CSI 700 – Numerical Methods – Project Report

Introduction:

This project required finding the minimum of Rosenbrock's function :

$$f(x) = 100(y - x^2)^2 + (1 - x)^2$$

Using unconstrained optimization techniques. We implement and investigate the convergence and optimality of Newton's method, Broyden-Fletcher-Goldfarb-Shanno (BFGS) update method, Steepest Descent method, Conjugate Gradient method. The methods are tested against the points [-1, 1] [0,1] and [3,1].

Analytical Solution:

Analytically, we can find the minimum through the following steps:

1. Solve $\nabla f(x, y) = 0$, to get the critical points of the function. (*first-order necessary condition*)
2. Find the Hessian Matrix of $f(x^*, y^*)$, where $x^*, y^*$are the critical points.
3. The function takes a minimum value if $H_f(x^*, y^*)$ is positive definite. (*second-order sufficient condition.*)

Applying these conditions to the Rosenbrock function we obtain:

$$\nabla f(x, y) = \begin{matrix} -400x(y - x^2) + 2(1 - x) \\ 200(y - x^2) \end{matrix}$$

The critical point we obtain is (1,1).

The Hessian matrix for the function is:

$$H_f = \begin{matrix} -400[(y - x^2) - 2x^2] + 2 & -400x \\ -400x & 200 \end{matrix}$$

At the critical points: $H_f = \begin{matrix} 802 & -400 \\ -400 & 200 \end{matrix}$

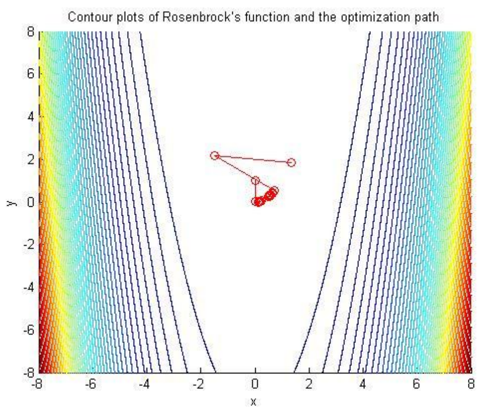
This matrix has positive eigen values and hence is a positive definite matrix.

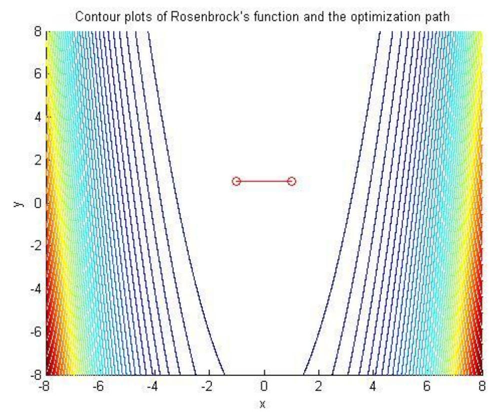
Hence function has minimum value at (1, 1).

Numerical Methods:

1. Newtons Method

The simulation results for various inputs are given below:

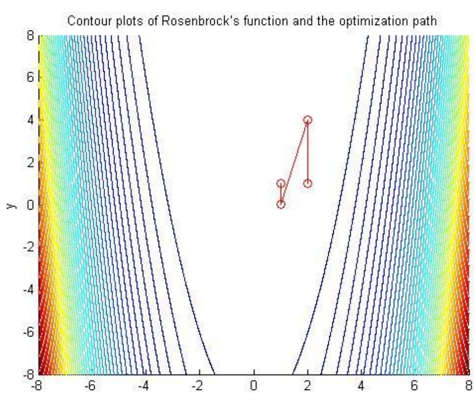


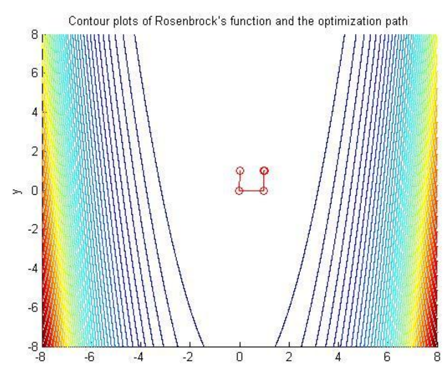


The first figure is the newtons method for input values 0, 1. And the second figure is for the input values -1,1. The solution doesn't converge to the analytic solution for the values 0,1 and 2,1.

2. BFGS Method

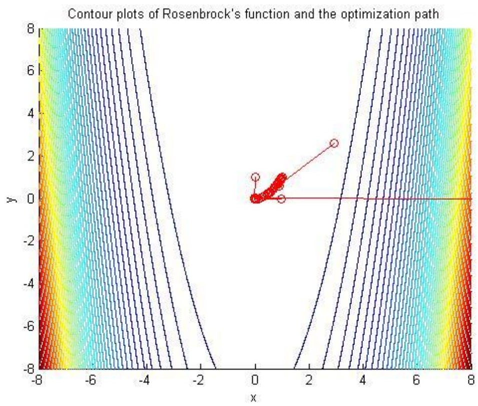
This is the fastest method among those implemented. It converges within 4 steps for the given inputs.

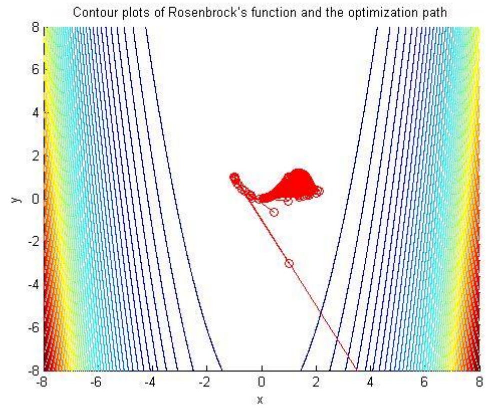




3. Conjugate Gradient Method

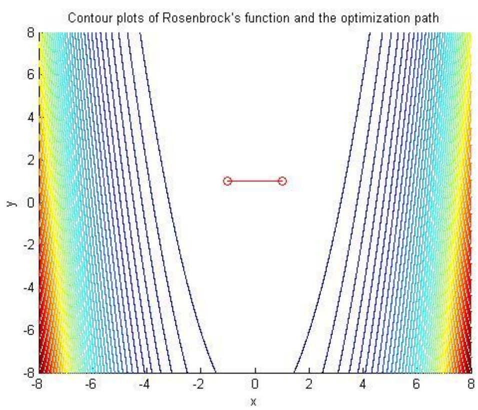
This method was implemented by using the linear line search for optimization. The Rosenbrock's function was changed in terms of α and optimized using the library function fminsearch.

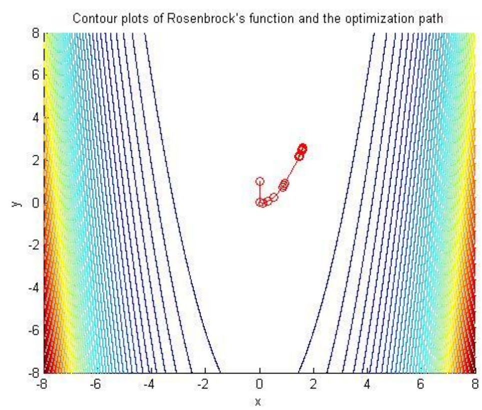




4. Steepest Descent Method:

This method also used a line search method coded similar to the Conjugate Gradient Method. It converges to the analytic solution.





5. Direct Search Method:

Used the library routine fminsearch to implement this method which, in turn, implements the Nealder-Mead Simplex Method.