CSI 701 – Assignment 3 – Elliptic PDE Based Model (FEM)

Introduction:

In this assignment, we are required to calculate the gradient, velocity and the pressure due to a velocity potential field around a cylinder using an unstructured grid. The unstructured grid co-ordinates and connectivity information are provided and Finite Element Method is implemented to find the required quantities at each node point.

The program implements code for the following tasks:

- Read the grid/mesh information.
- Calculate the shape functions (triangular grid) and element contributions to each of the grid node points.
- Form a lumped mass matrix for the LHS.
- Calculate the potential field gradient for RHS.
- Invert and multiply the lumped mass matrix with the RHS to get velocity; and thus obtain pressure.

Numerical Method:

Finite Element Method is used to solve the model with some changes to improve performance. The lumped mass matrix is formed directly instead of scatter adding the element contributions to the global matrix.

Simulation Results:

Grid 3:

Grid 3 has 2137 nodes and 4025 elements. The simulation results in the following output:

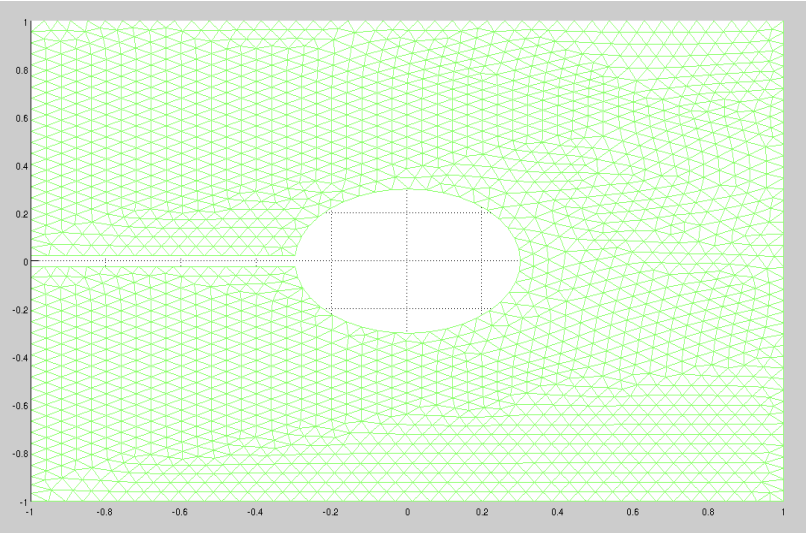

Illustration 1: Grid 3 - Mesh

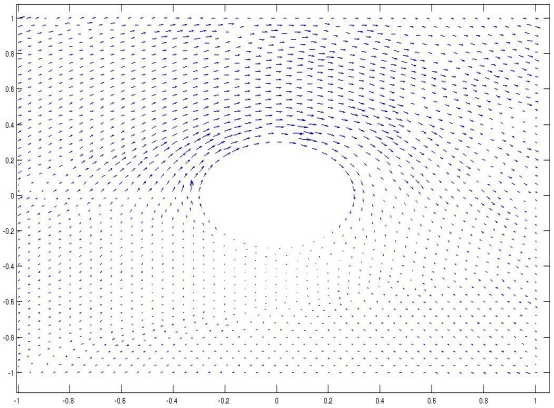

Illustration 2: Grid 3 - Velocity

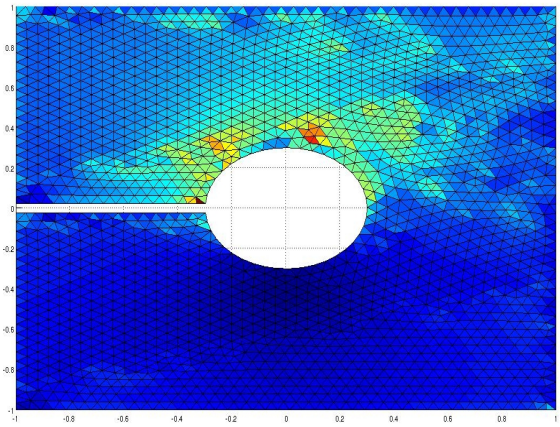

Illustration 3: Grid 3 - Pressure

Grid 4:

Grid 4 had 3838 nodes and 7329 elements with one velocity potential field. The simulation results are as follows:

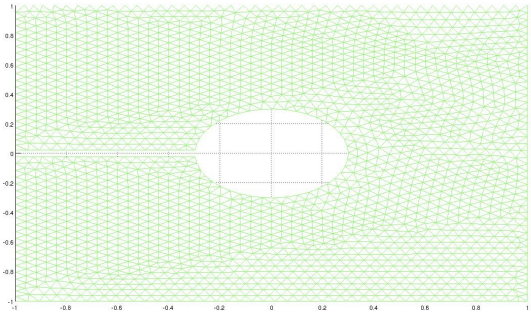

Illustration 4: Grid 4 - Mesh

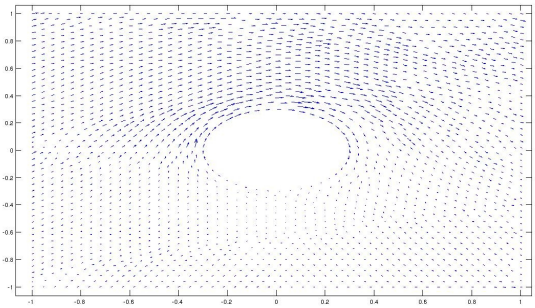

Illustration 5: Grid 4 - Velocity

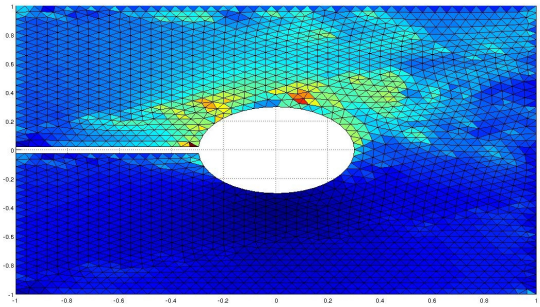

Illustration 6: Grid 4 - Pressure

Learnings & Conclusion:

Finite Element Method was very interesting to learn and implement. The assignment gave me a rudimentary knowledge of the stages of FEM analysis – from discretization, mesh generation, calculating the shape functions, element contributions and assimilating the approximations for the final result. I find it interesting to learn more of these in detail and also how this method can be implemented in a parallel computing frameworks.