Statistical Decision Theory

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***Abstract*—This manual provides an introduction to statistical decision theory.**

1 Introduction

1.1 Let

$$Y = f(\mathbf{X}) \tag{1.1}$$

The *mean square error* (MSE) is defined as

$$MSE(f) = E\left[Y - f(\mathbf{X})\right]^2 \tag{1.2}$$

Show that

$$MSE(f) = E_{\mathbf{X}}\left\{E_Y\left[Y - f(\mathbf{x})\right]^2 |\mathbf{X} = \mathbf{x}\right\} \tag{1.3}$$

1.2 Let

$$c = f(\mathbf{x}) \tag{1.4}$$

Using (1.3)

$$\begin{aligned} \min MSE(f) &= \min MSE(f)|X \\ &= \min_c E_Y\left\{\left[Y - c\right]^2 |\mathbf{X} = \mathbf{x}\right\} \end{aligned} \tag{1.5}$$

Show that

$$\begin{aligned} MSE(f)|X &= E_Y\left\{\left[Y - c\right]^2 |\mathbf{X} = \mathbf{x}\right\} \\ &= -2cE_Y\left\{Y|\mathbf{X} = \mathbf{x}\right\} + E_Y\left\{Y^2|\mathbf{X} = \mathbf{x}\right\} + c^2 \end{aligned} \tag{1.6}$$

1.3 $MSE(f)$ is minimum when

$$\frac{d}{dc}MSE(f)|X = 0. \tag{1.7}$$

Show that this results in

$$c = f(\mathbf{x}) = E\left[Y|X = \mathbf{x}\right] \tag{1.8}$$

f is known as the *regression* function.

2 Applications

2.1 Explain how (1.8) can be used to obtain the Nearest Neighbour approximation.

2.2 Repeat the exercise for the least squares method.

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