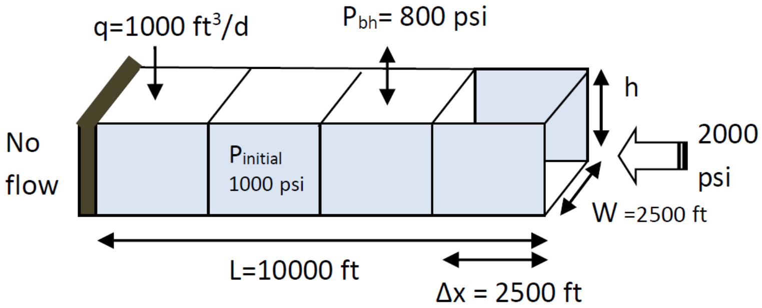
Example 8. Implicit Solution to 1D flow with a constant BHP well

Consider a 1D reservoir with the following properties:



$\phi = 0.2$
k = 50 mD
$\mu = 1$ cp
$B_w = 1$ rb/stb (assume constant)
$c_t = 10^{-6}$ psi^{-1}.
$L = 10000$ ft (length)
A = 200,000 ft^2 (Cross Sectional Area)

The initial condition is $P = 1000$ psi. The boundary conditions are no flow ($q = 0$) at $x = 0$ and constant pressure, $P = 2000$ psi, at $x = L$. There is a constant-rate injection well of 1000 ft^3/day at $x = 0$. There is a constant BHP producer with $P = 800$ psi at $x = 6250$ feet.

Determine the pressure field in the reservoir using 4 uniform blocks. Use a time step of $\Delta t = 1$ days. Assume the radius of both wells is 0.25 feet, that skin factor is negligible, and that the reservoir thickness is 250 feet.

Solution:
The pressure is governed by the 1D diffusivity equation with sources and sinks and the following boundary conditions:

$$\frac{1}{\alpha}\frac{\partial p}{\partial t} = \frac{\partial^2 p}{\partial x^2} + q; \quad \alpha = \left(\frac{k}{\mu \phi c_t}\right)$$

$$IC: p(x,0) = 1000\ psi$$

$$BC1: \frac{\partial p}{\partial x}(0,t) = 0$$

$$BC2: p(L,t) = 2000\ psi$$

Note that this problem is similar to example #5 in terms of boundary conditions (no flow at $x = 0$ and constant P at $x = L$). Also like example #5 we have 2 wells in the problem, but now the producer is a constant BHP well of 800 psi at x= 6250 feet. For the 4-block system here, the constant BHP well is in block #3.

We need to calculate the productivity index (J) of the BHP well in block #3, where the height can be found from the area and thickness and r_0 is found from the Peaceman correction

$$h = \frac{A}{W} = \frac{200000}{2500} = 80\,ft$$

$$r_{eq} \approx 0.2\Delta x = 0.2 \cdot 2500\,ft = 500\,ft$$

$$J_i^w = \frac{2\pi kh}{\mu B_w \ln\left(\frac{r_{eq}}{r_w} + s\right)} = \frac{2\pi\left(50mD\right)\left(80\,ft\right)}{1\text{ cp} \cdot 1 \cdot \ln\left(\frac{500}{0.25} + 0\right)} = 3306.5\frac{mD-ft}{cp}$$

Recall the *Control Volume* approach shows that a mass balance on each block is given by:

$$T\left(P_{i-1} - P_i\right) - T\left(P_i - P_{i+1}\right) = \frac{B}{\Delta t}\left(P_i^{n+1} - P_i^n\right) + q_i$$

