Date

Page No.

Competitive programming
- Elydeck

→ Prime sieve

Sieve of erathestheres

Make an array (bool) of size n.
Mark all true.

T	T	T	T	T	T	T	T	T	T	T
0	1	2	3	4	5	6	7	8	9	10

A[0] = false; A[1] = false;

We need to find all primes from 2 to n.

```
for (i = 2; i*i <= n; i++)
  {
      if (A[i] == true) {
          for (j = i*i; j <= n; j += i)
            {
                A[j] = false;
            }
      }
  }
```

print those index which are marked true.

Time-complexity = n log log n

Finding GCD

Euclid's algorithm

$gcd(a, b) = gcd(b, a \% b)$ when $a > b$

Learn it as "bab"

$gcd(a, 0) = a$

→ Extended euclid's algorithm

$gcd(a, b) = ax + by$

we need to find x & y.

$ax + by = gcd(a, b)$ —— i)

Also, $gcd(a, b) = gcd(b, a \% b)$ —— ii)

From i) & ii)

$ax + by = gcd(b, a \% b)$ —— iii)

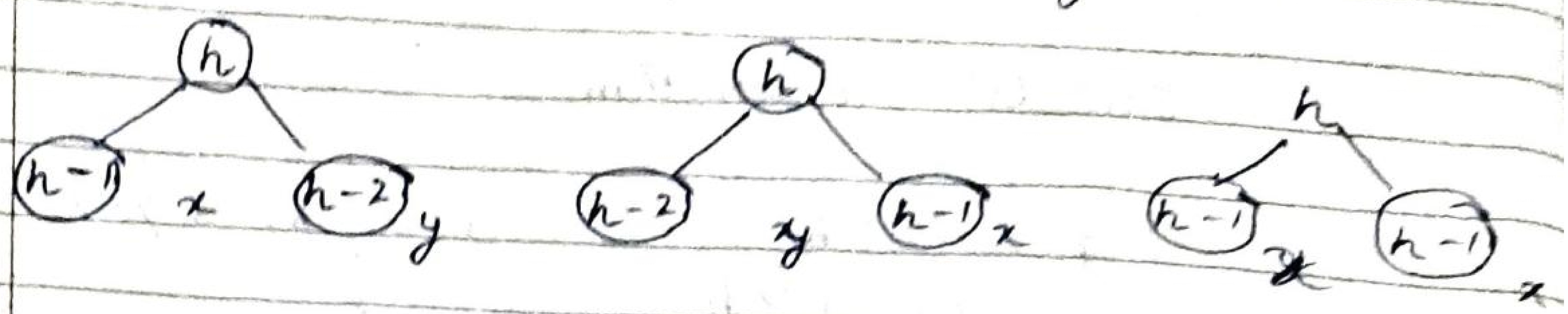
$\Rightarrow bx_1 + (a \% b) y_1 = gcd(a, b) = ax + by$

$\Rightarrow bx_1 + \left\{a - \left[\frac{a}{b}\right] \cdot b\right\} y_1 = ax + by$

$\Rightarrow$ On comparing, we get

$\boxed{x = y_1}$, $\boxed{y = x_1 - \left[\frac{a}{b}\right] y_1}$

→ Number of balanced Binary Tree



$\cancel{ans = 2x + y}$

```
int balanceBT(int h) {
    if (h==0 || h==1) {
        return 1;
    }
    int m = pow(10, 9) + 7;
    int x = balanceBT(h-1);
    int y = balanceBT(h-2);
    long res1 = (long) x*x;
    long res2 = (long) x*y*2;
    int ans1 = (int)(res1 % m);
    int ans2 = (int)(res2 % m);
    return (ans1 + ans2) % m;
}
```

Balance = $|LH - RH| \leq 1$

$(h-1) \longrightarrow x$

$(h-2) \longrightarrow y$

$ans = x^* x + x^* y + x^* y$

$= x^2 + 2xy$

modulo inverse

$$a \times b = 1$$

$$b = \frac{1}{a}$$ (multiplicative inverse)

Multiplicative mod inverse

$(A \cdot B) \% m = 1$

We have to find B.

