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Linear Regression

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Contents

***Abstract*—This manual provides an introduction to linear methods in regression.**

1 Least Squares

1.1 Show that

$$\hat{\mathbf{w}} = \min_{\mathbf{w}} \|\mathbf{y} - \mathbf{X}\mathbf{w}\|^2 \tag{1.1}$$

$$= \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y} \tag{1.2}$$

1.2 Using the Gram-Schmidt orthogonalization procedure, show that

$$\mathbf{X} = \mathbf{Q}\mathbf{R} \tag{1.3}$$

where $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$ and $\mathbf{R}$ is upper triangular.

1.3 Show that

$$\hat{\mathbf{w}} = \mathbf{R}\mathbf{Q}^T\mathbf{y} \tag{1.4}$$

1.4 Find $\hat{\mathbf{y}}$

2 Ridge Regression

2.1 The ridge problem is defined as

$$\hat{\mathbf{w}} = \min_{\mathbf{w}} \|\mathbf{y} - \mathbf{X}\mathbf{w}\| \tag{2.1}$$

$$\text{s.t } \|\mathbf{w}\|^2 \leq t \tag{2.2}$$

Using the Lagrangian, show that

$$\hat{\mathbf{w}} = \left(\mathbf{X}^T\mathbf{X} + \lambda\mathbf{I}\right)^{-1}\mathbf{X}^T\mathbf{y} \tag{2.3}$$

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3 The Lasso

3.1 The Lasso is defined as

$$\hat{\mathbf{w}} = \min_{\mathbf{w}} \|\mathbf{y} - \mathbf{X}\mathbf{w}\| \tag{3.1}$$

$$\text{s.t} \sum_i |w_i| \leq t \tag{3.2}$$

Obtain the corresponding Lagrangian.

3.2 Show that this is a quadratic programming problem and find a suitable algorithm.