Challenge Problem

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Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

Given a point P outside a circle whose equation is known. Find the equation(s) of the tangent(s) and the the distance from P to the point(s).

2 Explanation

Let from point $\mathbf{P}$ there are 2 tangents T_1 and T_2 on the circle that touch the circle at $\mathbf{q_1}$ and $\mathbf{q_2}$ points.At $\mathbf{q_1}$, the equation of the normal is:

$$\mathbf{n_1} = \mathbf{V}\mathbf{q_1} + \mathbf{u} \tag{2.0.1}$$

and at $\mathbf{q_2}$, the equation of the normal is:

$$\mathbf{n_2} = \mathbf{V}\mathbf{q_2} + \mathbf{u} \tag{2.0.2}$$

The equation of a circle is given by:

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.3}$$

and for circle $\mathbf{V} = \mathbf{I}$ Now the equation of tangent T_1 is given by:

$$\mathbf{n_1}^T(\mathbf{x} - \mathbf{q_1}) = 0 \tag{2.0.4}$$

$$\implies (\mathbf{q_1} + \mathbf{u})^T(\mathbf{x} - \mathbf{q_1}) = 0 \tag{2.0.5}$$

$$\implies (\mathbf{q_1} + \mathbf{u})^T\mathbf{x} = \|\mathbf{q_1}\|^2 + \mathbf{u}^T\mathbf{q_1} \tag{2.0.6}$$

Similarly, for tangent T_2 the equation will be:

$$\mathbf{n_2}^T(\mathbf{x} - \mathbf{q_2}) = 0 \tag{2.0.7}$$

$$\implies (\mathbf{q_2} + \mathbf{u})^T(\mathbf{x} - \mathbf{q_2}) = 0 \tag{2.0.8}$$

$$\implies (\mathbf{q_2} + \mathbf{u})^T\mathbf{x} = \|\mathbf{q_2}\|^2 + \mathbf{u}^T\mathbf{q_2} \tag{2.0.9}$$

So, these are the equations of the tangents. Now,

$$\mathbf{q} = \mathbf{V}^{-1}(k\mathbf{n} - \mathbf{u}) \tag{2.0.10}$$

Where

$$k = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\mathbf{n}^T V\mathbf{n}}} \tag{2.0.11}$$

For circle it will be:

$$k = \sqrt{\frac{\|u\|^2 - f}{\|n\|^2}} \tag{2.0.12}$$

$$\mathbf{q} = (k\mathbf{n} - \mathbf{u}) \tag{2.0.13}$$

Thus we can get:

$$k_1 = \pm\sqrt{\frac{\|u\|^2 - f}{\|n_1\|^2}} \tag{2.0.14}$$

$$\mathbf{q_1} = (k_1\mathbf{n_1} - \mathbf{u}) \tag{2.0.15}$$

$$k_2 = \sqrt{\frac{\|u\|^2 - f}{\|n_2\|^2}} \tag{2.0.16}$$

$$\mathbf{q_2} = (k_2\mathbf{n_2} - \mathbf{u}) \tag{2.0.17}$$

As the co-ordinate of the point P is given as (x,y), so we can say that the distance of P to q_1 will be $\|\mathbf{P} - \mathbf{q_1}\|$ and the distance between P and q_2 will be $\|\mathbf{P} - \mathbf{q_2}\|$.