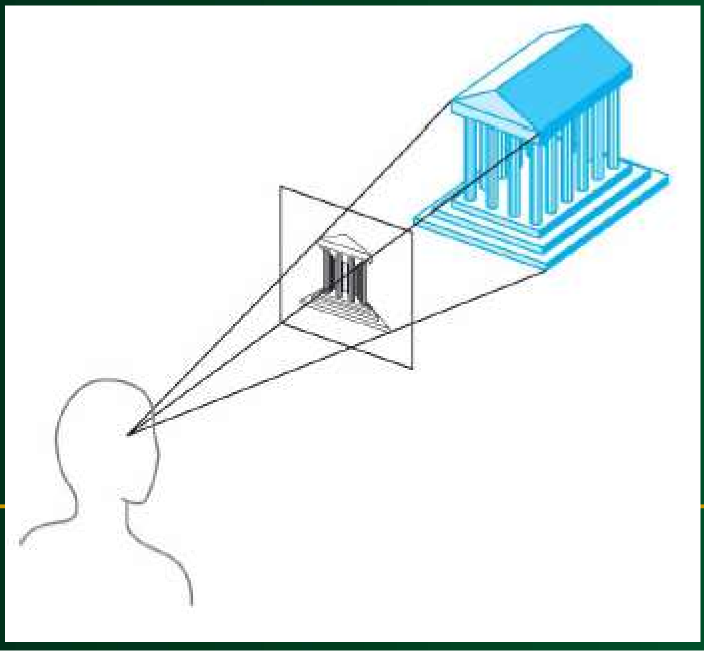
Perspective Projection

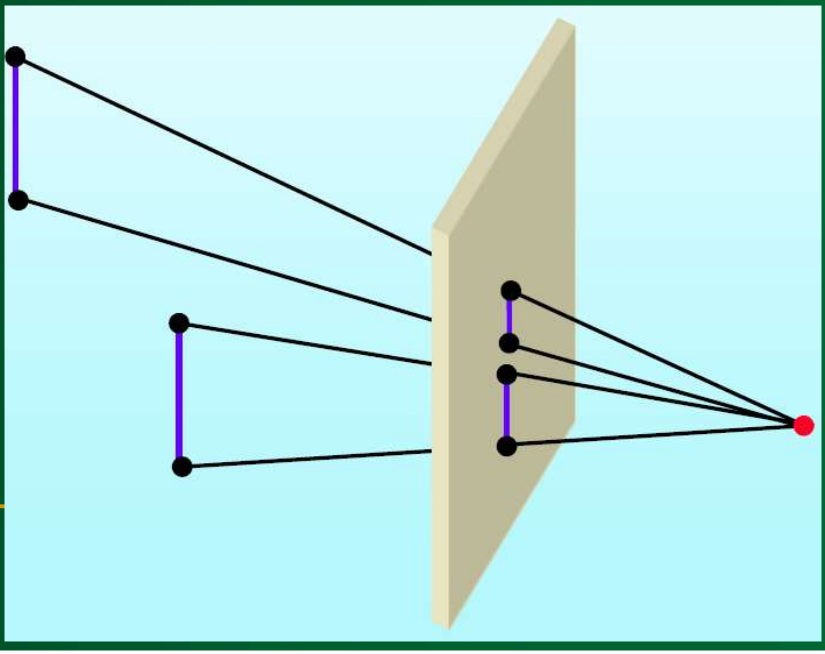
Perspective Projection



Perspective Projection

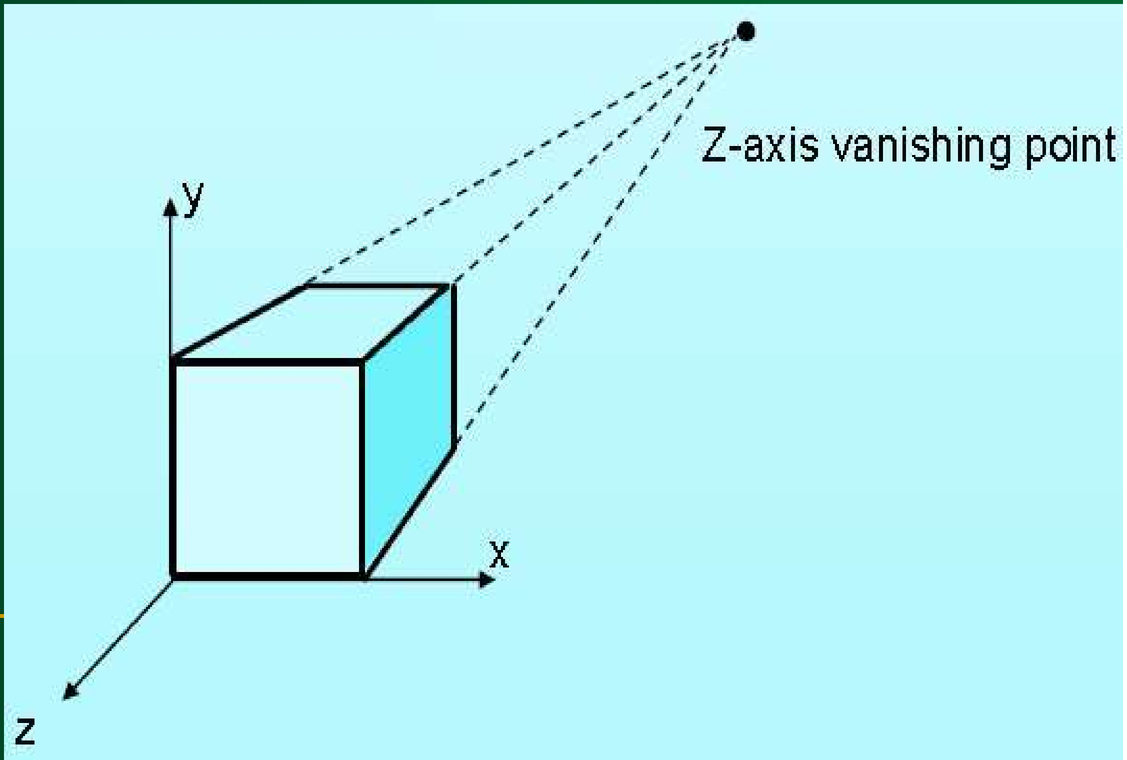
- In a perspective projection, the center of projection is at a finite distance from the viewing plane.
- Produces realistic views but does not preserve relative proportion of objects
- The size of a projection object is inversely proportional to its distance from the viewing plane.





