Assignment6_Quant

Jayasri

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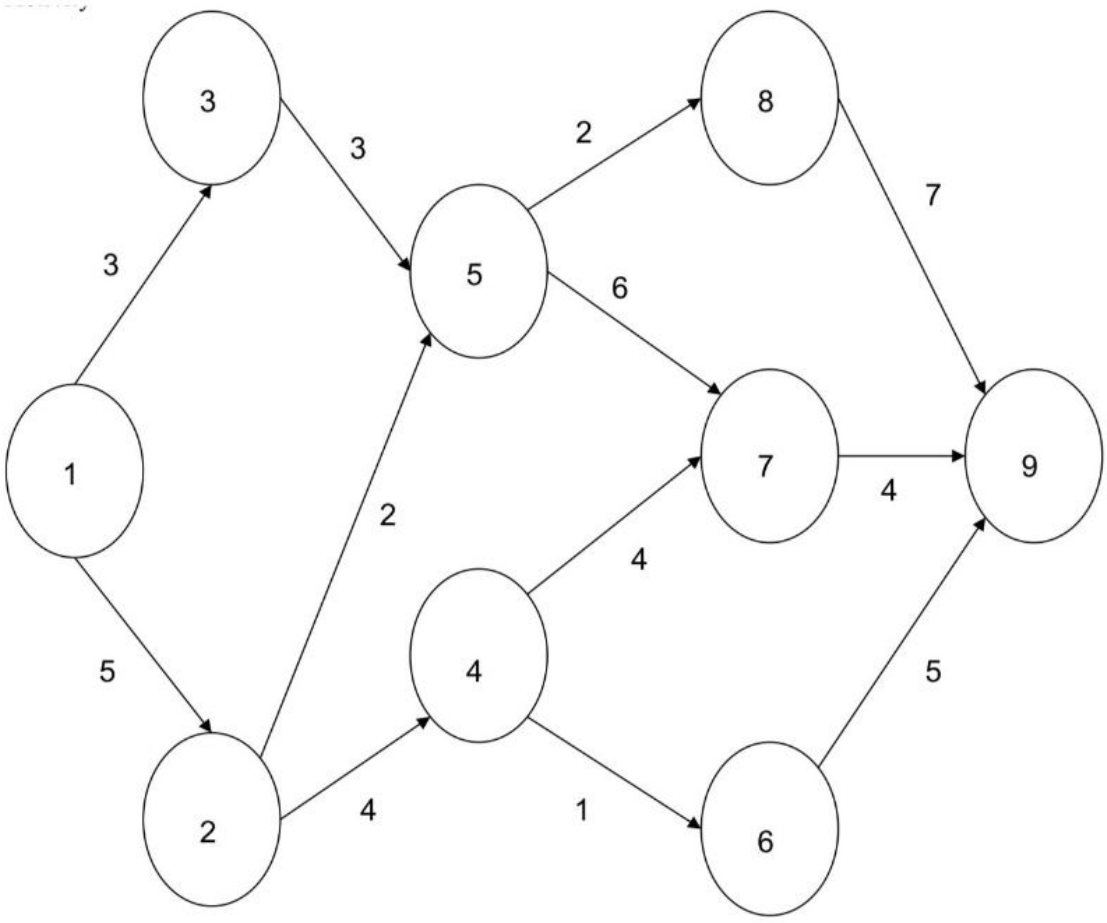
INTEGER PROGRAMMING

Consider the following activity-on-arc project network, where the 12 arcs (arrows) represent the 12 activities (tasks) that must be performed to complete the project and the network displays the order in which the activities need to be performed. The number next to each arc (arrow) is the time required for the corresponding activity. Consider the problem of finding the longest path (the largest total time) through this network from start (node 1) to finish (node 9), since the longest path is the critical path.



QUESTION 1:

Formulate and solve the binary integer programming (BIP) model for this problem using library lpsolve or equivalent in R.

Mathematical Formulation:

Let xij represents the arc from node i to node j

where

```
i = 1,2,3,4,5,6,7,8

j = 2,3,4,5,6,7,8,9
```

Max = 5x12 + 3x13 + 3x35 + 4x24 + 2x25 + 4x47 + x46 + 2x58 + 6x57 + 5x69 + 4x79 + 7x89 ;

S.T)

Starting Node:

```
x13 + x12 = 1;
```

Intermediate Node:

```
x12 - x25 - x24 = 0;
x13 - x35 = 0;
x25 + x35 - x57 - x58 = 0;
x24 - x46 - x47 = 0;
x58 - x89 = 0;
x46 - x69 = 0;
x57 + x47 - x79 = 0;
```

