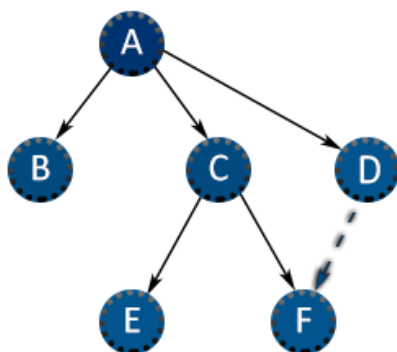


Overview of important Algorithms

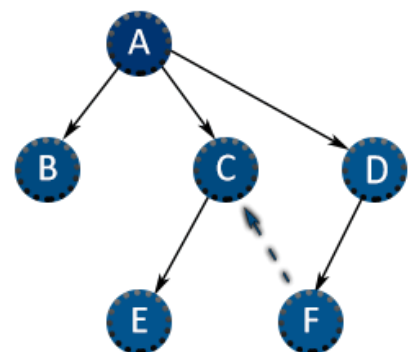
- Searching
 - Binary Search
 - Depth First Search for trees and graphs
 - start from the top of a tree and go as deep as possible along the same branch
 - once you are at the bottom then go to nearest unvisited node usually a sibling of the deepest node
 - this process is called **Backtracking**
 - used to solve a maze
 - $O(\text{number of nodes} + \text{number of branches})$
 - Breadth First Search for trees and graphs
 - you don't go to deepest point like DFS
 - instead you make sure that the sibling node has been visited
 - once you are on a node look at its children and add them to a queue and then you visit the node in the queue and add them to visited array and remove them from sibling queue
 - if the node in the queues has more children then add them to queue when marking it visited
 - used in chess
 - $O(\text{number of nodes} + \text{number of branches})$

BFS



A B C D E F

DFS



A D F C E B

- Sorting
 - Insertion Sort

- compares the n th element with $(n+1)$ th element and swaps them if n th element is larger
- best case $O(n)$ if everything is already sorted
- worst case $O(n^2)$ when nothing is sorted beforehand
- Merge Sort
 - divide and conquer and conquer by divide and conquer and so on
 - recursion is used
 - splits array in half till we have pairs of 2
 - then all pairs of 2 are sorted and then 2 pairs of 2 are merged and sorted till the array is completely merged back again
 - best and worst case are same $O(n \log n)$
- Quick Sort
 - recursive like merge sort so divides and conquers
 - we choose a pivot element of the array which is closest to the median of the array elements
 - then we split the lists into 2 such that one list has elements smaller than the pivot element and one where all elements are greater than the pivot element
 - we repeat the same on these 2 lists
 - we move the pivot element to the end of the list
 - we place 2 pointers one on the 0th index and the 2nd on the 2nd last element and compare the two if the 0th one is larger we swap
 - keep doing it till the 2 pointers meet
 - when they meet replace that element with the last one
 - we now have 2 lists like we wanted and we can do the same thing on them individually
 - best case $O(n \log n)$
 - worst case $O(n^2)$
 - still can be 2 to 3 times faster than merge sort by reducing the chances of worst case
 - needs less memory $O(\log n)$ than merge sort $O(n)$
- Greedy Algorithm
 - it makes the best possible decision at every local step
 - when not to be greedy
 - not meant for efficiency
 - when to be greedy

- when you don't want to find the most efficient way out of millions of permutations then greedy might be a good enough solution
- when optimal solution not possible and brute force is not acceptable become greedy

Recursion

- a recursive function should have a terminating condition also called as a base condition
- the values in the scope of the function can be used before(ascending) or after(descending) the termination condition and recursive call

```
#include <iostream>
using namespace std;

void head(int n)
{
    if (n > 0)
    {
        head(n - 1);
        cout << n << " ";
    }
}

void tail(int n)
{
    if (n > 0)
    {
        cout << n << " ";
        tail(n - 1);
    }
}

int main()
{
    head(10);
    cout << endl;
    tail(10);

    return 0;
}
```

```
}  
// 1 2 3 4 5 6 7 8 9 10  
// 10 9 8 7 6 5 4 3 2 1
```

