Linear Methods

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***Abstract*—This manual provides an introduction to linear lethods in regression.**

1 Introduction

1.1 Let

$$\mathbf{X}^T(\mathbf{y}-\mathbf{X}\mathbf{w}) = 0 \tag{1.1}$$

$$\hat{\mathbf{w}} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y} \tag{1.2}$$

where $\mathbf{w}$ is $p \times 1$ and $\mathbf{X}$ is $N \times p$. Show that

$$E(\hat{\mathbf{w}}) = \mathbf{w} \tag{1.3}$$

1.2 If the covariance matrix of $\mathbf{y}$ is

$$\mathbf{C}_{\mathbf{y}} = \sigma^2\mathbf{I} \tag{1.4}$$

show that

$$\mathbf{C}_{\mathbf{w}} = \sigma^2\left(\mathbf{X}^T\mathbf{X}\right)^{-1} \tag{1.5}$$

1.3 Let

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\mathbf{w}} \tag{1.6}$$

$$\hat{\sigma}^2 = \frac{1}{N-p}\|\mathbf{y}-\hat{\mathbf{y}}\|^2 \tag{1.7}$$

$$\mathbf{y}-\hat{\mathbf{y}} \sim \mathcal{N}\left(\mathbf{0},\sigma^2\mathbf{I}\right) \tag{1.8}$$

Show that

$$(N-p)\hat{\sigma}^2 \sim \sigma^2\chi^2_{N-p} \tag{1.9}$$

1.4 Let

$$\hat{z} = \frac{w_j}{\hat{\sigma}\sqrt{v_j}} \tag{1.10}$$

where v_j is the diagonal element of $\left(\mathbf{X}^T\mathbf{X}\right)^{-1}$. If $w_j = 0$, show that z_j has a t_{N-p} distribution.

1.5 Plot $\Pr(|Z| > z)$ for t_{30}, t_{100} and the standard normal distribution.

2 Applications

2.1 Explain how (**??**) can be used to obtain the Nearest Neighbour approximation.

2.2 Repeat the exercise for the least squares method.

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