End Node:

```
x69 + x79 + x89 = 1;
```

Binary and Non Negativity Constraints:

```
bin xij,  xij>=0;
```

LP File

```
Max : 5 x12 + 3 x13 + 3 x35 + 4 x24 + 2 x25 + 4 x47 + 1 x46 + 2 x58 + 6 x57 + 5 x69 + 4 x79 + 7 x89 ;

/* Constraints */

/* Starting Node */
x13 + x12 = 1;

/* Intermediate Node */
x12 - x25 - x24 = 0;
x13 - x35 = 0;
x25 + x35 - x57 - x58 = 0;
x24 - x46 - x47 = 0;
x58 - x89 = 0;
x46 - x69 = 0;
x57 + x47 - x79 = 0;


/* End Node */
x69 + x79 + x89 = 1;

bin x12,x13,x25,x24,x35,x47,x46,x58,x57,x79,x69,x89;
```

Problem Solution using R

```
library(lpSolveAPI)

arcs<- read.lp("Arcs.lp")

solve(arcs)
```

```
## [1] 0
```

```
#Print the model

arcs
```

```
## Model name:
##   a linear program with 12 decision variables and 9 constraints
```

```
#To identify the Optimal Solution
get.objective(arcs)
```

```
## [1] 17
```

The optimal solution of the problem is 17.

```
#To Identify the variables
Values <-get.variables(arcs)

# Assigning Rownames to the values
Nodes_names <- c("x12","x13","x35","x24","x25","x47","x46","x58","x57","x69","x79","x89")
as.data.frame(cbind(Nodes_names,Values))
```

```
##    Nodes_names Values
## 1          x12      1
## 2          x13      0
## 3          x35      0
## 4          x24      0
## 5          x25      1
## 6          x47      0
## 7          x46      0
## 8          x58      0
## 9          x57      1
## 10         x69      0
## 11         x79      1
## 12         x89      0
```

```
# To display the constraint values
get.constraints(arcs)
```

```
## [1] 1 0 0 0 0 0 0 0 1
```

OBSERVATIONS:

The critical Path is (which is the longest path): From Node 1 to 2

From Node 2 to 5

From Node 5 to 7

From Node 7 to 9

The time required for the corresponding activity is : 17

QUESTION 2:

Selecting an Investment Portfolio An investment manager wants to determine an optimal portfolio for a wealthy client. The fund has $2.5 million to invest, and its objective is to maximize total dollar return from both growth and dividends over the course of the coming year. The client has researched eight high-tech companies and wants the portfolio to consist of shares in these firms only. Three of the firms (S1 – S3) are primarily software companies, three (H1–H3) are primarily hardware companies, and two (C1–C2) are internet consulting companies. The client has stipulated that no more than 40 percent of the investment be allocated to any one of these three sectors. To assure diversification, at least $100,000 must be invested in each of the eight stocks. Moreover, the number of shares invested in any stock must be a multiple of 1000.

The table below gives estimates from the investment company's database relating to these stocks. These estimates include the price per share, the projected annual growth rate in the share price, and the anticipated annual dividend payment per share.

	Stock							
	S1	S2	S3	H1	H2	H3	C1	C2
Price per share	$40	$50	$80	$60	$45	$60	$30	$25
Growth rate	0.05	0.10	0.03	0.04	0.07	0.15	0.22	0.25
Dividend	$2.00	$1.50	$3.50	$3.00	$2.00	$1.00	$1.80	$0.00

Determine the maximum return on the portfolio. What is the optimal number of shares to buy for each of the stocks? What is the corresponding dollar amount invested in each stock?

Mathematical Formulation:

Let S1,S2,S3,H1,H2,H3,C1,C2 be the no. of shares for each stock(firms)

As, it is mentioned that the number of shares invested in any stock must be a multiple of 1000. So, lets define each stock has 1000 shares.

Maximize : 4000S1+ 6500S2+ 5900S3+5400H1+ 5150H2+ 10000H3+ 8400C1+ 6250C2;

S.T)

The fund has $2.5 million to invest

Invest_Fund: 40(1000*S1)+ 50(1000*S2)+ 80(1000*S3)+ 60(1000*H1)+ 45(1000*H2)+ 60(1000*H3)+ 30(1000*C1)+ 25(1000*C2) <= 2500000;

```
40S1 + 50S2 + 80S3 + 60H1 + 45H2 + 60H3 + 30C1 + 25C2 <= 2500
```

The client has stipulated that no more than 40 percent of the investment be allocated to any one of these three sectors

S Firms: 40S1+ 50S2+ 80S3 <= 1000;

H Firms: 60H1+ 45H2+ 60H3 <= 1000;

C Firms: 30C1+ 25C2 <= 1000;

To assure diversification, at least $100,000 must be invested in each of the eight stocks.

