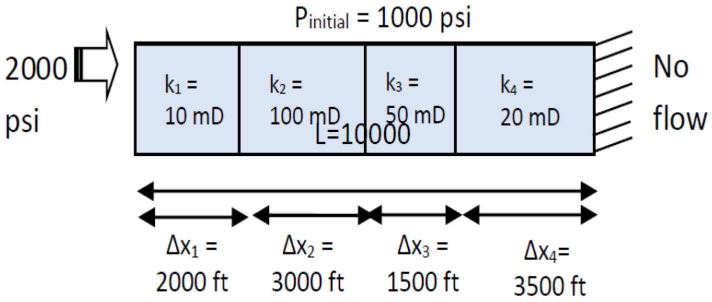
Example 7. Implicit solution to heterogeneous reservoir with variable grid sizes

$\phi = 0.2$
L = 10000 ft
A = 200,000 ft^2.
μ = 1 cp,
B_w = 1 rb/stb (assume constant)
$c_t = 10^{-6}$ psi^{-1}.



Heterogeneous permeability (k_1 = 10 mD; k_2 = 100 mD; k_3 = 50 mD; k_4 = 20 mD)
Non-uniform uniform-sized blocks (Δx_1 = 2000 ft; Δx_2= 3000 ft; Δx_3= 1500 ft; Δx_4= 3500 ft).

The initial condition is P = 1000 psi. The boundary conditions are P = 2000 psi at x = 0 and no flow (q = 0) x = L. Determine the pressure field in the reservoir. Use a time step of Δt = 1.0 days.

Note: the permeability is a function of position, k(x), so the PDE must keep the permeability inside the derivative.

$$\phi c_t \frac{\partial p}{\partial t} = \frac{\partial}{\partial x}\left(\frac{k}{\mu} \frac{\partial p}{\partial x} \right)$$

$$IC : \mathrm{p}(x,0) = 1000\ psi$$

$$BC1 : \mathrm{p}(0,t) = 2000\ psi$$

$$BC2 : \frac{\partial p}{\partial x}(L,t) = 0$$

The harmonic mean of permeabilities now involves weighting of grid block sizes

$$k_{3/2} = \left(\frac{\Delta x_1 + \Delta x_2}{\frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2}} \right) = \left(\frac{2000 + 3000}{\frac{2000}{10} + \frac{3000}{100}} \right) = 21.74 \text{ mD}$$

$$k_{5/2} = \left(\frac{\Delta x_2 + \Delta x_3}{\frac{\Delta x_2}{k_2} + \frac{\Delta x_3}{k_3}} \right) = \left(\frac{3000 + 1500}{\frac{3000}{100} + \frac{1500}{50}} \right) = 75 \text{ mD}$$

$$k_{7/2} = \left(\frac{\Delta x_3 + \Delta x_4}{\frac{\Delta x_3}{k_3} + \frac{\Delta x_4}{k_4}} \right) = \left(\frac{150 + 350}{\frac{1500}{50} + \frac{3500}{20}} \right) = 24.39 \text{ mD}$$

Half transmissibilities can now be computed:

$$T_{3/2} = \frac{k_{3/2} A}{\mu B_w \left(\Delta x_1 + \Delta x_2 \right)/2} = \frac{21.7 \cdot 200000}{1 \cdot 1 \cdot 2500} = 1739 \ \frac{\text{mD-ft}}{\text{cp}}$$

$$T_{5/2} = \frac{k_{5/2} A}{\mu B_w \left(\Delta x_2 + \Delta x_3 \right)/2} = \frac{75 \cdot 200000}{1 \cdot 1 \cdot 2250} = 6666.7 \ \frac{\text{mD-ft}}{\text{cp}}$$

$$T_{7/2} = \frac{k_{7/2} A}{\mu B_w \left(\Delta x_3 + \Delta x_4 \right)/2} = \frac{24.4 \cdot 200000}{1 \cdot 1 \cdot 2500} = 1951 \ \frac{\text{mD-ft}}{\text{cp}}$$

The boundary transmissibilities can be calculated using the boundary conditions:

(1)At $x = 0$, the pressure is constant. $T_{1/2}=2T_1 = 2000$ mD-ft/cp

(2)At $x = L$, there is no flow (no transmissibility). Therefore $T_{9/2} = 0$

So the matrices and vectors can now be computed. Note that the "B" matrix has also changed because Δx (and therefore block volume) varies.

$$\mathbf{T} = \begin{pmatrix} 2\cdot 1000+1739 & -1739 & 0 & 0 \\ -1739 & 1739+6667 & -6667 & 0 \\ 0 & -6667 & 6667+1951 & -1951 \\ 0 & 0 & -1951 & 1951 \end{pmatrix} \times 6.33E-3\frac{ft^3}{day-psi}$$

$$\mathbf{B} = \begin{pmatrix} 80 & & & \\ & 120 & & \\ & & 60 & \\ & & & 140 \end{pmatrix}\frac{ft^3}{psi};\quad \mathbf{P}^0 = \begin{pmatrix} 1000 \\ 1000 \\ 1000 \\ 1000 \end{pmatrix} psi;\quad \mathbf{Q} = \begin{pmatrix} 2\cdot 1000\cdot 2000\cdot 6.33E-03 \\ 0 \\ 0 \\ 0 \end{pmatrix}\frac{ft^3}{day}$$

Table 7.1 Summary of block pressures for example 7 using implicit method

	Method	**Block #1 (psi) 1000 ft**	**Block #2 (psi) 3500 ft**	**Block #3 (psi) 5750 ft**	**Block #4 (psi) 8250 ft**
Initial		1000	1000	1000	1000
1 day	Implicit	1123.0	1008.6	1003.2	1000.3
	CMG	1122.9	1008.6	1003.2	1000.3
2 days	Implicit	1219.4	1022.3	1010	1001.1
	CMG	1219.3	1022.3	1010	1001
3 days	Implicit	1295.6	1039.1	1019.9	1002.6
	CMG	1295.4	1039.1	1019.9	1002.6
∞		2000	2000	2000	2000