$(A \cdot B) \% m = 1$

$\Rightarrow ((A \% m) \cdot (B \% m)) \% m = 1$

$\Rightarrow \because (a \cdot b) \% m = ((a \% m) \cdot (b \% m)) \% m$

$\Rightarrow 1 \leq B \leq m-1$

$(a \cdot b) \% m = 1$

$\Rightarrow \cancel{(a \cdot b) \% m - 1 = 0}$

$a \cdot b \equiv 1$

$\Rightarrow a \cdot b - 1 \equiv 0$ (multiple of m)

$= a \cdot b - 1 \equiv mq$

$= a \cdot b + mQ = 1$

$= a \cdot b + mQ = 1$

$= gcd(a, m) = 1$

$(ax + by) = gcd(a, b)$

Date			
Page No			

mod inverse (a, m)

$= b$

matlab a me kya multiply kre aur m se mod kre ki ans = 1.

modinverse $(a, m) = b$

$$\Rightarrow \underbrace{(a \times ?)}_{} \ \% \ m = 1$$

we have to find this only

→ Hard question

$ax + by = d$

a & b item se d banana hai
Need to find number of pairs possible of (x, y) for such

$ax + by = d$

$ax = \underbrace{d - by}$

This term must be divisible by x.

$0 \leq y \leq \frac{d}{b}$, $0 \leq x \leq \frac{d}{a}$

Date			
Page No.			

$$ax = d - by$$
$$\downarrow$$

Let us assume y_1 to be minimum value for such condition to be hold.

$d \rightarrow$ fixed

$b \rightarrow$ fixed

$y \rightarrow$ can change.

$$ax = d - by_1$$

Next term, $d - b(y_1 + a)$

$d - b(y_1 + 2a)$

$d - b(y_1 + 3a)$

$\vdots$

$d - b(y_1 + na)$

On comparing with maximum value of y

$$y_1 + na = \frac{d}{b}$$

$$na = \frac{d}{b} - y_1$$

$$\boxed{n = \frac{\frac{d}{b} - y_1}{a}}$$

Total terms $= 1 + n$

Now we need to find y_1

$$(d - by_1) \% a = 0$$

$\Rightarrow$ $$d \% a - (by_1) \% a = 0$$

$\Rightarrow$ $$d \% a - [(b \% a) \cdot (y_1 \% a)] \% a = 0 \quad \text{— i)}$$

y_1

$$ax + by = d$$

$$y = \frac{d - ax}{b}$$

For smallest value,

$$y_1 \% a = \left(\frac{d - ax}{b}\right) \% a$$

$$= \left(\frac{d}{b}\right) \% a - \frac{(ax)}{b} \% a$$

$$y_1 \% a = \left(\frac{d}{b}\right) \% a \quad \text{— ii)}$$

i) & ii)

$\Rightarrow$ $$d \% a - \left[[b \% a] \cdot \left(\frac{d}{b}\right) \% a \right] \% a = 0$$

$\Rightarrow$ $$d \% a - \left[\left(b \cdot \frac{d}{b}\right) \% a \right] \% a = 0$$

$$d \% a - (d \% a) \% a = 0$$

this holds

$$y_1 = \left(\frac{d}{b}\right) \% a$$

$$= d * \text{mod inverse}(b, a)$$

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Date
Page No.

```
... g = gcd (a,b);
if (d%g ==1) {
    cout << 0 << endl;
}
if (d ==0) {
    cout << 1 << endl;
}
    a/=g;
    b/=g;
    d%=g;
long value= ((d%a) * modInverse(b,a)) %a;
long firstvalue = d/b
if (d < y1 *b) {
    cout << 0 << endl;
}
else {
    long n= (firstValue -y1) / a;
    cout << 1+n << endl;
}
```

$$y_1 = \left(\frac{d}{b}\right) \% a$$

$$ax + by = d$$

$$\frac{ax}{g} + \frac{by}{g} = \frac{d}{g}$$

$$Ax + By = D$$

Date | Page No.

