Least Squares Method

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Contents

***Abstract*—This manual introduces the least squares method.**

1 Algebra

1.1 Find the equation of the plane P containing the vectors

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{a}_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \tag{1.1}$$

1.2 Show that the vector

$$\mathbf{b} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \tag{1.2}$$

lies outside P.

1.3 Find the point $\mathbf{b}_0 \in P$ closest to $\mathbf{b}$.

2 Application

2.1 The Steinhart–Hart equation is a model of the resistance of a thermistor at different temperatures. The equation is given by

$$\frac{1}{\tau} = w_1 + w_2 \ln(R) + w_3[\ln(R)]^3 \tag{2.1}$$

Let

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ \ln(R_1) \\ [\ln(R_1)]^3 \end{pmatrix} \tag{2.2}$$

$$y_1 = \frac{1}{\tau_1} \tag{2.3}$$

$$\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \tag{2.4}$$

Show that

$$y_1 = \mathbf{a}_1^T \mathbf{w} \tag{2.5}$$

2.2 Suppose for $n > 3$

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, A^T = \begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \vdots & \mathbf{a}_n \end{pmatrix}, \tag{2.6}$$

show that

$$\mathbf{y} = A\mathbf{w} \tag{2.7}$$

2.3 For $\tau = 10^{\circ}C - 100^{\circ}C$, use the PT-100 resistance table in

https://github.com/gadepall/EE1390/blob/master/refs/5pt100sensoren_e.pdf?raw=true

to estimate $\mathbf{w}$ using the relation

$$\hat{\mathbf{w}} = \left(A^T A\right)^{-1} A^T \mathbf{y} \tag{2.8}$$

2.4 Verify your result by finding the temperature when the resistance is 175.86Ω.

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