

# Convex functions: Gradient Boost

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**Abstract**—This manual provides an introduction to the LMS algorithm.

## 1 CONVEX FUNCTIONS

A single variable function  $f$  is said to be convex if

$$f[\lambda x + (1 - \lambda)y] \leq \lambda f(x) + (1 - \lambda)f(y), \quad (1.1)$$

for  $0 < \lambda < 1$ .

1.1 Download and execute the following python script. Is  $\ln x$  convex or concave?

[https://raw.githubusercontent.com/gadepall/EE1390/master/manuals/supervised/gradient\\_boost/codes/1.1.py](https://raw.githubusercontent.com/gadepall/EE1390/master/manuals/supervised/gradient_boost/codes/1.1.py)

1.2 Modify the above python script as follows to plot the parabola  $f(x) = x^2$ . Is it convex or concave?

[https://raw.githubusercontent.com/gadepall/EE1390/master/manuals/supervised/gradient\\_boost/codes/1.2.py](https://raw.githubusercontent.com/gadepall/EE1390/master/manuals/supervised/gradient_boost/codes/1.2.py)

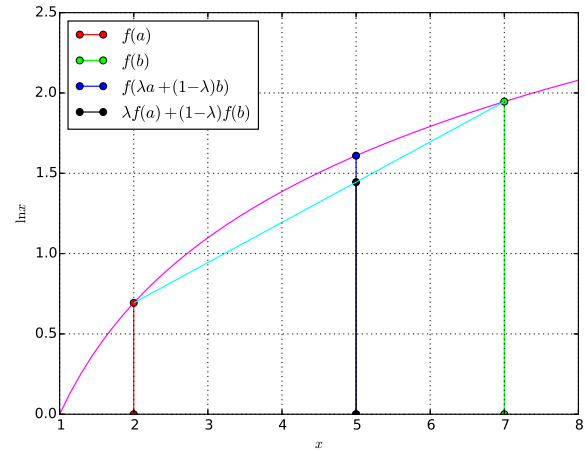


Fig. 1.1:  $\ln x$  versus  $x$

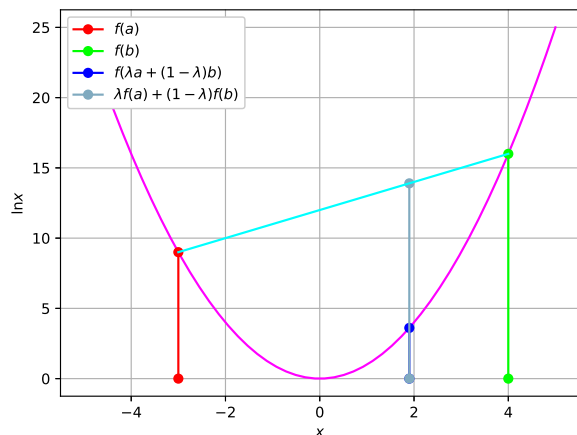


Fig. 1.2:  $x^2$  versus  $x$

1.3 Execute the following script to obtain Fig. 1.3. Comment.

<https://raw.githubusercontent.com/gadepall/EE1390/master/manuals/supervised/>

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gradient\_boost/codes/1.3.py

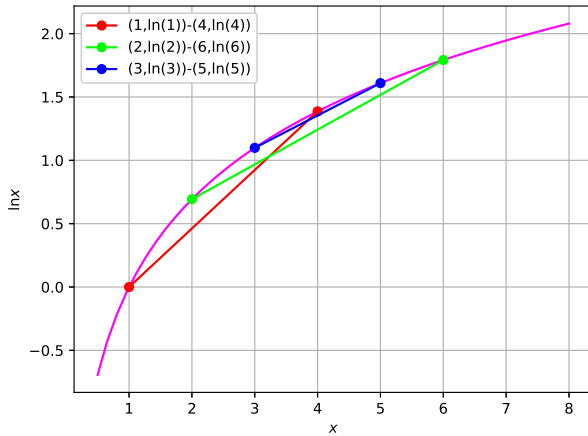


Fig. 1.3: Segments are below the curve

1.4 Modify the script in the previous problem for  $f(x) = x^2$ . What can you conclude?

1.5 Let

$$f(\mathbf{x}) = x_1 x_2, \quad \mathbf{x} \in \mathbf{R}^2 \quad (1.2)$$

Sketch  $f(\mathbf{x})$  and deduce whether it is convex. Can you theoretically explain your observation using (1.1)?

