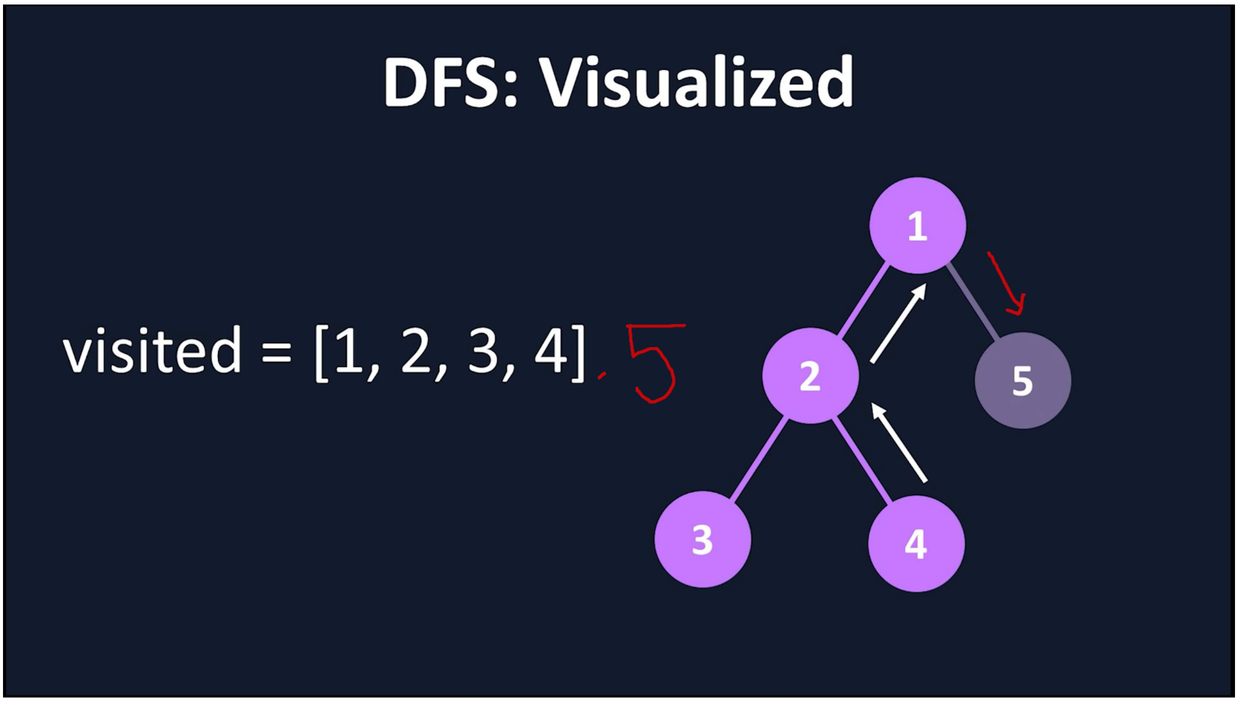
Overview of important Algorithms

- Searching
  - Binary Search
  - Depth First Search for trees and graphs
    - start from the top of a tree and go as deep as possible along the same branch
    - once you are at the bottom then go to nearest unvisited node usually a sibling of the deepest node
      - this process is called **Backtracking**
    - used to solve a maze
    - `O(number of nodes + number of branches)`
    - 

  - Breadth First Search for trees and graphs
    - you don't go to deepest point like DFS
    - instead you make sure that the sibling node has been visited
    - once you are on a node look at its children and add them to a queue and then you visit the node in the queue and add them to visited array and remove them from sibling queue
    - if the node in the queues has more children then add them to queue when marking it visited
    - used in chess
    - `O(number of nodes + number of branches)`
- Sorting