→ Advanced GCD

$gcd(a, b) = gcd(b, a \% b)$

both must be integers

$gcd(10^{240}, 40) = gcd(40, 10^{240} \% 40)$

Input as string.

ex- $(23567) \% 40$

$= (0 * 10 + 2) \% 40 = 2$

$(2 * 10 + 3) \% 40 = 23$

$(23 * 10 + 5) \% 40 =$

$2356 \% 40 = (235 * 10 + 6) \% 40$

$= ((235 * 10) \% 6 + 6 \% 40) \% 40$

$= (\text{from previous} + 6 \% 40) \% 40$

```
ll gcdLarge (ll a, string b) {
    ll mod = 0;
    for (int i=0; i < b.size(); i++) {
        mod = (mod * 10 + b[i] - '0') % a;
    }
    return mod;
}
```

ICPC question

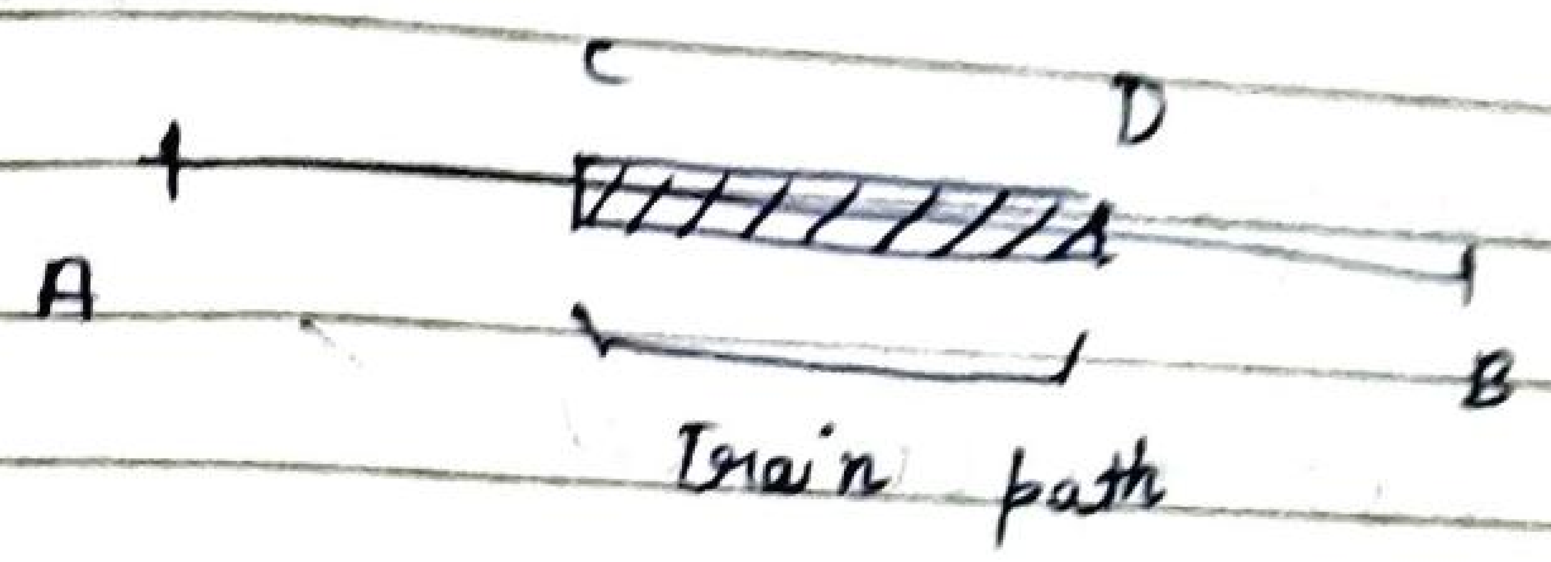
Train or walk

$v_{walk} = \left(\frac{1}{p}\right) m/s$

$v_{train} = \left(\frac{1}{q}\right) m/s$ train starts at $x = ys$

(From C to D)

$\Downarrow$



Path 1: A to B without train

distance = $|x_A - x_B|$

$S = \frac{D}{t}$

$t = \frac{|x_B - x_A|}{\left(\frac{1}{p}\right)} = p(x_B - x_a)$

Path 2: A to C and go to D then B.

distance = $|x_C - x_A| + |x_D - x_C| + |x_B - x_D|$

$t = p|x_C - x_A| + q|x_D - x_C| + p|x_B - x_D|$

ans = min (t1 , t2);

But train starts at t = y.

