EE5600 Assignment 3

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***Abstract*—This document contains the solution to a Gradient decent question.**

1 Problem

Find the maximum and minimum values, if any of the following function
$f(x) = -(x-1)^2 + 10$

2 Solution

A function is said to be convex if following inequality is true:

$$\lambda f(x_1) + (1-\lambda)f(x_2) \geq f(\lambda x_1 + (1-\lambda)x_2) \tag{2.0.1}$$

and for $\lambda \in [0,1]$

$$f(x) = -(x-1)^2 + 10 \tag{2.0.2}$$

$$\lambda(-(x_1-1)^2 + 10) + (1-\lambda)(-(x_2-1)^2 + 10) \geq -(\lambda x_1 + (1-\lambda)(x_2-1))^2 + 10 \tag{2.0.3}$$

$$-\lambda(x_1-1)^2 + 10\lambda - (x_2-1)^2 + 10 + \lambda(x_2-1)^2 - 10\lambda \geq -(\lambda(x_1-1)^2 + (1-\lambda)(x_2-1))^2 + 10 \tag{2.0.4}$$

$$-\lambda(x_1-1)^2 - (x_2-1)^2 + \lambda(x_2-1)^2 \geq -(\lambda^2(x_1-1)^2 + (1-\lambda)^2(x_2-1)^2 + 2\lambda(1-\lambda)(x_1-1)(x_2-1) \tag{2.0.5}$$

$$-\lambda(x_1-1)^2 - (x_2-1)^2 + \lambda(x_2-1)^2 \geq -(x_1-1)^2(-\lambda^2 + \lambda) - (x_2-1)^2(-\lambda^2 + \lambda) - 2\lambda(1-\lambda)(x_1-1)(x_2-1) \tag{2.0.6}$$

$$0 \geq (\lambda^2 - \lambda)((x_1-1)^2 + (x_2-1)^2 + 2\lambda(1-\lambda)(x_1-1)(x_2-1) \tag{2.0.7}$$

$$0 \geq \lambda(1-\lambda)((x_1-1) - (x_2-1))^2 \tag{2.0.8}$$

The above inequality is not true for all values from domain. Hence the given function f(x) is concave.So we gonna have only maximum wrt to given domain.
we can find the maximum as following, Since derivative of f(x) at maximum is zero.

$$f'(x) = -2(x-1) \tag{2.0.9}$$

$$0 = -2(x-1) \tag{2.0.10}$$

$$x = 1 \tag{2.0.11}$$

Thus maximum occurs at x = 1

$$x = 1 \tag{2.0.12}$$

$$f(x) = -(x-1)^2 + 10 \tag{2.0.13}$$

$$f(1) = -(1-1)^2 + 10 \tag{2.0.14}$$

$$f(1) = 10 \tag{2.0.15}$$

Thus maximum attainable value of f(x) is 10 at x = 1

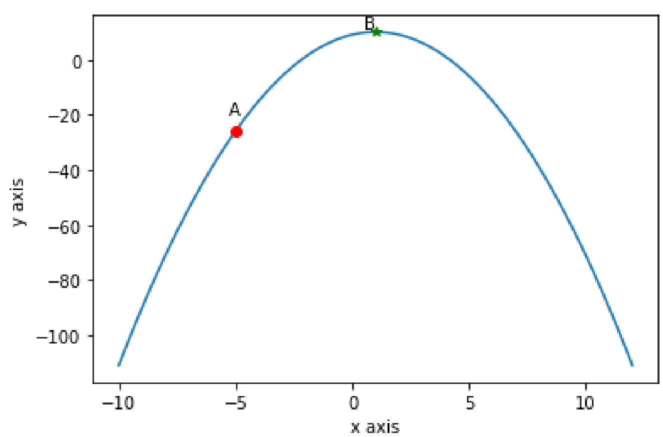

Fig. 2.0: Here B is maximum point and A is intial answer in gradient ascent method

The solution can found using gradient ascent method is as follows:
Let us Suppose at x = -5 (initial guess)be point where maximum value was attained,thus $x_0 =$

−5. Our Update Equation will be:

$$x_1 = x_0 + f'(x_0) \times \mu \tag{2.0.16}$$
$$x_1 = x_0 - 2\mu(x - 1) \tag{2.0.17}$$
$$error = |(x_0 - x_1)| \tag{2.0.18}$$
$$x_0 = x_1 \tag{2.0.19}$$

Here, μ is rate of convergence
we do update our x_0 until error between x_0 and x_1 becomes less than 0.000000001.
In this case using using $\mu = 0.1$ and code.py ,we got answer after 95 iterations.we got maximum value attained is 10.0 occured at x = 0.9999999962700757 .

Download all python codes from

https://github.com/Jayanth9969/EE5600/blob/master/Assignment4/code.py