- Insertion Sort
    - compares the nth element with (n+1)th element and swaps them if nth element is larger
    - best case `O(n)` if everything is already sorted
    - worst case `O(n^2)` when nothing is sorted beforehand
- Merge Sort
    - divide and conquer and conquer by divide and conquer and so on
    - recursion
    - splits array in half till we have pairs of 2
    - then all pairs of 2 are sorted and then 2 pairs of 2 are merged and sorted till the array is completely merged back again
    - best and worst case are same `O(n log n)`
- Quick Sort
    - recursive like merge sort so divides and conquers
    - we choose a pivot element of the array which is closest to the median of the array elements
    - then we split the lists into 2 such that one list has elements less than the pivot element and one where all elements are greater than the pivot element
    - we repeat the same on these 2 lists
    - we move the pivot element to the end of the list
    - we place 2 pointers one on the 0th index and the 2nd on the 2nd last element and compare the two if the 0th one is larger we swap
    - deep doing it till the 2 pointers meet
    - when they meet replace that element with the last one
    - we know have 2 lists like we wanted and we can do the same thing on them individually
    - best case `O(n log n)`
    - worst case `O(n^2)`
    - still can be 2 to 3 times faster than merge sort by reducing the chances of worst case
    - needs less memory `O(log n)` than merge sort `O(n)`

- Greedy Algorithm
    - It makes the best possible decision at every local step
    - when not to be greedy
        - not meant for efficiency

- when to be greedy
  - when you don't want to find the most efficient way out of millions of permutations then greedy might be a good enough solution
  - when optimal solution not possible and brute force is not acceptable become greedy

Recursion

- a recursive function should have a terminating condition also called as a base condition

  - the values in the scope of the function can be used before(ascending) or after(descending) the termination condition and recursive call

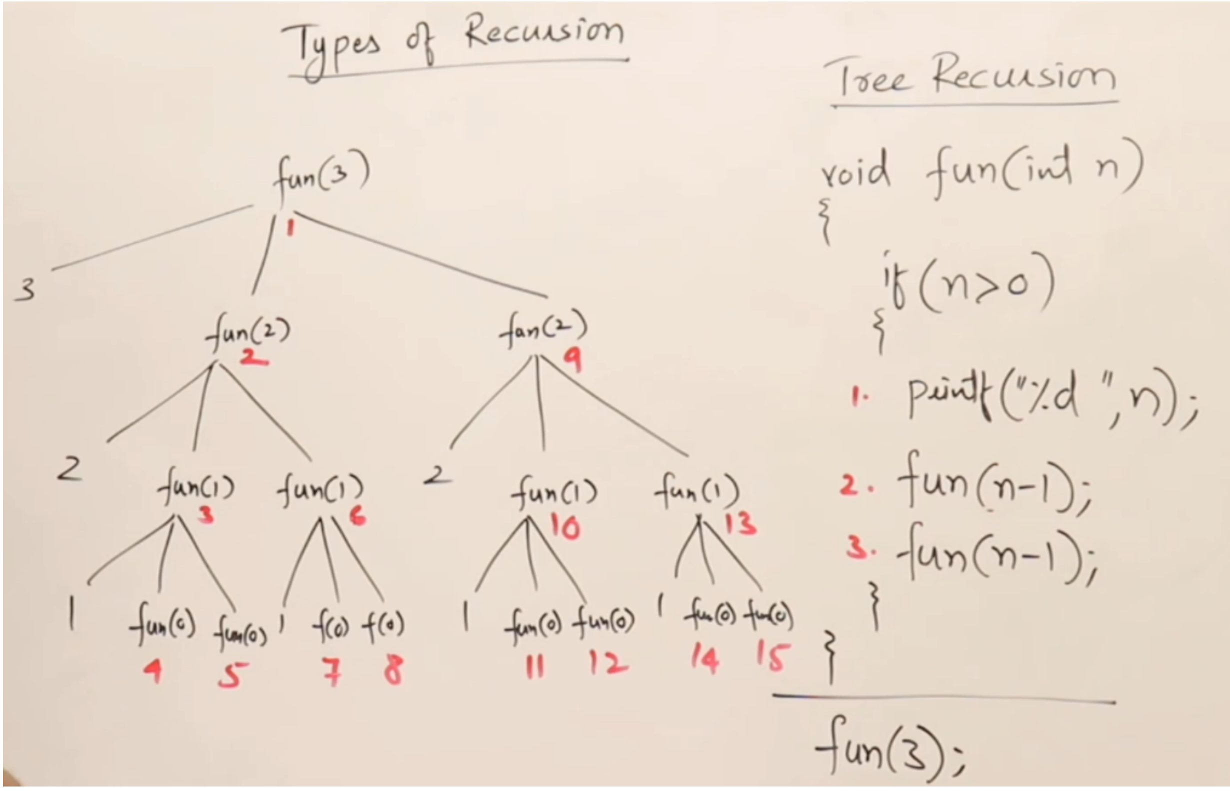
```cpp
#include <iostream>
using namespace std;

void head(int n)
{
    if (n > 0)
    {
        head(n - 1);
        cout << n << " ";
    }
}

void tail(int n)
{
    if (n > 0)
    {
        cout << n << " ";
        tail(n - 1);
    }
}

int main()
{
    head(10);
    cout << endl;
    tail(10);
```

```
    return 0;
}
// 1 2 3 4 5 6 7 8 9 10
// 10 9 8 7 6 5 4 3 2 1
```

