CSI 701: Assignment 1 – ODE Based Model

Introduction:

Problem requires modeling and solving two coupled ODEs using numerical methods to answer specific conclusions using the outcome of simulations of the model.

The given system of coupled ODEs for the conventional moel can be reduced to the form:

y'' = a*b*y + g'(t) – b*f(t) ; similarly x'' = a*b*x-b*b*g'(t)+f(t)

Here, the ODE is of the form y'' = b*y with b>0 (considering constant reinforcement functions)

Hence, the solution for Y and X will be of the form

$$Y = C1\,e^{kt} + C2\,e^{-kt} \qquad X = C\,e^{mt} + D\,e^{-mt}$$

Numerical Method:

The code for numerically solving these set of coupled ODEs is based on Runge-kutta 4th Order Method. The method gives an accuracy of O(h^4) based the time step h.

Development:

The code was designed in a top-down approach. It has three main functions: the driver function – takes care of incrementing time step, iterating and stopping simulation; Runge-Rutta method – that implements numerical approximations to the actual result; and the function definitions of the two coupled ODE models (conventional model and guerrilla model).

The code was first tested on simpler functions (linear, x^2 etc) for verification before trying to implement the ODE model. The Total error = computational error + propagation error in these cases were not more than the 4th order.

Simulation Results:

1. <u>Conventional vs Guerrilla </u>:

The simulation had the following parameters:

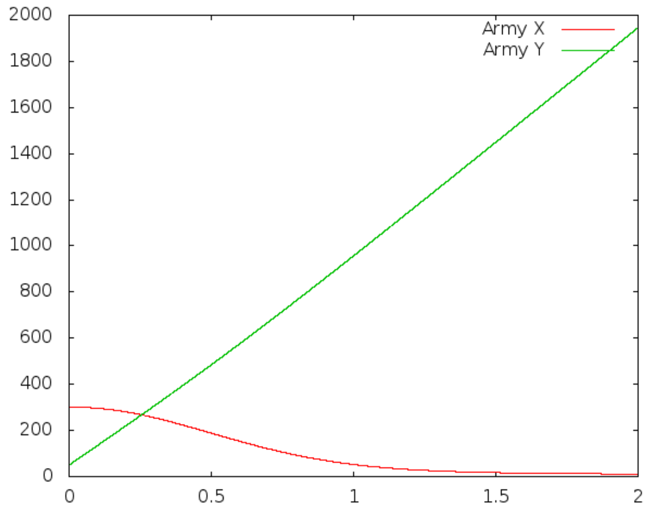
* Initial value of X (conventional army) = 300, initial value of Y (guerrilla army) = 50;

* Combat effectiveness of X = 0.5, combat effectiveness of Y = 0.005

* Both have a constant (and equal) reinforcements

This led to the following conclusion:

A smaller army can combat effectively against a conventional army using guerrilla methods rather than conventional methods.



2. <u>Large reinforcement rate vs. Large Combat effectiveness</u>:

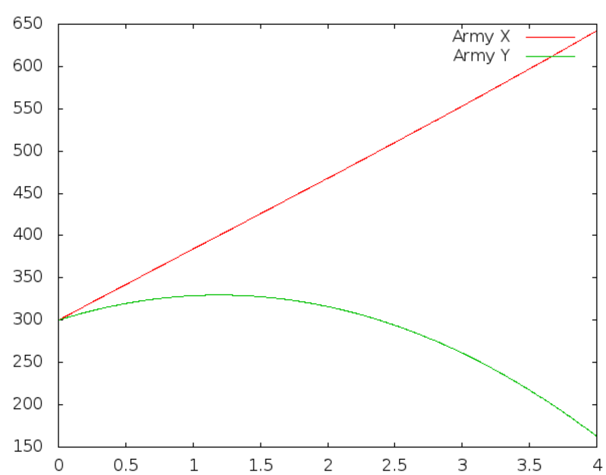
This simulation was run with the following parameters:

* Initial values of X : 300, Initial values of Y: 300,

* Combat effectiveness of X = **0.5**, combat effectiveness of Y = **0.05**,

* Reinforcement of X = **100** per time step, reinforcement of Y = **1000** per time step.

The simulation led to show that a higher combat effectiveness is more effective than a higher rate of reinforcement as shown in the graph below:



3. <u>Periodic reinforcements vs. non-periodic reinforcements</u>:

The parameters for this simulation was as follows:

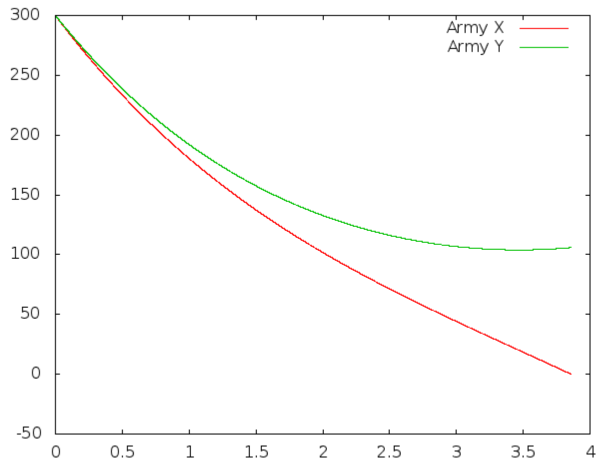
* Initial values of X : 300, Initial values of Y : 300,

* Combat effectiveness of X = 0.5, combat effectiveness of Y = 0.5,

* Reinforcement rate of X = 1000 per 10 time steps, Reinforcement rate of Y = 100 (constantly)

[giving the same amount of reinforcements for both armies in 10 time steps.]

A constant reinforcement is more effective than a periodic reinforcement of the same magnitude.



Conclusion:

The exercise was helpful to understand how a mathematical equation can model a given changing and continuous scenario and also how it can be helpful in predicting many questions that can be modeled by changing various parameters of the model.