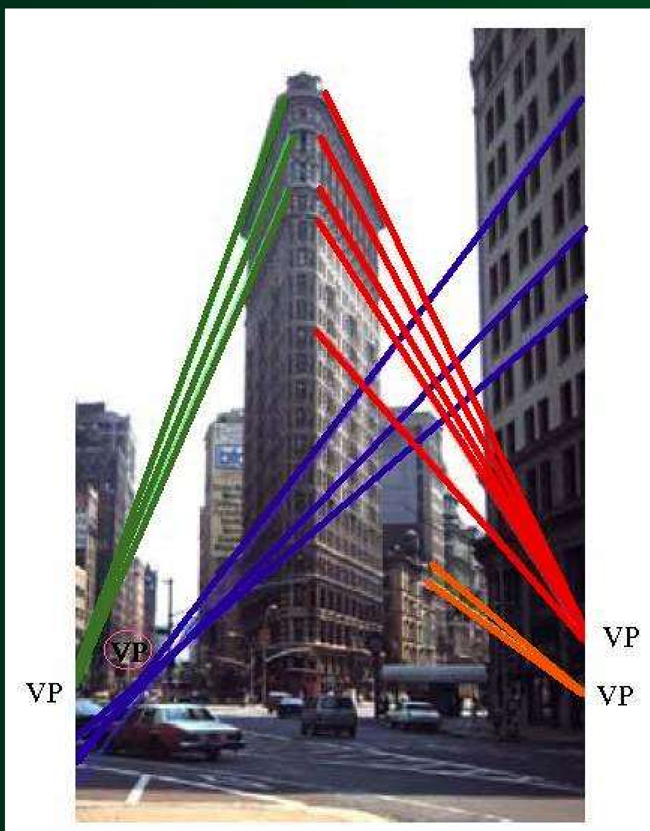
Perspective Projection

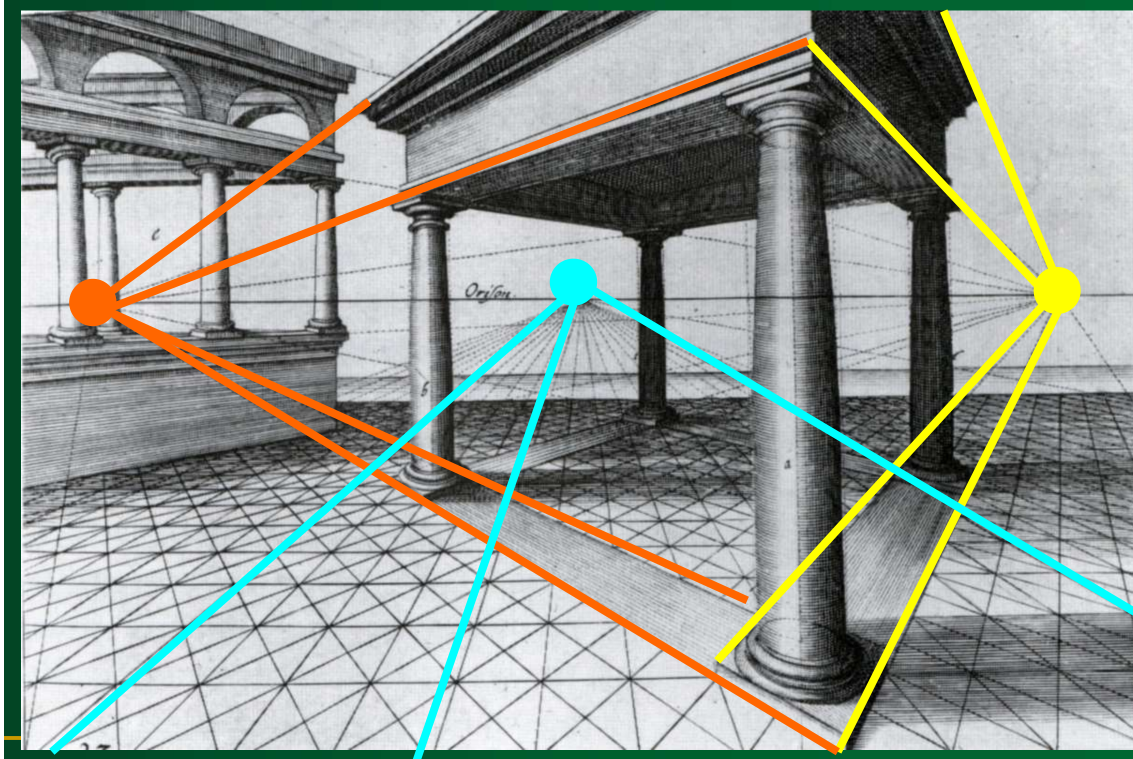
- Parallel lines that are not parallel to the viewing plane, **converge** to a ***vanishing point***.
- A vanishing point is the projection of a point at infinity.



