Lab 7 – Z Transform

7.1 LTI system transfer function

In this problem we will analyse LTI systems from their system function or transfer function. Specifically we consider systems for which transfer function is ratio of polynomials in z, i.e. $H(z) = \frac{N(z)}{D(z)}$, where zeros are given by $N(z) = 0$, and poles are given by $D(z) = 0$.

Consider the LTI system given by

$$H(z) = \frac{z}{z-p} = \frac{1}{1-pz^{-1}}, \qquad p \in (-1,1)$$

(a) Use the matlab function `zplane()` to get pole-zero plot of this system for p = 0.9.
(b) Use `freqz()` to plot frequency response (DTFT) of this filter. Plot in the frequency range $[-3\pi, 3\pi]$.
(c) For various values of $p \in (-1,1)$ how does the frequency response change?
(d) Use `impz()` to plot the impulse response of this filter. How many different impulse responses are possible for given system function? Which one is plotted by `impz()`?

Repeat the above parts (a)-(c) for the system given by

$$H(z) = \frac{z-p^{-1}}{z-p}, \qquad p \in (0,1)$$

(e) What is the kind of filter represented by this system?

7.2 LTI system with complex poles

Consider the transfer function given by

$$H(z) = \frac{z}{z^2 - (2r\cos\theta)z + r^2}, \qquad r \in (0,1), \theta \in [0,\pi]$$

(a) Where are the poles and zeros of this transfer function? Verify your answer in matlab using the `zplane()` function.
(b) For fixed r $(= 0.95)$, analyse using `freqz()` the behaviour of the system as θ is changed from 0 to π.
(c) For fixed θ $(= 60^o)$, how does the filter frequency response change with r?

7.3 Difference equations, pole-zero cancellation

Consider the LTI system described by the constant coefficient difference equation

$$y[n] = 2.5y[n-1] - y[n-2] + x[n] - 5x[n-1] + 6x[n-2]$$

(a) What are the poles and zeros corresponding to this system function?
(b) Can you propose a reduced-order difference equation for this system based on your answer to part (a)?
(c) Find the impulse response of a causal system which has above difference equation using partial fraction method and verify your answer using `impz()`

For answer to theory questions above write them down in your notebook and upload their scan on moodle along with the code.