CSI 701 – Assignment 2 – Hyperbolic PDE Based Model (FDM)

Introduction:

In this assignment, the convection-diffusion equation is modeled using Upwind scheme for the convection term and Central Difference Scheme for the diffusion term. The simulation is modeled by executing the following set of tasks:

- Map the given measurements to a 100 X 100 grid.
- Calculate the step size for x and y
- Calculate the time step using the stability condition: $k\frac{\Delta t}{(\Delta x)^2}<\frac{1}{2}$.
- Iterate through the scenario according to the boundary conditions, convection & diffusion components.
- Check for possibility of convergence every 500 time-steps, and exit if converging.

Numerical Method:

We use upwind scheme for the convective term and central difference scheme for the diffusion term.

Convergence criteria:

The condition set in the code for checking convergence is to look at the change of temperature at each time-step, and when its magnitude starts decreasing beyond a threshold (less than 10% of the maximum change so far), we can assume that the simulation is converging.

$$T^{n+1}-T^{n}<\frac{max(T^{n+1}-T^{n})}{10}$$

Simulation Results:

1. Base simulation:

The simulation of the basic scenario is carried out with the following inputs:

- Size of the tissue: 50 by 50
- Size of the needle: 2 by 2
- Position of the needle: 25, 25 (Center)
- Position of the blood vessel : 40 units along y-axis
- Size of the vessel: 50 units along x and 5 units along y

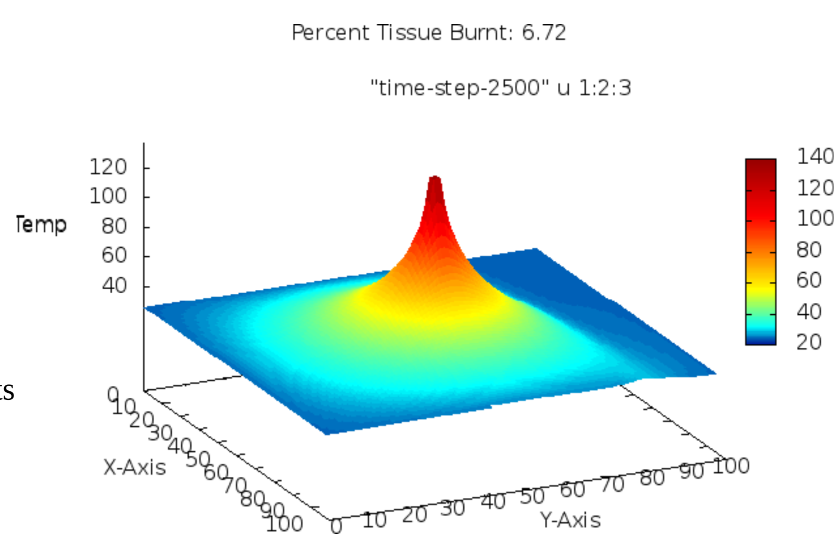

Illustration 1: With Blood Vessel along grid-point 80 on Y-Axis

- Temperatures of the tissue, needle and burn temperature: 25, 137, 60

We can notice that temperature drop near the blood vessel (at grid point 80 along Y-Axis), and the burn region (Yellow ring in the plot).

2. Size of burnt region with no vessel:

We run the simulation with same inputs as in the base scenario, but with no blood vessel near the needle. That is with the size and position of the vessel set to 0.

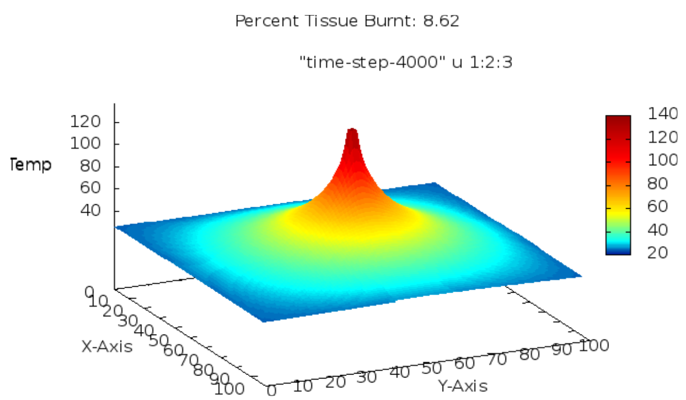

Illustration 2: With no blood vessel

Scenario conclusion: We can see the uniform distribution of temperature and burnt region in the tissue.