Vanishing Points

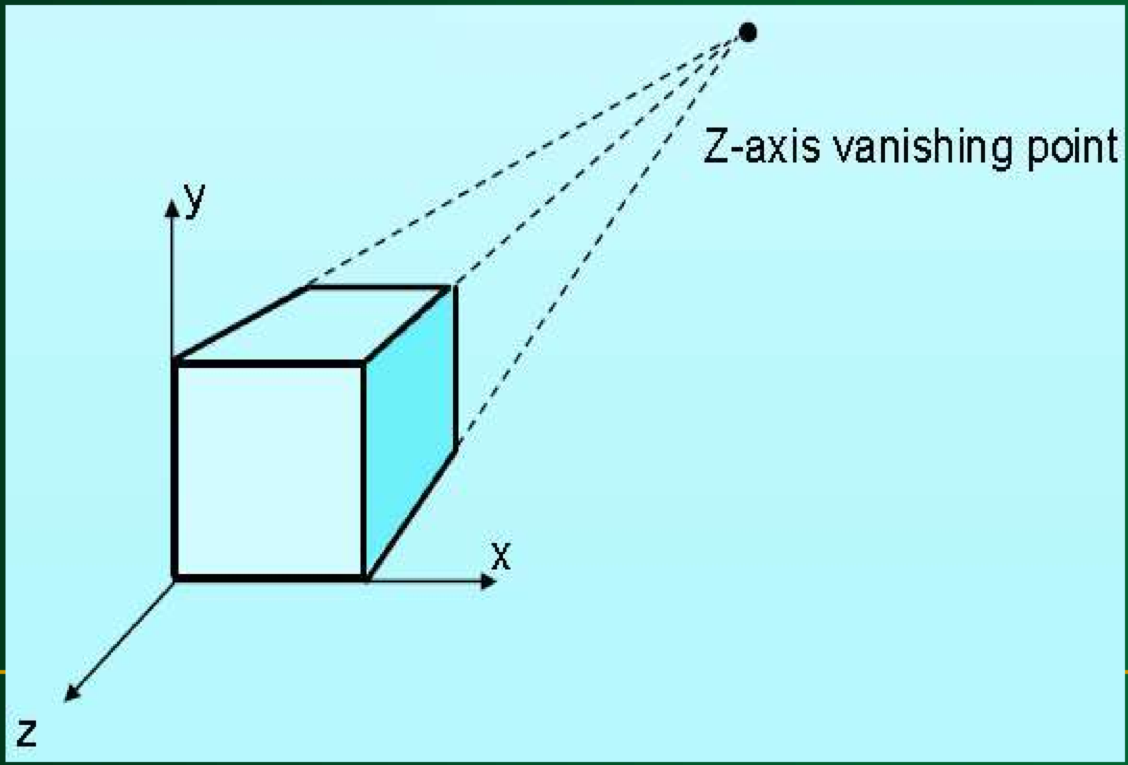
- Each set of projected parallel lines will have a separate vanishing points.





Perspective Projection

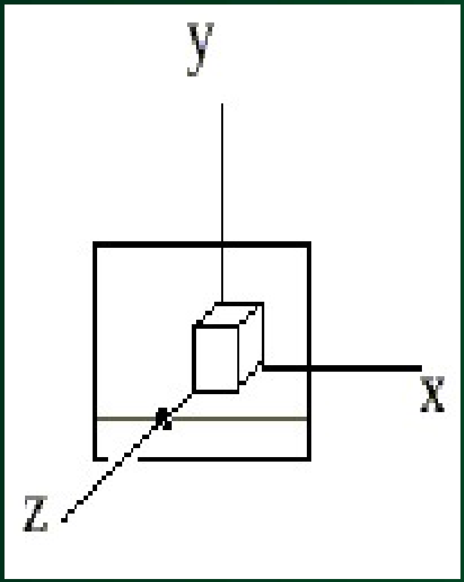
- The vanishing point for any set of lines that are parallel to one of the principal axes of an object is referred to as a **principal vanishing point**.
- We control the number of principal vanishing points (one, two, or three) with the orientation of the projection plane.

