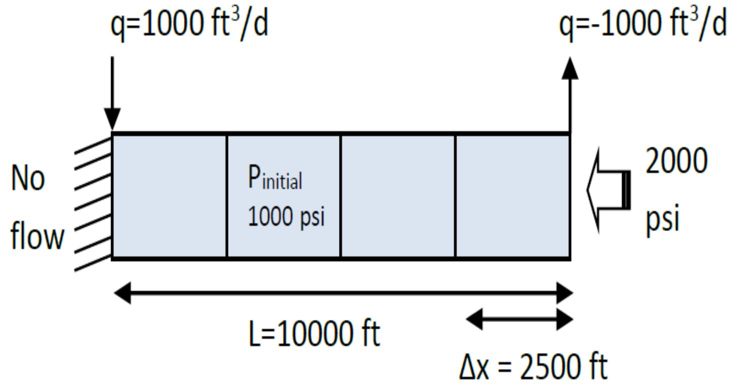
Example 5. Implicit Solution to 1D flow with constant rate wells.

Consider a 1D reservoir with the following reservoir and fluid properties:

$\phi = 0.2$
k = 50 mD
μ =1 cp
B_w = 1 rb/stb (assume constant)
$c_t = 10^{-6}$ psi^{-1}.
L= 10,000 ft (reservoir length)
A = 200,000 ft^2 (Cross Sectional Area)



The initial condition is P = 1000 psi. The boundary conditions are no flow (q = 0) at x = 0 and P = 2000 psi at x = L (note: these boundary conditions are different than in the previous examples). There is an injection well of 1000 ft^3/day at x = 0 and a producer of 1000 ft^3/day at x = L. Determine the pressure field in the reservoir using 4 uniform blocks. Use a time step of Δt = 1.0 days.

<u>Solution:</u>
The pressure is governed by the 1D diffusivity equation with sources and sinks and the following boundary conditions:

$$\frac{1}{\alpha}\frac{\partial p}{\partial t} = \frac{\partial^2 p}{\partial x^2} + q; \quad \alpha = \left(\frac{k}{\mu\phi c_t}\right)$$

$$IC : \mathrm{p}(x,0) = 1000\ psi$$

$$BC1 : \frac{\partial p}{\partial x}(0,t) = 0$$

$$BC2 : \mathrm{p}(L,t) = 2000\ psi$$

There are 2 key differences between this problem and the previous examples (1-4).

(1) The boundary conditions are flipped; no flux at $x = 0$ and constant pressure at $x = L$. This will affect both the T matrix and Q vector

(2) There are constant rate wells (sources and sinks) in the reservoir. These are treated by including in the source (Q) vector

Recall the Control Volume approach shows that a mass balance on each block is given by:

$$T\left(P_{i-1} - P_i\right) - T\left(P_i - P_{i+1}\right) = \frac{B}{\Delta t}\left(P_i^{n+1} - P_i^n\right) + q_i$$

Writing the equation implicitly for each block we get a system of equations:

$$T\left(P_0^{n+1}-P_1^{n+1}\right)+T\left(P_2^{n+1}-P_1^{n+1}\right)=\frac{B}{\Delta t}\left(P_1^{n+1}-P_1^{n}\right)+q_1$$

$$T\left(P_1^{n+1}-P_2^{n+1}\right)+T\left(P_3^{n+1}-P_2^{n+1}\right)=\frac{B}{\Delta t}\left(P_2^{n+1}-P_2^{n}\right)+q_2$$

$$T\left(P_2^{n+1}-P_3^{n+1}\right)+T\left(P_4^{n+1}-P_3^{n+1}\right)=\frac{B}{\Delta t}\left(P_3^{n+1}-P_3^{n}\right)+q_3$$

$$T\left(P_3^{n+1}-P_4^{n+1}\right)+T\left(P_5^{n+1}-P_4^{n+1}\right)=\frac{B}{\Delta t}\left(P_4^{n+1}-P_4^{n}\right)+q_4$$

Sources and sinks

The position of the wells is used to determine which blocks they reside in. Here, there is an injector in block #1 and a producer in block #4. Therefore q_1=-q_4 = 1000 ft^3/day (Recall our convention that an injector is positive). There are no wells in block #2 or #3 (q_2=q_3=0).

Boundary conditions

In block #1 we have a no-flow boundary condition at x = 0. That means that the first term of equation # vanishes, i.e.

$$T\left(P_0^{n+1}-P_1^{n+1}\right)=0$$

In block #4 we have a constant pressure at the boundary. We don't know P_5 (block #5 doesn't exist), but we know P = P_{out} at the boundary. The boundary is "half-way" in between blocks 4 and 5 and that would mean the effective transmissibility is twice as large. Therefore we can say:

$$T\left(P_4^{n+1}-P_5^{n+1}\right)=2T\left(P_4^{n+1}-P_{out}\right)=0$$

In matrix form we get the usual:

$$\left(\mathbf{T}+\frac{\mathbf{B}}{\Delta t}\right)\mathbf{P}^{n+1}=\frac{\mathbf{B}}{\Delta t}\mathbf{P}^{n}+\mathbf{Q}$$

Where

$$\mathbf{T}=\begin{pmatrix} 4000 & -4000 & 0 & 0 \\ -4000 & 8000 & -4000 & 0 \\ 0 & -4000 & 8000 & -4000 \\ 0 & 0 & -4000 & 12000 \end{pmatrix}\times 6.33E-3\frac{ft^3}{day-psi}$$

$$\mathbf{B}=\begin{pmatrix} 100 & & & \\ & 100 & & \\ & & 100 & \\ & & & 100 \end{pmatrix}\frac{ft^3}{psi};\quad \mathbf{P}^0=\begin{pmatrix} 1000 \\ 1000 \\ 1000 \\ 1000 \end{pmatrix}psi;\quad \mathbf{Q}=\begin{pmatrix} 1000 \\ 0 \\ 0 \\ 6.33\text{E-}03\cdot 2\cdot 2000\cdot 4000-1000 \end{pmatrix}\frac{ft^3}{day}$$

Table 5.1 Summary of block pressures for example 5

	Method	**Block #1 (psi) 1250 ft**	**Block #2 (psi) 3750 ft**	**Block #3 (psi) 6250 ft**	**Block #4 (psi) 8750 ft**
Initial		1000	1000	1000	1000
1 day	Implicit	1010	1010.1	1050.3	1289.4
	CMG	1010	1010.1	1050.3	1289.2
2 days	Implicit	1022	1030	1116.3	1463.3
	CMG	1022	1030	1116.3	1463
3 days	Implicit	1037.1	1056.9	1182.9	1571.7
	CMG	1037	1056.8	1182.8	1571.4

The pressure in block #1 begins to rise immediately above the initial pressure (1000 psi) as a result of the injector well at $x = 0$. Eventually it also begins to feel the effects of the constant pressure boundary condition at $x = L$. At $x = L$, the pressure rises quickly because if the P = 2000 psi boundary condition, but the increase is somewhat mitigated by the producer well also at $x = L$.

The implicit solution obtained by hand is again almost identical to CMG, with very small differences near the dirichlet boundary condition.