# EE1390

## BFSK (Binary Frequency Shift Keying)

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# Introduction

## Frequency Shift Keying

Digital information is transmitted through discrete frequency changes of carrier signal

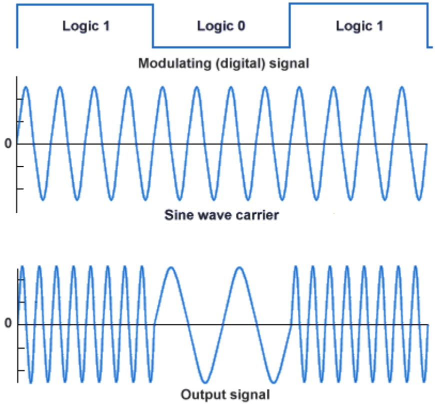



Figure: Example of Binary FSK

$x_0(t)$ to represent 0
$x_1(t)$ to represent 1

# Theory

Let $s_0$ be the component of signal for 0 in the received signal
Let $s_1$ be the component of signal for 1 in the received signal

**Ideal case**

*For 0*

$$\begin{pmatrix} s_0 \\ s_1 \end{pmatrix} = \begin{pmatrix} \sqrt{A} \\ 0 \end{pmatrix}$$

*For 1*

$$\begin{pmatrix} s_0 \\ s_1 \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{A} \end{pmatrix}$$

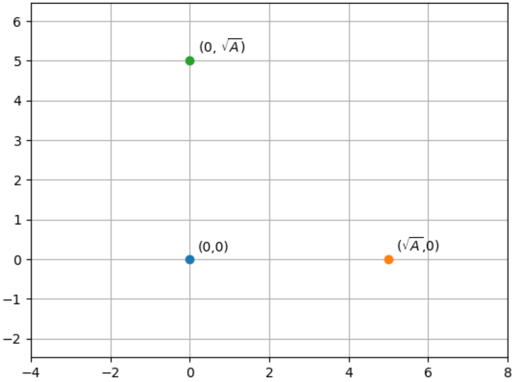



Figure: Ideal case

But normally, the channel adds noise to the signal.
Suppose the noise has the form of Additive White Gaussian Noise

$$\mathcal{N}(0, N)$$

The received components can then be modelled as

$$\mathbf{y} = \mathbf{s} + \begin{pmatrix} n_0 \\ n_1 \end{pmatrix}$$

where $N_0$ and $N_1$ are also Gaussian with 0 mean and finite variance.
Let us assume variance as 1 for convenience, i.e.,

$$N_0, N_1 \sim \mathcal{N}(0, 1)$$

*If 0 is sent*

$$\begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \begin{pmatrix} \sqrt{A} + n_0 \\ n_1 \end{pmatrix}$$

*If 1 is sent*

$$\begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \begin{pmatrix} n_0 \\ \sqrt{A} + n_1 \end{pmatrix}$$

# Simulation

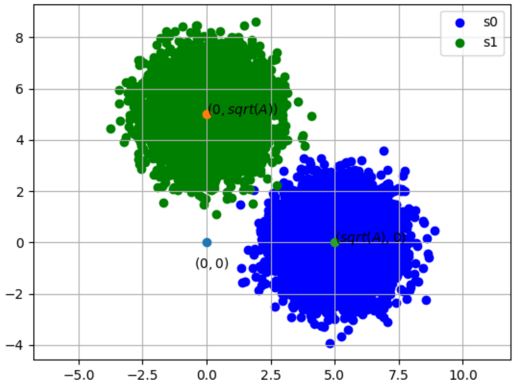



Figure: Simulation after addition of noise, if 0 and 1 are equi-probable

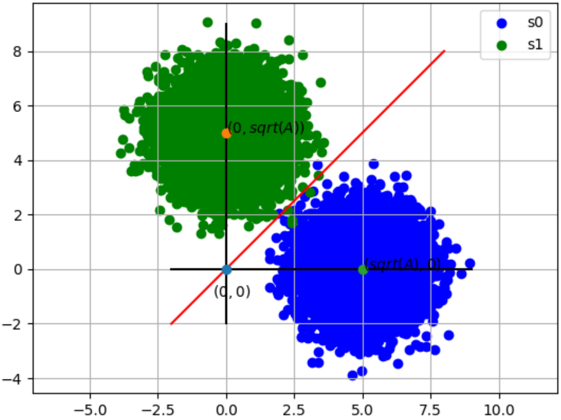
## Code

```
simlen = 1e4
a=5

s0 = np.array([1,0])
y0 = [((a*s0[j])+np.random.normal(0, 1, int(simlen))) for j in
range(2)]
# y0 is 2 x simlen array = ( sqrt(A)+n1 , n2)

s1 = np.array([0,1]) y1 = [((a*s1[j])+np.random.normal(0, 1,
int(simlen))) for j in range(2)]
# y1 is 2 x simlen array = ( n1 , sqrt(A)+n2)
```

# Guessing y



# Decision

**How do you guess s, i.e, $\hat{s}|$ y=?**

If $|y - s_0| < |y - s_1|$, decide s=$s_0$,
else, $s = s_1$

# Mathematical Derivation

## Multivariate Gaussian PDF

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp -\frac{1}{2\sqrt{1-\rho^2}} \times \frac{x-\mu_x{}^2}{\sigma_x^2} + \frac{y-\mu_y{}^2}{\sigma_y^2} - \frac{2\rho x-\mu_x y-\mu_y}{\sigma_x\sigma_y} \quad (1)$$

P(r|$s_o$)=P($r_1, r_2|s_o$)=P($\sqrt{A}$+n$_1, n_2$)
Let $\sqrt{(A)} = a$
E($r_1$) $= E(a+n_1) = a + E(n_1) = a$
E($r_2$) $= E(n_2) = E(n_2) = 0$

# cntd

$\sigma_{r_1} = \sigma_{a+n_1} = \sigma_{n_1} = \sigma$
$\sigma_{r_2} = \sigma_{n_2} = \sigma_{n_2} = \sigma$
Substituting the above values keeping in mind n1,n2 are independent
we get $\rho = 0$

$$p(r|s0) = \frac{1}{2\pi\sigma^2}\exp\frac{-1}{2} \times \frac{r_1 - a^2}{\sigma^2} + \frac{{r_2}^2}{\sigma^2} \quad (2)$$

# Decision

## Detecting s0

$\mathrm{P}(s_0|r) > P(s_1|r)$
Now apply Baye's conditional probility

$\mathrm{P}(s_0)\frac{P(r|s_0)}{P(r)} > P(s_1)\frac{P(r|s_1)}{P(r)}$

If $s_0$ and $s_1$ are equiprobable, the above expression reduces to
$\mathrm{P}(r|s_0) > \mathrm{P}(r|s_1)$

i.e, $-(r1-a)^2 - r2^2 > -(r1)^2 - (r2-a)^2$

$2r_1a > 2r_2a$

$r_2 < r_1$ for detecting s0