- use static variables in recursive function if you need a counter and don't want the counter to reset on every recursive call
    - static variable will have a single copy for all recursive calls and will not be a local variable of the scope of a recursive function
    - it is like global but more restrictive
- types of recursion
    - `tail`
        - when the function calls itself in the last line of the function
        - easier to convert recursive logic to iterative
    - `head`
        - when the function calls itself in the first line of the function
        - harder to convert recursive logic to iterative
    - `tree`
        - opposite of tree recursion is linear recursion when the recursive function calls itself only one time
        - in tree recursion the recursive function calls itself more than one times

```
#include <iostream>
using namespace std;
void tree(int n)
{
    if (n > 0)
    {
        cout << n << " ";
        tree(n - 1);
        tree(n - 1);
    }
```

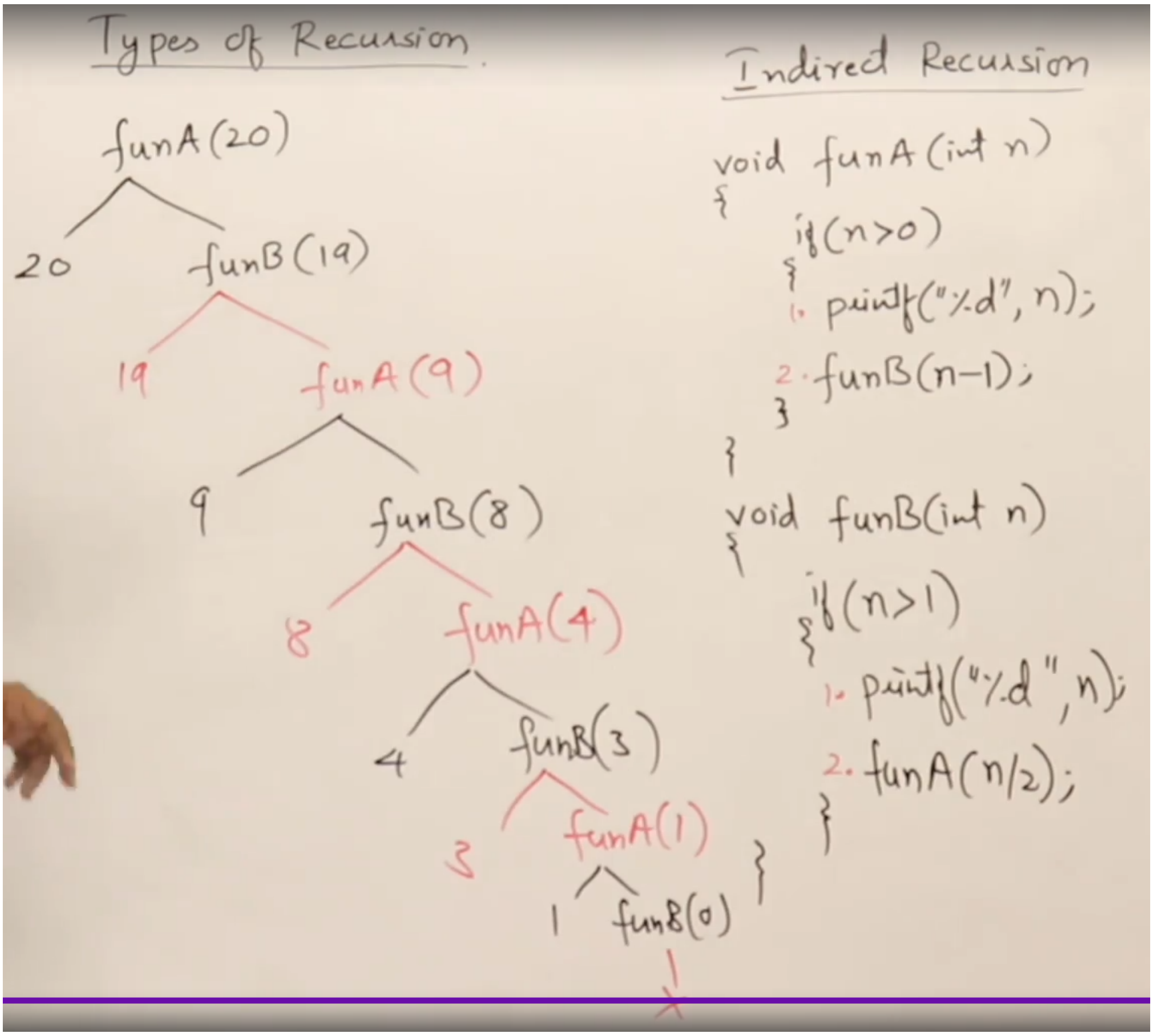
```
}

int main()
{
    tree(3);
    return 0;
}
// 3 2 1 1 2 1 1
// Time O(2^n)
// Space O(n)
```

- 

- indirect
    - when a function A calls B and B calls C and C calls A

```
#include <iostream>
using namespace std;
void funB(int n);

void funA(int n)
{
    if (n > 0)
    {
        cout << n << " ";
        funB(n - 1);
    }
}
```