And for a constant BHP well we have:

$$q_i = J_i^w\left(P_i - P_w\right)$$

Writing for each block and noting we have wells in blocks 1 and 3

$$T\left(P_0^{n+1}-P_1^{n+1}\right)+T\left(P_2^{n+1}-P_1^{n+1}\right)=\frac{B}{\Delta t}\left(P_1^{n+1}-P_1^{n}\right)+q_1$$

$$T\left(P_1^{n+1}-P_2^{n+1}\right)+T\left(P_3^{n+1}-P_2^{n+1}\right)=\frac{B}{\Delta t}\left(P_2^{n+1}-P_2^{n}\right)+0$$

$$T\left(P_2^{n+1}-P_3^{n+1}\right)+T\left(P_4^{n+1}-P_3^{n+1}\right)=\frac{B}{\Delta t}\left(P_3^{n+1}-P_3^{n}\right)+J_3^{w}\left(P_3^{n+1}-P_w\right)$$

$$T\left(P_3^{n+1}-P_4^{n+1}\right)+T\left(P_5^{n+1}-P_4^{n+1}\right)=\frac{B}{\Delta t}\left(P_4^{n+1}-P_4^{n}\right)+0$$

The boundary conditions are treated as usual for constant pressure and no flux. In matrix form we get the usual matrix equations:

$$\left(\mathbf{T}+\mathbf{J}+\frac{\mathbf{B}}{\Delta t}\right)\mathbf{P}^{n+1}=\frac{\mathbf{B}}{\Delta t}\mathbf{P}^{n}+\mathbf{Q}$$

Where

$$\mathbf{T} = \begin{pmatrix} T & -T & 0 & 0 \\ -T & 2T & -T & 0 \\ 0 & -T & 2T & -T \\ 0 & 0 & -T & 3T \end{pmatrix} = \begin{pmatrix} 4000 & -4000 & 0 & 0 \\ -4000 & 8000 & -4000 & 0 \\ 0 & -4000 & 8000 & -4000 \\ 0 & 0 & -4000 & 12000 \end{pmatrix} \times 6.33E-3$$

$$\mathbf{J} = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 3306.5 & \\ & & & 0 \end{pmatrix} \times 6.33E-3; \ \mathbf{B} = \begin{pmatrix} 100 & & & \\ & 100 & & \\ & & 100 & \\ & & & 100 \end{pmatrix} \frac{ft^3}{psi}; \quad P^0 = \begin{pmatrix} 1000 \\ 1000 \\ 1000 \\ 1000 \end{pmatrix} psi;$$

$$\mathbf{Q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 + J_3^w P_w \\ q_4 + 2TP_{out} \end{pmatrix} = \begin{pmatrix} 1000 \\ 0 \\ 3306.5 \cdot 800 \cdot 6.33E-03 \\ 2 \cdot 4000 \cdot 2000 \cdot 6.33E-03 \end{pmatrix} \frac{ft^3}{day}$$

Table 8.1 Summary of block pressures for example 8 using various solution techniques

	Method	**Block #1 (psi) 1250 ft**	**Block #2 (psi) 3750 ft**	**Block #3 (psi) 6250 ft**	**Block #4 (psi) 8750 ft**
Initial		1000	1000	1000	1000
1 day	Implicit	1008.9	1004.7	1019.2	1290.6
	CMG	1008.9	1004.7	1019.1	1290.3
2 days	Implicit	1018.3	1015.8	1057.2	1461.1
	CMG	1018.3	1015.8	1057.1	1460.8
3 days	Implicit	1029	1031.6	1096.8	1563.8
	CMG	1029	1031.6	1096.6	1563.4
∞		1441.2	1401.7	1362.2	1787.4

The pressure in the reservoir increases with time as a result of the injector well in block #1 and the constant pressure boundary condition (2000 psi) at $x = L$. However, this pressure increase is somewhat offset by the low-pressure bottomhole pressure well (800 psi) that acts as a producer. The wells prevent the steady solution from equilibrating at 2000 psi. Notice that at steady state, the pressure is highest in block #4 which is adjacent to the 2000 psi boundary and the second highest pressure is in block #1 which contains an injector well. The pressure is lowest in block #3 which contains the 800 psi BHP well.

