CSI 747 - HW6 - Submission

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Exercise 7.4: <u>PET Image reconstruction Problem</u>:

The model used to solve this problem is as follows:

```
PET_Model.mod
```

```
param n; param N;
set voxels := {1..n}; set lines := {1..N};
param y{lines};
param C{voxels,lines};

var x{voxels} >=0, :=0.001;  #Constraint & Initial guess
var y_cap{j in lines} = sum {i in voxels} C[i,j]*x[i];
var logy{j in lines} = log(y_cap[j]);

maximize X_cap: sum{j in lines} (-(y_cap[j])+y[j]*logy[j]);
```

(I) The model was run on a problem with 9 variables and with N=33 detector pairs. The data file used was the following:

```
param N := 9;

param C :

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 :=

1 0.18 0.017 0.18 0 0 0 0 0 0 0 0 0 0.18 0.017 0.18 0 0 0 0 0 0 0 0.18 0.017 0.18 0 0 0 0 0 0 0 0

2 0 0.18 0.017 0.18 0 0 0 0 0 0 0 0 0.18 0.017 0.18 0 0 0 0 0 0 0 0.18 0.017 0.18 0 0 0 0 0 0 0

3 0 0 0.18 0.017 0.18 0 0 0 0 0 0 0 0 0.18 0.017 0.18 0 0 0 0 0 0 0 0.18 0.017 0.18 0 0 0 0 0 0

4 0 0 0 0.18 0.017 0.18 0 0 0 0 0 0 0 0.18 0.017 0.18 0 0 0 0 0 0 0 0.18 0.017 0.18 0 0 0 0 0

5 0 0 0 0 0.18 0.017 0.18 0 0 0 0 0 0 0 0.18 0.017 0.18 0 0 0 0 0 0 0 0.18 0.017 0.18 0 0 0 0
```

param y:= 0 0 1 19 27 30 40 50 35 15 1 0 0 1 7 20 38 56 55 38 20 7 1 0 1 3 17 38 40 20 7 1 0;

The run yielded the following result:

Output:

ampl: model PET_Model.mod;

MINOS 5.51: optimal solution found.

35 iterations, objective 1395.000296

Nonlin evals: obj = 81, grad = 80.

ampl: display x;

x[*] :=

- 1 1.68482
- 2 0
- 3 3.89696
- 4 51.3017
- 5 109.66
- 6 142.701
- 7 128.025
- 8 67.9574
- 9 14.6668

;

2. The model was run on a problem with n=1080 variables and N=2164 detector pairs. The matrix C was constructed using MATLAB as specified in the problem with a=0.15 and b=0.05. The resulting first few lines of the output obtained is as follows:

```
x[*] :=
 1 0.001 217 0.001 433 0.001
                                649 0.001
                                           865 0.001
 2 0.001 218 0.001
                    434 0.001
                                650 0.001
                                           866 0.001
         219 0.001
 3 0.001
                    435 0.001
                                651 0.001
                                           867 0.001
   0.001
          220 0.001
                     436 0.001
                                652 0.001
                                           868 0.001
          221 0.001
   0.001
                    437 0.001
                                653 0.001
                                           869 0.001
   0.001 222 0.001 438 0.001
                                654 0.001 870 0.001
   0.001 223 0.001
                    439 0.001
                                655 0.001 871 0.001
   0.001
          224 0.001
                    440 0.001
                                656 0.001 872 0.001
          225 0.001
   0.001
                    441 0.001
                                657 0.001 873 0.001
          226 0.001
    0.001
                     442 0.001
                                 658 0.001 874 0.001
 11 0.001
          227 0.001
                     443 576.569
                                 659 0.001 875 0.001
 12 0.001
          228 0.001
                     444 0.001
                                660 0.001 876 0.001
 13 0.001
          229 0.001
                     445 0.001
                                 661 0.001
                                           877 0.001
 14 0.001
          230 0.001
                                 662 0.001
                     446 0.001
                                           878 0.001
 15 0.001
          231 0.001
                     447 0.001
                                 663 0.001
                                           879 0.001
 16 0.001
          232 0.001
                                 664 0.001
                     448 0.001
                                            880 0.001
 17 0.001
          233 0.001
                     449 0.001
                                 665 0.001
                                           881 0.001
 18 423.426
           234 0.001 450 0.001
                                 666 0.001
                                            882 0.001
 19 0.001
          235 0.001
                     451 0.001
                                 667 0.001
                                            883 0.001
   0.001
          236 0.001
                                 668 0.001
                     452 0.001
                                            884 0.001
 21 0.001
          237 0.001
                                 669 0.001
                     453 0.001
                                            885 0.001
 22 0.001
          238 0.001
                     454 0.001
                                670 0.001
                                            886 0.001
 23 0.001
          239 0.001
                     455 0.001
                                671 0.001
                                            887 369.998
 24 0.001
          240 0.001
                     456 0.001
                                672 0.001
                                            888 0.001
 25 0.001
          241 0.001 457 0.001
                                673 0.001
                                           889 0.001
 26 0.001
          242 0.001
                     458 0.001 674 0.001
                                            890 0.001
 27 0.001
          243 0.001
                     459 544.283 675 0.001 891 0.001
    0.001
          244 0.001
                     460 0.001 676 0.001
                                           892 0.001
          245 0.001 461 0.001 677 0.001 893 0.001
   0.001
```

3. The image obtained after processing the output from AMPL using MATLAB was as follows:

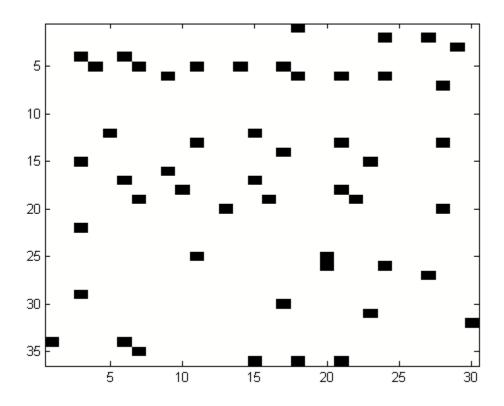


Illustration 1: PET Image - Constructed from Problem 2