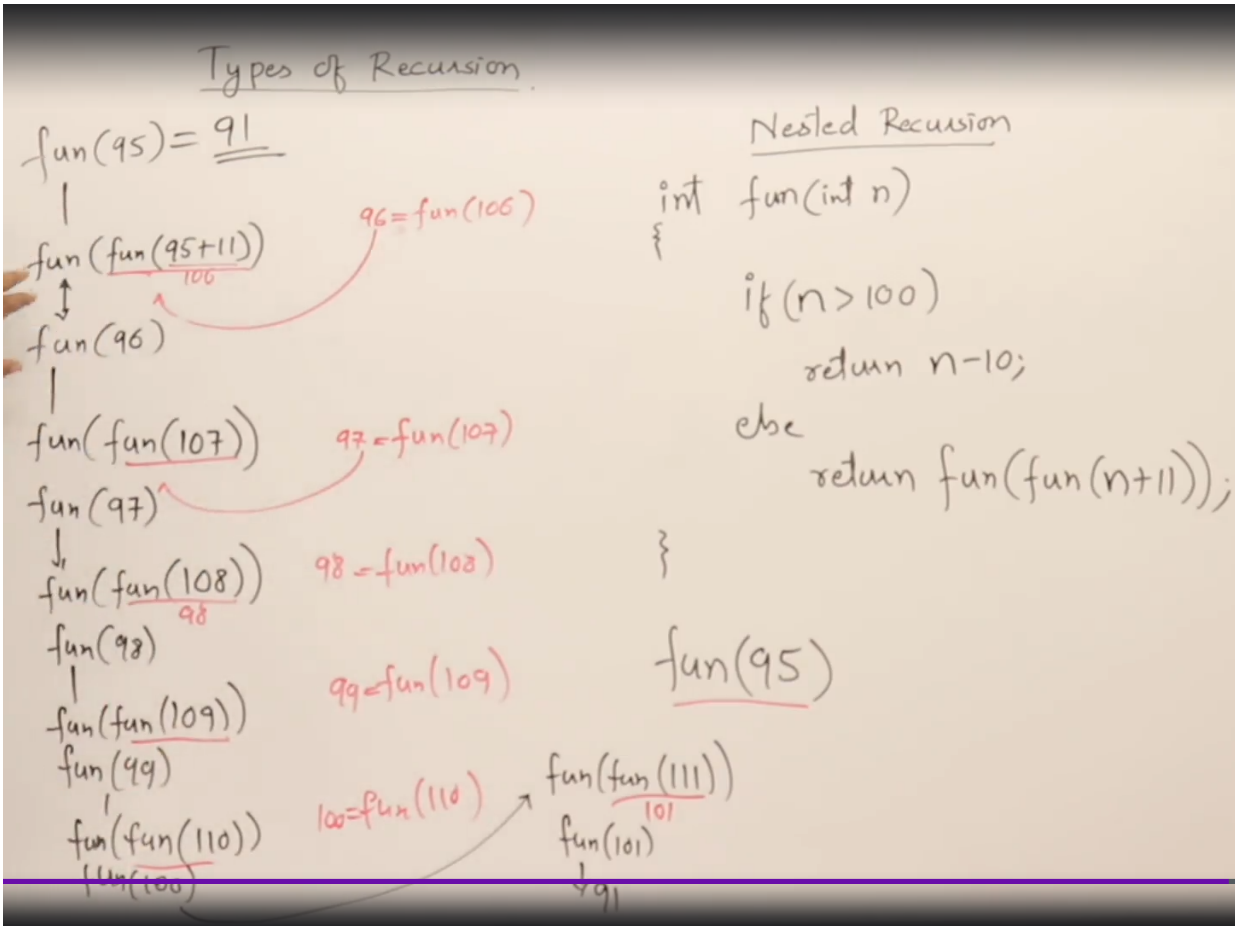
```cpp
void funB(int n)
{
    if (n > 1)
    {
        cout << "\n"
             << n << " ";
        funA(n / 2);
    }
}

int main()
{
    funA(20);
    return 0;
}
// 20
// 19 9
// 8 4
// 3 1
```

- 

- nested

- parameter of a recursive call function is the same function

```cpp
#include <iostream>
using namespace std;
int fun(int n)
{
    if (n > 100)
        return n - 10;
    return fun(fun(n + 11));
}
int main()
{
    cout << fun(95); // 91
        return 0;
}
```

- 

- Implementing `pow` function from `cmath` using recursion

```cpp
#include <iostream>
using namespace std;
int pow(int k, int p) { return p == 0 ? 1 : pow(k, p - 1) * k; }
int main()
{
    cout << "Enter constant and power: ";
```

```
    int con, pwr;
    cin >> con >> pwr;
    cout << con << "^" << pwr << " = " << pow(con, pwr);
    return 0;
}
```

- optimization: for 2^8 instead of multiplying 2 8 times shouldn't we half and square like (2^2)^4 = 4^4
  - this way we can reduce the stack height and increase memory efficiency
  - so if the power is even we half it and then we square the constant
  - else if the power is odd like 2^9 we can still do 2 x 2^8 and so on

```
#include <iostream>
using namespace std;
int pow(int k, int p) { return p == 0 ? 1 : p % 2 == 0 ? pow(k
* k, p / 2)
                                                                  : k *
pow(k * k, (p - 1) / 2); }
int main()
{
    cout << "Enter constant and power: ";
    int con, pwr;
    cin >> con >> pwr;
    cout << con << "^" << pwr << " = " << pow(con, pwr);
    return 0;
}
```

