$\boxed{6.2}$

a) Let $x[n] = e^{j\omega_0 n}$

Let o/p of LTI = $y[n]$.

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k].$$

$$= \sum_{-\infty}^{\infty} e^{+j\omega_0(n-k)}\, h[k]$$

$$= \sum_{k=-\infty}^{\infty} e^{j\omega_0 n} \cdot e^{-j\omega_0 k}\, h[k]$$

$$= e^{+j\omega_0 n} \underbrace{\sum_{-\infty}^{\infty} e^{-j\omega_0 k}\, h[k]}_{\text{DTFT of } h[k] = H(e^{j\omega_0})}$$

$$\therefore y[n] = e^{j\omega_0 n} \cdot \underbrace{H(e^{j\omega_0})}_{\text{constant}}$$

$\therefore e^{j\omega_0 n}$ is an eigen function of LTI systems.

b) $y[n] = x[n] * h[n].$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (x[n] * h[n])\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k]\, h[n-k]\right) e^{-j\omega n}$$

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Re arranging summation terms,

$$Y(e^{j\omega}) = \sum_{k=-\infty}^{\infty}\left(x[k]\sum_{n=-\infty}^{\infty} h[n-k]e^{-j\omega n}\right)$$

$$h[n] \xleftrightarrow{DTFT} H(e^{j\omega})$$

$$h[n-k] \xleftrightarrow{DTFT} e^{-j\omega k} H(e^{j\omega})$$

$$Y(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k].\left[e^{-j\omega k}\, H(e^{j\omega})\right]$$

$$= H(e^{j\omega}).\sum_{k=-\infty}^{\infty}\left(x[k]\, e^{-j\omega k}\right)$$

$$Y(e^{j\omega}) = \underline{\underline{H(e^{j\omega}).X(e^{j\omega})}}$$

Hence proved.

$\boxed{6.3}$

b) $$Y(e^{j\omega}) = \sum_{m=0}^{M} b_m e^{-j\omega m} X(e^{j\omega}) - \sum_{l=1}^{L} a_l e^{-j\omega l}\, Y(e^{j\omega})$$

$$Y(e^{j\omega})\left[1 + \sum_{l=1}^{L} a_l e^{-j\omega l}\right] = \sum_{m=0}^{M} b_m e^{-j\omega m} X(e^{j\omega})$$

&

$$\boxed{\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{m=0}^{M} b_m e^{-j\omega m}}{1 + \sum_{l=1}^{L} a_l e^{-j\omega l}}}$$

(c) $y[n] = x[n] + 0.9y[n-1]$

$$Y(e^{j\omega}) = X(e^{j\omega}) + 0.9e^{-j\omega}Y(e^{j\omega})$$

$$\boxed{\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1-0.9e^{-j\omega}} = H(e^{j\omega})}$$

(d) $y[n] = x[n] - 0.9y[n-1]$

$$Y(e^{j\omega}) = X(e^{j\omega}) - 0.9e^{-j\omega}Y(e^{j\omega})$$

$$\boxed{H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1+0.9e^{-j\omega}}}$$

e)

$\cancel{y[n] = x[n] +}$

Part (c) : Low Pass Filter
Part (d) : High Pass Filter

(f) $$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{e^{j\omega n}}{1+ae^{-j\omega}}\,d\omega$$

where $a = -0.9, 0.9$

But we know if
$h[n] = a^n u[n]$

$$H(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}}$$

If $\underline{a = 0.9}$

$$H(e^{j\omega}) = \frac{1}{1 - 0.9e^{-j\omega}}$$

$$\therefore h[n] = (0.9)^n u[n]$$

Similarly, if $a = -0.9$

$$H(e^{j\omega}) = \frac{1}{1 + 0.9e^{-j\omega}}$$

$$h[n] = (-0.9)^n u[n]$$

Impulse Response of

(c) $h[n] = (0.9)^n u[n]$