Well Rates and Pressures

We can also calculate the production rate in the constant BHP well and the bottom hole pressure in the constant rate wells using the productivity index.

$$Q_i = J_i \left(P_{BHP} - P_i \right)$$

<u>Block #3 constant BHP well (calculate rates)</u>

$$\text{t} = 0\text{:} \quad Q_i = J_i \left(P_{BHP} - P_i \right) = 3306.5 \frac{\text{mD-ft}}{\text{cp}} \times 6.33E - 03 \times \left(800 \text{ psi} - 1000 \text{ psi} \right) = -4186.03 \frac{\text{ft}^3}{\text{day}}$$

$$\text{t} = 1\text{:} \quad Q_i = J_i \left(P_{BHP} - P_i \right) = 3306.5 \frac{\text{mD-ft}}{\text{cp}} \times 6.33E - 03 \times \left(800 \text{ psi} - 1019.2 \text{ psi} \right) = -4587.89 \frac{\text{ft}^3}{\text{day}}$$

$$\text{t} = 2\text{:} \quad Q_i = J_i \left(P_{BHP} - P_i \right) = 3306.5 \frac{\text{mD-ft}}{\text{cp}} \times 6.33E - 03 \times \left(800 \text{ psi} - 1057.2 \text{ psi} \right) = -5383.23 \frac{\text{ft}^3}{\text{day}}$$

$$\text{t} = 3\text{:} \quad Q_i = J_i \left(P_{BHP} - P_i \right) = 3306.5 \frac{\text{mD-ft}}{\text{cp}} \times 6.33E - 03 \times \left(800 \text{ psi} - 1096.8 \text{ psi} \right) = -6212.07 \frac{\text{ft}^3}{\text{day}}$$

$$\text{t} = \infty\text{:} \quad Q_i = J_i \left(P_{BHP} - P_i \right) = 3306.5 \frac{\text{mD-ft}}{\text{cp}} \times 6.33E - 03 \times \left(800 \text{ psi} - 1362.2 \text{ psi} \right) = -11766.90 \frac{\text{ft}^3}{\text{day}}$$

Block #1 constant rate well (calculate BHP)

$$t = 0:\quad P_{BHP} = \frac{Q_i}{J_i} + P_i = \frac{1000\,\mathrm{ft}^3/\mathrm{day}}{3306.5\,\mathrm{mD\text{-}ft}/\mathrm{cp} \times 6.33E-03} + 1000\ \mathrm{psi} = 1047.78\ \mathrm{psi}$$

$$t = 1:\quad P_{BHP} = \frac{Q_i}{J_i} + P_i = \frac{1000\,\mathrm{ft}^3/\mathrm{day}}{3306.5\,\mathrm{mD\text{-}ft}/\mathrm{cp} \times 6.33E-03} + 1008.9\ \mathrm{psi} = 1056.68\ \mathrm{psi}$$

$$t = 2:\quad P_{BHP} = \frac{Q_i}{J_i} + P_i = \frac{1000\,\mathrm{ft}^3/\mathrm{day}}{3306.5\,\mathrm{mD\text{-}ft}/\mathrm{cp} \times 6.33E-03} + 1018.3\ \mathrm{psi} = 1066.08\ \mathrm{psi}$$

$$t = 3:\quad P_{BHP} = \frac{Q_i}{J_i} + P_i = \frac{1000\,\mathrm{ft}^3/\mathrm{day}}{3306.5\,\mathrm{mD\text{-}ft}/\mathrm{cp} \times 6.33E-03} + 1029.0\ \mathrm{psi} = 1076.78\ \mathrm{psi}$$

$$t = \infty:\quad P_{BHP} = \frac{Q_i}{J_i} + P_i = \frac{1000\,\mathrm{ft}^3/\mathrm{day}}{3306.5\,\mathrm{mD\text{-}ft}/\mathrm{cp} \times 6.33E-03} + 1441.2\ \mathrm{psi} = 1488.98\ \mathrm{psi}$$

