Part - C

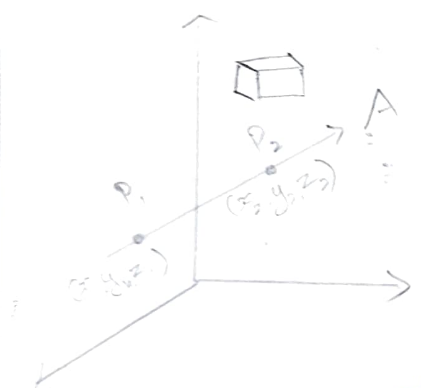
(a) Given a cube in 3D space

$\Rightarrow$ Axis : $P_1(x_1, y_1, z_1)$ $P_2(x_2, y_2, z_2)$

$\rightarrow$ Not parallel to any of the axis

$\Rightarrow$ Rotate cube about θ about axis P_1P_2

$\Rightarrow$ Rotation along arbitrary axis $P_1 P_2$



Axis vector, $V = P_2 - P_1$

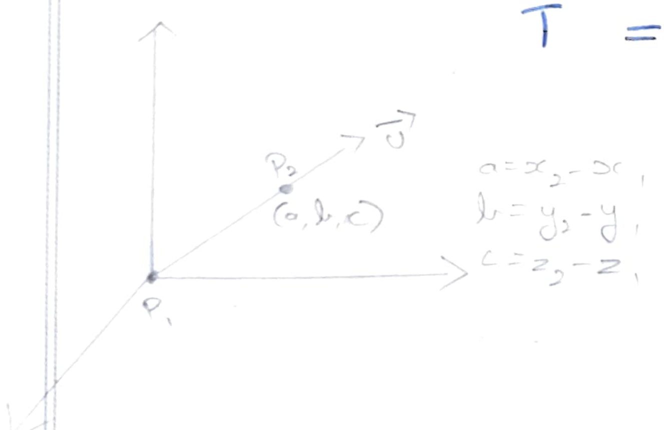
$= (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

Normal vector of axis $= U = \dfrac{V}{|V|}$

Unit

Now we are going to do composite transformation

Step 1: Move P_1 to origin



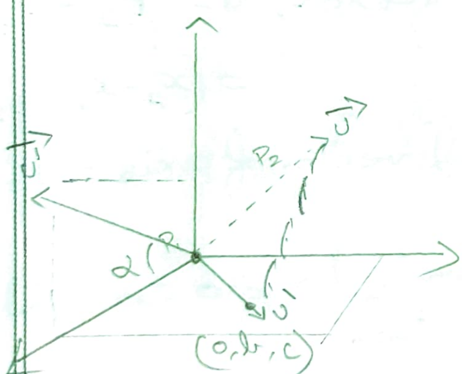
$$T = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 2: Move the axis vector along z axis

Here we need 2 transformation

a) Rotate anticlockwise in x axis by α to bring $\vec{u}$ to xz plane

b) Rotate clockwise in y axis by β to bring $\vec{u}$ to z axis



a) Rotate α along x axis

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α - angle between the projection in yz plane & z axis

$\therefore$ $\alpha \Rightarrow$ angle between $\vec{u'}$ & u_z

$U \cdot V = U V \cos\alpha$

$$\cos\alpha = \frac{U \cdot V}{|U||V|}$$

$$= \frac{(0, b, c) \cdot (0, 0, c)}{|U||V|}$$

$U \times V = U V \sin\alpha$

$$\sin\alpha = \frac{U \times V}{|U||V|}$$

$$= \frac{(0, b, c) \times (0, 0, c)}{|U|\,|V|}$$

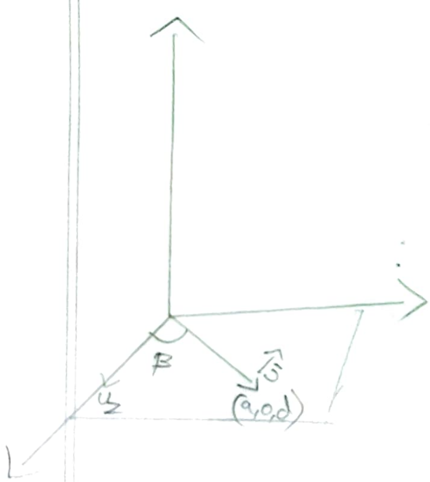
$\cos\alpha = \dfrac{c}{d}$ $\qquad$ $\sin\alpha = \dfrac{b}{d}$

$$d = \sqrt{b^2 + c^2}$$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) Rotate β along y-axis $\quad \theta = -\beta$
$\Rightarrow$ clockwise

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$\cos\beta = d$ $\qquad$ $\sin\beta = a$

$$R_y(\beta) = \begin{bmatrix} d & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 3: Now rotate the cube along $\vec{u_z}$ (z axis)

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 4: Now move back the coordinate to original position

(ie) Rotate along y in β in anticlockwise
Rotate along x in α in clockwise
~~Final Tran~~ ~~Rotat~~ Translate P_1 back

Final Transformation matrix :

$$T = T(x_1, y_1, z_1)\, R_x(-\alpha)\, R_y(-\beta)\, R_z(\theta)\, R_y(\beta)\, R_x(\alpha)\, T(-x_1, -y_1, -z_1)$$