Predator Prey Model Report

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Introduction

Predator-Prey models are considered the canonical foundation of biological systems since, every species require life resources to survive. In a balanced eco system, this usually involves competing species and their struggle for co-existence. A closer inspection of such interactions often reveals the underlying predator-prey mechanisms in play. More specifically, how one species interacted with other species within the context of its surroundings. Understanding more about such population dynamics has been the interest of study for ecologists and mathematicians alike. One of the earliest known effort in this direction was that of the British Mathematician Malthus (1798), when he published his work titled "Essay of the Principle of population". It dealt with population dynamics of a single species; how resources limit such a species' exponential growth. Years later another mathematician Verhulst (1833) modified this relationship into a mathematical model which is now referred to as the logistic equation for a population model (Barryman, 1992).

$$\frac{dN}{dt} = aN\left(1 - \frac{N}{K}\right) \tag{1.1}$$

Where N is the population density of a species,

a is rate of change of the population,

K is the carrying capacity of the population

Lotka Volterra Differential Equations

The formulation cited above remained popular until Lotka (1920) & Volterra (1926) proposed the trophic model which included an interaction between species using the principle of mass action (thus involving product of populations) and modelled them into the same differential equations (Barryman, 1992). Their work gave birth to the famous Lotka Volterra population model given below,

$$\frac{dN}{dt} = a_{Birth}N - b_{Cons}NP \tag{2.1}$$

$$\frac{dP}{dt} = c_{Death}NP - d_{Food}P$$

Where N is the population density of the prey,

P is the population density of the predator,

a_{Birth} is the birth rate of the prey,

b_{Cons} is the consummation rate of the prey,

c_{Death} is the death rate of the predator,

d_{Food} is the rate at which predator eats the prey

Related Research

Many interactions in biology, healthcare, chemistry, business, meteorology, oceanology and even warfare have been modelled using the predator prey relationship. Choquenot, Hone and Saunders (1999) used a predator prey model to evaluate the helicopter shooting of feral pigs in Australia. Rassoulzadegan and Sheldon (1986) explored the interactions of Nano zooplankton with bacteria using a similar model. Gracia (2005) provided alternate explanations for the market bubbles and business cycles mechanism using a predator prey model.

Some researchers also explored a three species interaction model. One predator two prey model involving lions, wildebeest and zebras was studied extensively by Fay and Greeff (2006). Similarly, Hsu and Hubbell (1979) analyzed, a variant involving two species and two predators to understand more about species packing.

Finnish Parliamentary Elections Predator Prey Model

A Democratic nation is one that is built by and for the citizens of a country. The elected leaders of democratic countries are often regarded as the chosen 'representative' of the people. Consequently, a political scenario may find many parties offering their candidacy for leading the country. In most of the elections, voters are permitted to elect for a single representative only and each such vote casted allowed for the claim of a seat by associated political party. In effect, it wasn't uncommon to witness political clashes and aggressive tactics within such systems.

In the context of a democratic country like Finland, which allowed for proportionate representation, several parties existed, both large and small, 'dominant' and less 'dominant'. As a consequence, political structure, agenda, goodwill and other social factors affected the votes received by parties during election times (Durand and Durand, 1992). Considering the competition involved in procuring such votes, the political electoral scenario in Finland can be imagined representing a form of predator prey relationship and thus be modelled using the Lotka Volterra Equation.

One of Finland's foremost political parties is the communist led electoral alliance ("SKDL") (Durand and Durand, 1992). Having shown success in many consecutive years, this party is modelled as the 'predator' for other parties within the Finnish parliamentary elections. All the other parties ("Others") collectively form the 'prey' for "SKDL" (Durand and Durand, 1992). The table below shows the electoral votes received by both parties from a time period ranging from 1954 to 1983.

Election year	SKDL Vote (in thousands)	Vote for All Other (in thousands)
1954	433	1575
1958	450	1494
1962	507	1795
1966	502	1868
1970	421	2115
1972	439	2139
1975	519	2231
1979	518	2376
1983	401	2579

Fig 1.1 Electoral votes received by “SKDL” & “Other” parties

In modelling the ‘predator prey’ relationship within the Finnish political system, the usual parameters in the Lotka Volterra Equations in Equation 2.1, takes up the following roles.

a_{Birth} relates to the rate at which votes are accumulated by “Other” party
b_{Cons} relates to the consummation rate of votes of “Other” party
c_{Death} relates to the rate at which votes for “SKDL” are depleting
d_{Food} relates to the rate at which “SKDL” is ‘eating’ away votes of “Other” party

One assumption used by this model was that other extraneous environment do not affect the number of votes obtained by either party.