Path 2

Train ke liye wait kara

Train chut gayi

if AC(time) > y :
can't catch train

Path 1

waiting_time = $y - p|x_C - x_A|$

t2 += waiting_train

ans = min (t1, t2)

```
cin >> n >> A >> B >> C >> D >> P >> Q >> Y;
int A[n];
for (int i=0; i<n; i++)  cin >> A[i];
int t1 = p * (abs ( A[a] - A[b]) );
int tAC = p * (abs ( A[c] - A[a]));
 if tAC > y {
    // do nothing;    t2 = INT_MAX;
    }
else {
   waiting_time = y - tAC;
     t2 += waiting_time;
 }
cout << min(t1, t2) << endl;
```

Modified GCD - Codeforces

a = 24, b = 36

gcd = 12

1 2 3 4 6 → all divides gcd.

Range - low & high which divides both a and b.

Making list of divisors

```
for (i=1; i*i<=n; i++) {
    if (gcd % i == 0) {
            v.pb (i); }
        if (i*i != gcd) {
            v.pb (gcd/i);
        }
    }
    res = -1;
for (d : div) {
    if (d >= L && d <= H) {
            res = max(res, d);
        }
    }
cout << res;
```

Zero remainder array - Codeforces

ex- $k = 5$

4 4 4 → same group (same remainder)

map <int, int> m;
m [remainder] = no. of same;

4 4 4 for $k = 5$

$\underbrace{x_1 = 1, \quad x_2}_{5} = \underbrace{6, \quad x_3}_{5} = 11$

$x_1 \% 5 = x_2 \% 5 = x_3 \% 5$

Claim- Sum of 2 numbers is divisible by k, iff sum of their remainder is divisible by k.

$r_1 = 4 \% 5 = 4$
$r_2 = 1 \% 5 = 1$ } $s = 5 \% 5 = 0$

$r_1 = 4 \% 5 = 4$
$r_2 = 6 \% 5 = 1$ } $s = 5 \% 5 = 0$

⋮

ex- $k = 6$

8 7 1 8 3 7 5 10 8 9

$m[1] = 3$, $x_1 = 5, 11, 17$
$m[2] = 3$, $x_2 = 4, 10, 16$
$m[3] = 2$, $x_3 = 3, 9$
$m[4] = 1$, $x_4 = 2$
$m[5] = 1$, $x_5 = 1$

ans = 17 + 1

Chinese Remainder theorem

2 pack of chocolates
1 pack = 5 people (3 rem)
1 pack = 6 people (2 rem)

$x \equiv 3 \pmod 5$
$x \equiv 2 \pmod 6$

$x = n5 + 3 \Rightarrow x = 3, \boxed{8}, 13, 18, 23$
$x = m6 + 2 \Rightarrow x = 2, \boxed{8}, 14, 20, 26$

$x = 8, 38, 68$

$\underbrace{8, \quad 38}_{30}, \underbrace{\quad 68}_{30}$

?

LCM (5, 6)

CRT,

$a \equiv a_1 \pmod{p_1}$
$a \equiv a_2 \pmod{p_2}$
$\vdots$
$a \equiv a_k \pmod{p_k}$

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$x = \sum (rem[i] * pp[i] * inv[i]) \% prod$

$pp[i] = \dfrac{prod}{num[i]}$

$inv[i] =$ modInv(pp[i], num[i]);

Code

```
for (i=0; i<k; i++){
        int pp= prod / num[i];
        result += rem[i] * inv(pp, num[i]) *
                                            pp;
   }
return result % prod;
```

TC = n log n

Date
Page No.