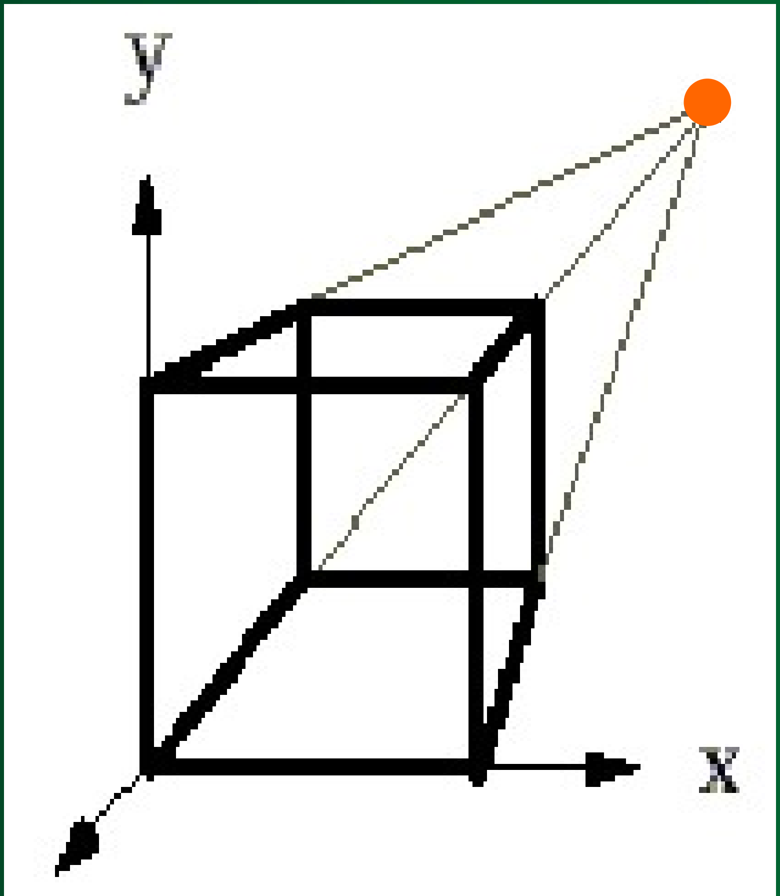


Perspective Projection

- The number of principal vanishing points in a projection is determined by the number of principal axes **intersecting** the view plane.

Perspective Projection

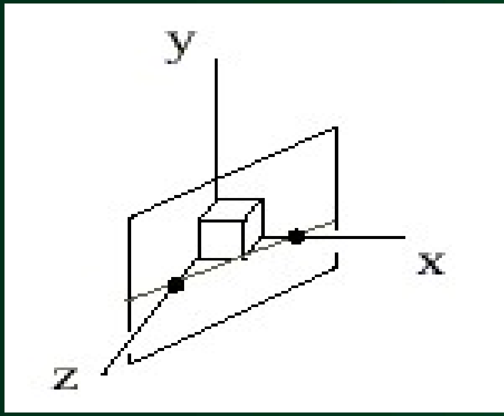


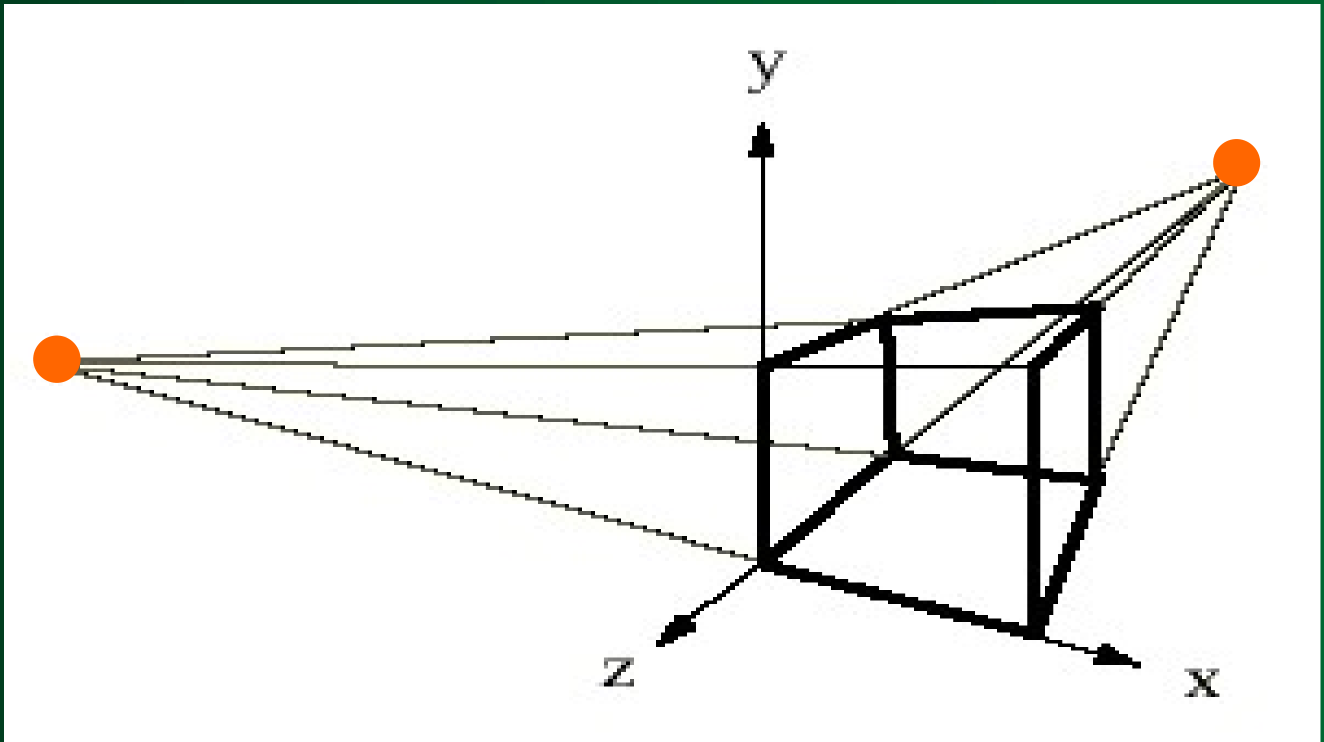


One Point Perspective

(*z*-axis vanishing point)