Qualitative studies and numerical approximations helped obtain the coefficient parameters mentioned above. The coefficients obtained were,

$a_{Birth} = 0.6$
$b_{Cons} = 0.0013$
$c_{Death} = 0.39$
$d_{Food} = 0.00022$

The initial vote counts were taken to be those from the year 1954. Thus, the final Lotka Volterra Equation can be modelled as an initial value problem shown below,

Differential Equations:

$$\frac{dP}{dt} = 0.6P - 0.0013PS \qquad (4.1)$$

$$\frac{dS}{dt} = -0.39S + 0.00022SP \qquad (4.2)$$

$$\text{Where P denotes "Other" parties' current number of votes,}$$

$\frac{dP}{dt}$, its rate of change over time

S, refers to SKDL's number of votes,

$\frac{dS}{dt}$, its rate of change over time

Initial Values:

$$P(0) = 1575 \tag{4.3}$$

$$S(0) = 433 \tag{4.4}$$

Another parameter of interest would be the equilibrium or critical points of the system of differential equations. These are the number of votes accumulated by either parties after which there will not be any change to their values over time. In other words, their rates of changes of will be zero,

$$\frac{dP}{dt} = 0 \tag{4.5}$$

$$\frac{dS}{dt} = 0 \tag{4.6}$$

The Lotka Volterra equations thus reduces to

$$0.6P - 0.0013PS = 0$$

$$-0.39S + 0.00022SP = 0$$

Which gives two sets of equilibrium or critical points as,

$$P = 0 \;\&\; S = 0 \qquad \text{(trivial solution)}$$

$$P = 1772.75 \;\&\; S = 461.5 \tag{4.7}$$

Implementing Finnish Parliamentary Elections Predator Prey Model in Python

The predator prey model was implemented in python and solved using the 'integrate.ode' function of the 'scipy' package. A ten thousand step time discretization was used to obtain the solution curves of the system of differential equations.

Further, plot functions of python were used to sketch two kinds of plots; a component plot to graph the vote counts of the two parties over time and phase plane plot to track their mutual relationship over the same time period.

Results & Conclusion

The component plots were obtained as below,

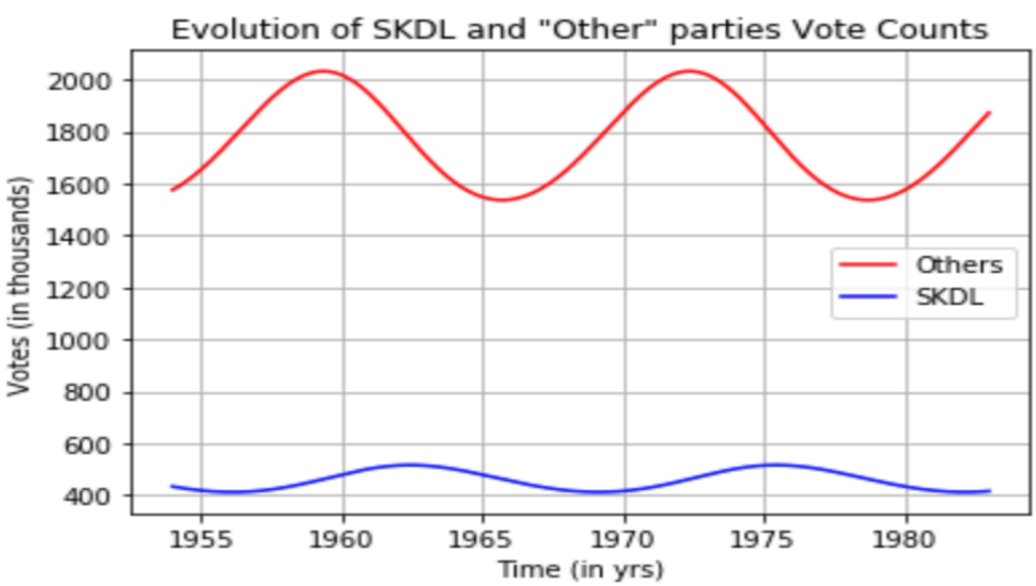

Fig 1.2 Combined component plots representing votes of “SKDL” & “Other” parties over time

A periodic pattern, reflecting the cyclic nature of votes accumulation is observed for the both the “SKDL” and “Others” party groups.