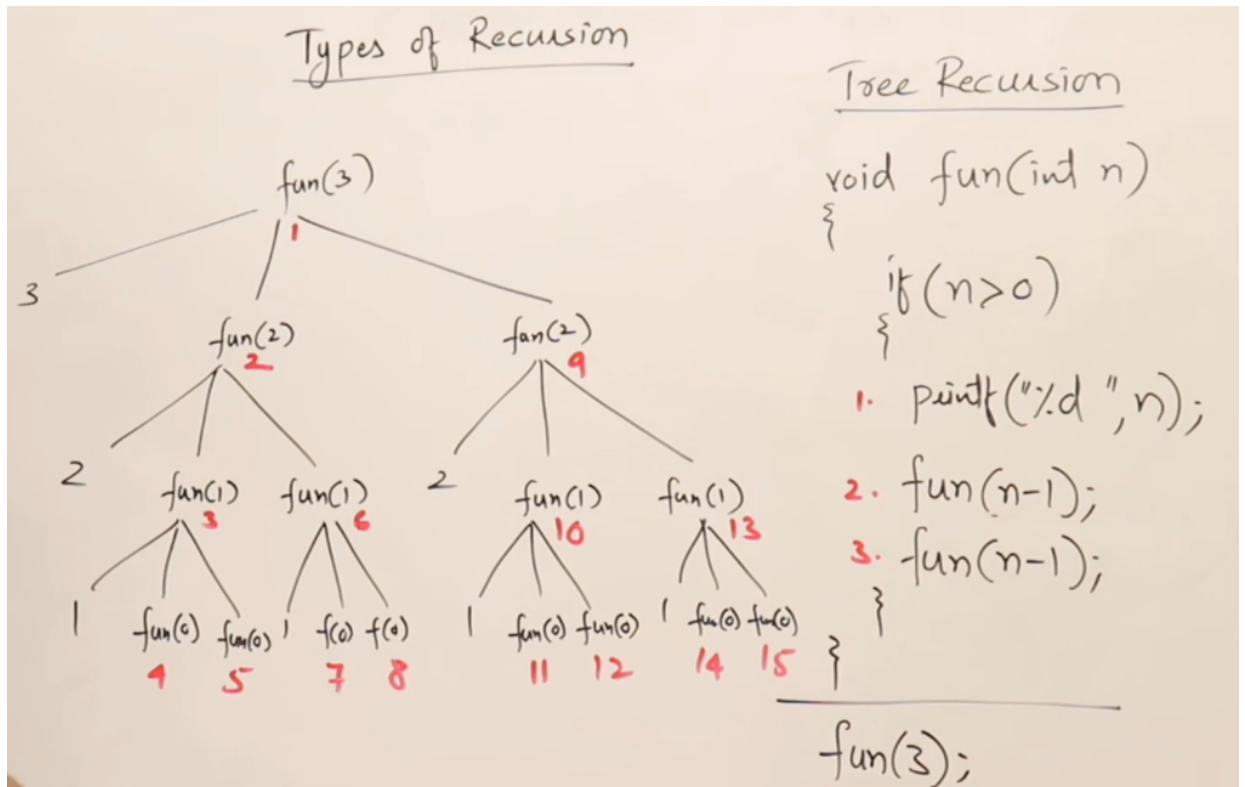
- use static variables in recursive function when you need a counter and don't want the counter to reset on every recursive call
 - static variable will have a single copy for all recursive calls and will not be a local variable of the scope of a recursive function
 - it is like global but more restricted
- types of recursion
 - tail
 - when the function calls itself in the last line of the function
 - easier to convert recursive logic to iterative
 - head
 - when the function calls itself in the first line of the function
 - harder to convert recursive logic to iterative
 - tree
 - opposite of tree recursion is linear recursion when the recursive function calls itself only one time
 - in tree recursion the recursive function calls itself more than one times

```
#include <iostream>  
using namespace std;  
void tree(int n)  
{  
    if (n > 0)  
    {  
        cout << n << " ";  
        tree(n - 1);  
        tree(n - 1);  
    }  
}
```

```

int main()
{
    tree(3);
    return 0;
}
// 3 2 1 1 2 1 1
// Time  $O(2^n)$ 
// Space  $O(n)$ 

```



- indirect

- when a function A calls B and B calls C and C calls A

```

#include <iostream>
using namespace std;
void funB(int n);

void funA(int n)
{
    if (n > 0)
    {
        cout << n << " ";
        funB(n - 1);
    }
}

void funB(int n)