Perspective Projection

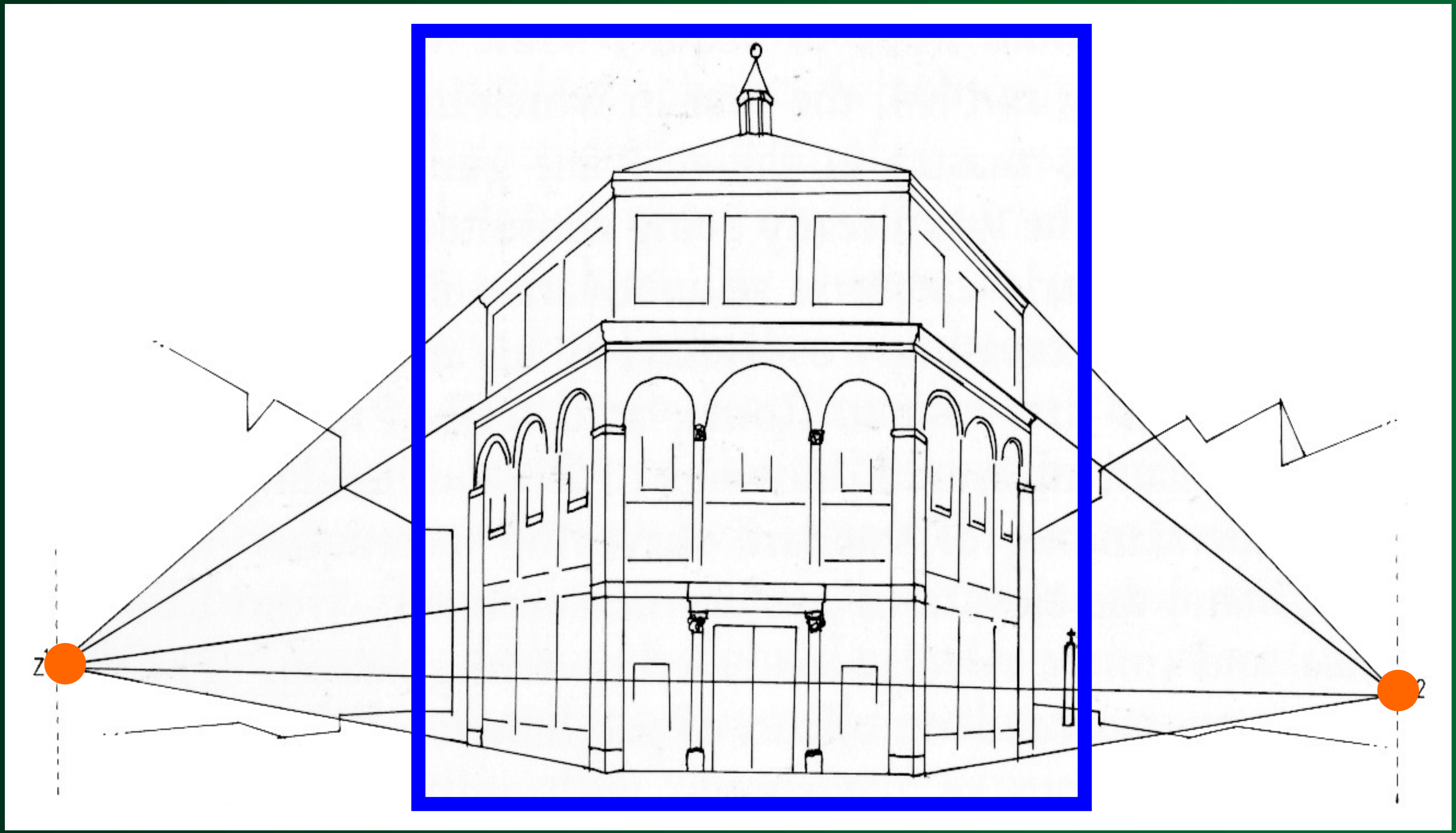




Two Point Perspective

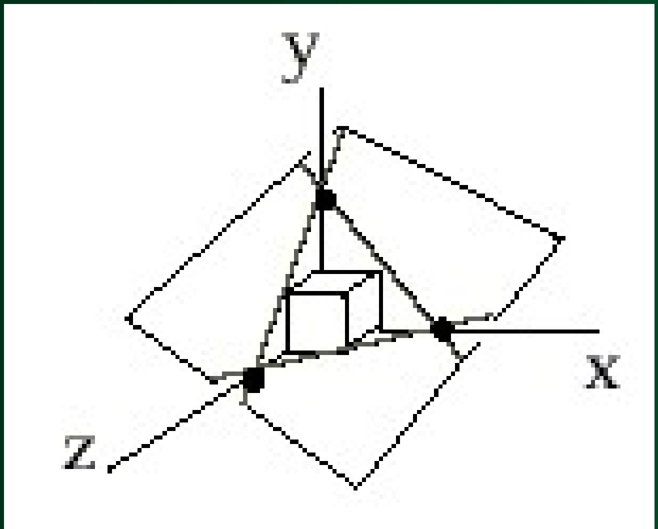
(z, and x-axis vanishing points)

