Supervised Learning

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Contents

***Abstract*—This manual introduces various methods in supervised learning.**

1 Least Squares Method

1.1 Application

1.1.1 The Steinhart–Hart equation is a model of the resistance of a thermistor at different temperatures. The equation is given by

$$\frac{1}{\tau} = w_1 + w_2 \ln(R) + w_3[\ln(R)]^3 \tag{1.1}$$

Let

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ \ln(R_1) \\ [\ln(R_1)]^3 \end{pmatrix} \tag{1.2}$$

$$y_1 = \frac{1}{\tau_1} \tag{1.3}$$

$$\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \tag{1.4}$$

Show that

$$y_1 = \mathbf{a}_1^T \mathbf{w} \tag{1.5}$$

1.1.2 Suppose for $n > 3$

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, A^T = \begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \vdots & \mathbf{a}_n \end{pmatrix}, \tag{1.6}$$

show that

$$\mathbf{y} = A\mathbf{w} \tag{1.7}$$

1.1.3 For $\tau = 10^\circ C - 100^\circ C$, use the PT-100 resistance table in

https://github.com/gadepall/EE1390/blob/master/refs/5pt100sensoren_e.pdf?raw=true

to estimate $\mathbf{w}$ using the relation

$$\hat{\mathbf{w}} = \left(A^T A\right)^{-1} A^T \mathbf{y} \tag{1.8}$$

1.1.4 Verify your result by finding the temperature when the resitance is 175.86Ω.

1.2 Calculus

1.2.1 Find

$$\|\mathbf{y} - A\mathbf{w}\|^2 \tag{1.9}$$

Solution:

$$\|\mathbf{y} - A\mathbf{w}\|^2 = (\mathbf{y} - A\mathbf{w})^T (\mathbf{y} - A\mathbf{w}) \tag{1.10}$$

$$= \|\mathbf{y}\|^2 - \mathbf{w}^T A^T \mathbf{y} - \mathbf{y}^T A\mathbf{w} + \mathbf{w}^T A^T A\mathbf{w} \tag{1.11}$$

1.2.2 Assuming 2×2 matrices and 2×1 vectors, show that

$$\frac{\partial}{\partial \mathbf{w}}\mathbf{w}^T A^T \mathbf{y} = \frac{\partial}{\partial \mathbf{w}}\mathbf{y}^T A\mathbf{w} = \mathbf{y}^T A \tag{1.12}$$

1.2.3 Show that

$$\frac{\partial}{\partial \mathbf{w}}\mathbf{w}^T A^T A\mathbf{w} = 2\mathbf{w}^T \left(A^T A\right) \tag{1.13}$$

1.2.4 Find

$$\min_{\mathbf{w}} \|\mathbf{y} - A\mathbf{w}\|^2 \tag{1.14}$$

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