```

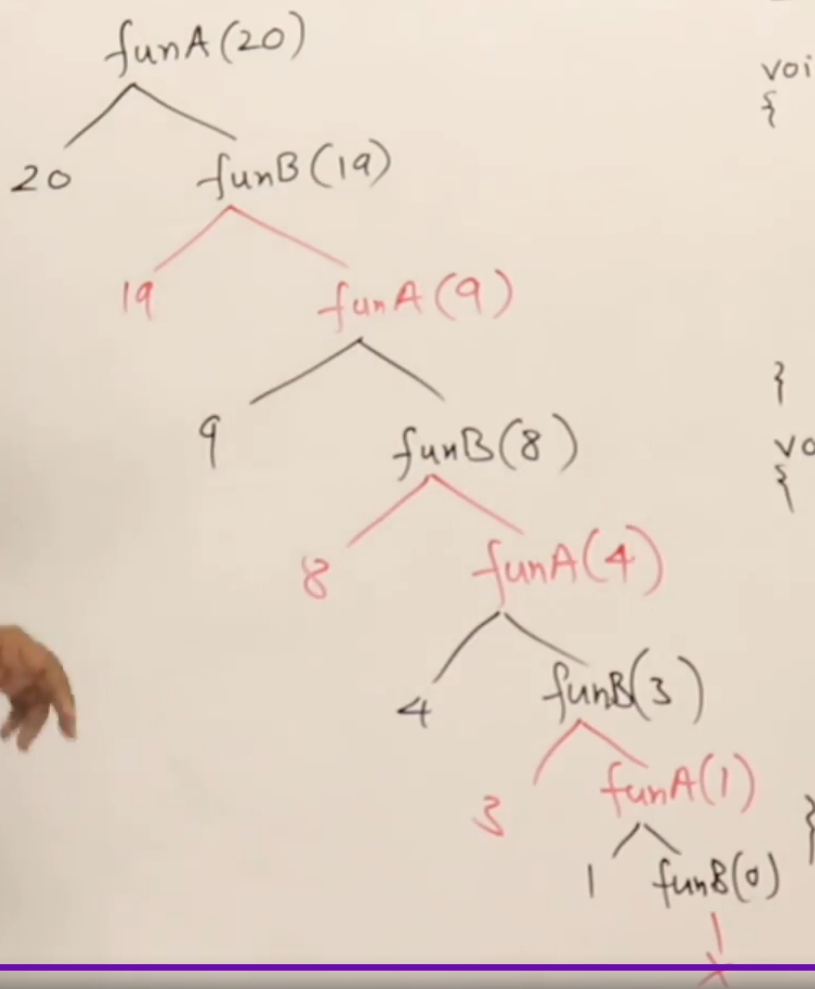
```

{
    if (n > 1)
    {
        cout << "\n"
              << n << " ";
        funA(n / 2);
    }
}

int main()
{
    funA(20);
    return 0;
}
// 20
// 19 9
// 8 4
// 3 1

```

Types of Recursion.



Indirect Recursion

```

void funA(int n)
{
    if(n > 0)
    {
        1. printf("%d", n);
        2. funB(n-1);
    }
}

void funB(int n)
{
    if(n > 1)
    {
        1. printf("%d", n);
        2. funA(n/2);
    }
}

```

- nested

- parameter of a recursive call function is the same function

```
#include <iostream>
using namespace std;
int fun(int n)
{
    if (n > 100)
        return n - 10;
    return fun(fun(n + 11));
}
int main()
{
    cout << fun(95); // 91
    return 0;
}
```



- Implementing pow function from cmath using recursion

```
#include <iostream>
using namespace std;
int pow(int k, int p) { return p == 0 ? 1 : pow(k, p - 1) * k; }
int main()
{
    cout << "Enter constant and power: ";
}
```

```

int con, pwr;
cin >> con >> pwr;
cout << con << "^" << pwr << " = " << pow(con, pwr);
return 0;
}

```

- **optimization** for 2^8 instead of multiplying 2 8 times shouldn't we half and square like $(2^2)^4 = 4^4$
 - this way we can reduce the stack height and increase memory efficiency
 - so if the power is even we half it and then we square the constant
 - else if the power is odd like 2^9 we can still do 2×2^8 and so on

```

#include <iostream>
using namespace std;
int pow(int k, int p) { return p == 0 ? 1 : p % 2 == 0 ? pow(k
                                                                : k *
pow(k * k, (p - 1) / 2); }
int main()
{
    cout << "Enter constant and power: ";
    int con, pwr;
    cin >> con >> pwr;
    cout << con << "^" << pwr << " = " << pow(con, pwr);
    return 0;
}

```

- **Taylor Series using recursion is a combination of sum till n, power, factorial using recursion**
 - to print $e^x = 1 + x/1 + x^2/2! + x^3/3! + x^4/4! + \dots$ till n terms
 - we need to use static variables as 3 variables are involved but we can return only one
 - the program will be less efficient if we don't use power and factorial as static variables as we will have to calculate the complete factorial over and over again
 - if factorial would have been static we just need to multiply a new number with the factorial of the previous number as $n! = n \times$

(n-1)!