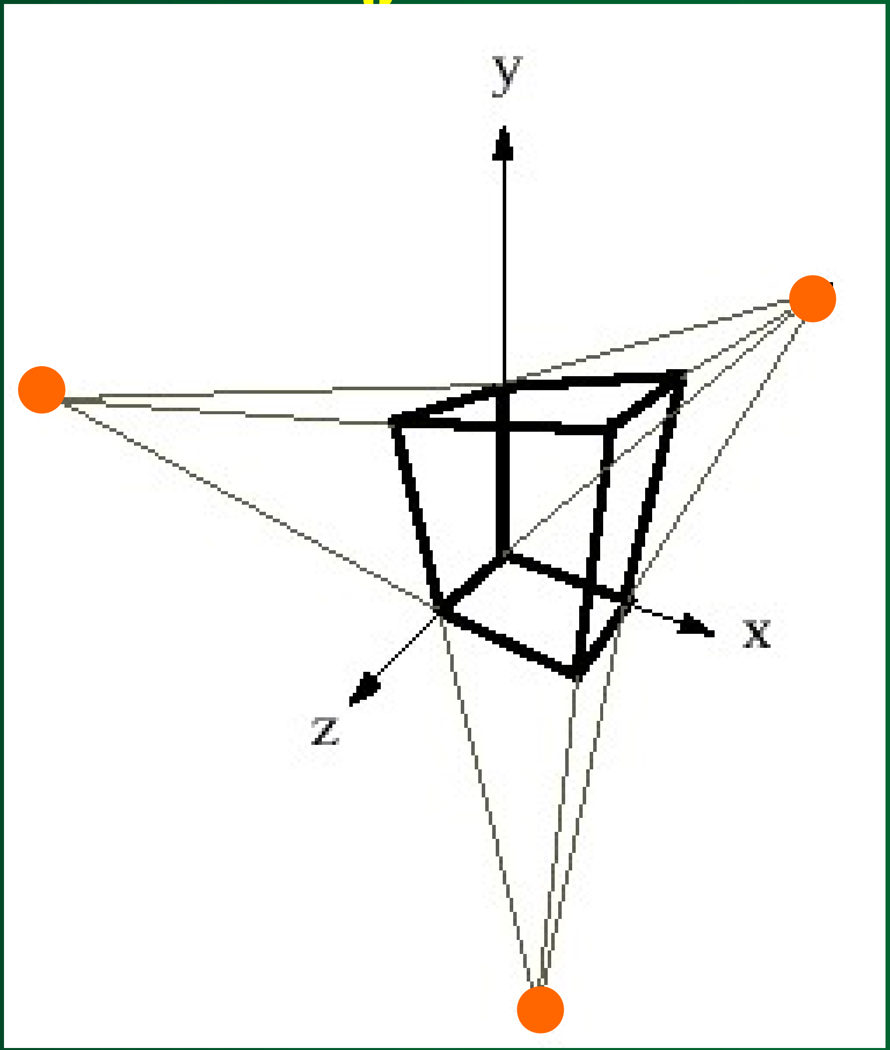
Perspective Projection



Two Point Perspective

Perspective Projection

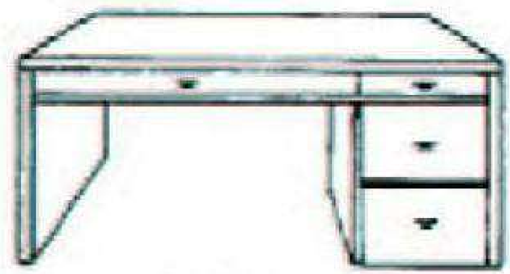




Three Point Perspective

(z, x, and y-axis vanishing points)