- Taylor Series using recursion is a combination of sum till n, power, factorial using recursion

  - to print e^x = 1 + x/1 + x^2/2! + x^3/3! + x^4/4! + ... till n terms
  - we need to use static variables as 3 variables are involved but we can return only one
    - the program will be less efficient if we don't use power and factorial as static variables as we will have to calculate the complete factorial over and over again
      - if factorial would have been static we just need to multiply a new number with the factorial of the previous number as n! = n x (n-1)!

- similarly we have to find x^n every time but if static we can store x^(n-1) and multiply x once

```cpp
#include <iostream>
using namespace std;
float e(int x, int n)
{
    if (n == 0)
        return 1;

    static float pwr = 1, fac = 1;
    float res = e(x, n - 1);
    pwr *= x;
    fac *= n;
    return res + pwr / fac;
}
int main()
{
    cout << "Enter x and n: ";
    int x, n;
    cin >> x >> n;
    cout << "e^" << x << " till n precision is " << e(x, n);
    return 0;
}
```

- optimizing using Horner's Rule
    - earlier the number of times we were multiplying was `O(n^2)` but using Horner's Rule it can be `O(1)`
    - to print e^x = 1 + x/1(1 + x/2(1 + x/3(1 + x/4 + ... till n terms)))
        - we keep taking commons out and this reduces number of multiplications that are needed to be performed
        - we find the value for the innermost bracket lets say (1+ x/4) here and multiply it with the common multiple x/3 and add 1 to it and go on recursively
        - using iteration

```cpp
#include <iostream>
using namespace std;
float e(int x, int n)
{
    int res = 1;
    for (; n > 0; n--)
```

```
        res = 1 + x * res / n;
    return res;
}
int main()
{
    cout << "Enter x and n: ";
    int x, n;
    cin >> x >> n;
    cout << "e^" << x << " till n precision is " << e(x,
n);
    return 0;
}
```

