Voice recognition Algorithm

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$$y' = sigmoid(W.X + B)$$

$$y'i = sigmoid(\sum_{i} Wi * xi + b)$$

$$sigmoid(x) = 1/(1 + e^{-x})$$

$$sigmoidprime(x) = \frac{\partial sigmoid(x)}{\partial x}$$

$$= \frac{\partial}{\partial x} \frac{1}{1 + e^{-x}}$$

$$= \frac{e^{-x}}{(1 + e^{-x})^2}$$

$$= \frac{e^{-x}}{1 + e^{-x}} \frac{1}{1 + e^{-x}}$$

$$= \frac{e^{-x}}{1 + e^{-x}} \frac{1}{1 + e^{-x}}$$

$$= \frac{1 - 1 + e^{-x}}{1 + e^{-x}} \frac{1}{1 + e^{-x}}$$

$$= (1 - \frac{1}{1 + e^{-x}}) \frac{1}{1 + e^{-x}}$$

$$= (1 - sigmoid(x))(sigmoid(x))$$

1 Gradient Descent method

$$J(W, b) = \frac{1}{2} \|y - \hat{y}\|^2$$
$$Wi := Wi - \alpha \frac{\partial J}{\partial W}$$
$$b := b - \alpha \frac{\partial J}{\partial b}$$

Partial differentiation terms are evaluated as follows:

$$\frac{\partial J}{\partial W} = \frac{\partial}{\partial W} \frac{1}{2} \|y - \hat{y}\|^2$$

$$\frac{\partial J}{\partial W} = \frac{\partial}{\partial W} \frac{1}{2} (y - \hat{y})^T (y - \hat{y})$$
$$\frac{\partial J}{\partial W} = \frac{1}{2} \frac{\partial}{\partial W} (\|y\|^2 - y^T \hat{y} - y \hat{y}^T + \|\hat{y}\|^2)$$

$$\begin{split} \frac{\partial \hat{y}}{\partial W} &= \frac{\partial}{\partial W} sigmoid(W*x+b) \\ \frac{\partial \hat{y}}{\partial W} &= sigmoidprime(W*x+b)x \\ \frac{\partial \hat{y}}{\partial b} &= sigmoidprime(W*x+b) \end{split}$$

Property:

$$\frac{\partial(xa)}{\partial x} = a$$
$$\frac{\partial(x^T x)}{\partial x} = 2x^T$$

$$= \frac{1}{2} \left(-\frac{\partial \hat{y}}{\partial W} y^T - \frac{\partial \hat{y}^T}{\partial W} y + \frac{\partial \hat{y}^T \hat{y}}{\partial W} \right)$$

$$= \frac{1}{2} \left(-2 \frac{\partial \hat{y}}{\partial W} y^T + 2 \frac{\partial \hat{y}}{\partial W} \hat{y}^T \right)$$

$$= \frac{\partial \hat{y}}{\partial W} \left(-y^T + \hat{y}^T \right)$$

$$= sigmoidprime(W * x + b)x(-y + \hat{y})^T$$

$$= sigmoidprime(W * x + b)((-y + \hat{y})x^T)^T$$

$$= ((-y + \hat{y})x^T)^T sigmoidprime(W * x + b)$$

where

$$sigmoidprime(x) = sigmoid(x) * (1 - sigmoid(x))$$

Similarly for b matrix coefficients:

$$\frac{\partial J}{\partial b} = \frac{1}{2} \frac{\partial}{\partial b} (\|y\|^2 - y^T \hat{y} - y \hat{y}^T + \|\hat{y}\|^2)$$

$$= \frac{1}{2} (-\frac{\partial \hat{y}}{\partial b} y^T - \frac{\partial \hat{y}^T}{\partial b} y + \frac{\partial \hat{y}^T \hat{y}}{\partial b})$$

$$= \frac{1}{2} (-2\frac{\partial \hat{y}}{\partial b} y^T + 2\frac{\partial \hat{y}}{\partial b} \hat{y}^T)$$

$$= \frac{\partial \hat{y}}{\partial b} (-y^T + \hat{y}^T)$$

$$= sigmoidprime(W * x + b)(-y + \hat{y})^T$$

$$= (-y + \hat{y})^T sigmoidprime(W * x + b)$$

for these gradient descent method are:

$$W := W - \alpha((-y + \hat{y})x^T)^T sigmoidprime(W * x + b)$$

$$b := b - \alpha(-y + \hat{y})^T sigmoidprime(W * x + b)$$

$$W := W + \alpha((y - \hat{y})x^T)^T sigmoidprime(W * x + b)$$
$$b := b + \alpha(y - \hat{y})^T sigmoidprime(W * x + b)$$