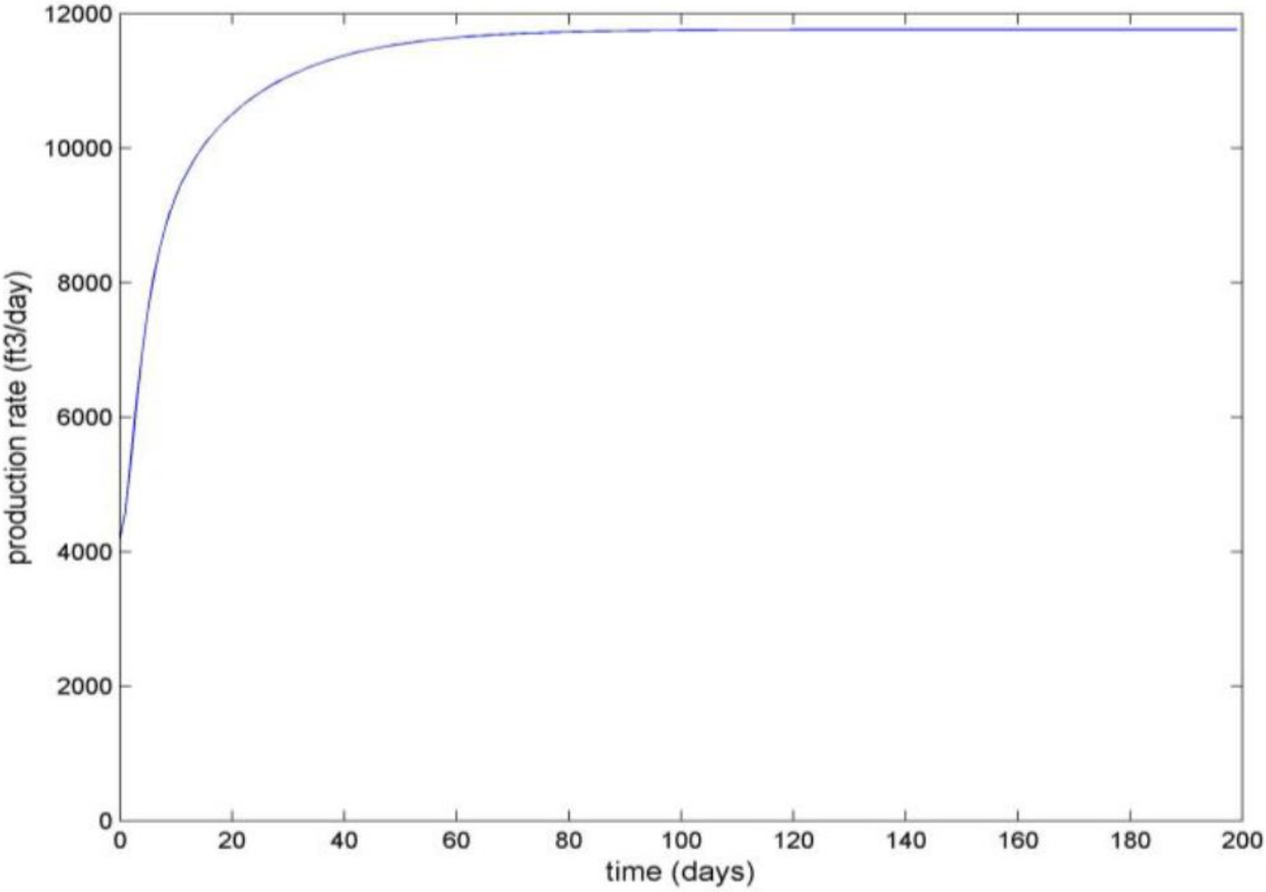


Figure 8.1. Production rate of the constant BHP well placed at 6250 feet (block #3). The rate of the well increases with time because the reservoir pressure increases with time. Fluids are being injected both through the injector well ($x = 0$ feet) and from the boundary condition ($x = L$). Eventually, the reservoir reaches steady state; fluid produced equals that injected and no change/increase in rate are observed with time