## 2 GRADIENT DESCENT METHOD

Consider the problem of finding the square root of a number  $c$ . This can be expressed as the equation

$$x^2 - c = 0 \quad (2.1)$$

2.1 Sketch the function for different values of  $c$

$$f(x) = x^3 - 3xc \quad (2.2)$$

and comment upon its convexity.

2.2 Show that (2.1) results from

$$\min_x f(x) = x^3 - 3xc \quad (2.3)$$

2.3 Find a numerical solution for (2.1).

**Solution:** A numerical solution for (2.1) is obtained as

$$x_{n+1} = x_n - \mu f'(x) \quad (2.4)$$

$$= x_n - \mu (3x_n^2 - 3c) \quad (2.5)$$

where  $x_0$  is an initial guess.

2.4 Write a program to implement (2.5).

**Solution:** Download and execute

```
wget
https://raw.githubusercontent.com/gadepall/
EE1390/master/manuals/supervised/
gradient_boost/codes/square_root.py
```

## 3 AUDIO SOURCE FILES

### 3.1 Get the **audio\_source**

```
svn
checkout https://github.com/gadepall/EE1390/
trunk/manuals/supervised/gradient_boost/
codes/audio_source
cd audio_source
```

3.2 Play the **signal\_noise.wav** and **noise.wav** file. Comment.

**Solution:** **signal\_noise.wav** contains a human voice along with an instrument sound in the background. This instrument sound is captured in **noise.wav**.

## 4 PROBLEM FORMULATION

4.1 See Table 4.1. The goal is to extract the human voice  $e(n)$  from  $d(n)$  by suppressing the component of  $\mathbf{X}(n)$ . Formulate an equation for this.

**Solution:** The maximum component of  $\mathbf{X}(n)$  in

Signal	Label	Type	Filename
Known	$d(n)$	Human+Instrument	signal_noise.wav
	$\mathbf{X}(n)$	Instrument	noise.wav
Unknown	$e(n)$	Human estimate	
	$\mathbf{W}(n)$	Weight Vector	

TABLE 4.1

$d(n)$  can be estimated as

$$\mathbf{W}^T(n)\mathbf{X}(n) \quad (4.1)$$

where

$$\mathbf{W}(n) = \begin{bmatrix} w_1(n) \\ w_2(n) \\ w_3(n) \\ \vdots \\ w_{n-M+1}(n) \end{bmatrix}_{MX1} \quad (4.2)$$

Intuitively, the human voice  $e(n)$  is obtained after removing as much of  $\mathbf{X}(n)$  from  $d(n)$  as

possible. The first step in this direction is to estimate  $\mathbf{W}$  in (4.1) using the metric

$$\min_{\mathbf{W}(n)} \|d(n) - \mathbf{W}^T(n)\mathbf{X}(n)\|^2 \quad (4.3)$$

The human voice can be then obtained as

$$e(n) = d(n) - \mathbf{W}^T(n)\mathbf{X}(n) \quad (4.4)$$

## 5 LMS ALGORITHM

5.1 Show using (4.4) that

$$\nabla_{\mathbf{W}(n)} e^2(n) = \frac{\partial e^2(n)}{\partial \mathbf{W}(n)} \quad (5.1)$$

$$= -2\mathbf{X}(n)d(n) + 2\mathbf{X}(n)\mathbf{X}^T(n)\mathbf{W}(n) \quad (5.2)$$

5.2 Use the gradient descent method to obtain an algorithm for solving (4.3)

**Solution:** The desired algorithm can be expressed as

$$\mathbf{W}(n+1) = \mathbf{W}(n) - \bar{\mu}[\nabla_{\mathbf{W}(n)} e^2(n)] \quad (5.3)$$

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu\mathbf{X}(n)e(n) \quad (5.4)$$

where  $\mu = \bar{\mu}$ .

5.3 Write a program to suppress  $\mathbf{X}(n)$  in  $d(n)$ .

**Solution:** Execute

```
https://raw.githubusercontent.com/gadepall/
EE1390/master/manuals/supervised/
gradient_boost/codes/
LMS_NC_SPEECH.py
```