Taylor series and random variables

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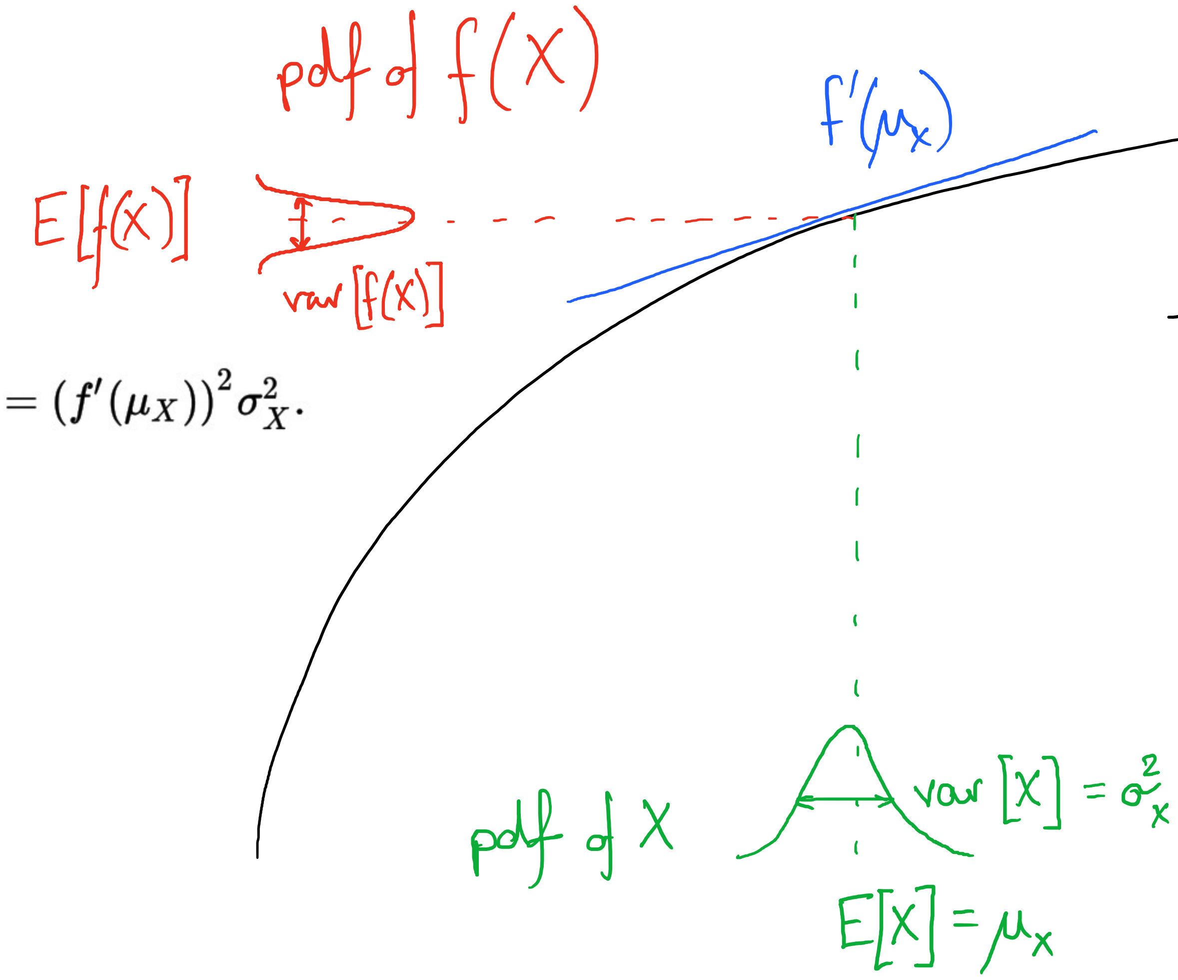
See: [Taylor expansions for the moments of functions of random variables]

$$\mathbf{E}[f(X)] \approx f(\mu_X) + \frac{f''(\mu_X)}{2}\sigma_X^2$$

$$\mathbf{var}[f(X)] \approx (f'(\mathbf{E}[X]))^2\,\mathbf{var}[X] = (f'(\mu_X))^2\sigma_X^2.$$



$f(x) = \log(x)$

$f'(x) = \frac{1}{x}$

The Taylor series approximation of $f(x)$ at $x = \mu_X$ "linearises" the curve.

So, if the probability density function of the random variable X has the first and second moments μ_X and σ_X^2 then the first and second moments of $f(X)$ are $E[f(X)]$ and $\mathrm{var}[f(X)]$.

When $f(x) = \log(x)$, $f'(x) = \frac{1}{x}$ so as μ_X gets smaller $f'(\mu_X)$ gets larger!