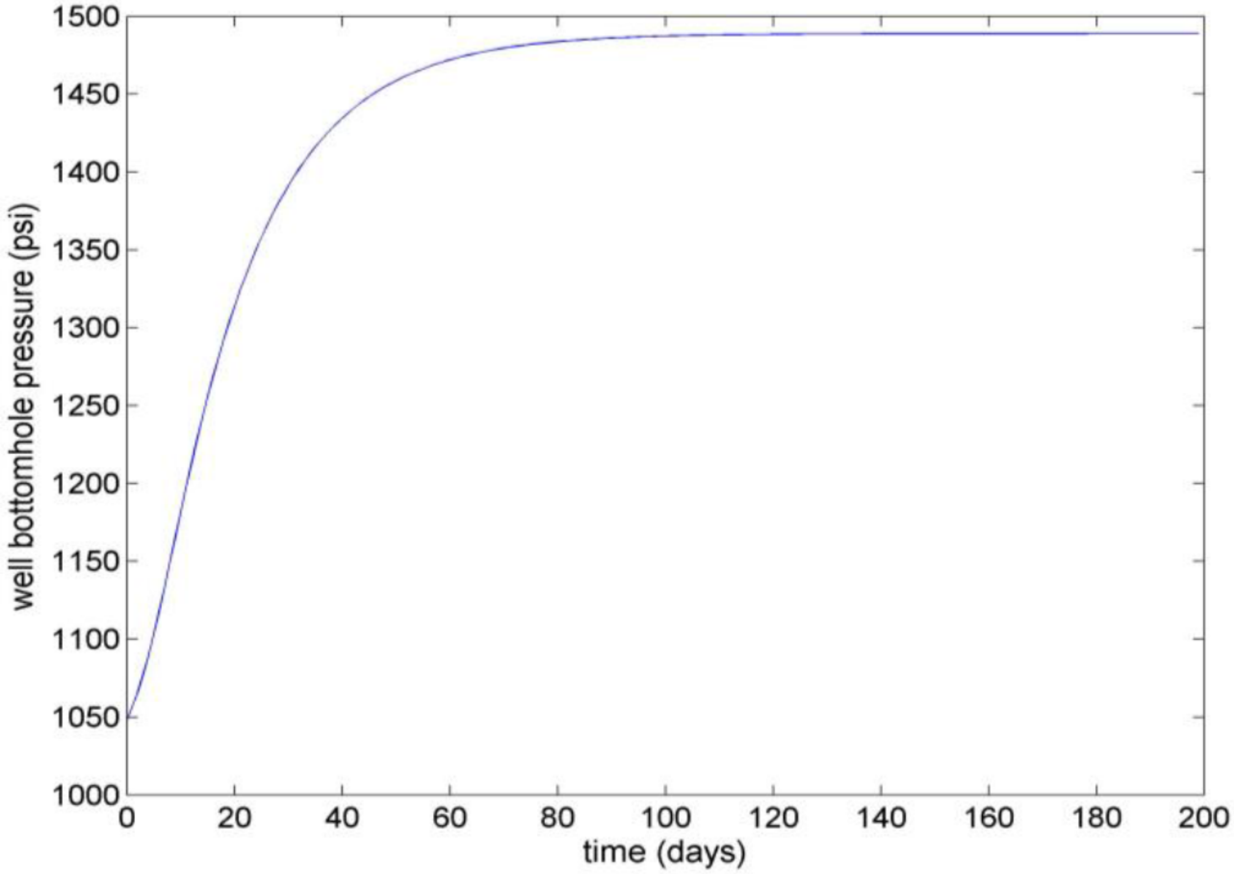


Figure 8.2. Well pressure of the constant rate well placed at 0 feet (block #1). Since the reservoir pressure (and block #1 pressure) increase with time, the well BHP must increase with time to maintain the constant rate of 1000 ft^3/day. The bottomhole pressure reaches "steady state" at late times.