The “SKDL” group’s votes approaches its first crest (peak) point at about 510,000 votes in the year 1962 and reaches the first trough (lowest) point of 400,000 votes in the year 1956. The approximate period for this vote cycle is observed to be close to 12.5 years while the amplitude of the periodic fluctuations is 55,000 votes.

On the other hand, the “Others” group approaches its crest value of around 2100,000 votes by the year 1958, while reaching the trough point of 1550,000 votes in the year 1966. The period in this case was 14 years, while an amplitude of 275,000 votes was recorded.

The phase plane plots gave the following results,

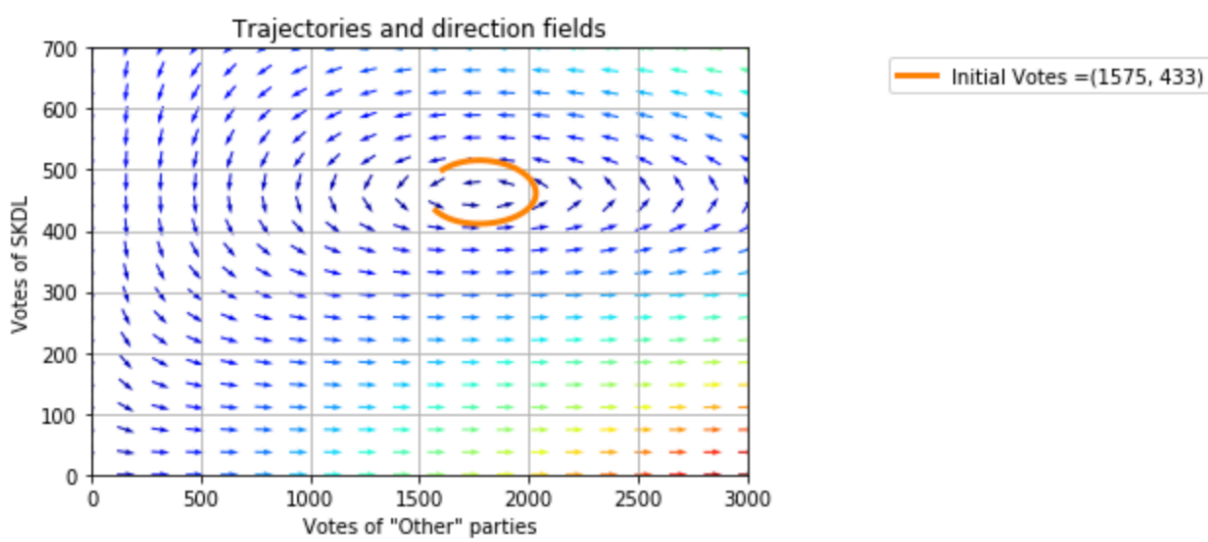

Fig 1.3 Phase plane plots representing votes of “SKDL” & “Other” parties over time

The phase plane plotted shows how the system of differential equations converges to the equilibrium points. Depending on the initial values for the differential equation, we can track a trajectory for the initial value, observing how it converges to the critical point. Note that the normalized arrows changes direction depending on the relative sign of the rates of changes of "SKDL" & "Others" vote counts. For e.g. if $\frac{dS}{dt} < 0$ & $\frac{dP}{dt} > 0$, there will be a gradual decline in the votes for "SKDL" and an increase in that for "Others". The arrows will consequently shift their direction to reflect this change.

For this predator prey model, we have assumed an initial vote count as stated in Equation 4.3, Since this is different from the critical point obtained in Equation 4.7, we note the initial path line as a yellow trajectory line. This line gradually moves in the direction governed by the normalized arrows and eventually help converge to the equilibrium state.

Overall, the results obtained tallied well with the experimental results published in Durand and Durand's (1992) work.

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