S1: 40S1>=100;

S2: 50S2>=100;

S3: 80S3>=100;

H1: 60H1>=100;

H2: 45H2>=100;

H3: 60H3>=100;

C1: 30C1>=100;

C2: 25C2>=100;

Solution using R

LP File of the problem without integer restriction

```
/* objective function */
max: 4000S1+ 6500S2+ 5900S3+5400H1+ 5150H2+ 10000H3+ 8400C1+ 6250C2;


/* Constraints */
Invest_Fund: 40S1+ 50S2+ 80S3+ 60H1+ 45H2+ 60H3+ 30C1+ 25C2 <= 2500;
Software_Firms: 40S1+ 50S2+ 80S3 <= 1000;
Hardware_Firms: 60H1+ 45H2+ 60H3 <= 1000;
Consultancy: 30C1+ 25C2 <= 1000;
S1: 40S1>=100;
S2: 50S2>=100;
S3: 80S3>=100;
H1: 60H1>=100;
H2: 45H2>=100;
H3: 60H3>=100;
C1: 30C1>=100;
C2: 25C2>=100;
```

Solution of LP Model without using integer restriction

```
library(lpSolveAPI)

stock<- read.lp("Stock.lp")

solve(stock)
```

```
## [1] 0
```

```
#Print the model

stock
```

```
## Model name:
##                    S1     S2     S3     H1     H2     H3     C1     C2
## Maximize         4000   6500   5900   5400   5150  10000   8400   6250
## Invest_Fund        40     50     80     60     45     60     30     25  <=  2500
## Software_Firms     40     50     80      0      0      0      0      0  <=  1000
## Hardware_Firms      0      0      0     60     45     60      0      0  <=  1000
## Consultancy         0      0      0      0      0      0     30     25  <=  1000
## S1                 40      0      0      0      0      0      0      0  >=   100
## S2                  0     50      0      0      0      0      0      0  >=   100
## S3                  0      0     80      0      0      0      0      0  >=   100
## H1                  0      0      0     60      0      0      0      0  >=   100
## H2                  0      0      0      0     45      0      0      0  >=   100
## H3                  0      0      0      0      0     60      0      0  >=   100
## C1                  0      0      0      0      0      0     30      0  >=   100
## C2                  0      0      0      0      0      0      0     25  >=   100
## Kind              Std    Std    Std    Std    Std    Std    Std    Std
## Type             Real   Real   Real   Real   Real   Real   Real   Real
## Upper             Inf    Inf    Inf    Inf    Inf    Inf    Inf    Inf
## Lower               0      0      0      0      0      0      0      0
```

```
#To identify the Optimal Solution
get.objective(stock)
```

```
## [1] 487152.8
```

```
values <-get.variables(stock)
stocknames<-c("S1","S2","S3","H1","H2","H3","C1","C2")
d<-as.data.frame(cbind(stocknames,values*1000))
colnames(d)<- c("Stocknames","No. Of Shares")
d
```

```
##   Stocknames    No. Of Shares
## 1         S1             2500
## 2         S2             6000
## 3         S3             1250
## 4         H1 1666.66666666667
## 5         H2 2222.22222222222
## 6         H3 13333.3333333333
## 7         C1            30000
## 8         C2             4000
```

```
get.constraints(stock)
```

```
##  [1] 2500  500 1000 1000  100  300  100  100  100  800  900  100
```

OBSERVATIONS:

- The Maximum return on the Portfolio is : 487152.8

Optimal values without Integer Restriction			
No. of shares	**Price PerShare**	**Optimal Value**	**Dollar Amount Invested in each stock**
S1	40	2500	100000
S2	50	6000	300000
S3	80	1250	100000
H1	60	1666.67	100000.2
H2	45	2222.22	99999.9
H3	60	13333.33	799999.8
C1	30	30000	900000
C2	25	4000	100000

LP File of the problem with integer restriction

```
/* objective function */
max: 4000S1+ 6500S2+ 5900S3+5400H1+ 5150H2+ 10000H3+ 8400C1+ 6250C2;


/* Constraints */
Invest_Fund: 40S1+ 50S2+ 80S3+ 60H1+ 45H2+ 60H3+ 30C1+ 25C2 <= 2500;
Software_Firms: 40S1+ 50S2+ 80S3 <= 1000;
Hardware_Firms: 60H1+ 45H2+ 60H3 <= 1000;
Consultancy: 30C1+ 25C2 <= 1000;
S1: 40S1>=100;
S2: 50S2>=100;
S3: 80S3>=100;
H1: 60H1>=100;
H2: 45H2>=100;
H3: 60H3>=100;
C1: 30C1>=100;
C2: 25C2>=100;


int S1,S2,S3,H1,H2,H3,C1,C2;
```