Train partner - codechef

1	4	9	12
2	5	10	13
3	6	11	14
7	8	15	16

Remainder (% 8)	Solution	Coach
0	$n-1$	SL
7	$n+1$	SU
1	$n+3$	LB
4	$n-3$	LB
2	$n+3$	MB
5	$n-3$	MB
3	$n+3$	UB
6	$n-3$	UB

Date | Page No.

Matrix exponentiation

$A = \begin{bmatrix} \ \end{bmatrix}$ $B = \begin{bmatrix} \ \end{bmatrix}$

$$A \cdot B = \sum_{r=1}^{k} A_{ir} \times B_{rj}$$

We know, $f(n) = f(n-1) + f(n-2)$

$$M * \begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} f(n+1) \\ f(n \end{bmatrix}$$

We need to find this M

$M = 2 \times 2$

$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix}$$

$$\Rightarrow a * f(n) + b * f(n-1) = f(n+1)$$

Clearly, $a = 1$, $b = 1$

$$c * f(n) + d * f(n-1) = f(n)$$

Clearly, $c = 1$, $d = 0$

$$M = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

So, $M * \begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix}$

$$M * \begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} f(n+2) \\ f(n+1) \end{bmatrix}$$

$$M * M * \begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} f(n+2) \\ f(n+1) \end{bmatrix}$$

$$M^k * \begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} f(n+k) \\ f(n+k-1) \end{bmatrix}$$

Put $n = 1$
& $k = n-1$

$$M^{n-1} * \begin{bmatrix} f(1) \\ f(0) \end{bmatrix} = \begin{bmatrix} f(1+n-1) \\ f(1+n-1-1) \end{bmatrix}$$

$$M^{n-1} * \begin{bmatrix} f(1) \\ f(0) \end{bmatrix} = \begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix}$$

This is matrix exponentiation

$$M = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$M^{n-1} \rightarrow \log n$

$$A^n = \begin{cases} A^{n/2} \cdot A^{n/2}, & n = \text{even} \\ [A^{n/2} \cdot A^{n/2}] \cdot A & n = \text{odd} \end{cases}$$

Date | Page No.

Fibo sum

n & m

$S(i) = S_{i-1} + f(n)$

$f(n) = f(n-1) + f(n-2)$
$f(n-1) = f(n-2) + f(n-3)$
$\vdots$

$f(1) = 1$
(+)

$f(n) = f(n-2) + f(n-3) + f(n-4)$
$+ \ldots f(1) + 1$

$\Rightarrow$ $f(n) = S(n-1) + 1$

$S(n-1) = f(n) - 1$
$S(n) = f(n+2) - 1$

According to question,
$S(m) - S(n-1)$
$= f(m+2) - 1 - f(n+1) + 1$

$f(m+2) - f(n+1)$
$\downarrow$
ans $= [f(m+2) - f(n+1)]$ % m

Date | Page No.

<u>Game theory</u>
(2 player game)
Combinatarial game theory - Finite game

Imfartial game (No- limitation)

Partison Game (limitation) ex- chess

Game theory deals with this part mostly.

→ Game of Nim

3 4 5 8

Player 1 Player 2

Can remove any no. of file.
In the end who will be the winner

ex-

3	4	5
1	2	3

P1 - 2 → remove all

3	5
1	3

p2 → 1 → remove 1

2	5
1	3

P1 → 1 → remove all

5

$\boxed{3}$

P2 → 3 → remove all

Player 1 lost

Player 2 wins

Will player 1 always loose?

Not reccessary.