- similarly we have to find x^n every time but if static we can store $x^{(n-1)}$ and multiply x once

```
#include <iostream>
using namespace std;
float e(int x, int n)
{
    if (n == 0)
        return 1;

    static float pwr = 1, fac = 1;
    float res = e(x, n - 1);
    pwr *= x;
    fac *= n;
    return res + pwr / fac;
}
int main()
{
    cout << "Enter x and n: ";
    int x, n;
    cin >> x >> n;
    cout << "e^" << x << " till n precision is " << e(x, n);
    return 0;
}
```

- optimizing using Horner's Rule

- earlier the number of times we were multiplying was $O(n^2)$ but using Horner's Rule it can be $O(1)$
- to print $e^x = 1 + x/1(1 + x/2(1 + x/3(1 + x/4 + \dots \text{till } n \text{ terms})))$
 - we keep taking commons out and this reduces number of multiplications that are needed to be performed
 - we find the value for the innermost bracket lets say $(1 + x/4)$ here and multiply it with the common multiple $x/3$ and add 1 to it and go on recursively
 - using iteration

```
#include <iostream>
using namespace std;
float e(int x, int n)
```

```

    {
        int res = 1;
        for (; n > 0; n--)
            res = 1 + x * res / n;
        return res;
    }
int main()
{
    cout << "Enter x and n: ";
    int x, n;
    cin >> x >> n;
    cout << "e^" << x << " till n precision is " <<
e(x, n);
    return 0;
}

```

- using recursion

```

#include <iostream>
using namespace std;
float e(int x, int n)
{
    static int res = 1;
    if (n == 0)
        return res;
    res = 1 + x * res / n;
    return e(x, n - 1);
}
int main()
{
    cout << "Enter x and n: ";
    int x, n;
    cin >> x >> n;
    cout << "e^" << x << " till n precision is " <<
e(x, n);
    return 0;
}