3. Needle geometry :

In this simulation the needle is 5 units long and 2 units wide. The blood vessel (in the case it exists) is kept at 40 units (Grid point 80) along the Y-Axis. The resulting burn region and the resulting temperature distribution graphs are:

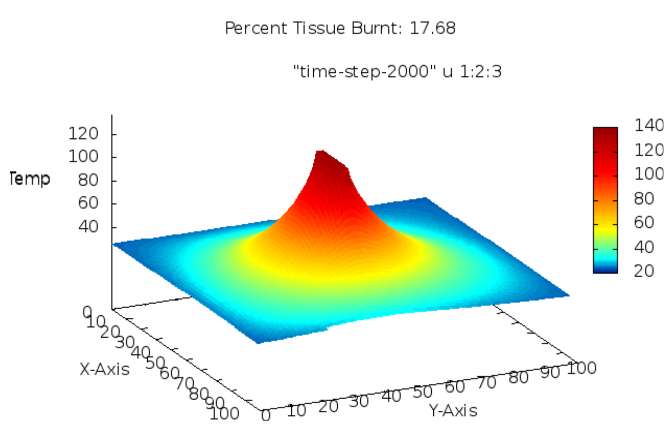

Illustration 3: Parallel Needle with no blood vessel

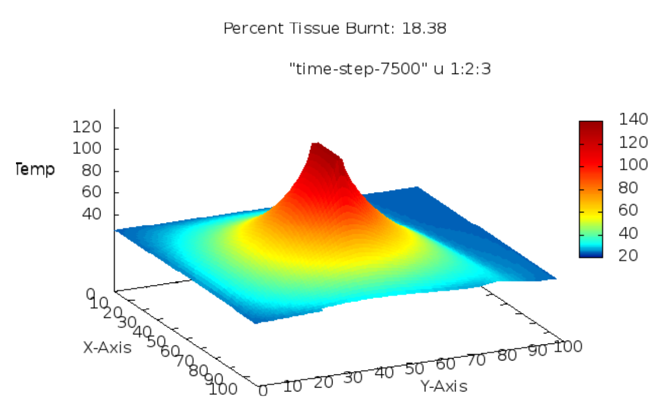

Illustration 4: Parallel Needle with blood vessel (along grid 80)

3. Dependence on Velocity:

The blood velocity was decreased to half in this scenario, and the output is compared against the base case.