Perspective Projection



One-Point Perspective Projection

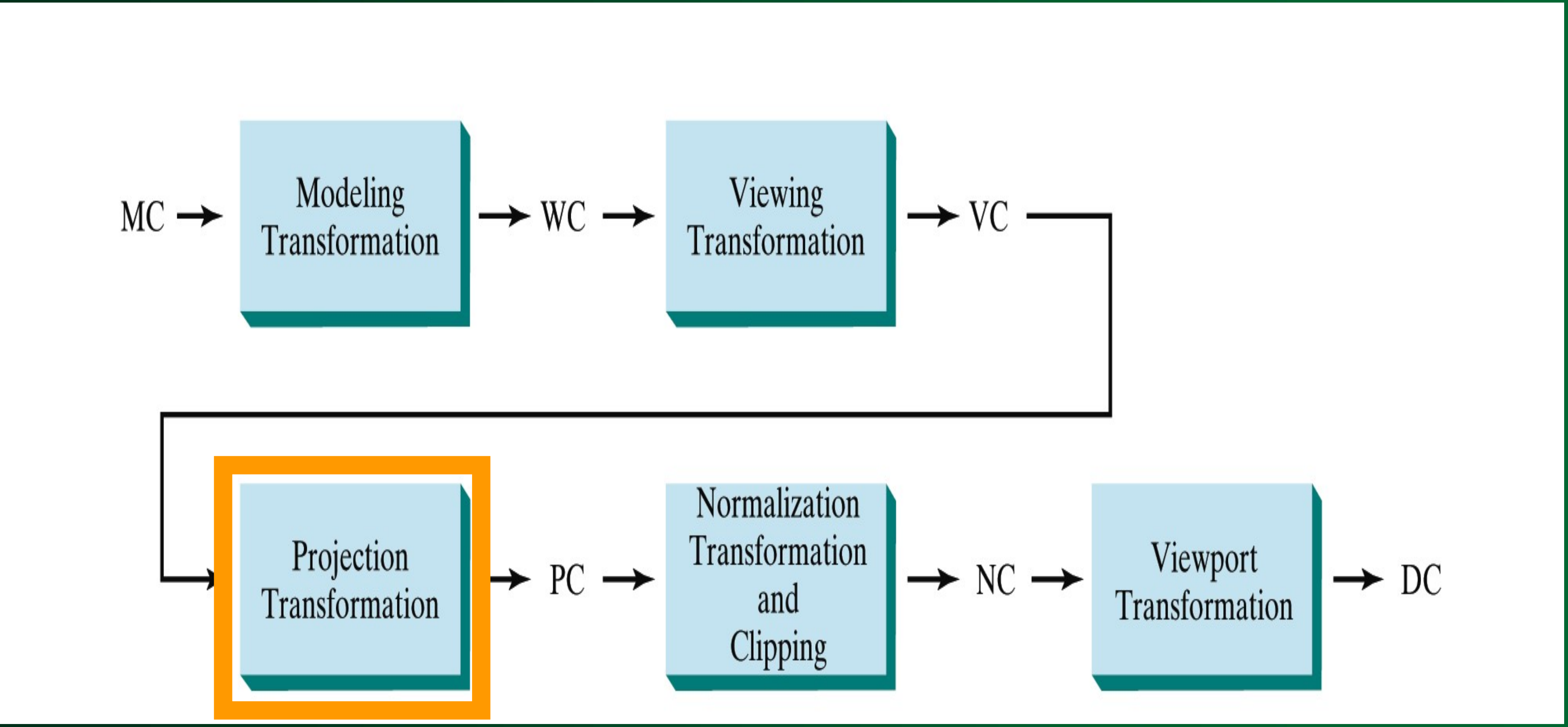
Two-Point Perspective Projection

Tree-Point Perspective Projection

Perspective Projection Transformation

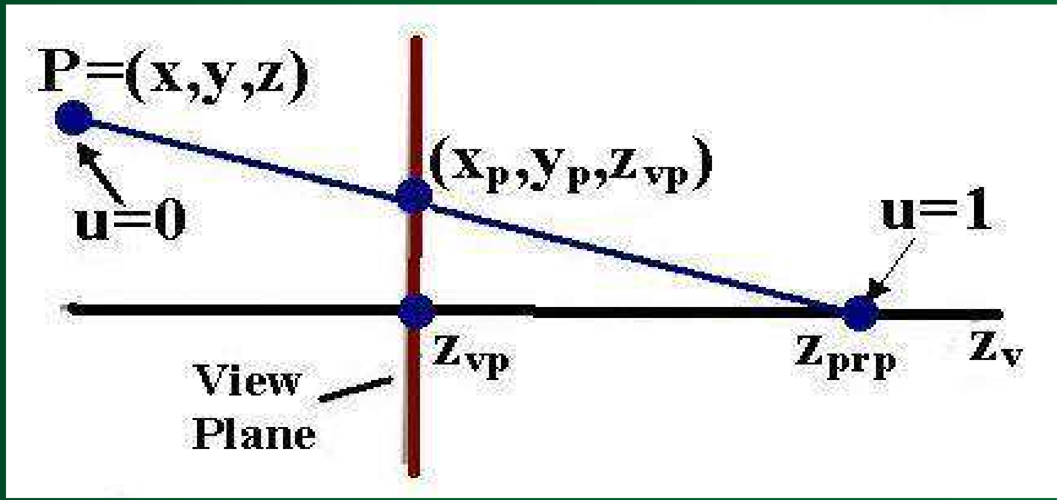
Perspective Projection Transformation

- Convert the **viewing coordinate** description of the scene to coordinate positions on the **perspective projection plane**.



Perspective Projection Transformation

- To obtain a perspective projection of a three dimensional object, transform points along the projection lines that meet a projection reference point.
- Set the projection reference point at position z_{prp} along the z_v axis.
- Set the view plane at z_{vp}.

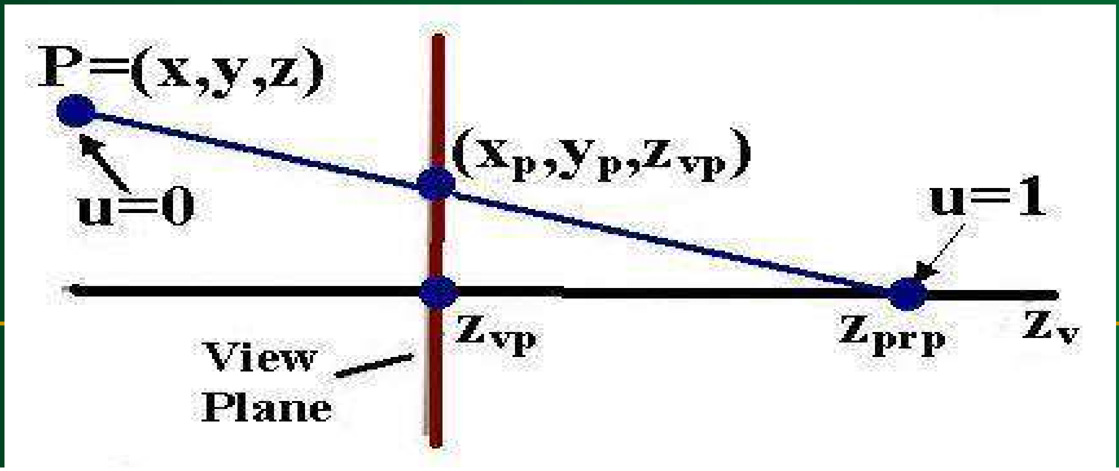


Perspective Projection Transformation

- Suppose the projection reference point at position z_{prp} along the z_v axis, and the view plane at z_{vp}.
- Equations of the perespective projection line in parametric form as

