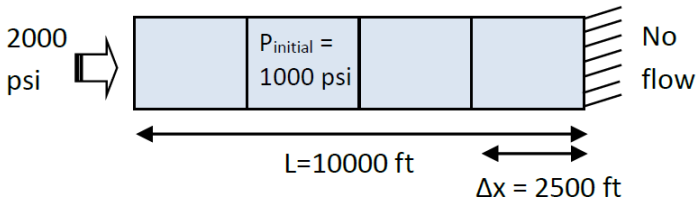
Lecture 18

Example 3.Explicit/Implicit Solution to 1D flow in terms of transmissibility (flow units)

Consider a 1D reservoir with the following reservoir and fluid properties:

$\phi = 0.2$
k = 50 mD
μ =1 cp
B_w = 1 rb/stb (assume constant)
$c_t = 10^{-6}$ psi^{-1}.
L= 10000 ft (reservoir length)
A = 200,000 ft^2 (Cross Sectional Area)



The initial condition is P = 1000 psi. The boundary conditions are P = 2000 psi at x = 0 and no flow (q = 0) at x = L. Determine the pressure field in the reservoir using 4 uniform blocks. Use a time step of Δt = 1 days.

Solution:
The pressure is governed by the 1D diffusivity equation with the following boundary conditions:

$$\frac{1}{\alpha}\frac{\partial P}{\partial t}=\frac{\partial^2 P}{\partial x^2};\quad \alpha=\left(\frac{k}{\mu\phi c_t}\right)$$

$$IC: P(x,0)=1000\ psi$$

$$BC1: P(0,t)=2000\ psi$$

$$BC2: \frac{\partial P}{\partial x}(L,t)=0$$

The explicit and implicit finite difference solution in flow rate units is given by the matrix equations

$$\mathbf{P}^{n+1}=\mathbf{P}^{n}-\Delta t\mathbf{B}^{-1}\left(\mathbf{T}\mathbf{P}^{n}+\mathbf{Q}\right)$$

$$\left(\mathbf{T}+\frac{\mathbf{B}}{\Delta t}\right)\mathbf{P}^{n+1}=\frac{\mathbf{B}}{\Delta t}\mathbf{P}^{n}+\mathbf{Q}$$

Or, (the implicit equations)

$$\left[\begin{pmatrix} 3T & -T & 0 & 0 \\ -T & 2T & -T & 0 \\ 0 & -T & 2T & -T \\ 0 & 0 & -T & T \end{pmatrix} + {}^{1}\!/\!_{\Delta t}\begin{pmatrix} B & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & B \end{pmatrix}\right]\begin{pmatrix} P_1^{n+1} \\ P_2^{n+1} \\ P_3^{n+1} \\ P_4^{n+1} \end{pmatrix} = {}^{1}\!/\!_{\Delta t}\begin{pmatrix} B & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & B \end{pmatrix}\begin{pmatrix} P_1^{n} \\ P_2^{n} \\ P_3^{n} \\ P_4^{n} \end{pmatrix} + \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{pmatrix}$$

Where:

$$T = \frac{kA}{\mu \Delta x} = \frac{50 \cdot 200000}{1 \cdot 1 \cdot 2500} = 4000 \frac{mD - ft}{cp}$$

$$B_i = V_i \phi c_t = \left(200000\, ft^2\right)\left(2500\, ft\right)\left(0.2\right)\left(1E-6\, psi^{-1}\right) = 100.0 \frac{ft^3}{psi}$$

$$Q_1 = 2TP_{in} = 2 \cdot 4000 \cdot 2000 = 1.6E7 \frac{mD - ft - psi}{cp}$$

Therefore the matrices and vectors can be written as:

$$\mathbf{T} = \begin{pmatrix} 12000 & -4000 & 0 & 0 \\ -4000 & 8000 & -4000 & 0 \\ 0 & -4000 & 8000 & -4000 \\ 0 & 0 & -4000 & 4000 \end{pmatrix} \times 6.33E-3 \frac{ft^3}{day - psi}$$

$$\mathbf{B} = \begin{pmatrix} 100 & & & \\ & 100 & & \\ & & 100 & \\ & & & 100 \end{pmatrix} \frac{ft^3}{psi}$$

$$\mathbf{P}^0 = \begin{pmatrix} 1000 \\ 1000 \\ 1000 \\ 1000 \end{pmatrix} psi; \quad \mathbf{Q} = \begin{pmatrix} 2 \cdot 4000 \cdot 2000 \\ 0 \\ 0 \\ 0 \end{pmatrix} \times 6.33E-03 \frac{ft^3}{day}$$

Note that T, B, and Q never change with time. Only the pressure from the previous time step changes. The solution for block pressures (explicitly or implicitly) is summarized in Table 3.1

Table 3.1 Summary of block pressures for various solution techniques

	Method	**Block #1 (psi) 1250 ft**	**Block #2 (psi) 3750 ft**	**Block #3 (psi) 6250 ft**	**Block #4 (psi) 8750 ft**
Initial		1000	1000	1000	1000
1 days	Explicit	1506.4	1000	1000	1000
	Implicit	1295.1	1051.1	1008.9	1001.8

	CMG	1294.9	1051.1	1008.9	1001.8
	Analytical	1482.3	1035.0	1000.4	1000.0
2 days	Explicit	1628.1	1128.2	1000	1000
	Implicit	1472.5	1117.9	1026.9	1006.9
	CMG	1472.2	1117.8	1026.8	1006.9
	Analytical	1619.3	1136.1	1013.0	1000.5
3 days	Explicit	1689.9	1222.3	1032.5	1000
	Implicit	1582.9	1184.9	1051.6	1015.9
	CMG	1582.6	1184.8	1051.5	1015.9
	Analytical	1685	1223.6	1042.5	1004.8
∞		2000	2000	2000	2000

A few important notes about table 3.1

- Solutions for explicit and implicit method are exactly the same as example 1 and 2, respectively. Example 3 just worked in different units (flow rate instead of pressure)
- We get different answers for the explicit and implicit solution, but neither is "more accurate" than the other.
- CMG is nearly identical to the solution obtained using the implicit method by hand. This is because CMG uses the implicit method and same equations. The very slight differences are a result of the fact that CMG doesn't allow for true "constant pressure" boundary conditions
- We could improve the accuracy of both the explicit and implicit method by using smaller time steps and smaller grids and both would converge to the analytical solution
- However, the explicit method would become unstable if the grid size were reduced without reducing the time steps in an appropriate fashion.