```

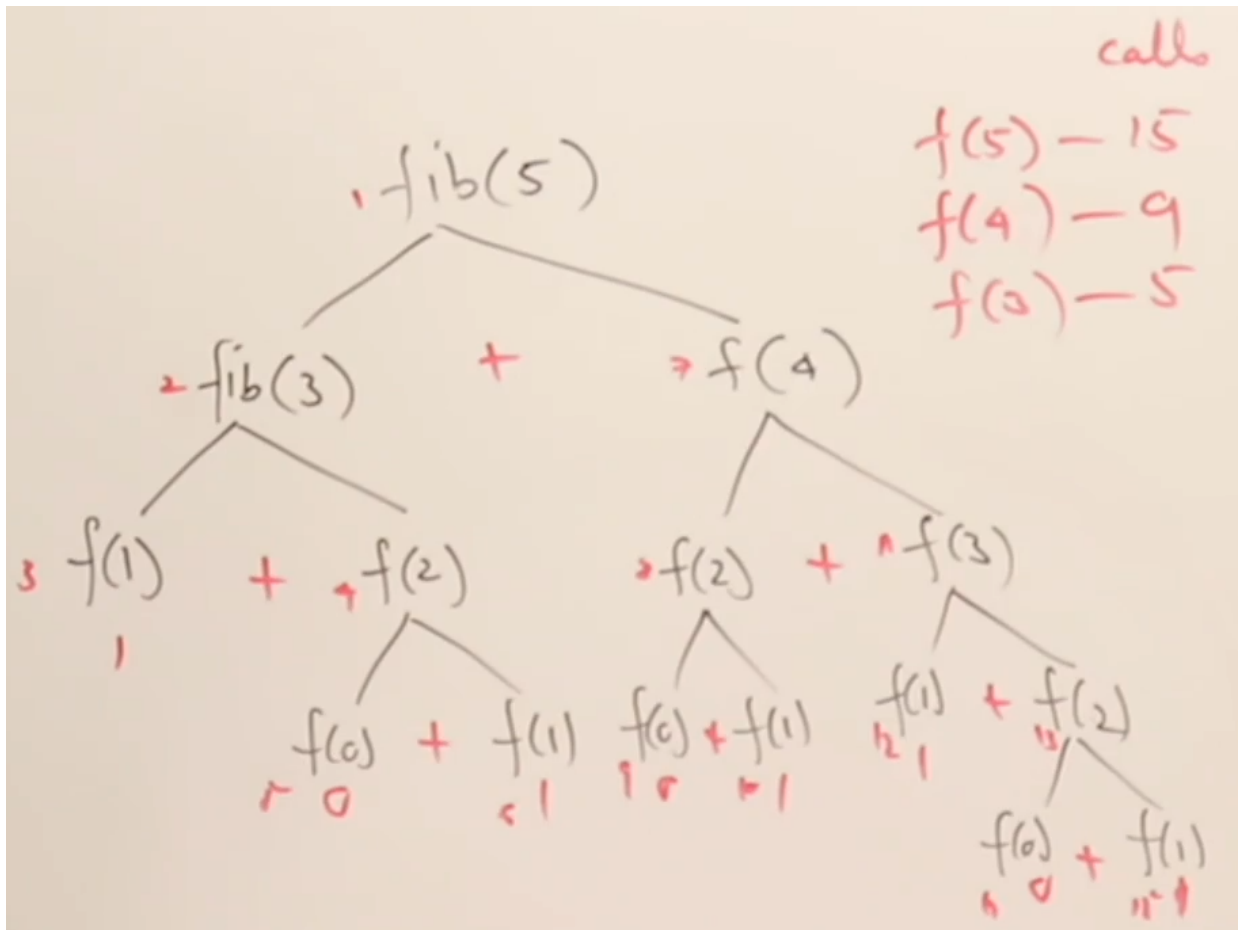
- Fibonacci Series

- using recursion $O(2^n)$

```

#include <iostream>
using namespace std;
int fibo(int n)
{
    if (n ≤ 1)
        return n;
    return fibo(n - 2) + fibo(n - 1); // as the function calls
    itself 2 times with n as arg so  $O(2^n)$ 
}

```



- here we can see that fib(3) and fib(2) get calculated over and over as the value is not stored
 - it is a case of excessive recursion and we can fix it by using static variables
 - we create a static array that stores the fib(n) at index n and the default values for all the elements is -1 so we can check do we need to find fib(n) at every step so $O(n)$
 - this process is called **memoization**

```

#include <bits/stdc++.h>
using namespace std;
int fibo(int n)
{

```

```

static vector<int> memo(n + 1, -1);
if (n ≤ 1)
{
    memo[n] = n;
    return n;
}
else if (memo[n - 2] == -1)
    memo[n - 2] = fibo(n - 2);
if (memo[n - 1] == -1)
    memo[n - 1] = fibo(n - 1);
return memo[n - 2] + memo[n - 1];
}

```

- using iteration $O(n)$

```

#include <iostream>
using namespace std;
int fibo(int n)
{
    if (n ≤ 1)
        return n;
    int t0 = 0, t1 = 1, s = 0;
    for (int i = 2; i ≤ n; i++)
    {
        s = t0 + t1;
        t0 = t1;
        t1 = s;
    }
    return s;
}

```

- $nCr = n! / (r! * (n-r)!)$

- using iteration

```

#include <iostream>
using namespace std;
int fac(int n)
{
    int res = 1;
    for (int i = 1; i ≤ n; i++)
        res *= i;
    return res;
}

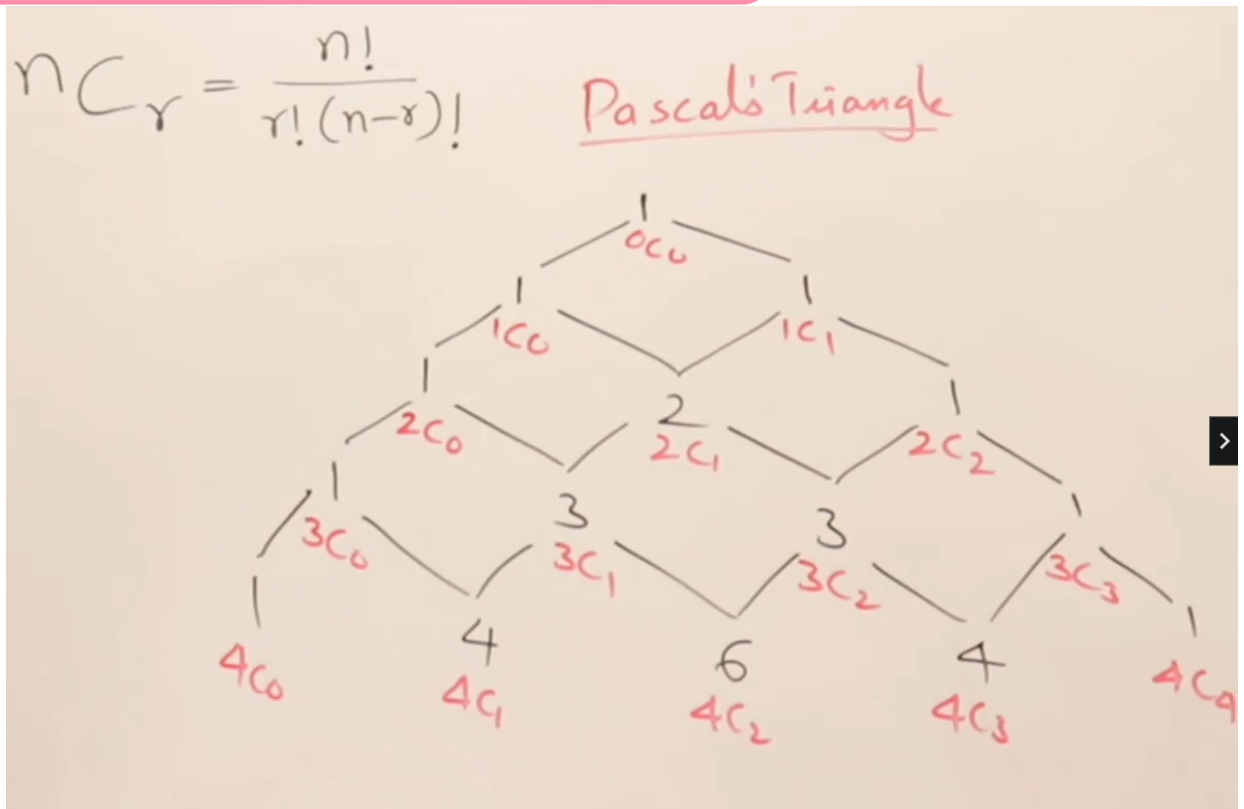
```

```

}
int main()
{
    cout << "Enter n and r: ";
    int n, r;
    cin >> n >> r;
    if (r > n)
    {
        cout << "Invalid Input " << r << " grtr than " << n;
        return -1;
    }
    cout << n << "C" << r << " = " << fac(n) / (fac(r) * fac(n
- r));
    return 0;
}