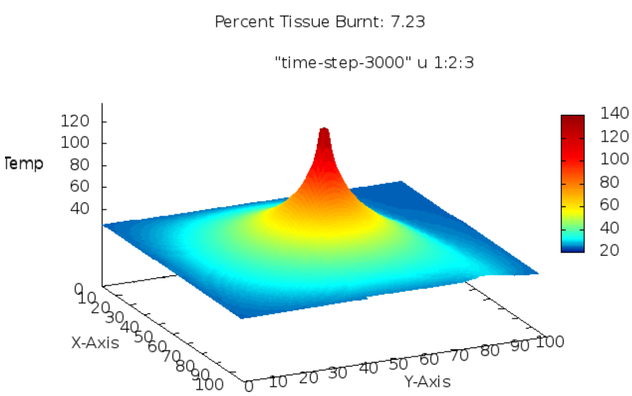

Illustration 6: Effects with half blood velocity of the base scenario

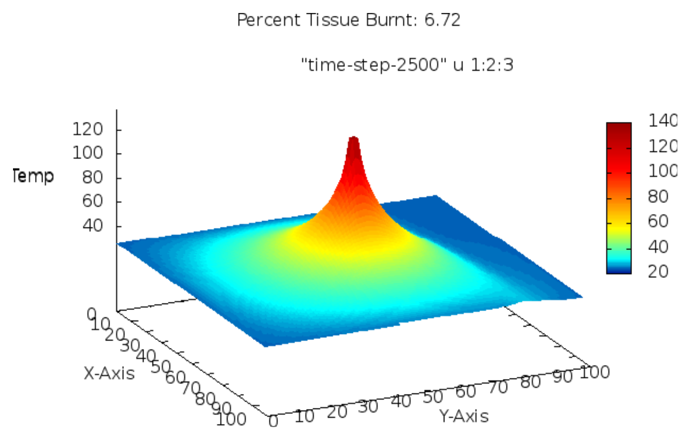

Illustration 5: Normal Blood Velocity (Base Case)

4. Dependence on vessel size:

The vessel size was decreased by half and other parameters are same as in the base scenario. The graphs obtained was as below:

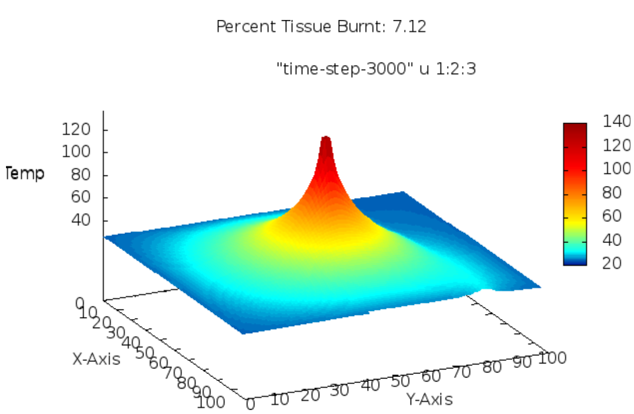

Illustration 8: Reduced Vessel Size

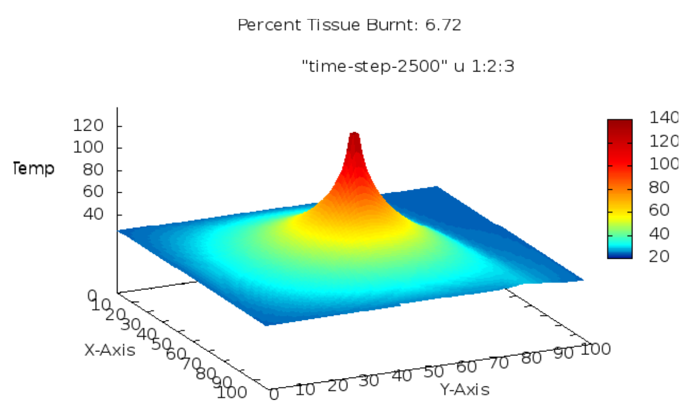

Illustration 7: Normal Vessel Size (Base Case)

5. Dependence of burn region with distance:

The blood vessel was moved closer to the needle (at grid point 60) and the effects of the simulation are as follows:

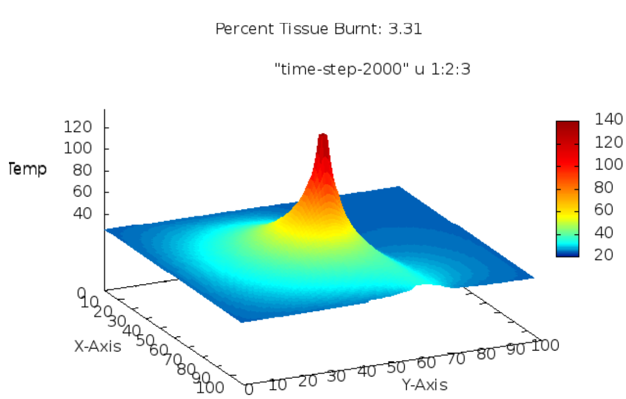

Illustration 9: Reduced Distance to the vessel

Learnings and Conclusion:

I could learn how to implement central difference and upwind schemes and also how to change them to achieve higher orders of accuracy and to higher dimensions. I also came to know who important determining the correct time-steps is to be able to run correctly and to converge to a solution.

In conclusion the model seems to be able to address most of the basic scenarios obtained by changing the different parameters of the convection-diffusion equation.