Solution of LP Model with integer restriction

```
#Solution of LP Model with using integer restriction
library(lpSolveAPI)

Stock_int<- read.lp("Stock_int.lp")

solve(Stock_int)
```

```
## [1] 0
```

```
#Print the model

Stock_int
```

```
## Model name:
##                    S1     S2     S3     H1     H2     H3     C1     C2
## Maximize         4000   6500   5900   5400   5150  10000   8400   6250
## Invest_Fund        40     50     80     60     45     60     30     25  <=  2500
## Software_Firms     40     50     80      0      0      0      0      0  <=  1000
## Hardware_Firms      0      0      0     60     45     60      0      0  <=  1000
## Consultancy         0      0      0      0      0      0     30     25  <=  1000
## S1                 40      0      0      0      0      0      0      0  >=   100
## S2                  0     50      0      0      0      0      0      0  >=   100
## S3                  0      0     80      0      0      0      0      0  >=   100
## H1                  0      0      0     60      0      0      0      0  >=   100
## H2                  0      0      0      0     45      0      0      0  >=   100
## H3                  0      0      0      0      0     60      0      0  >=   100
## C1                  0      0      0      0      0      0     30      0  >=   100
## C2                  0      0      0      0      0      0      0     25  >=   100
## Kind              Std    Std    Std    Std    Std    Std    Std    Std
## Type              Int    Int    Int    Int    Int    Int    Int    Int
## Upper             Inf    Inf    Inf    Inf    Inf    Inf    Inf    Inf
## Lower               0      0      0      0      0      0      0      0
```

```
#To identify the Optimal Solution
get.objective(Stock_int)
```

```
## [1] 477400
```

```
Vars<-get.variables(Stock_int)
stocknames<-c("S1","S2","S3","H1","H2","H3","C1","C2")
V<-as.data.frame(cbind(stocknames,Vars*1000))
colnames(V)<- c("Stocknames","No. Of Shares")
V
```

```
##   Stocknames No. Of Shares
## 1         S1          3000
## 2         S2          5000
## 3         S3          2000
## 4         H1          2000
## 5         H2          3000
## 6         H3         12000
## 7         C1         29000
## 8         C2          5000
```

```
get.constraints(Stock_int)
```

```
##  [1] 2500  530  975  995  120  250  160  120  135  720  870  125
```

OBSERVATIONS

- The Maximum return on the Portfolio is : 477400

Optimal values with Integer Restriction			
No. of shares	Price PerShare	Optimal Value	Dollar Amount Invested in each stock
S1	40	3000	120000
S2	50	5000	250000
S3	80	2000	160000
H1	60	2000	120000
H2	45	3000	135000
H3	60	12000	720000
C1	30	29000	870000
C2	25	5000	125000

All the constraints have been satisfied with the integer programming.

CONSTRAINTS	STOCKS	OPTIMAL VALUE	PERCENTAGE	COMMENTS
The fund has $2.5 million to invest	All Firms	2500000		Satisfied
The client has stipulated that no more than 40 percent of the investment be allocated to any one of these three sectors	S Firms	530000	21.2	
	H Firms	975000	39	
	C Firms	995000	39.8	Satisfied
To assure diversification,atleast $100,000 must be invested in each of the eight stock	S1	120000		
	S2	250000		
	S3	160000		
	H1	120000		
	H2	135000		
	H3	720000		
	C1	870000		
	C2	125000		Satisfied

Compare the solution in which there is no integer restriction on the number of shares invested. By how much (in percentage terms) do the integer restrictions alter the value of the optimal objective function? By how much (in percentage terms) do they alter the optimal investment quantities?

a) By how much (in percentage terms) do the integer restrictions alter the value of the optimal objective function?

Optimal value of Objective function	No Integer Restriction	Integer Restriction	Change	Change in %
	487152.8	477400	9752.8	2.002

b) By how much (in percentage terms) do they alter the optimal investment quantities?

stock Companies	Number of Shares		Difference	Difference in %
	No Integer Restriction	Integer Restriction		
S1	2500	3000	-500	-20
S2	6000	5000	1000	16.66666667
S3	1250	2000	-750	-60
H1	1666.67	2000	-333.33	-19.99976
H2	2222.22	3000	-777.78	-35.000135
H3	13333.33	12000	1333.33	9.9999775
C1	30000	29000	1000	3.333333333
C2	4000	5000	-1000	-25
Total	60972.22	61000	-27.78	-0.045561733

stock Companies	Invested quantities in $		Difference	Difference in %
	No Integer Restriction	Integer Restriction		
S1	100000	120000	-20000	-20
S2	300000	250000	50000	16.66666667
S3	100000	160000	-60000	-60
H1	100000.2	120000	-19999.8	-19.99976
H2	99999.9	135000	-35000.1	-35.000135
H3	799999.8	720000	79999.8	9.9999775
C1	900000	870000	30000	3.333333333
C2	100000	125000	-25000	-25
Total	2499999.9	2500000	-0.1	-4E-06