Statistical Decision Theory

G V V Sharma*

Contents

Abstract—This manual provides an introduction to statistical decision theory.

1 Introduction

1.1 Let

$$Y = f(\mathbf{X}) \qquad (1.1)$$

The *mean square error* (MSE) is defined as

$$MSE(f) = E[Y - f(\mathbf{X})]^2 \qquad (1.2)$$

Show that

$$MSE(f) = E_{\mathbf{X}}\left\{ E_Y[Y - f(\mathbf{x})]^2 | \mathbf{X} = \mathbf{x} \right\} \qquad (1.3)$$

1.2 Let

$$c = f(\mathbf{x}) \qquad (1.4)$$

Using (1.3)

$$\begin{aligned} \min MSE(f) &= \min MSE(f)|X \\ &= \min_c E_Y\left\{ [Y - c]^2 | \mathbf{X} = \mathbf{x} \right\} \end{aligned} \qquad (1.5)$$

Show that

$$\begin{aligned} MSE(f)|X &= E_Y\left\{ [Y - c]^2 | \mathbf{X} = \mathbf{x} \right\} \\ &= -2cE_Y\{Y|\mathbf{X} = \mathbf{x}\} + E_Y\left\{Y^2|\mathbf{X} = \mathbf{x}\right\} + c^2 \end{aligned} \qquad (1.6)$$

1.3 $MSE(f)$ is minimum when

$$\frac{d}{dc}MSE(f)|X = 0. \qquad (1.7)$$

Show that this results in

$$c = f(\mathbf{x}) = E[Y|X = \mathbf{x}] \qquad (1.8)$$

f is known as the *regression* function.

2 Applications

2.1 Explain how (1.8) can be used to obtain the Nearest Neighbour approximation.

2.2 Repeat the exercise for the least squares method.