$$\boxed{\begin{aligned} x' &= x - xu \\ y' &= y - yu \\ z' &= z - (z - z_{prp})u \end{aligned}}$$

$$0 \le u \le 1$$



Perspective Projection Transformation

At u=0, x'=x ; y=y'; z=z' at p

At u=1 , x'=0; y'=0; z'=Zprp at Zprp

On the view plane: $z' = z_{vp}$

$$u = \frac{z_{vp} - z}{z_{prp} - z} \qquad d_p = z_{prp} - z_{vp}$$

$$\boxed{\begin{aligned} x_p &= x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = x\left(\frac{d_p}{z_{prp} - z}\right) \\ y_p &= y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = y\left(\frac{d_p}{z_{prp} - z}\right) \end{aligned}}$$

dp is the distance of the view plane from the projection reference point

Perspective Projection Transformation

The perspective projection transformation of three-dimensional can be represented using homogeneous coordinates: $\boxed{x_p = x_h/h, \quad y_p = y_h/h}$

The homogeneous factor is:

$$h = \frac{z_{prp} - z}{d_p}$$

$$\boxed{\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -z_{vp}/d_p & z_{vp}\left(z_{prp}/d_p\right) \\ 0 & 0 & -1/d_p & z_{prp}/d_p \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}$$

Perspective Projection Transformation

Special Cases:If view plane is uv plane

$$z_{vp} = 0$$

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = x\left(\frac{d_p}{z_{prp} - z}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = y\left(\frac{d_p}{z_{prp} - z}\right)$$

$$x_p = x\left(\frac{z_{prp}}{z_{prp} - z}\right) = x\left(\frac{1}{1 - z/z_{prp}}\right)$$

$$y_p = y\left(\frac{z_{prp}}{z_{prp} - z}\right) = y\left(\frac{1}{1 - z/z_{prp}}\right)$$

Perspective Projection Transformation

Special Cases: The projection reference point is at the viewing coordinate origin:

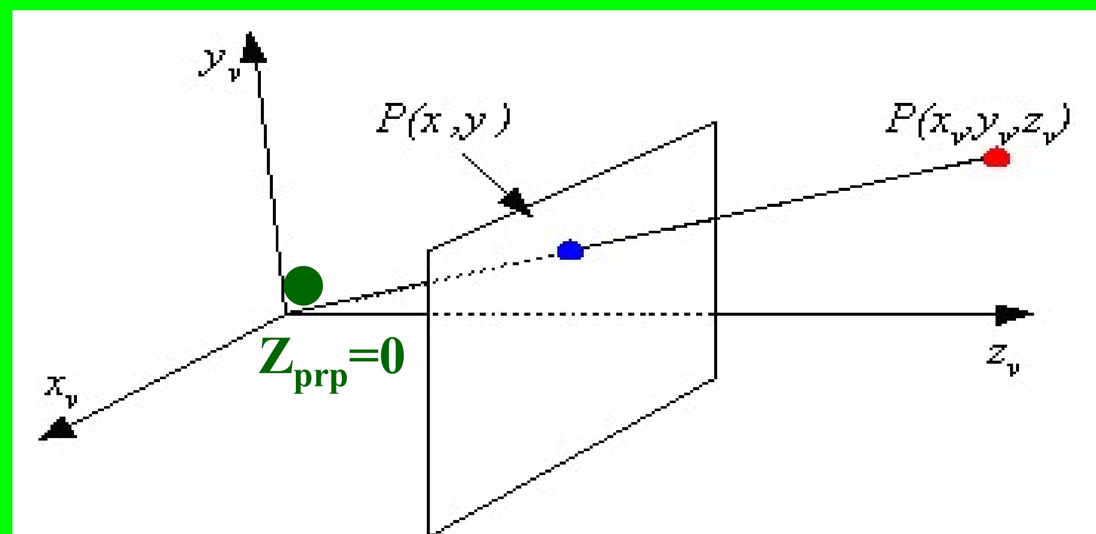
$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = x\left(\frac{d_p}{z_{prp} - z}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = y\left(\frac{d_p}{z_{prp} - z}\right)$$

$$z_{prp} = 0$$

$$x_p = x\left(\frac{z_{vp}}{z}\right) = x\left(\frac{1}{z/z_{vp}}\right)$$

$$y_p = y\left(\frac{z_{vp}}{z}\right) = y\left(\frac{1}{z/z_{vp}}\right)$$



Exercise

- Consider a 3D coordinate system where Y-axis is vertical and Z-axis is pointing towards the viewer. A line A(10,-10,10) B(10,-10,0) is viewed from the point P(0,0,20). Find where points A and B would be projected on the XY screen?

Summary