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Least Squares Method

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Contents

***Abstract*—This manual introduces the least squares method.**

1 Algebra

1.1 Find the equation of the plane P containing the vectors

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{a}_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \tag{1.1}$$

1.2 Show that the vector

$$\mathbf{b} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \tag{1.2}$$

lies outside P.

1.3 Find the point $\mathbf{b}_0 \in P$ closest to $\mathbf{b}$.

2 Calculus

2.1 Find

$$\|\mathbf{y} - A\mathbf{w}\|^2 \tag{2.1}$$

Solution:

$$\|\mathbf{y} - A\mathbf{w}\|^2 = (\mathbf{y} - A\mathbf{w})^T (\mathbf{y} - A\mathbf{w}) \tag{2.2}$$

$$= \|\mathbf{y}\|^2 - \mathbf{w}^T A^T \mathbf{y} \tag{2.3}$$

$$- \mathbf{y}^T A\mathbf{w} + \mathbf{w}^T A^T A\mathbf{w} \tag{2.4}$$

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2.2 Assuming 2×2 matrices and 2×1 vectors, show that

$$\frac{\partial}{\partial \mathbf{w}} \mathbf{w}^T A^T \mathbf{y} = \frac{\partial}{\partial \mathbf{w}} \mathbf{y}^T A\mathbf{w} = \mathbf{y}^T A \tag{2.5}$$

2.3 Show that

$$\frac{\partial}{\partial \mathbf{w}} \mathbf{w}^T A^T A\mathbf{w} = 2\mathbf{w}^T \left(A^T A\right) \tag{2.6}$$

2.4 Find

$$\hat{\mathbf{w}} = \min_{\mathbf{w}} \|\mathbf{y} - A\mathbf{w}\|^2 \tag{2.7}$$

2.5 Use the above result to verify your solution in the previous section

3 Application

3.1 The Steinhart–Hart equation is a model of the resistance of a thermistor at different temperatures. The equation is given by

$$\frac{1}{\tau} = w_1 + w_2 \ln(R) + w_3 [\ln(R)]^3 \tag{3.1}$$

Let

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ \ln(R_1) \\ [\ln(R_1)]^3 \end{pmatrix} \tag{3.2}$$

$$y_1 = \frac{1}{\tau_1} \tag{3.3}$$

$$\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \tag{3.4}$$

Show that

$$y_1 = \mathbf{a}_1^T \mathbf{w} \tag{3.5}$$

3.2 Suppose for $n > 3$

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, A^T = \begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \vdots & \mathbf{a}_n \end{pmatrix}, \tag{3.6}$$

show that

$$\mathbf{y} = A\mathbf{w} \tag{3.7}$$

3.3 For $\tau = 10^{\circ}C - 100^{\circ}C$, use the PT-100 resistance table in

https://github.com/gadepall/EE1390/blob/master/refs/5pt100sensoren_e.pdf?raw=true

to estimate $\mathbf{w}$ using the relation

$$\hat{\mathbf{w}} = \left(A^T A\right)^{-1} A^T \mathbf{y} \tag{3.8}$$

3.4 Verify your result by finding the temperature when the resistance is 175.86Ω.