```

- for recursion we need to use Pascal's Triangle



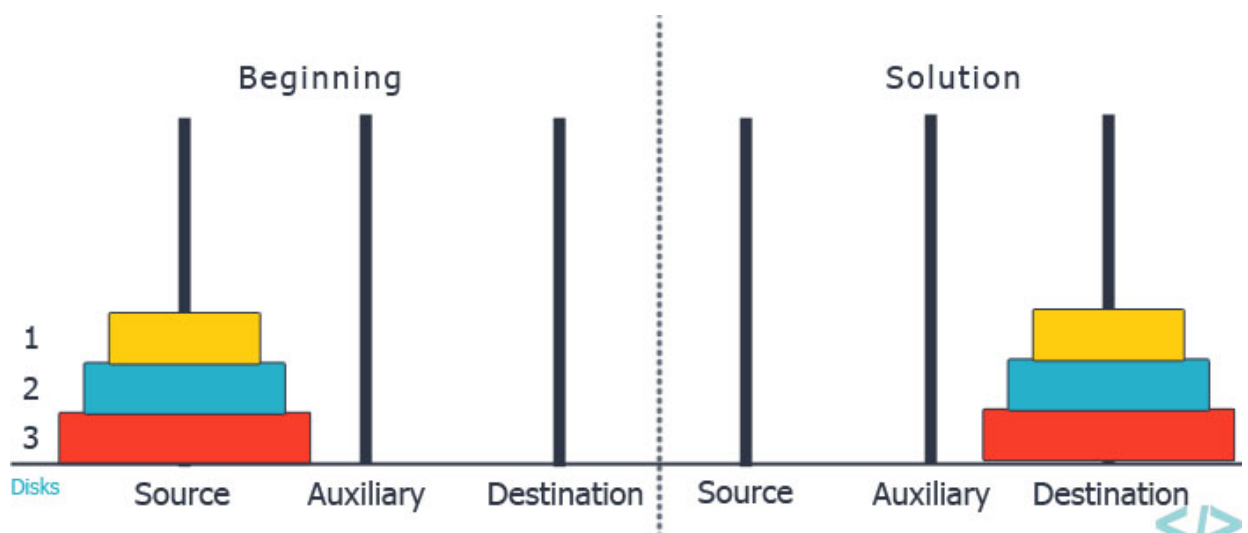
- here we have rows and cols and an element having both \ and / points to elements whose sum is equal to itself
 - observe how $2C_1$ points to above elements $1C_0$ and $1C_1$ and $2C_1 = 2 = 1C_0 + 1C_1 = 1 + 1$
 - so we can say 6 is $4C_2$ is obtained from $3+3$ of $3C_1$ and $3C_2$ which themselves are obtained from a sum
 - our base condition can be outlined by observing that the extreme leftmost and rightmost elements are always 1 and the topmost element is also always 1

- so $nCr = (n-1)C(r-1) + (n-1)Cr$
- we go bottom to up for recursion and then come down from top to bottom when returning