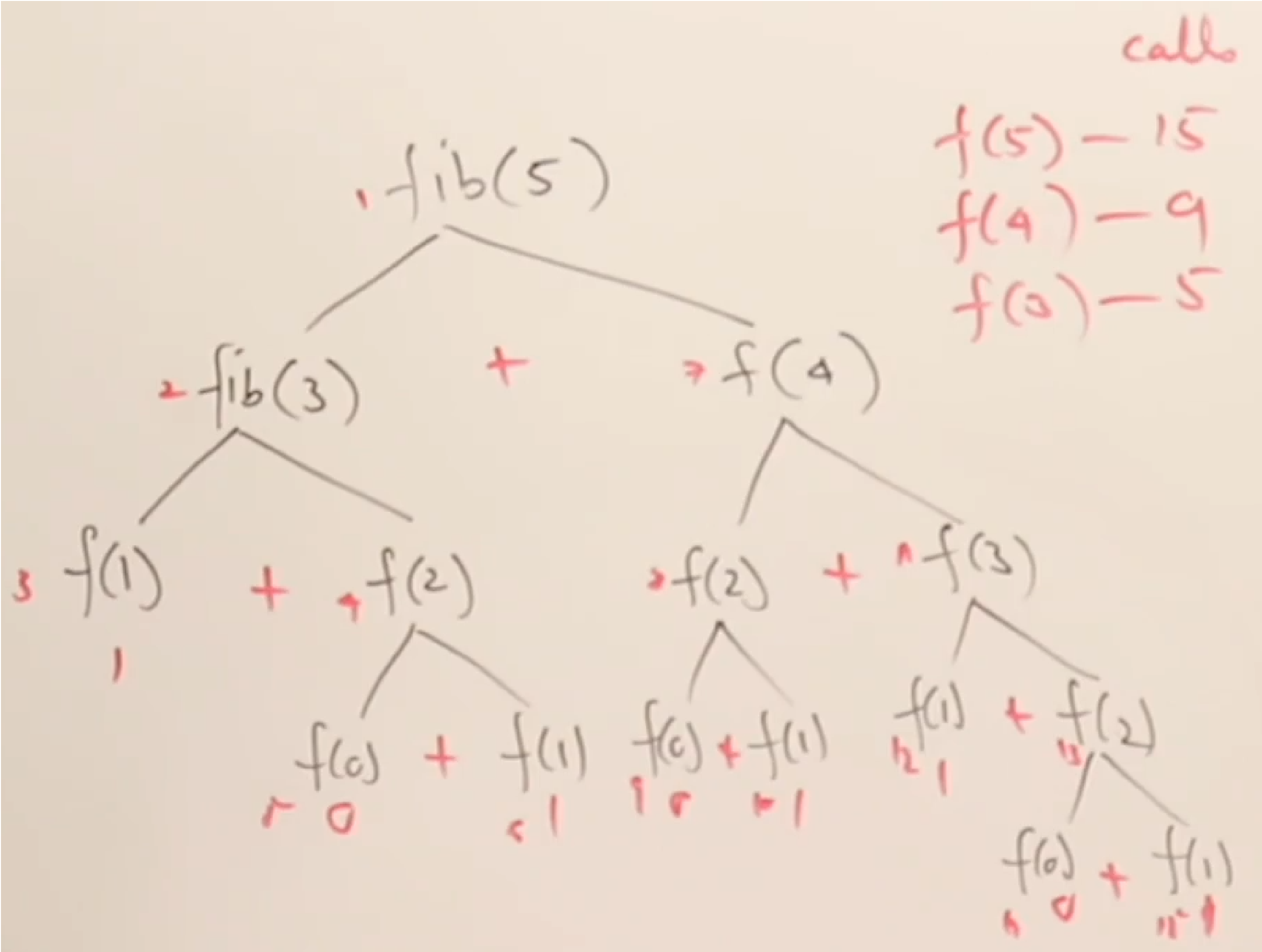
- using recursion

```
#include <iostream>
using namespace std;
float e(int x, int n)
{
    static int res = 1;
    if (n == 0)
        return res;
    res = 1 + x * res / n;
    return e(x, n - 1);
}
int main()
{
    cout << "Enter x and n: ";
    int x, n;
    cin >> x >> n;
    cout << "e^" << x << " till n precision is " << e(x,
n);
    return 0;
}
```

- Fibonacci Series
    - using recursion `O(2^n)`

```
#include <iostream>
using namespace std;
int fibo(int n)
{
```

```
    if (n <= 1)
        return n;
    return fibo(n - 2) + fibo(n - 1); // as the function calls itself 2 times with n as arg so O(2^n)
}
```

- 

    - here we can see that fib(3) and fib(2) get calculated over and over as the value is not stored
        - it is a case of excessive recursion and we can fix it by using static variables
            - we create a static array that stores the fib(n) at index n and the default values for all the elements is -1 so we can check do we need to find fib(n) at every step so `O(n)`
            - this process is called memoization

```
#include <bits/stdc++.h>
using namespace std;
int fibo(int n)
{
    static vector<int> memo(n + 1, -1);
    if (n <= 1)
    {
        memo[n] = n;
```

```
        return n;
    }
    else if (memo[n - 2] == -1)
        memo[n - 2] = fibo(n - 2);
    if (memo[n - 1] == -1)
        memo[n - 1] = fibo(n - 1);
    return memo[n - 2] + memo[n - 1];
}
```

- using iteration `O(n)`

```
#include <iostream>
using namespace std;
int fibo(int n)
{
    if (n <= 1)
        return n;
    int t0 = 0, t1 = 1, s = 0;
    for (int i = 2; i <= n; i++)
    {
        s = t0 + t1;
        t0 = t1;
        t1 = s;
    }
    return s;
}
```