

EE5600 Assignment 3

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Abstract—This document contains the solution to a Gradient decent question.

Download all python codes from

<https://github.com/Jayanth9969/EE5600/blob/master/Assignment4/code.py>

1 PROBLEM

Find the maximum and minimum values, if any of the following function

$$f(x) = -(x - 1)^2 + 10$$

2 SOLUTION

Given $f(x) = -(x - 1)^2 + 10$

A function is said to be convex if following inequality is true:

$$\lambda f(x_1) + (1 - \lambda)f(x_2) \geq f(\lambda x_1 + (1 - \lambda)x_2) \quad (2.0.1)$$

or we can check that is $f''(x) > 0$ then the function is convex else the function will be concave

$$f(x) = -(x - 1)^2 + 10 \quad (2.0.2)$$

$$f'(x) = -2(x - 1) \quad (2.0.3)$$

$$f''(x) = -2 \quad (2.0.4)$$

Since $f''(x) < 0$ so $f(x)$ is concave function. So we can find the maximum using gradient ascent method. Since function is concave it has only maximum and minimum will be -infinity we can also find the maximum as following, Since derivative of $f(x)$ at maximum is zero.

$$f'(x) = -2(x - 1) \quad (2.0.5)$$

$$0 = -2(x - 1) \quad (2.0.6)$$

$$x = 1 \quad (2.0.7)$$

Thus maximum occurs at $x = 1$

$$x = 1 \quad (2.0.8)$$

$$f(x) = -(x - 1)^2 + 10 \quad (2.0.9)$$

$$f(1) = -(1 - 1)^2 + 10 \quad (2.0.10)$$

$$f(1) = 10 \quad (2.0.11)$$

Thus maximum attainable value of $f(x)$ is 10 at $x = 1$

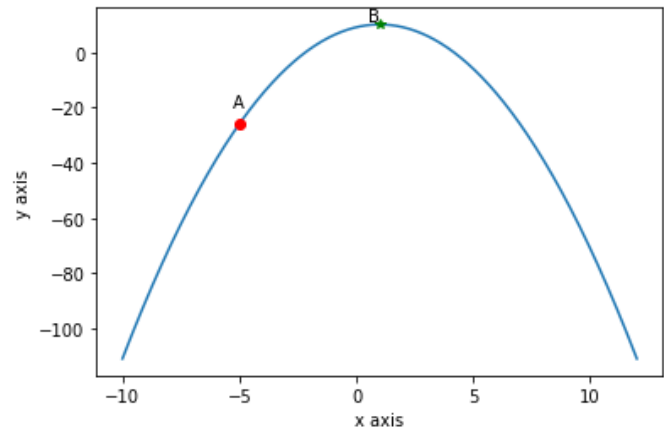


Fig. 2.0: Here B is maximum point and A is initial answer in gradient ascent method

The solution can found using gradient ascent method is as follows:

Let us Suppose at $x = -5$ (initial guess) be point where maximum value was attained, thus $x_0 = -5$. Our Update Equation will be:

$$x_1 = x_0 + f'(x_0) \times \text{rate} \quad (2.0.12)$$

$$\text{error} = |x_0 - x_1| \quad (2.0.13)$$

$$x_0 = x_1 \quad (2.0.14)$$

Here, rate = 0.1

we do update our x_0 until error between x_0 and x_1 becomes less than 0.000000001.

In this case using code.py, we got answer after 95 iterations. we got maximum value attained is 10.0 occurred at $x = 0.9999999962700757$.