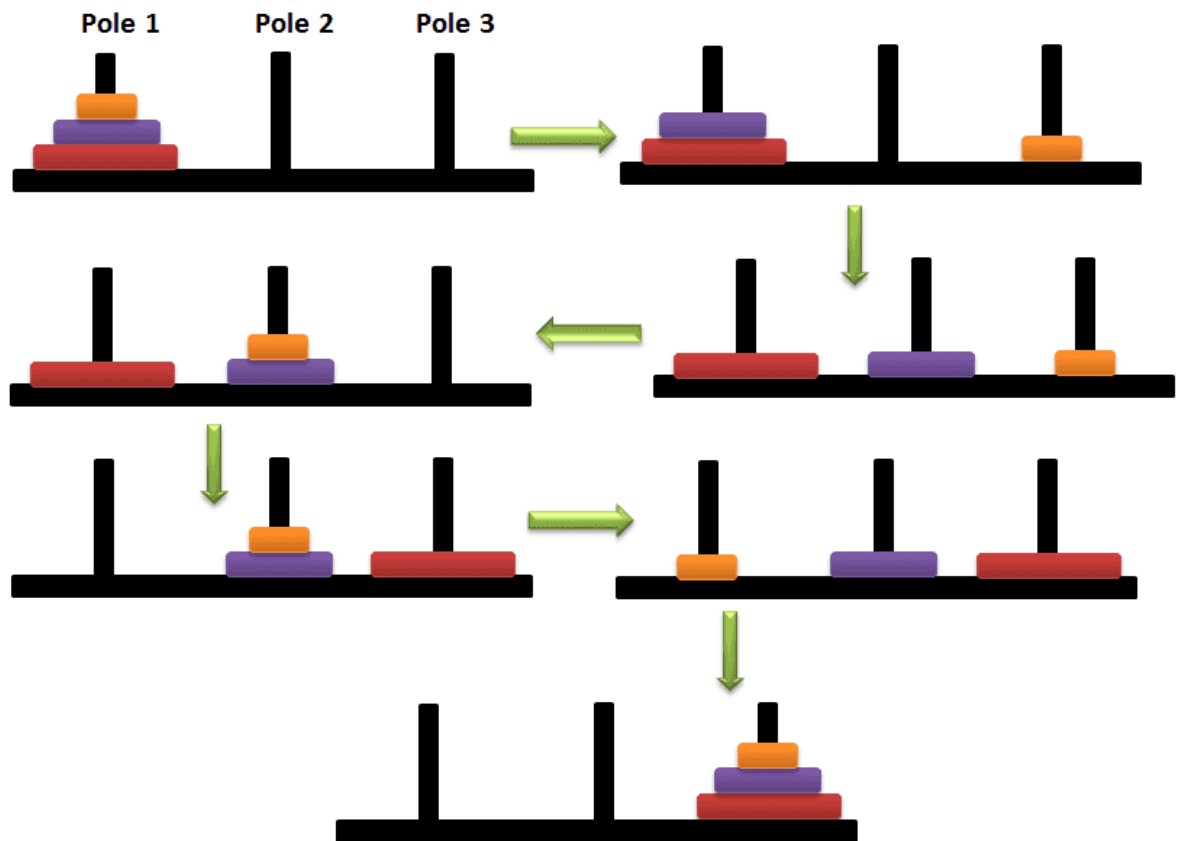
```
#include <iostream>
using namespace std;
int C(int n, int r)
{
    if (r == 0 || n == r)
        return 1; // extreme left and right base
    condition
    return C(n - 1, r - 1) + C(n - 1, r);
}
int main()
{
    cout << "Enter n and r: ";
    int n, r;
    cin >> n >> r;
    if (r > n)
    {
        cout << "Invalid Input " << r << " grtr than "
        << n;
        return -1;
    }
    cout << n << "C" << r << " = " << C(n, r);
    return 0;
}
```

- Tower of Hanoi

- Question:

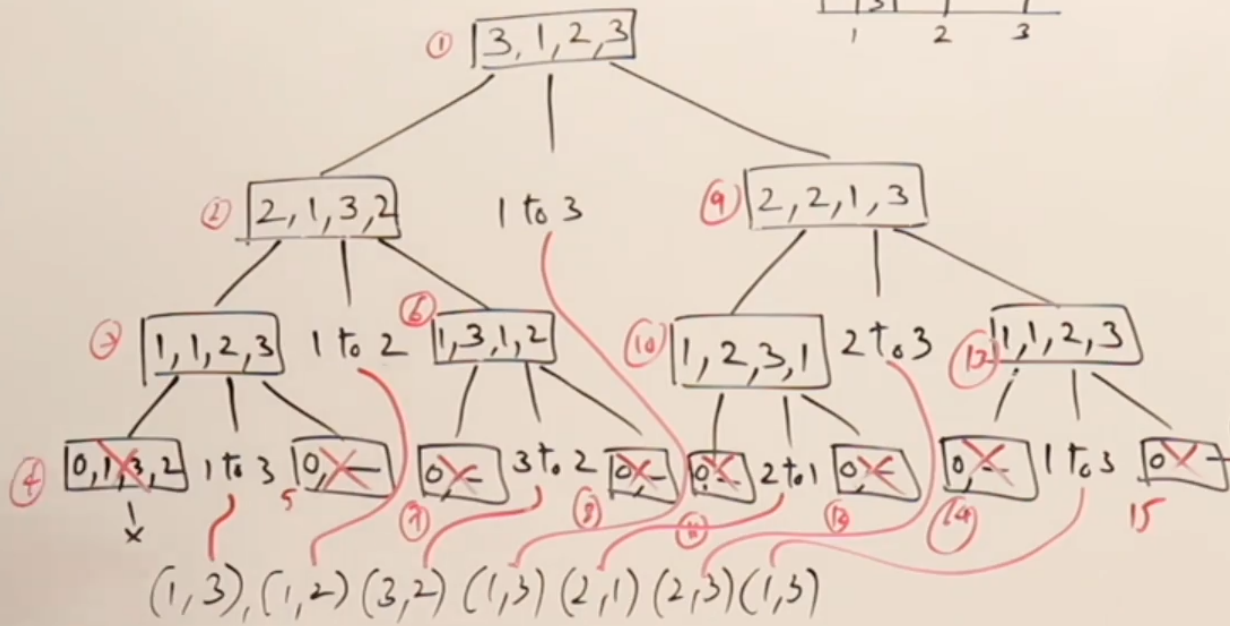
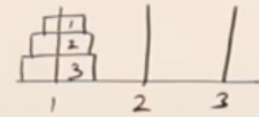


- Auxiliary pole is for helping like temp variable for swapping
 - discs are always sorted in the source pole
- Procedure to Solution: $O(2^n)$



- cases
 - if the number of discs is one then just move it from pole 1 to 3
 - if the number of discs are 2 then
 1. move the smaller disc to middle pole
 2. move the larger disc from 1st to the 3rd pole
 3. move the smallest disc from 2nd to 3rd pole
 - if number of discs are 3 then
 1. ignore the largest disc (3rd disc) and move the 2 discs to 2nd pole as if the number of discs were 2
 2. move the largest disc to 3rd pole
 3. move the 2 discs in 2nd pole to the 3rd pole as if the number of discs were 2
 - if number of discs are n then
 1. ignore the largest disc (nth disc) and move the n-1 discs to 2nd pole as if the number of discs were n and do this recursively
 2. move the largest disc (nth disc) to 3rd pole
 3. move the n-1 discs in 2nd pole to the 3rd pole as if the number of discs were n

Tower of Hanoi



```
#include <iostream>
using namespace std;
```

```
void toh(int n, int sp1, int mp2, int dp3)
```

```
// source pole 1, middle pole 2 and destination pole 3 are just
// pole numbers and n discs are stored in sp1 and we want to move it
// to dp3 in a specific order
```

```
{
    if (n < 0)
        return;
    static int ctr = 0;
    toh(n - 1, sp1, dp3, mp2); // move (n-1)th disc from sp1 to
    mp2 using dp3 as Auxiliary Pole
    ctr++;
    cout << "[Step " << ctr << "] move disc from " << sp1 << " to
    " << dp3 << endl; // print what was done in the above step
    toh(n - 1, mp2, sp1, dp3);
    // move (n-1)th disc from mp2 to dp3 using sp1 as Auxiliary Pole
    // when n=1 the topmost pole is moved and then n=2 so the one
    // below it is moved and so on
}
```

```
int main()
```

```
{
    toh(2, 1, 2, 3); // means sp1 is the 1st pole, mp2 is the 2nd
    pole and dp3 is the 3rd pole
    return 0;
}
```



```
}
```

```
/**
```

```
[Step 1] move disc from 1 to 3
```

```
[Step 2] move disc from 1 to 2
```

```
[Step 3] move disc from 3 to 2
```

```
[Step 4] move disc from 1 to 3
```

```
[Step 5] move disc from 2 to 1
```

```
[Step 6] move disc from 2 to 3
```

```
[Step 7] move disc from 1 to 3
```

```
*/
```