→ Nim sum theorem

Find Nim sum

i.e. Commulative XOR of all piles

$3 \wedge 4 \wedge 5.$

$$\text{Nim sum} = \begin{cases} 0, & \text{Player 1 looses} \\ \neq 0 & \text{Player 1 wins} \end{cases}$$

On condition that both plays optimally.

and if n → even, play 1 wins

Proof of Nim sum

$\boxed{A} \xrightarrow{\text{zero}} \boxed{B}$

Not needed

Max - minimum excluded set

max {0, 1, 3} , ans = 2

max {0, 1} ans = 2

Grundy numbers.

grundy numbers = max { grundy (n-1), grundy (n-2) }

grundy (0) = 0.

grundy (1) = { g(0) } = {0} = 1

grundy (2) = { g(0), g(1) }

= {0, 1} = 2

⋮

grundy (n) = { g(0), g(1), g(2) ... g(n-1) } = n

Grundy number is a no. that defines a state of a game.

$\boxed{n \text{ stones}} \rightarrow$ 1, 2, 3 can be removed.

grundy (n) = max { g(n-1), g(n-2), g(n-3) }

Here, g(0) = 0, g(1) = 1, g(2) = 2

g(3) = 3, g(4) = 0

g(5) = 1 , g(6) = 2 , g(7) = 3

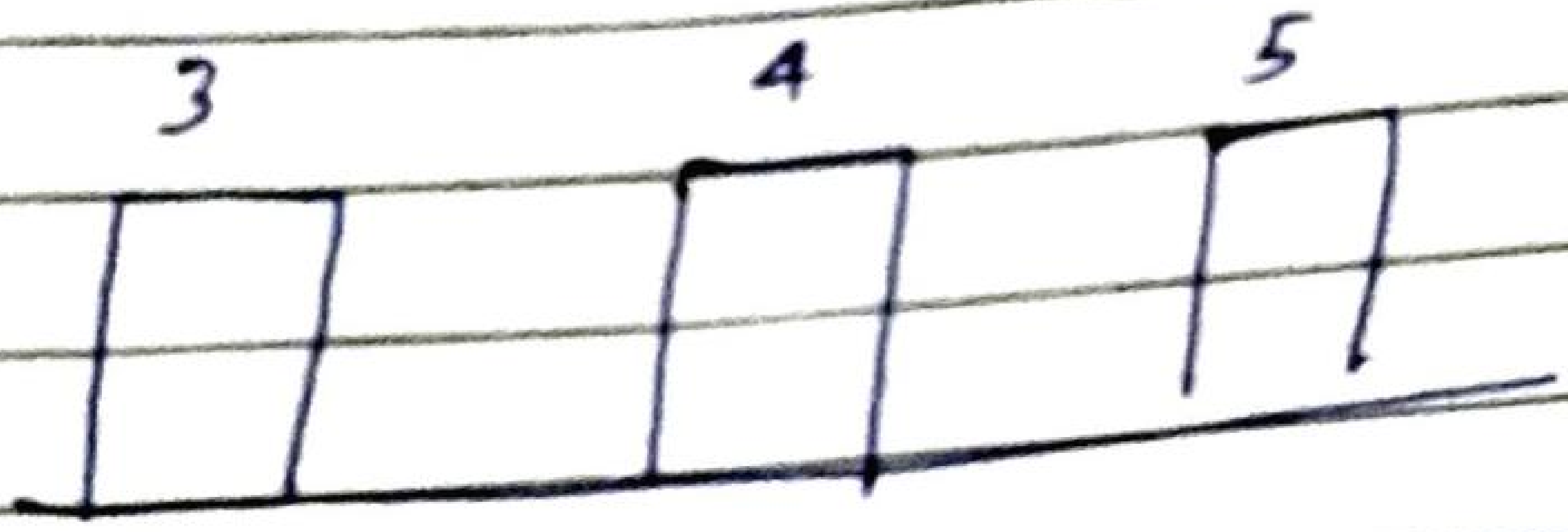
Sprague grundy theorem

Cumulative XOR = $(g(p_1) \wedge g(p_2) \wedge \ldots)$

$$XOR = \begin{cases} 0, & \text{Player 1 loose} \\ \neq 0 & \text{Player 1 wins} \end{cases}$$

on condition that both player plays optimally.

ex -



g	0	1	2	3	0	1	2	3
	0	1	2	3	4	5	6	7

$$XOR = g(3) \wedge g(4) \wedge g(5)$$
$$= 3 \wedge 0 \wedge 1$$